

# **Sunny.jl Tutorial**

**HFM 2025, Toronto**

**Kipton Barros  
Los Alamos National Lab**

# Installation

SunnyContributed / workshops / 2025\_05\_HFM





# Su(n)ny



**Kip Barros**  
**(LANL)**



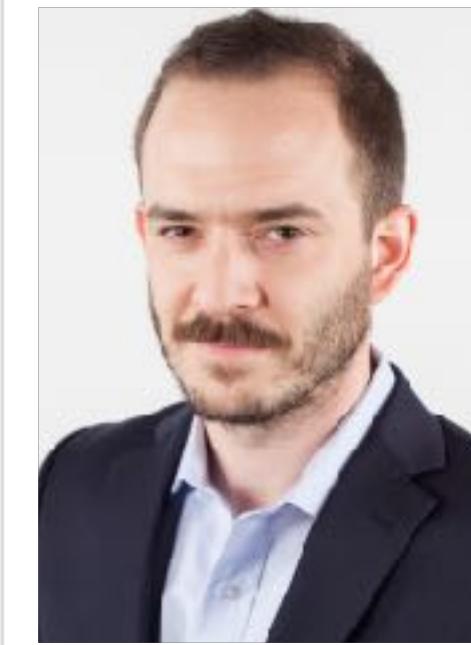
**Daniel Pajerovski**  
**(ORNL)**



**M. Mourigal**  
**(Georgia Tech)**



**X. Bai**  
**(LSU)**



**David Dahlbom**  
**(ORNL)**



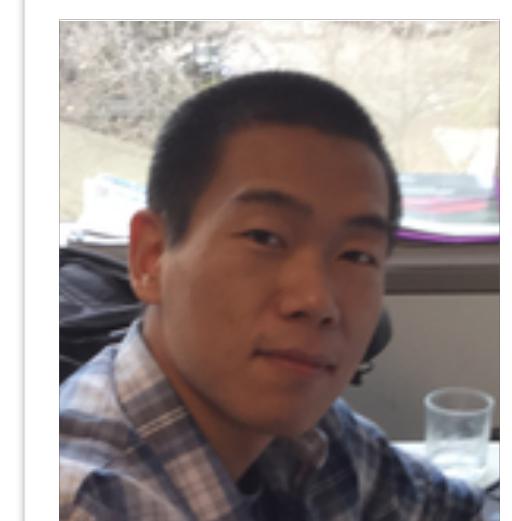
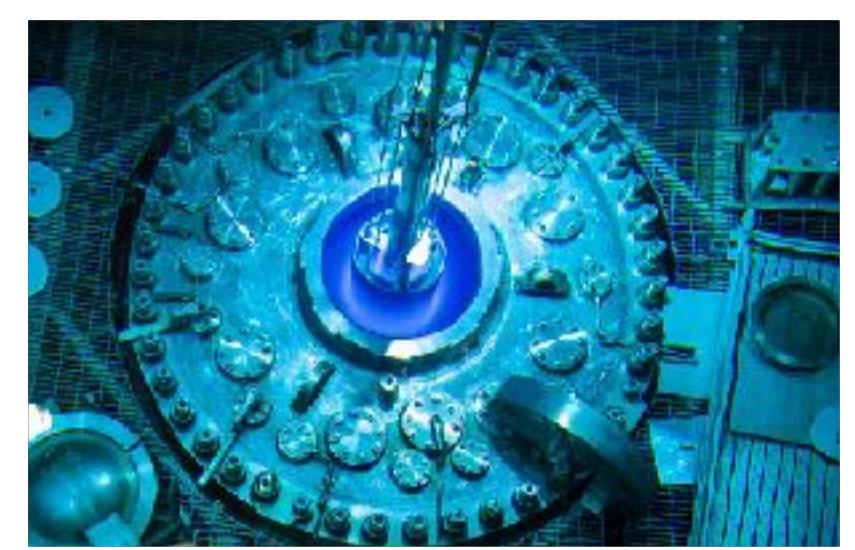
**C. D. Batista**  
**(UTK/ORNL)**



**Ying Wai Li**  
**(LANL)**



**Steve Hahn**  
**(ORNL)**



**Zhentao Wang**  
**(Zhejiang Univ.)**



**Matt Wilson**  
**(LANL)**



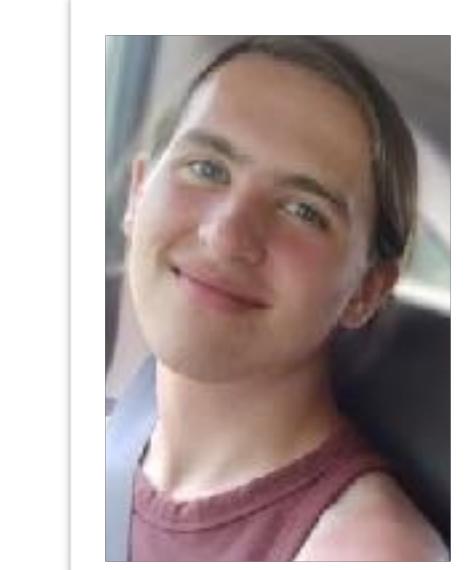
**Hao Zhang**  
**(LANL)**



**Sakib Matin**  
**(LANL)**



**Het Mankad**  
**(ORNL)**



**Sam Quinn**  
**(Georgia Tech)**



**Harry Lane**  
**(St. Andrews)**



**Alin Niraula**  
**(LSU)**

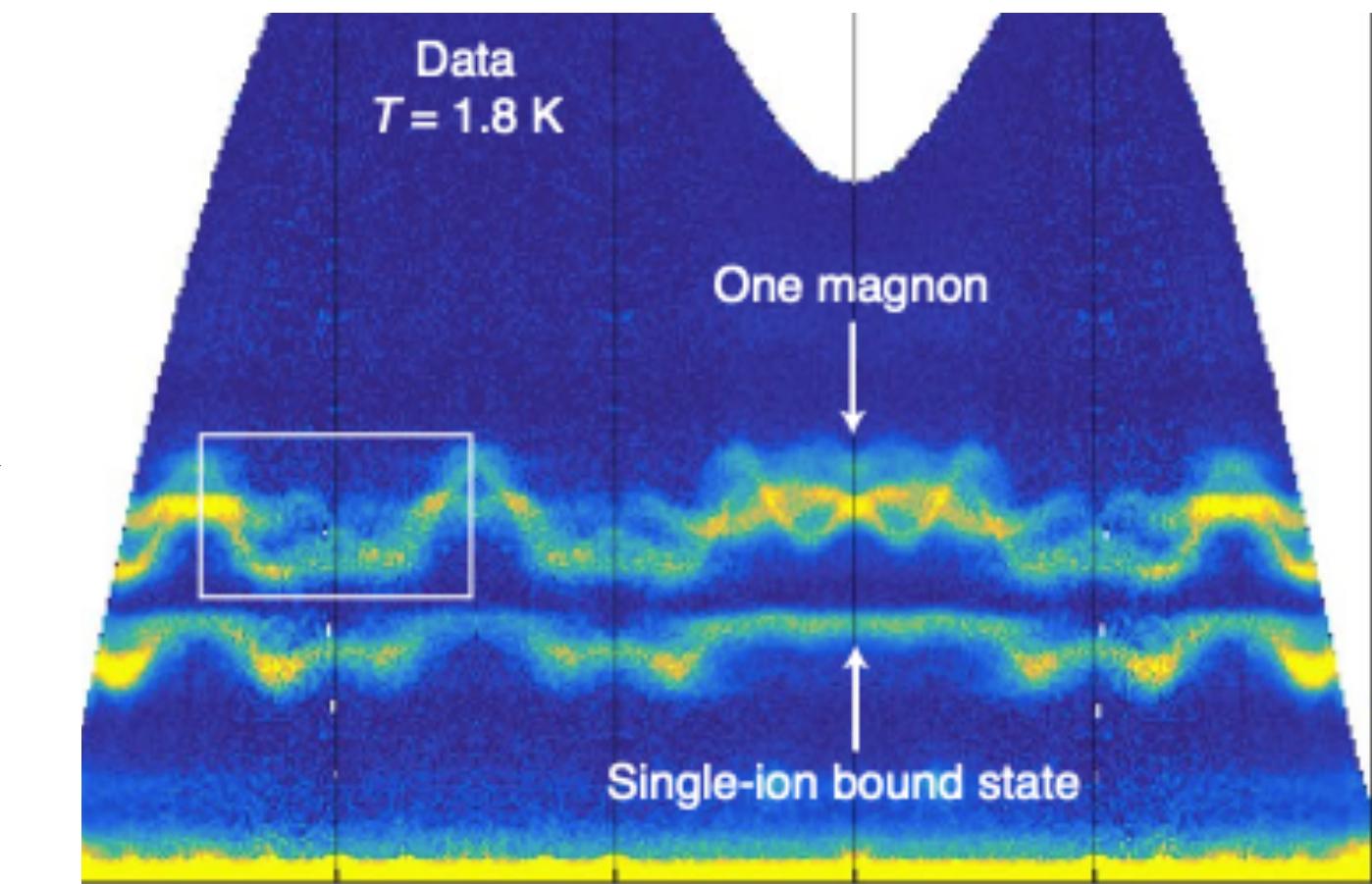
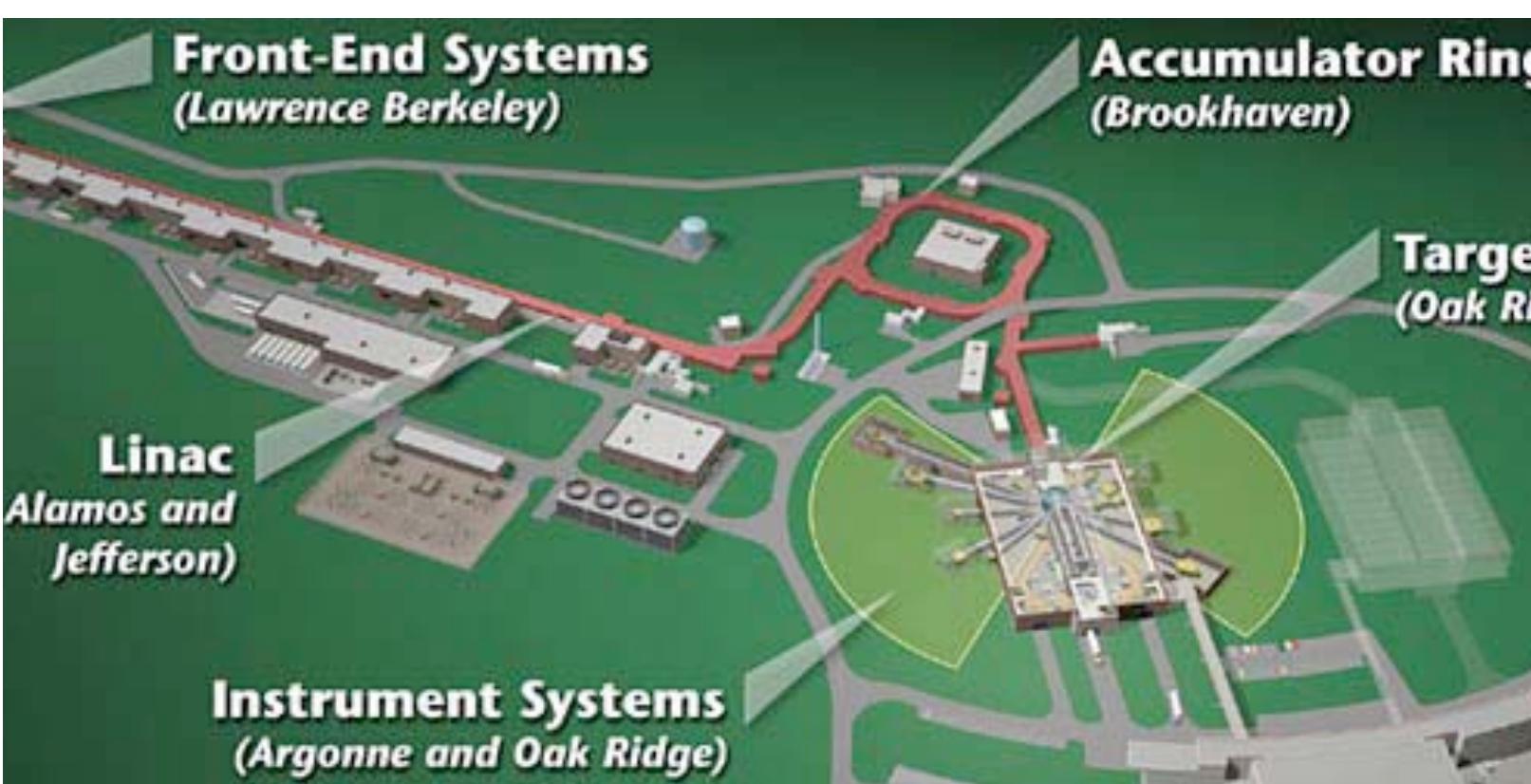


**Bhushan  
Thipe**  
**(LSU)**



**Cole Miles**  
**(Kodiak)**

# Experiment

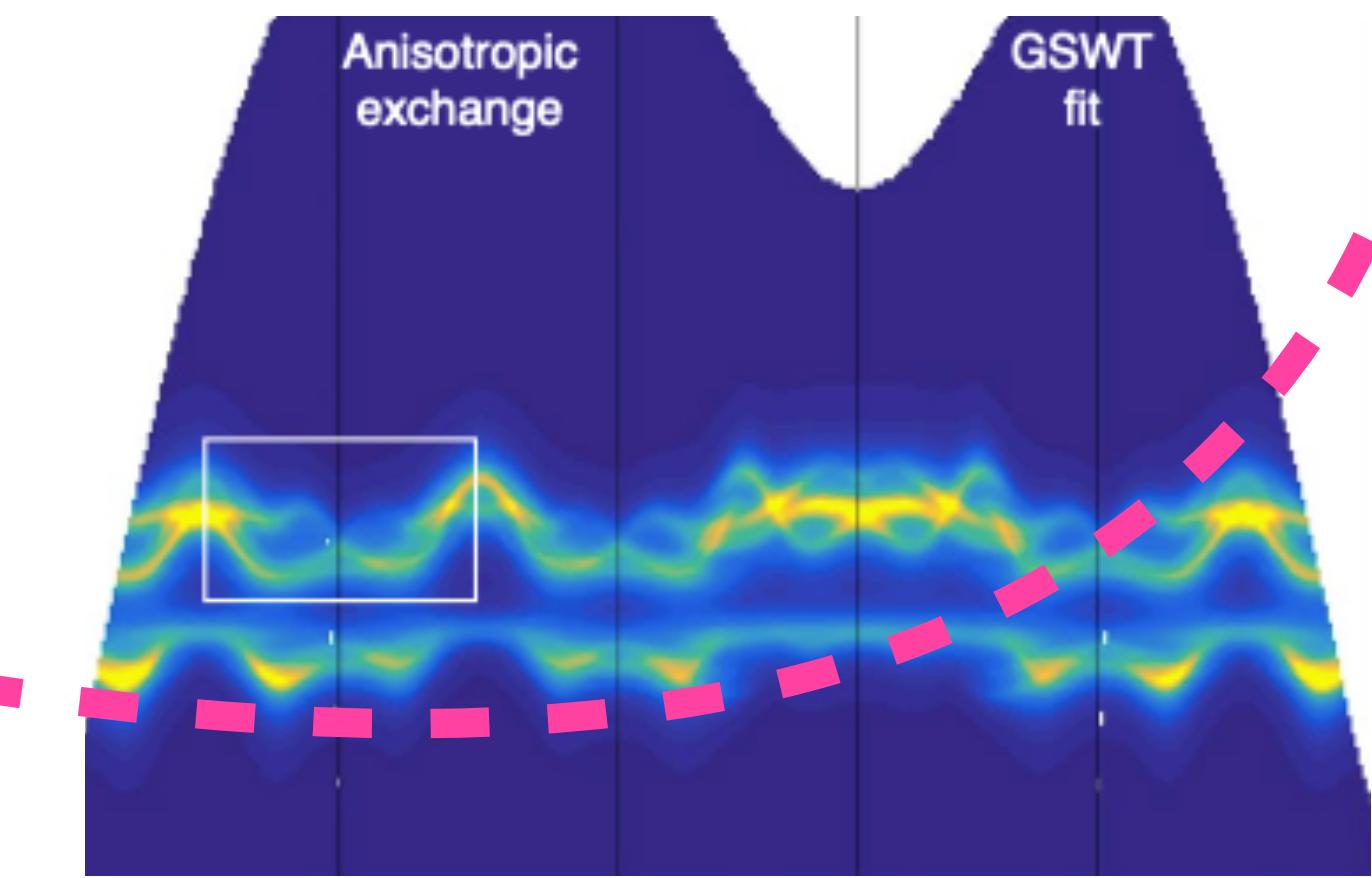


# Theory

$$\hat{\mathcal{H}}$$



$$i\hbar \frac{d}{dt} |\Psi\rangle = \hat{\mathcal{H}} |\Psi\rangle$$



# Landau-Lifshitz spin dynamics

$$\frac{d\mathbf{S}_j}{dt} = \mathbf{S}_j \times \frac{\partial H}{\partial \mathbf{S}_j}$$



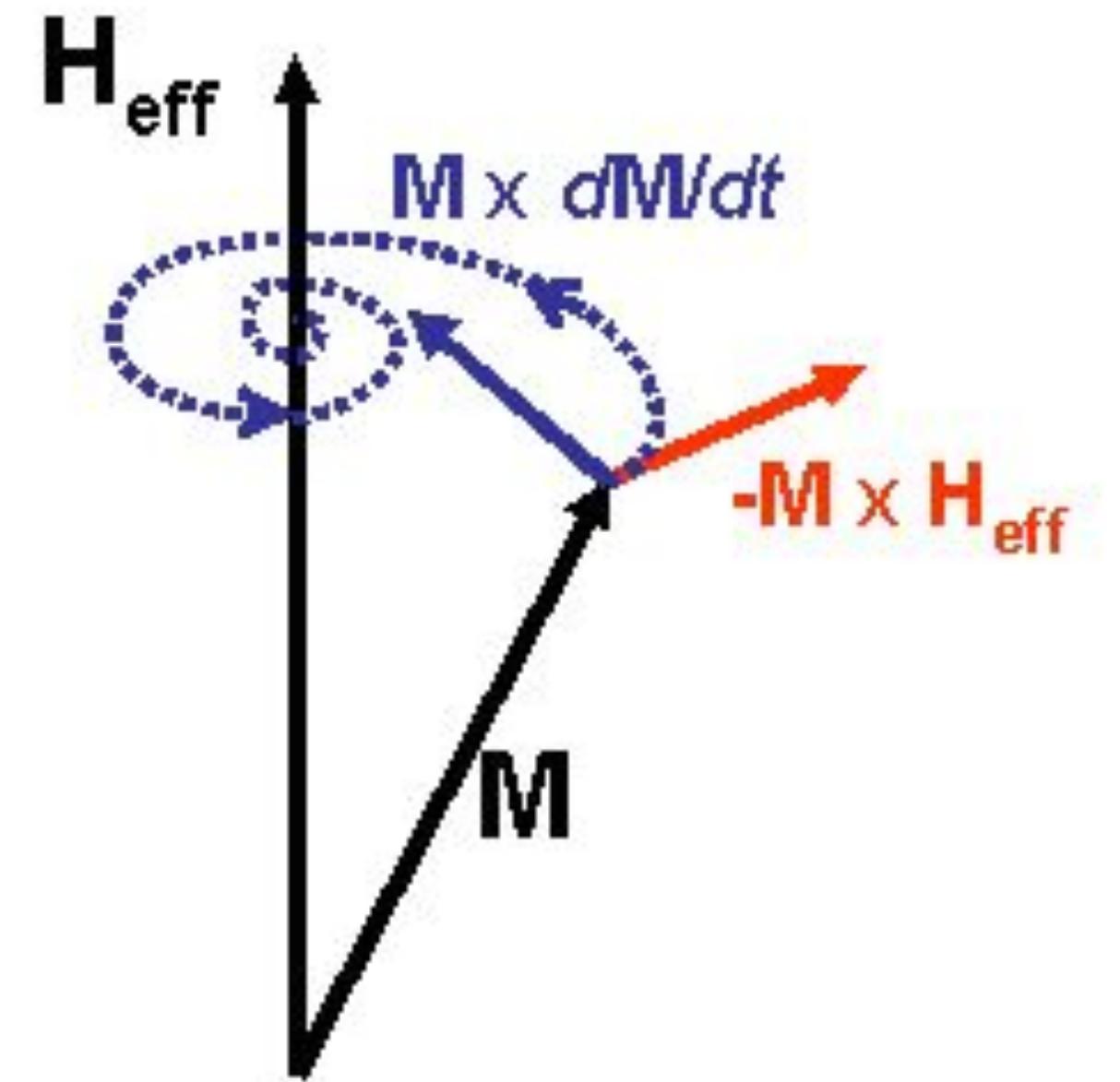
# Stochastic Landau-Lifshitz-Gilbert dynamics

$$\frac{d\mathbf{S}_j}{dt} = \mathbf{S}_j \times \mathbf{B}_j - \lambda \mathbf{S}_j \times (\mathbf{S}_j \times \mathbf{B}_j) + \mathbf{S}_j \times \boldsymbol{\xi}_j$$

Effective field  $\mathbf{B}_j = - \frac{\partial H}{\partial \mathbf{S}_j}$

Damping timescale  $\lambda$

White noise  $\langle \boldsymbol{\xi} \rangle = 0, \quad \langle \boldsymbol{\xi}^\alpha(t) \boldsymbol{\xi}^\beta(t)' \rangle = 2k_B T \lambda \delta_{\alpha\beta}$



SunnyContributed /  
workshops /  
2025\_05\_HFM

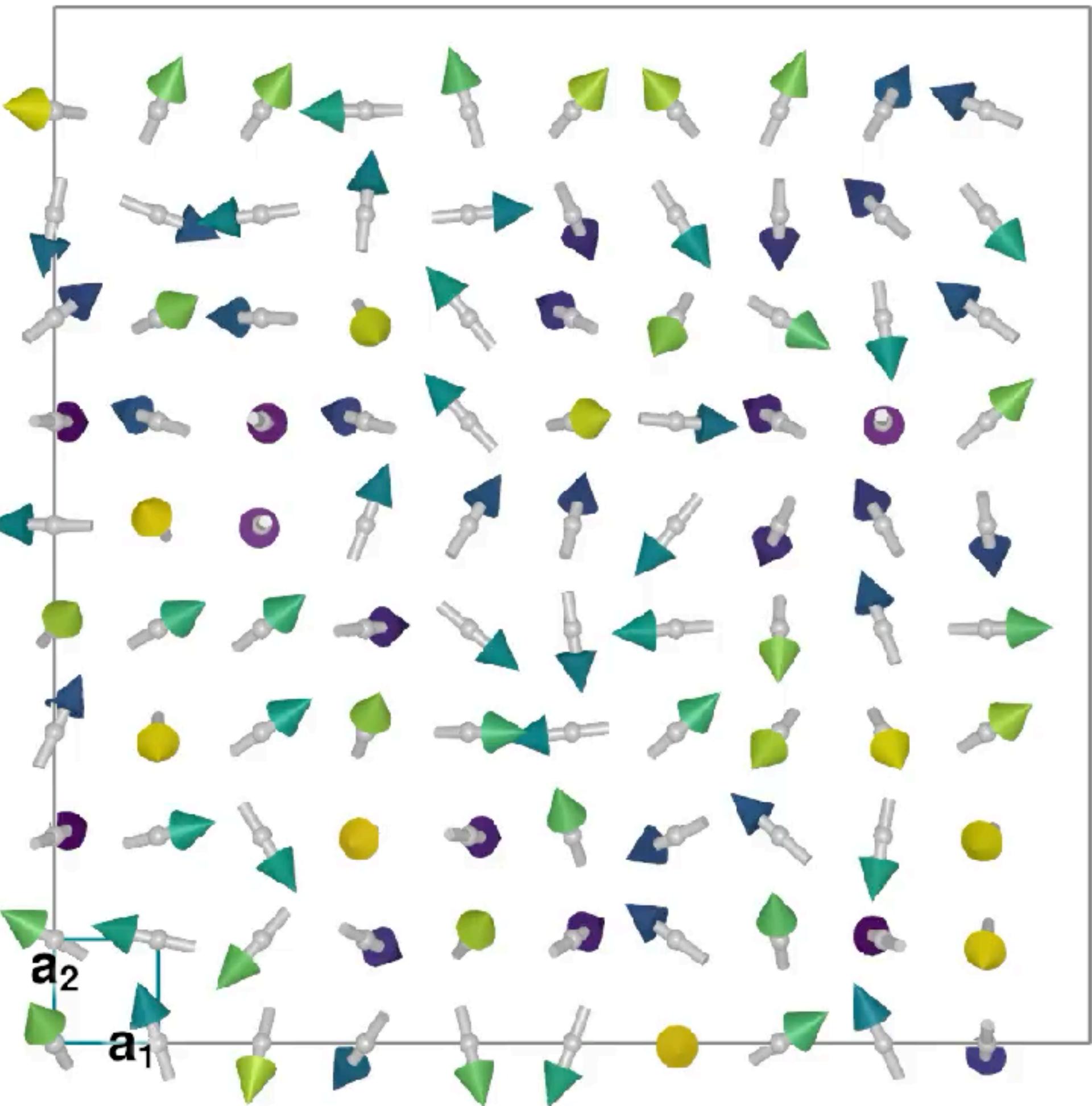


using Sunny, GLMakie

```
# build crystal
latvecs = lattice_vectors(1, 1, 10, 90, 90, 90)
cryst = Crystal(latvecs, [[0,0,0]])

# build spin system
sys = System(cryst, (10,10,1), [SpinInfo(1, S=1, g=2)], :dipole; seed=1)
set_exchange!(sys, -1.0, Bond(1, 1, (1, 0, 0)))
randomize_spins!(sys)

# time integrate
fig = plot_spins(sys)
langevin = Langevin(0.1; kT=0, λ=0.05)
for _ in 1:500
    for _ in 1:5
        step!(sys, langevin)
    end
    notify(fig)
    sleep(1/60)
end
```



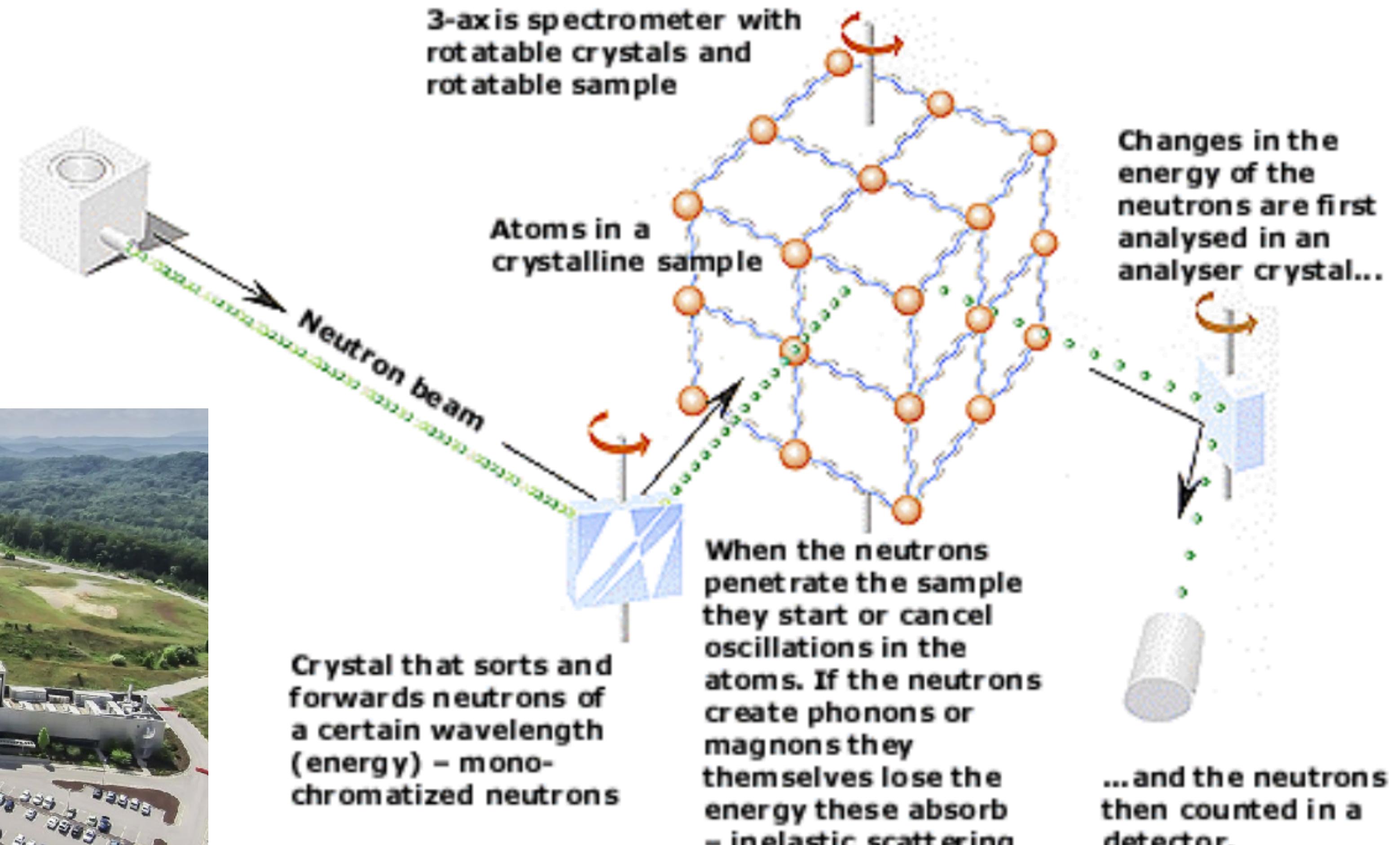
# Thoughts on Julia for a "production" scientific code

**Superpower:** Python flexibility and C++ performance

- Pros:**
- Express code like math (multi-method dispatch)
  - Interactive development experience
  - Built-in package manager
  - Easily bind C++/Fortran/Python libraries
  - "Drop down" to machine level (stack allocs / SIMD / JIT)
  - Ecosystem of enthusiasts 

- Cons:**
- Performance learning curve
  - JIT  $\Rightarrow$  Julia is a heavy dependency
  - Small community  $\Rightarrow$  you'll find bugs

# Neutrons an ideal probe for magnetism



# Spin order and dynamics in the diamond-lattice Heisenberg antiferromagnets **CuRh<sub>2</sub>O<sub>4</sub> and CoRh<sub>2</sub>O<sub>4</sub>**

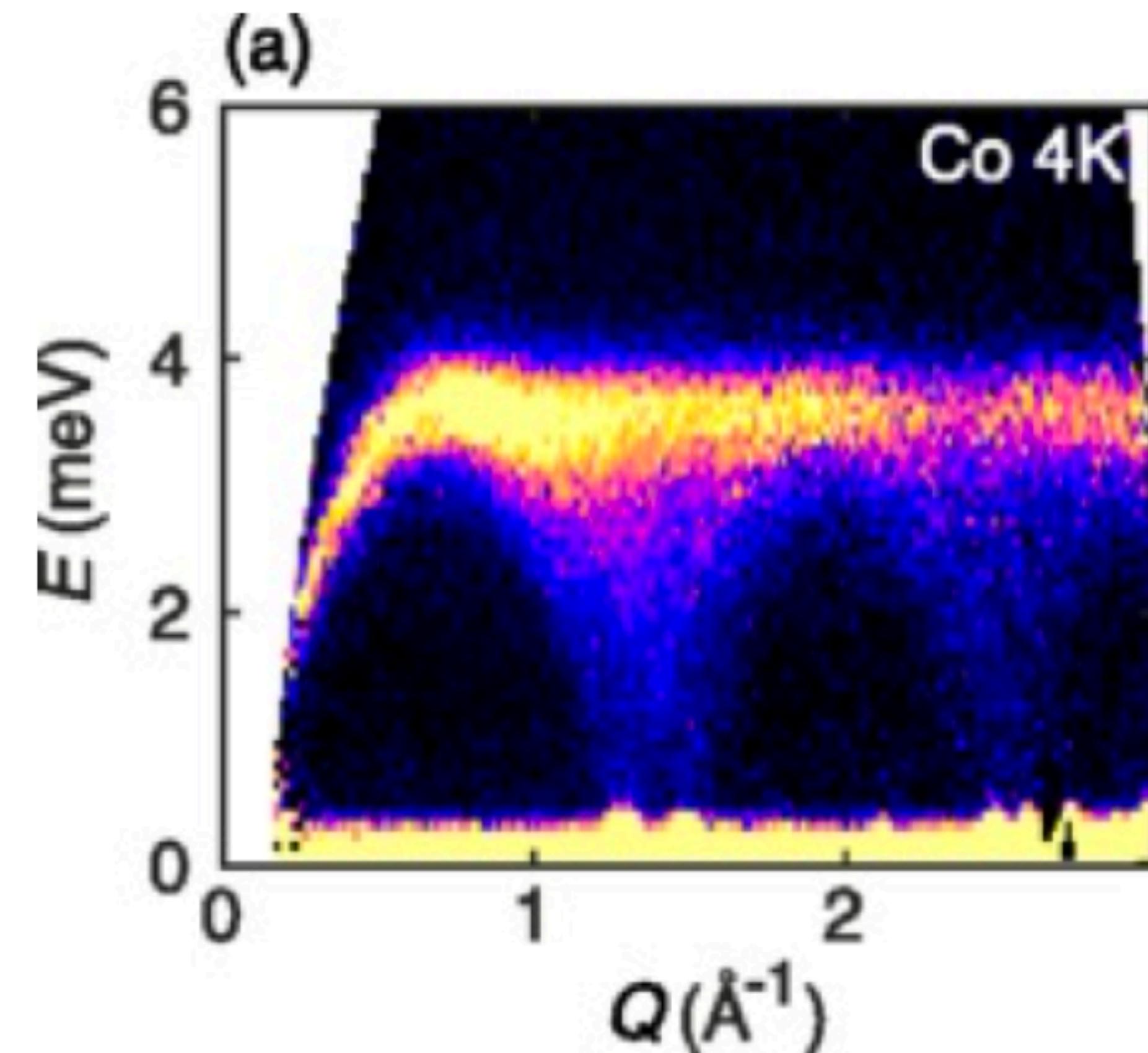
L. Ge<sup>1</sup>, J. Flynn<sup>2</sup>, J. A. M. Paddison<sup>1,\*</sup>, M. B. Stone<sup>3</sup>, S. Calder<sup>3</sup>, M. A. Subramanian<sup>2</sup>, A. P. Ramirez<sup>4</sup>, and M. Mourigal<sup>1</sup>

Phys. Rev. B **96**, 064413 – Published 9 August, 2017

$$\langle \hat{S}^\alpha(\mathbf{r}, t) \hat{S}^\beta(0,0) \rangle$$

→  $\mathcal{S}^{\alpha\beta}(\mathbf{k}, \omega) = \langle \hat{S}_{-\mathbf{k}, -\omega}^\alpha \hat{S}_{\mathbf{k}, \omega}^\beta \rangle$

$$\frac{d\mathbf{S}_j}{dt} = \mathbf{S}_j \times \frac{\partial H}{\partial \mathbf{S}_j}$$



```

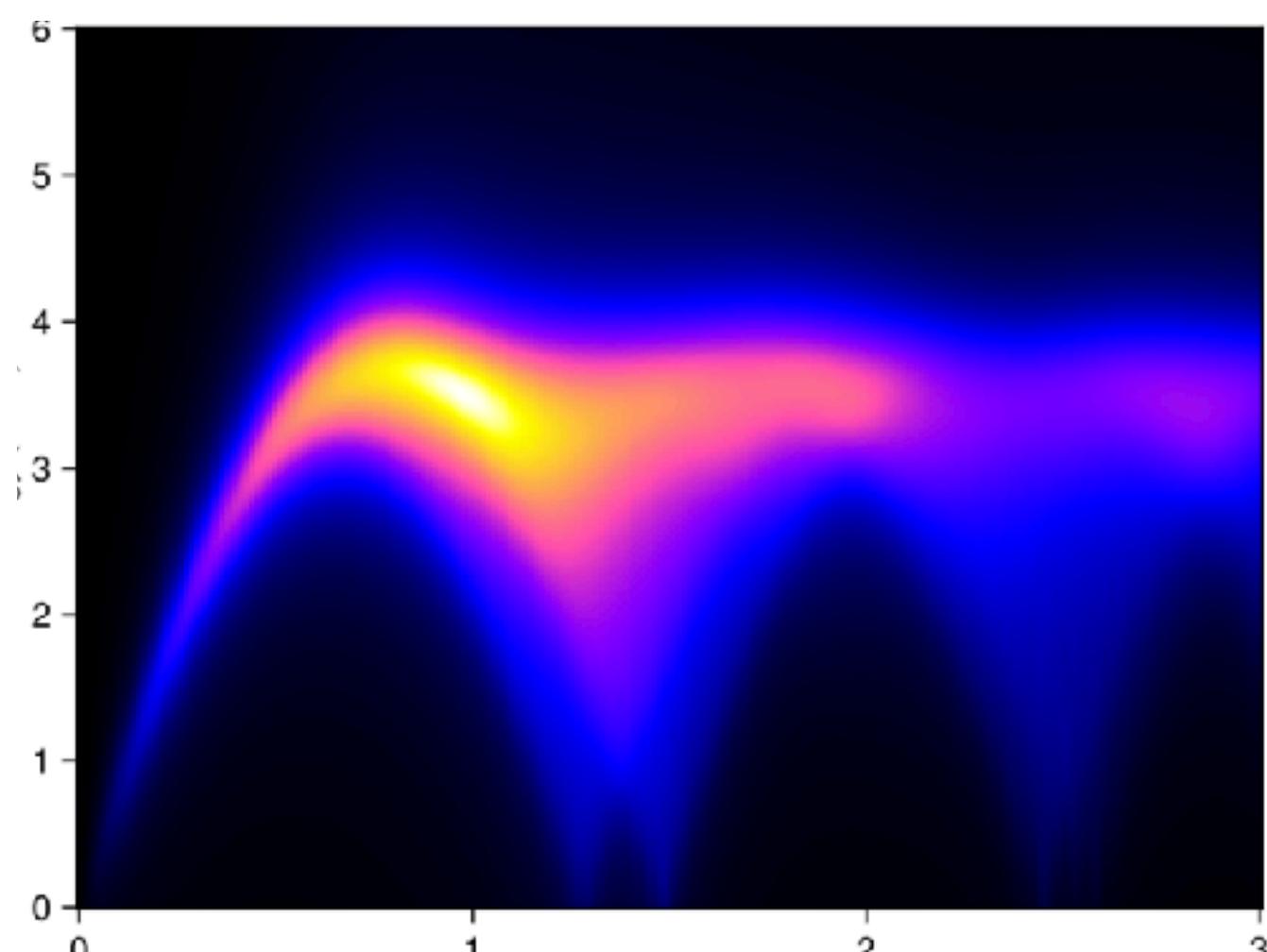
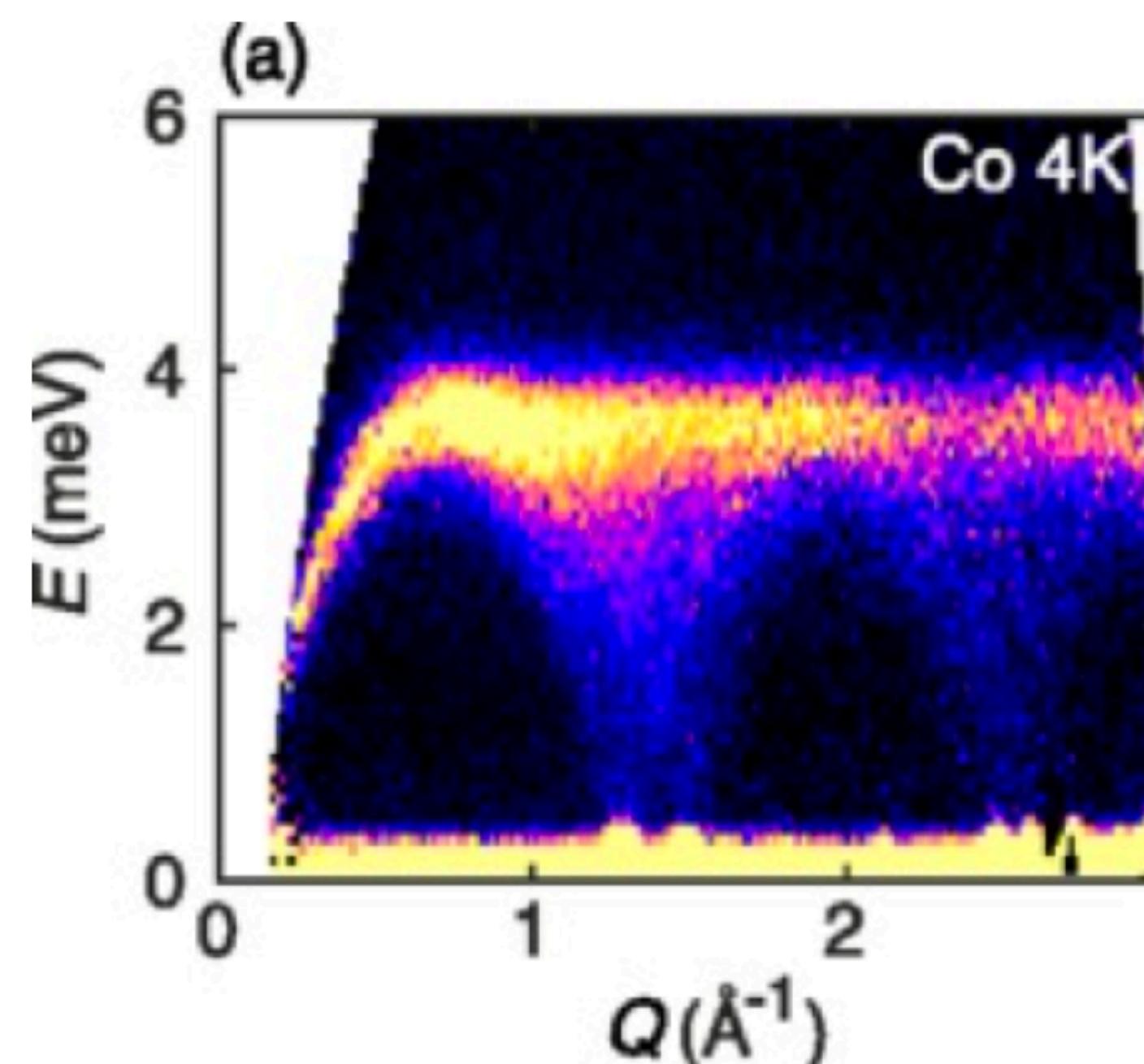
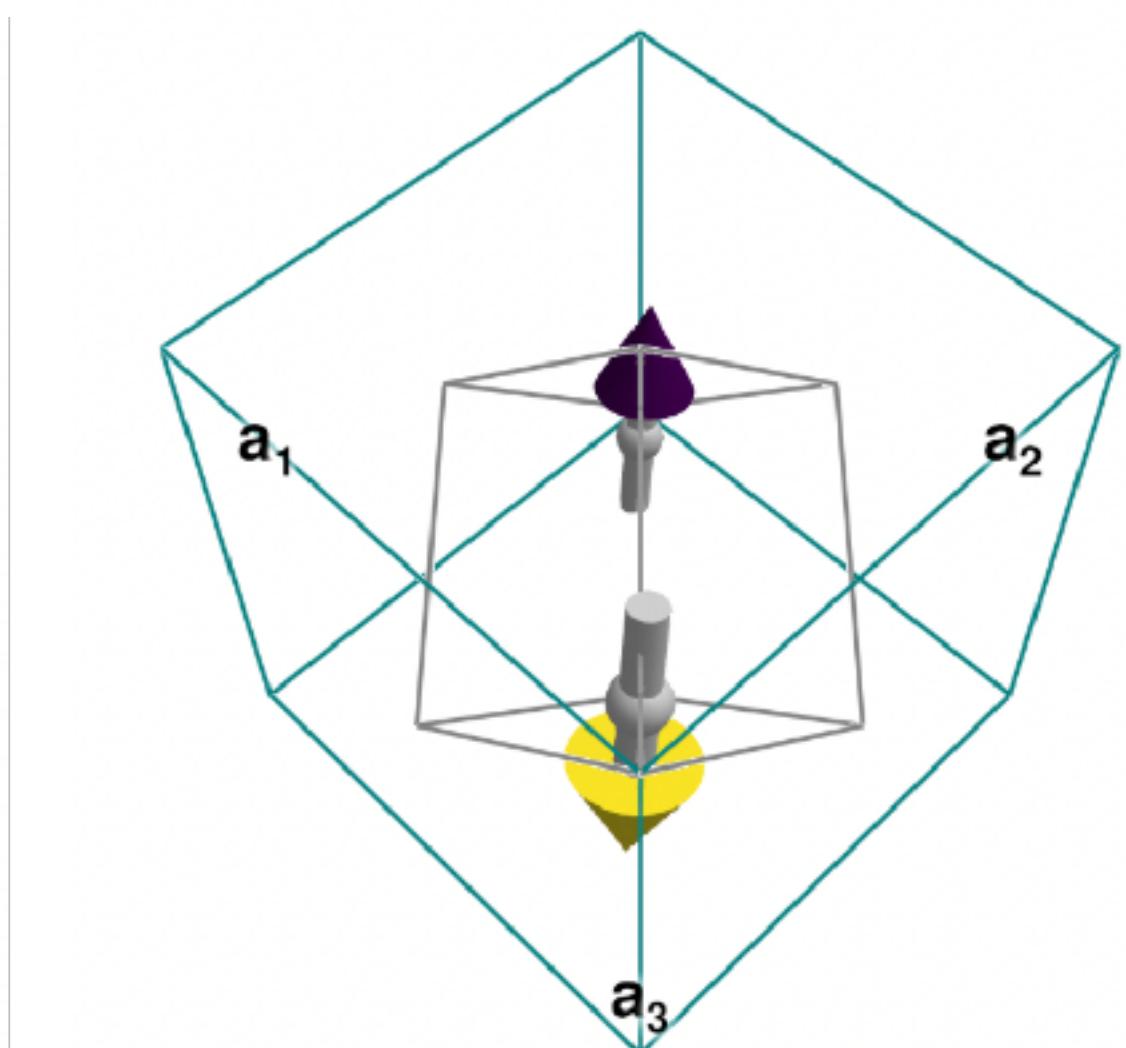
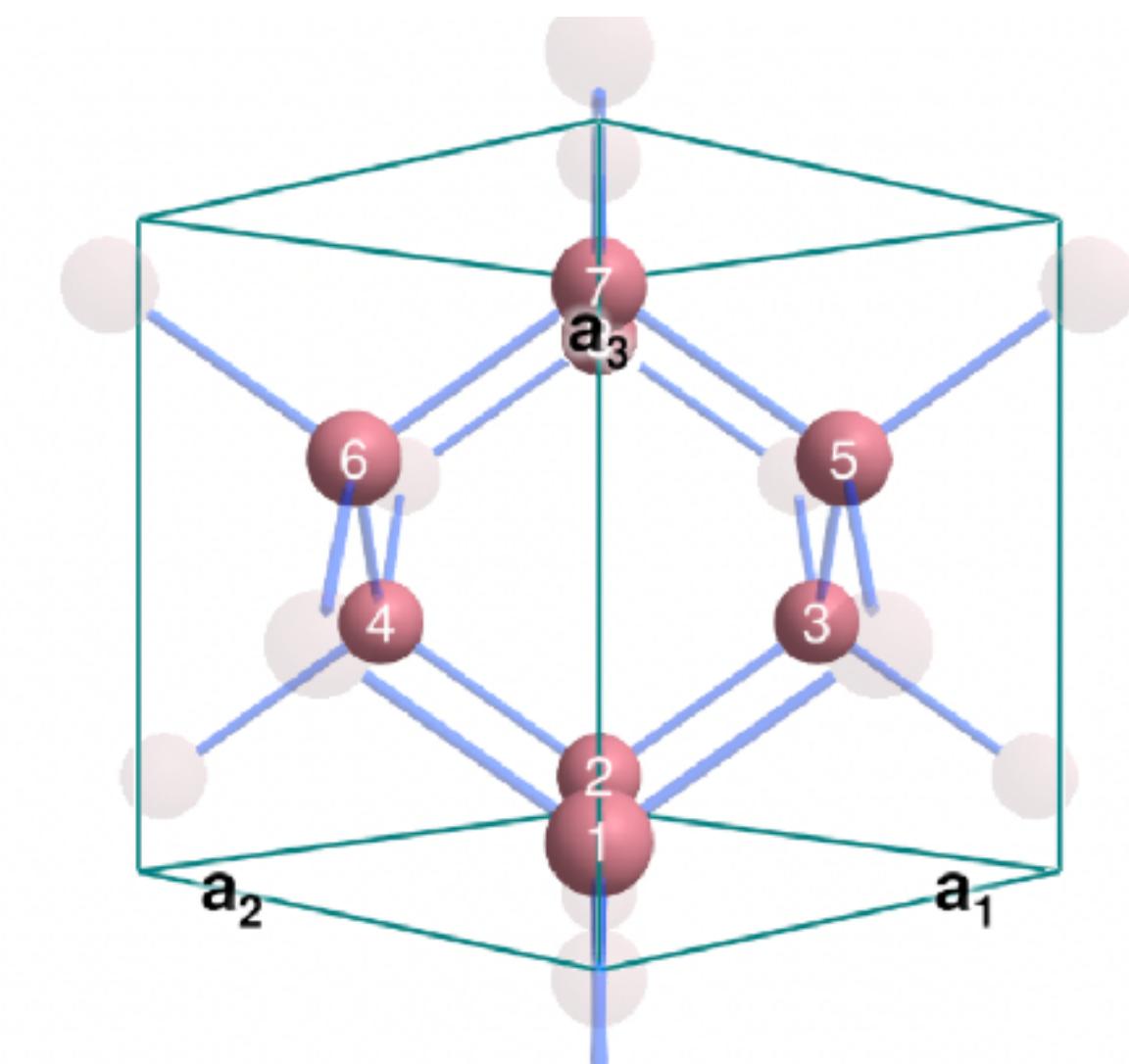
using Sunny, GLMakie
a = 8.5031 # (Å)
latvecs = lattice_vectors(a, a, a, 90, 90, 90)
positions = [[1/8, 1/8, 1/8]]
cryst = Crystal(latvecs, positions, 227; types=["Co"])
sys = System(cryst, [1 => Moment(s=3/2, g=2)], :dipole)
J = +0.63 # (meV)
set_exchange!(sys, J, Bond(2, 3, [0, 0, 0]))
randomize_spins!(sys)
minimize_energy!(sys)
sys_prim = reshape_supercell(sys, primitive_cell(cryst))

```

```

formfactors = [1 => FormFactor("Co2")]
measure = ssf_perp(sys_prim; formfactors)
swt = SpinWaveTheory(sys_prim; measure)
kernel = lorentzian(fwhm=0.8)
energies = range(0, 6, 300)
radii = range(0, 3, 200) # (1/Å)
res = powder_average(cryst, radii, 2000) do qs
    intensities(swt, qs; energies, kernel)
end
plot_intensities(res)

```



## Quantum to classical (large-s)

$$\langle \hat{\mathbf{S}}_j \hat{\mathbf{S}}_k \rangle \longrightarrow \langle \hat{\mathbf{S}}_j \rangle \langle \hat{\mathbf{S}}_k \rangle = \mathbf{S}_j \mathbf{S}_k$$

$$\hat{\mathcal{H}} = - \sum_{jk} J_{jk} \hat{\mathbf{S}}_j \cdot \hat{\mathbf{S}}_k \longrightarrow H = - \sum_{jk} J_{jk} \mathbf{S}_j \cdot \mathbf{S}_k$$

$$\frac{d\hat{\mathbf{S}}_j}{dt} = \hat{\mathbf{S}}_j \times \frac{\partial \hat{\mathcal{H}}}{\partial \hat{\mathbf{S}}_j} \longrightarrow$$

$$\frac{d\mathbf{S}_j}{dt} = \mathbf{S}_j \times \frac{\partial H}{\partial \mathbf{S}_j}$$

Landau-Lifshitz equation

Exact quantum spin Hamiltonian

$$\hat{\mathcal{H}} = - \sum_{jk} J_{jk} \hat{\mathbf{S}}_j \cdot \hat{\mathbf{S}}_k - D \sum_j (\hat{S}_j^z)^2$$

"Large- $S$ " approximation

$$\langle \hat{\mathcal{H}} \rangle = - \sum_{jk} J_{jk} \langle \hat{\mathbf{S}}_j \rangle \cdot \langle \hat{\mathbf{S}}_k \rangle - D \sum_j \langle \hat{S}_j^z \rangle^2$$

Neglect entanglement      Neglect quantum spin structure!

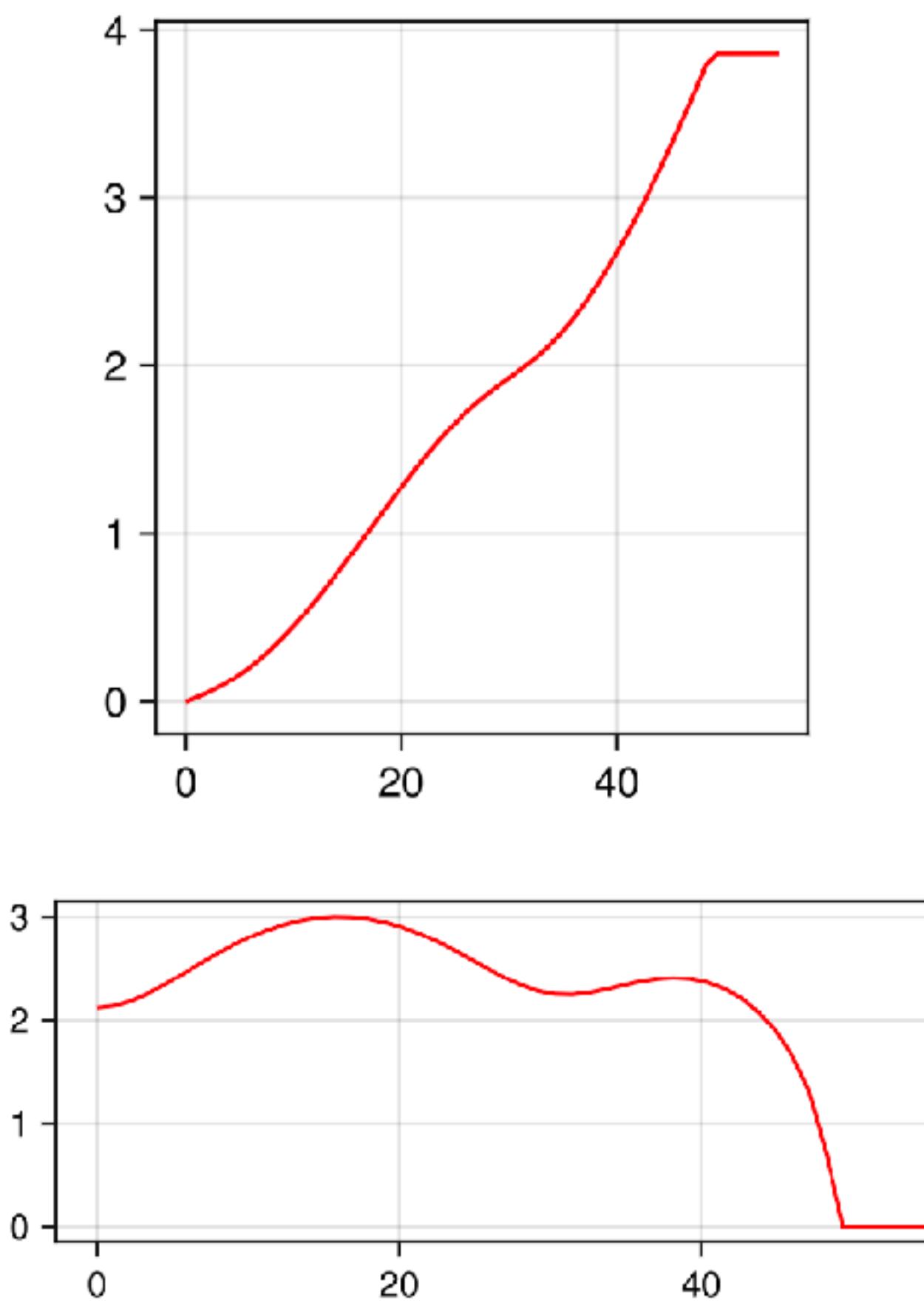
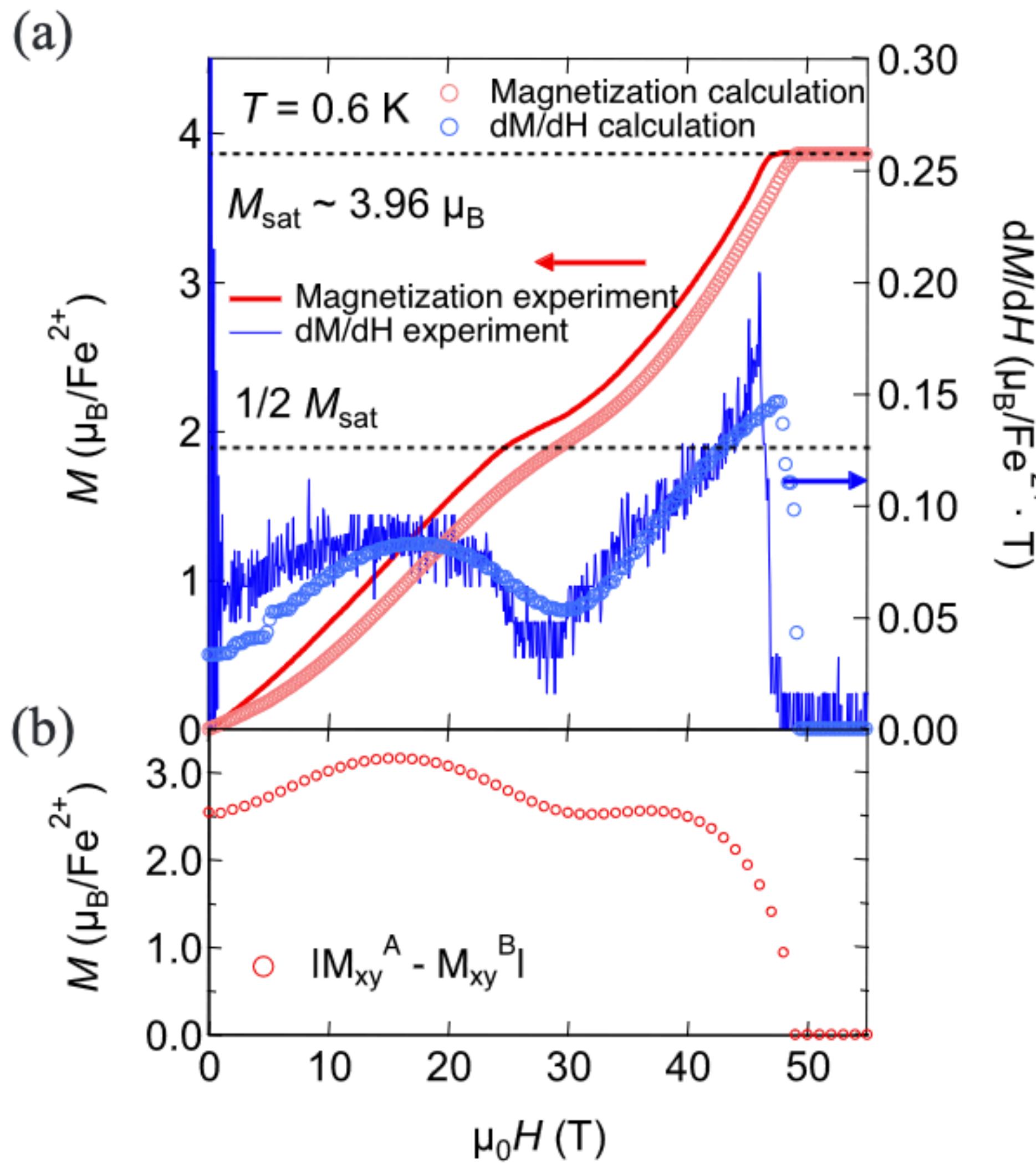
SU( $N$ ) treatment

$$\begin{aligned} \hat{T}^\alpha &= \{ \hat{\mathbf{S}}^x, \hat{\mathbf{S}}^y, \hat{\mathbf{S}}^z, && \text{Dipoles} \\ &(\hat{\mathbf{S}}^x)^2 - (\hat{\mathbf{S}}^y)^2, \dots \} && \text{Quadrupoles} \end{aligned}$$

$$\frac{d\hat{T}_j^\alpha}{dt} = f^{\alpha\beta\gamma} \hat{T}_j^\beta \frac{\partial \hat{\mathcal{H}}}{\partial \hat{T}_j^\gamma}$$

# SU(5) Application: Ba<sub>2</sub>FeSi<sub>2</sub>O<sub>7</sub>

Lee et al., PRB 107 144427 (2023)

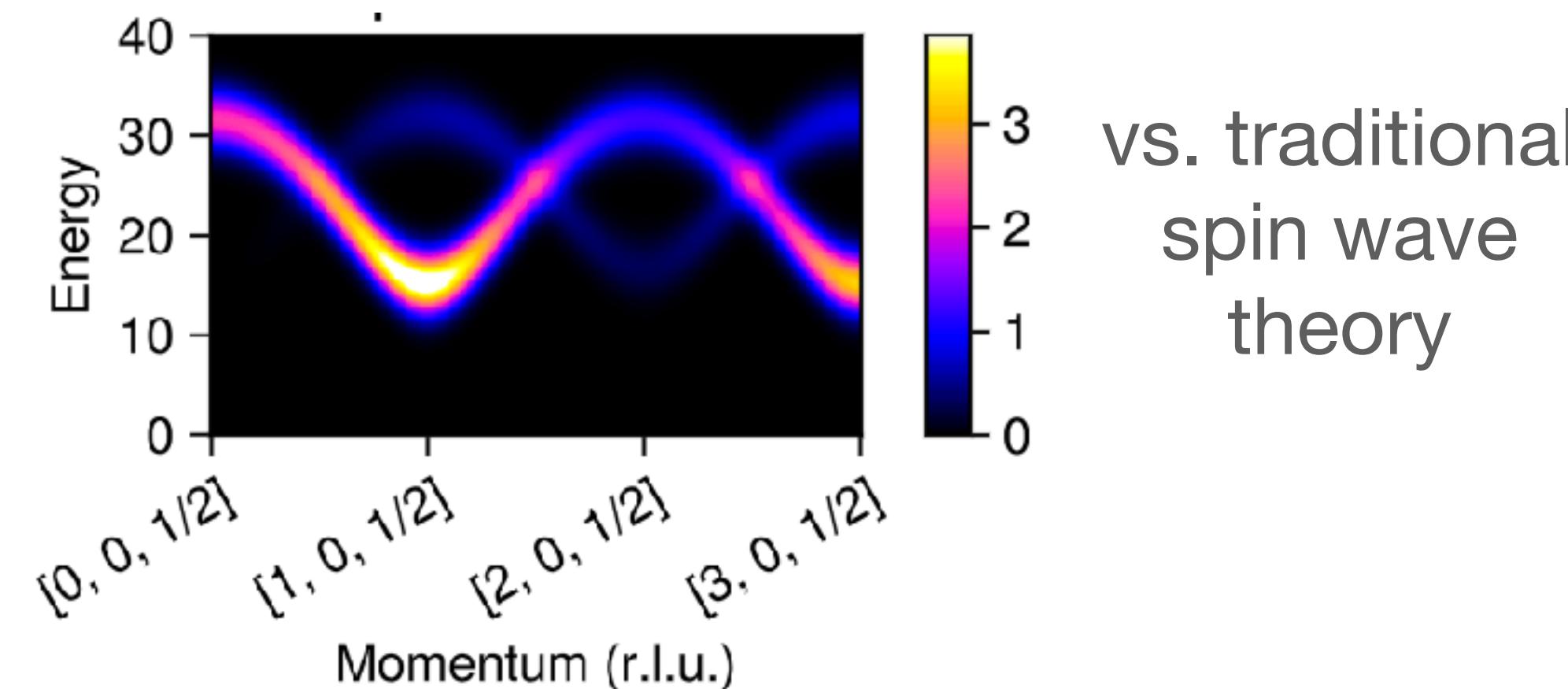
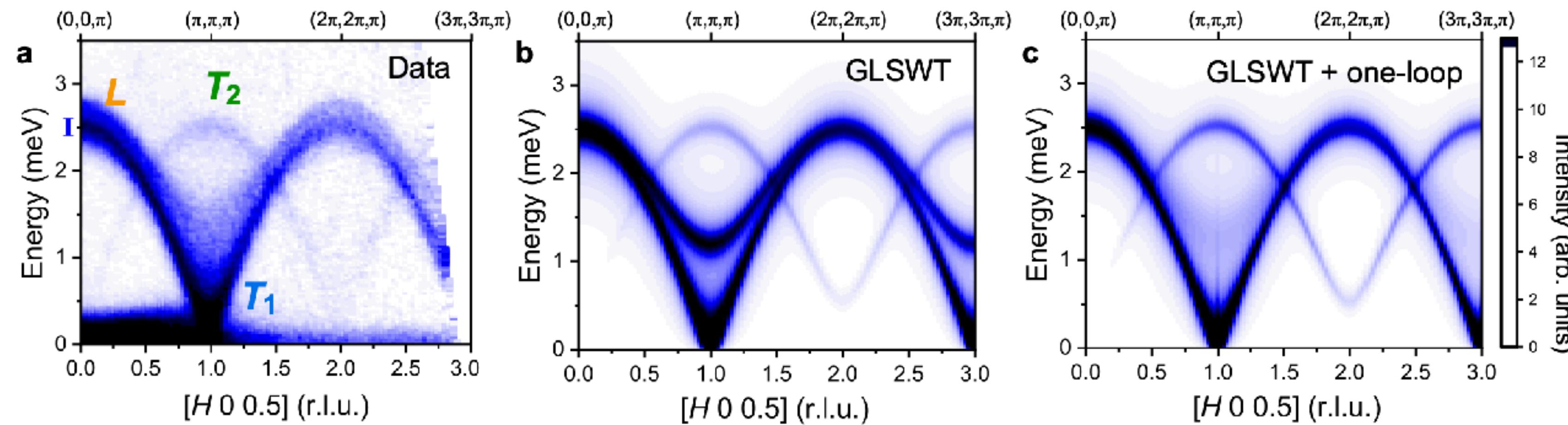


**Net  
magnetization**

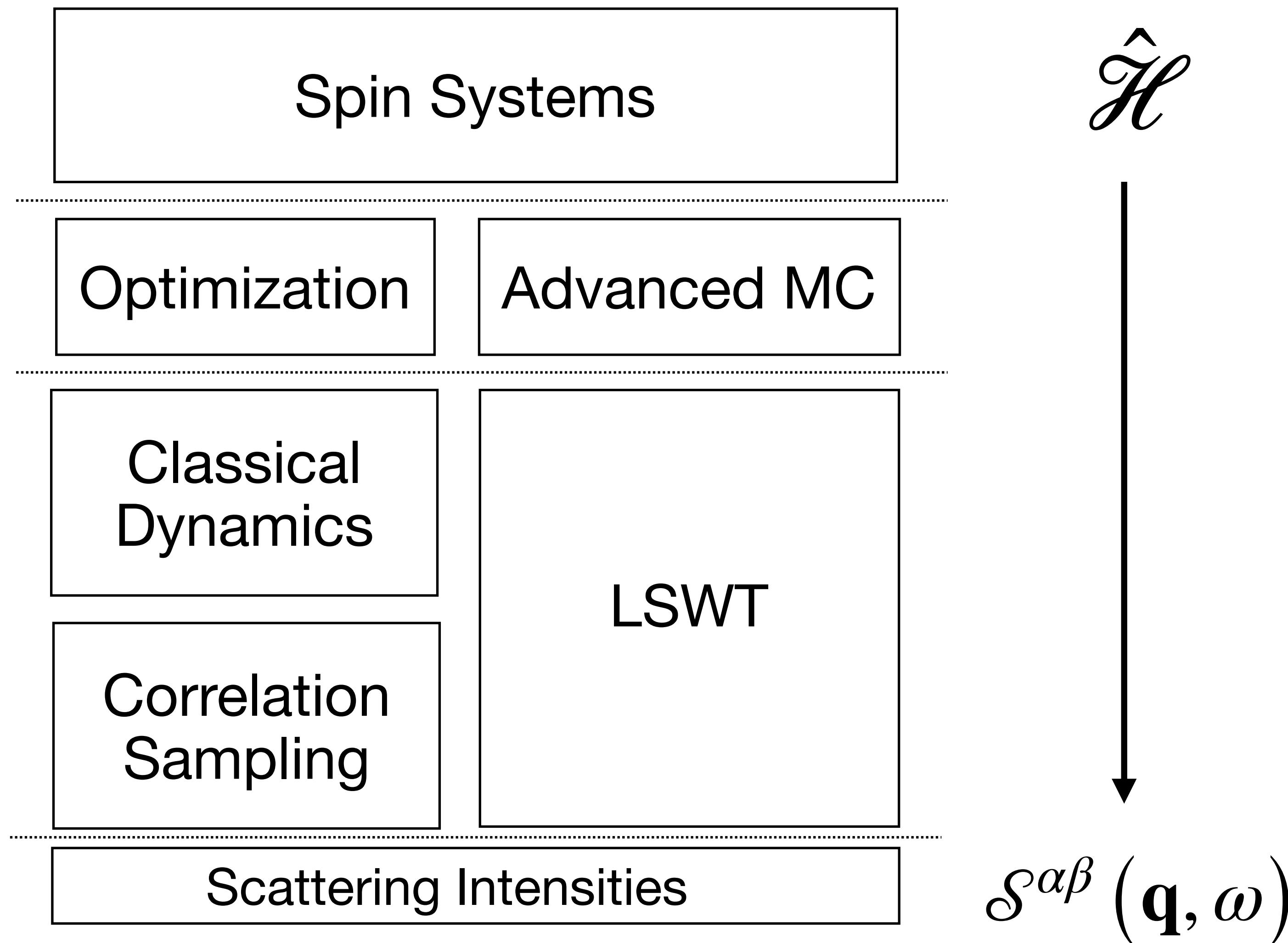
**Staggered  
magnetization**

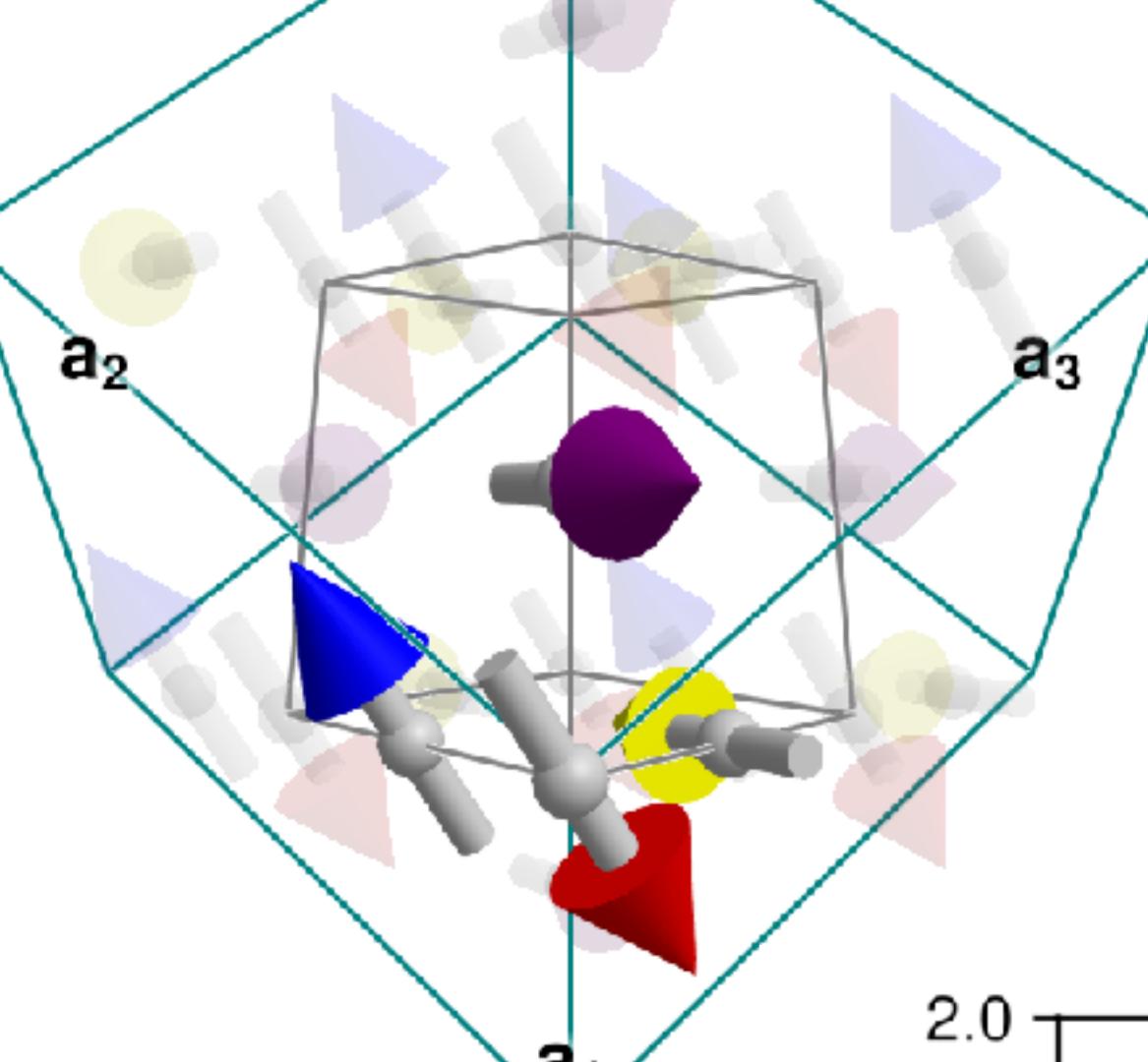
# SU(5) Application: Ba<sub>2</sub>FeSi<sub>2</sub>O<sub>7</sub>

Do et al., Nature Communications 12:5331 (2021)

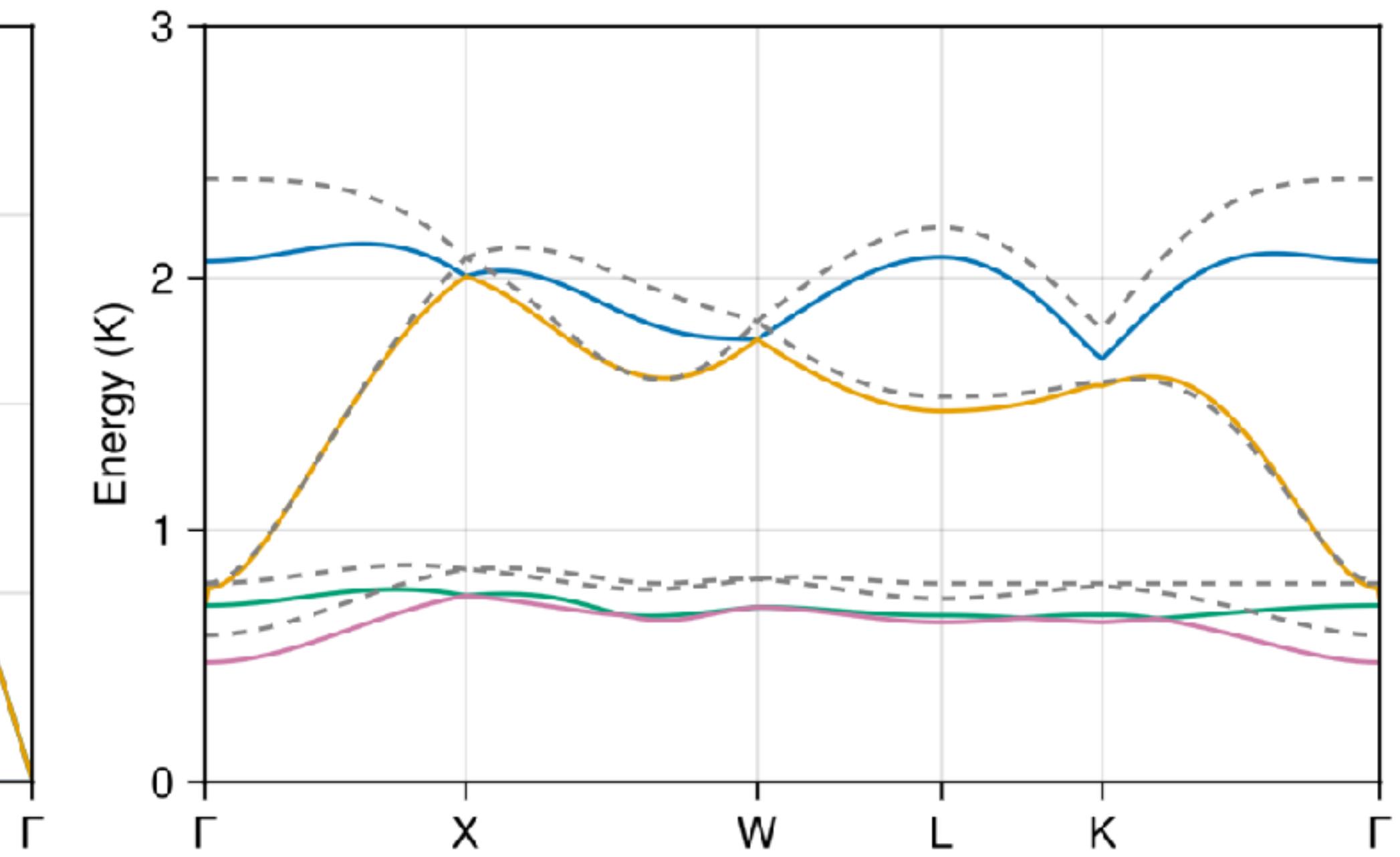
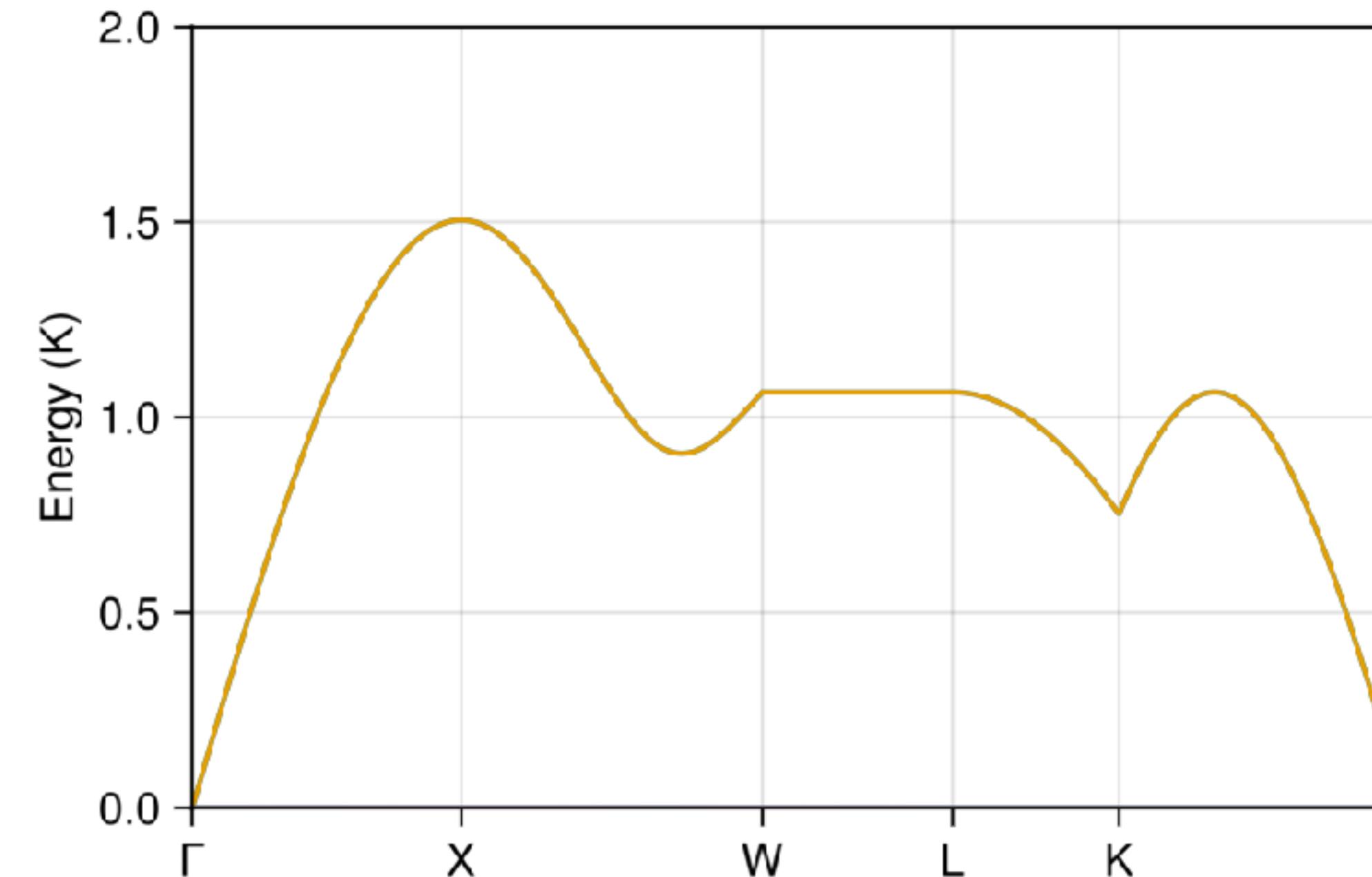


# Sunny structure

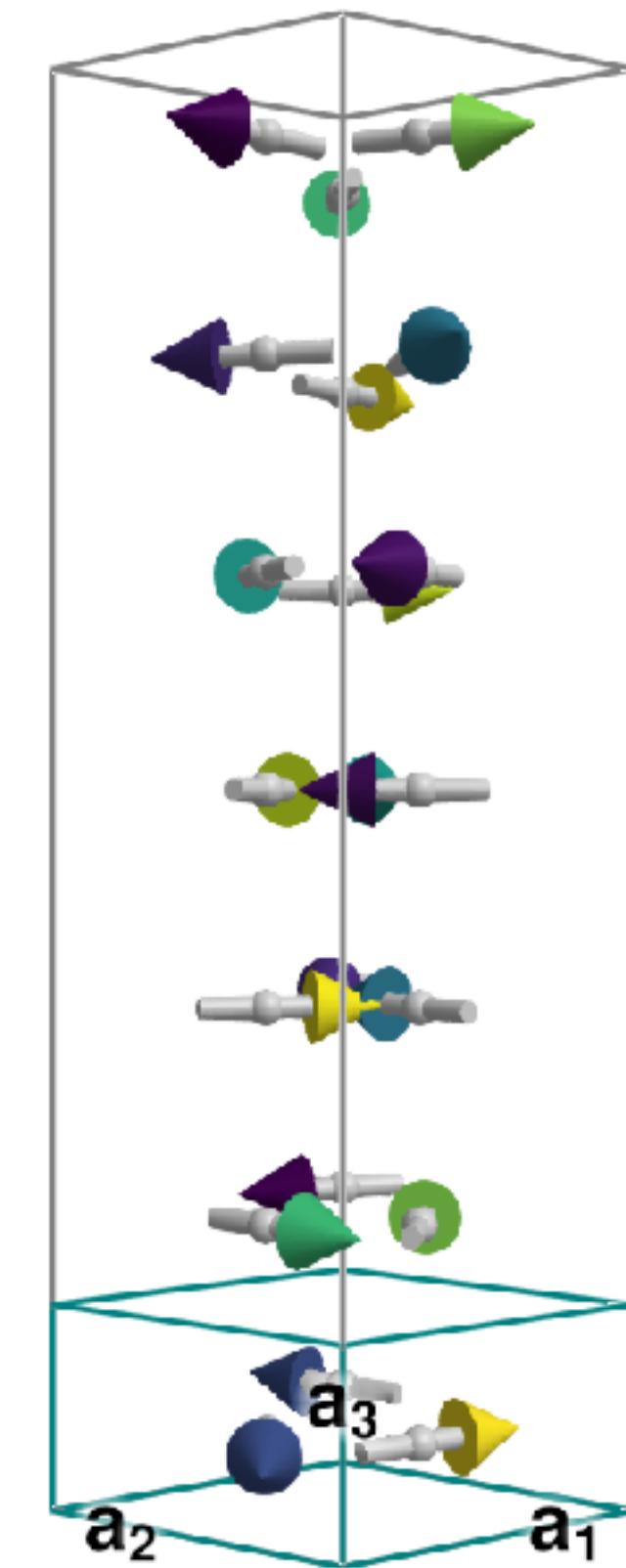
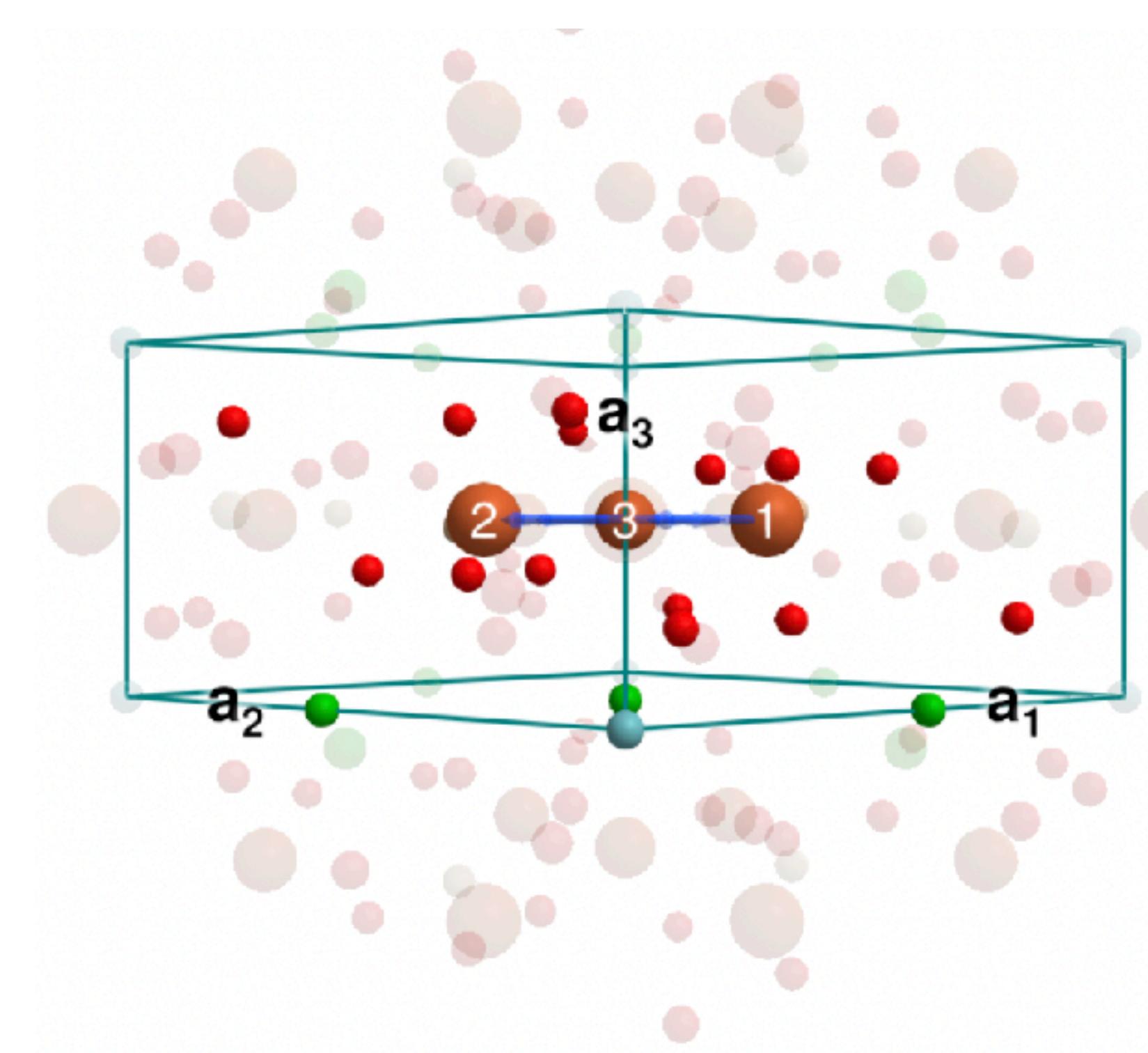




# Long-range dipole-dipole



# Incommensurate spiral order



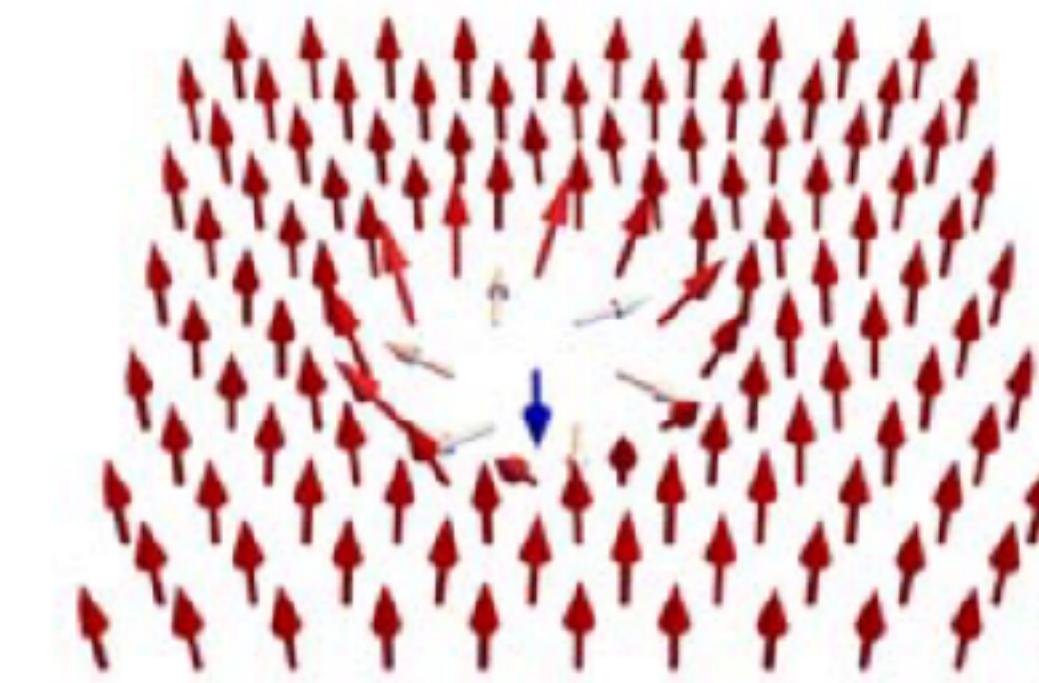
SpinW  
Su(n)ny

# Classical dynamics beyond spin waves

Zhang et al, Nature Commun. 14, 3626 (2023)

Baby  
skyrmions:

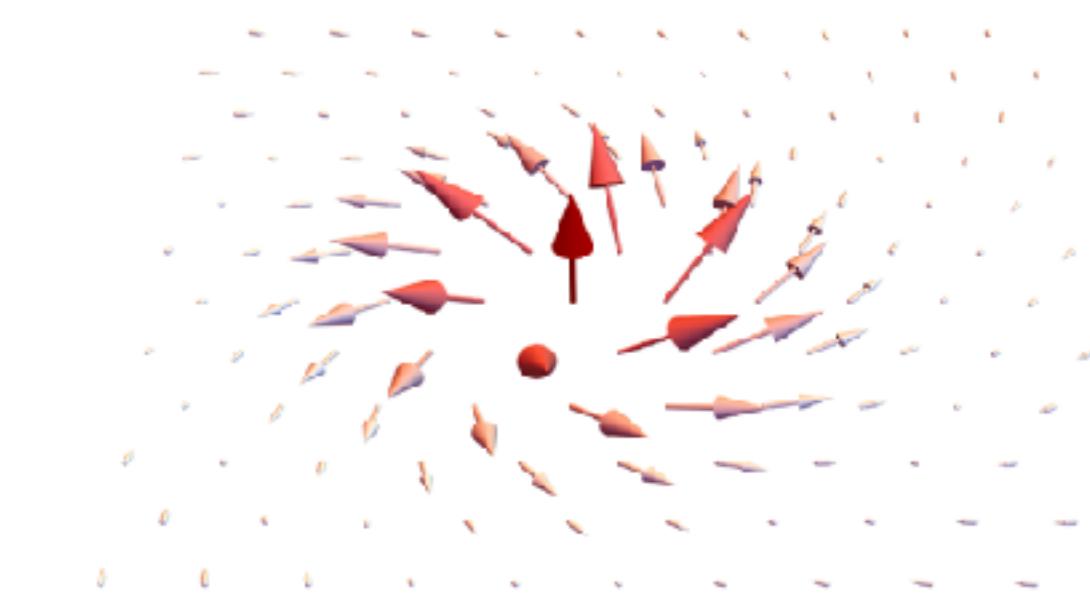
$$S^2 \mapsto S^2$$
$$\pi_2(S^2) \cong \mathbb{Z}$$



$\text{CP}^2$  skyrmions  
for spin-1:

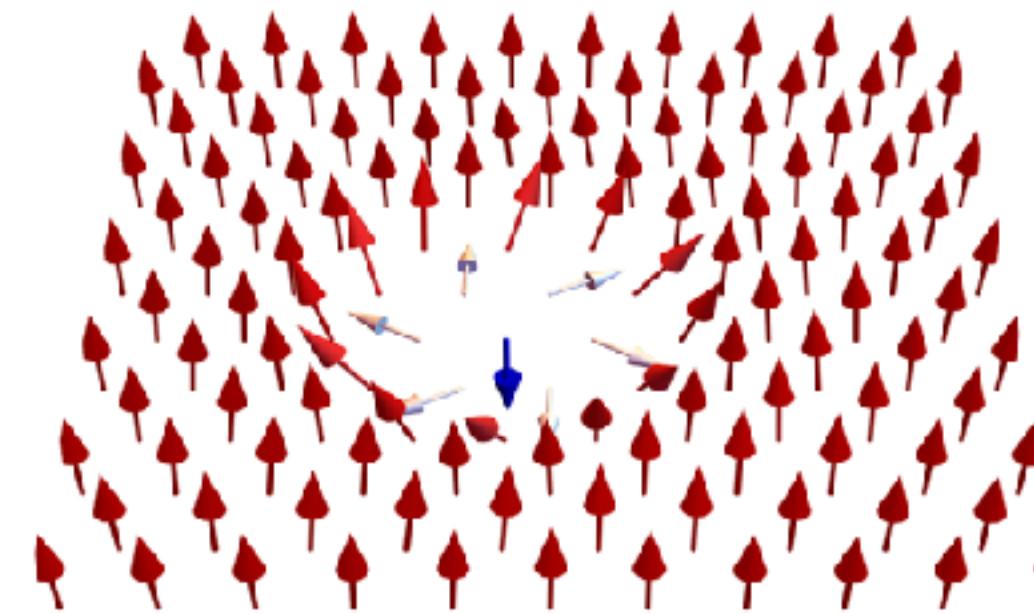
$$S^2 \mapsto \text{CP}^2$$
$$\pi_2(\text{CP}^2) \cong \mathbb{Z}$$

dipole + quadrupole  $\leftrightarrow$  3 level state

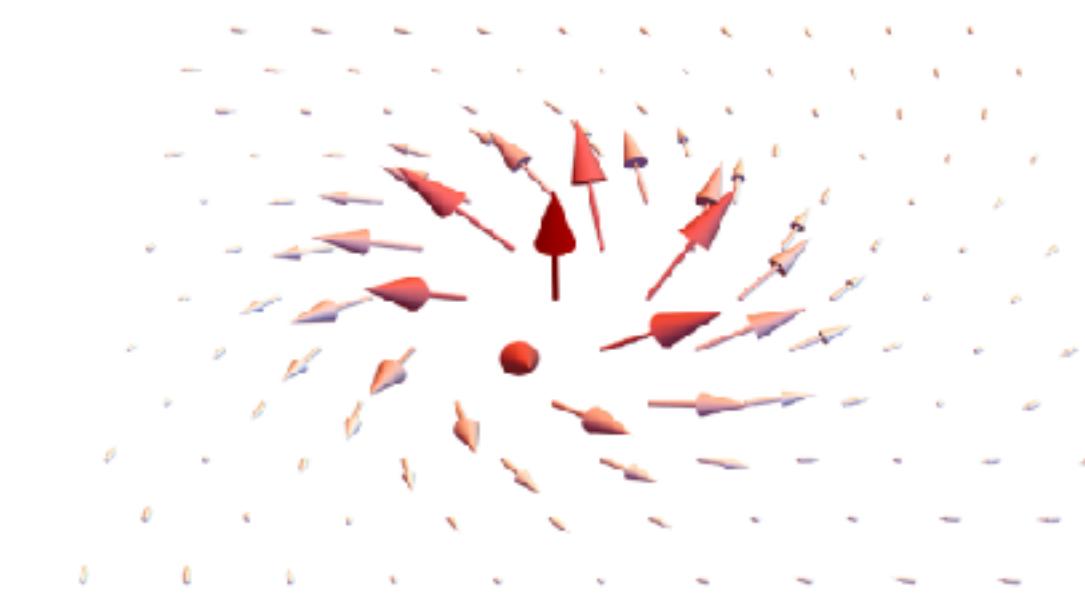


Topologically protected state on top of  
"boring" quantum paramagnet!

## Examples

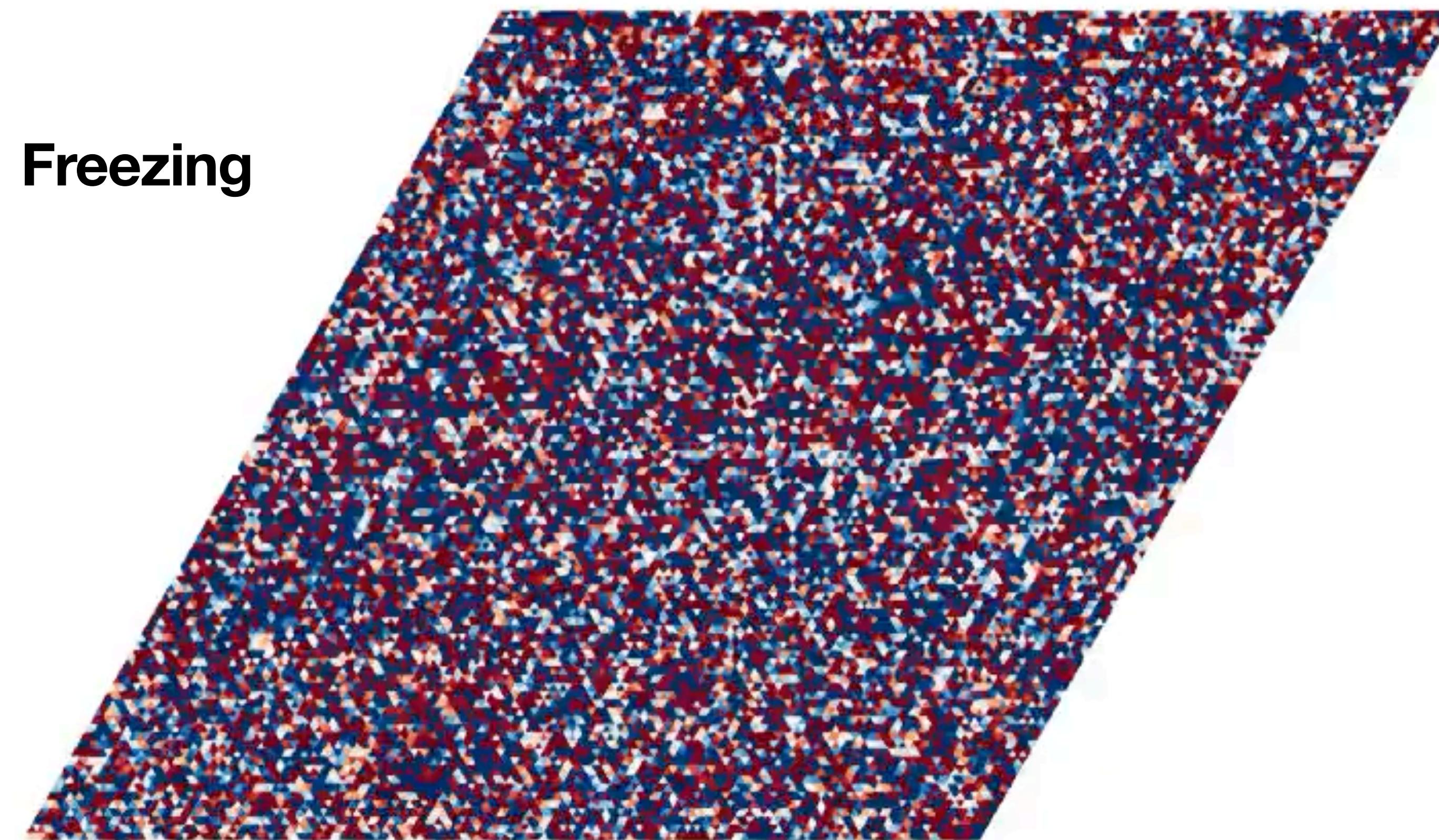


Traditional skyrmion

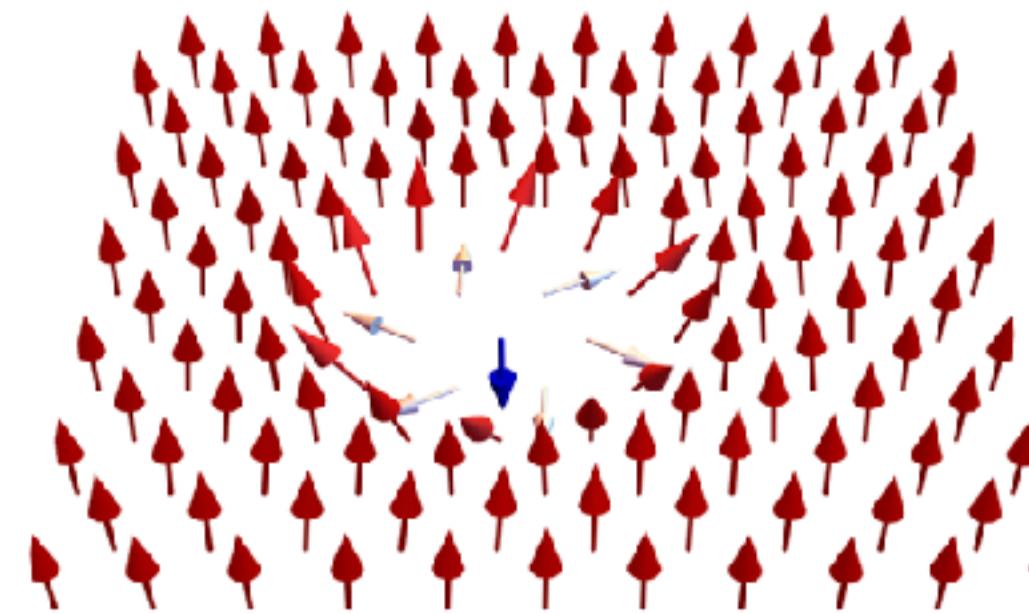


New  $\text{CP}^2$  skyrmion

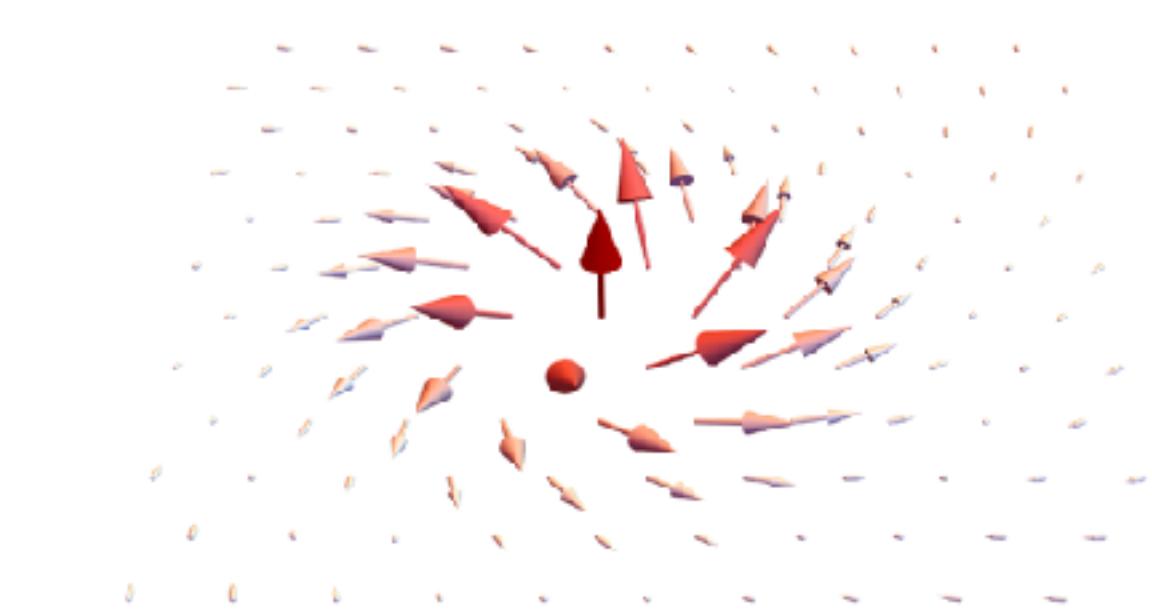
Freezing



## Examples

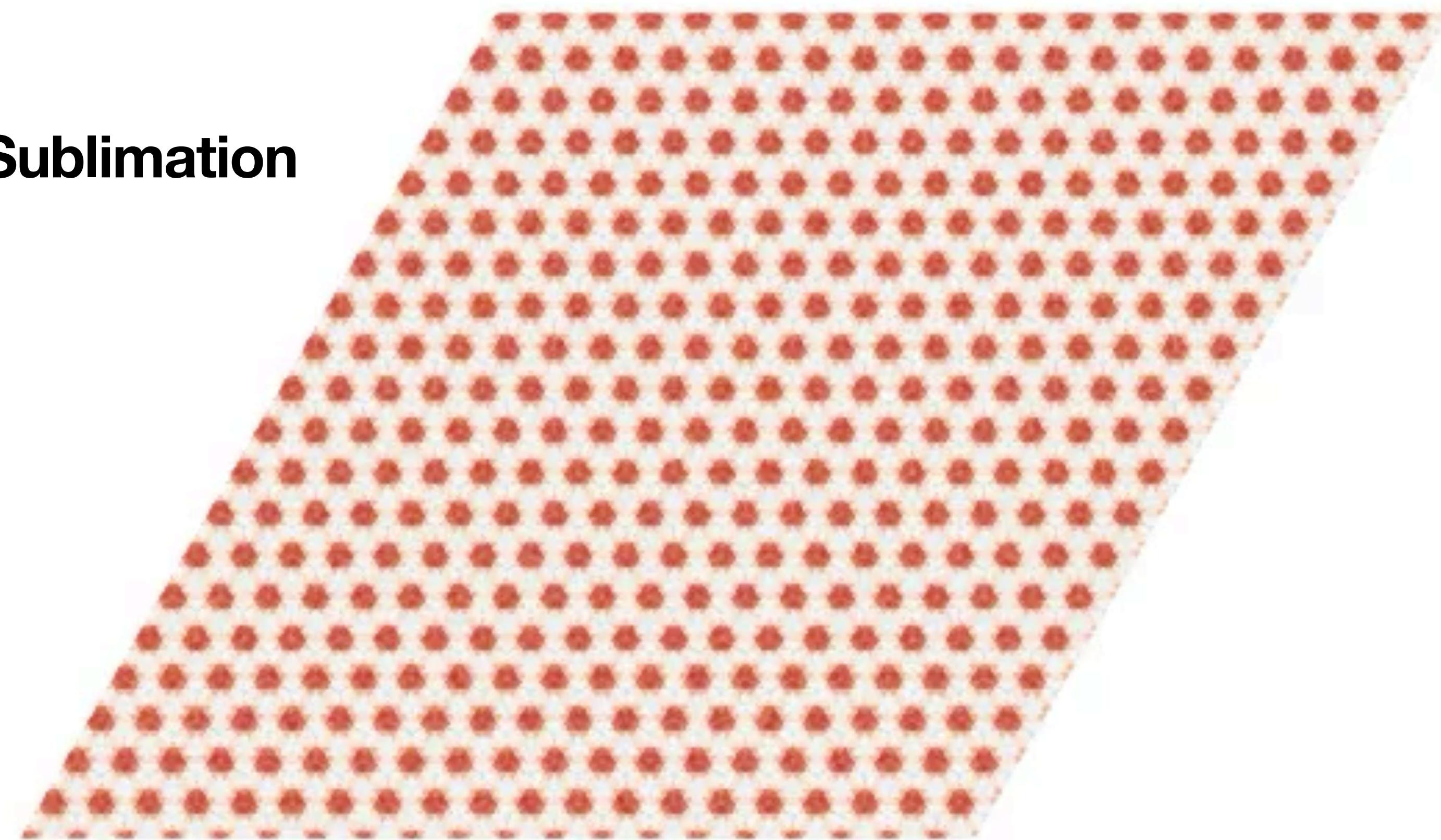


Traditional skyrmion



New  $\text{CP}^2$  skyrmion

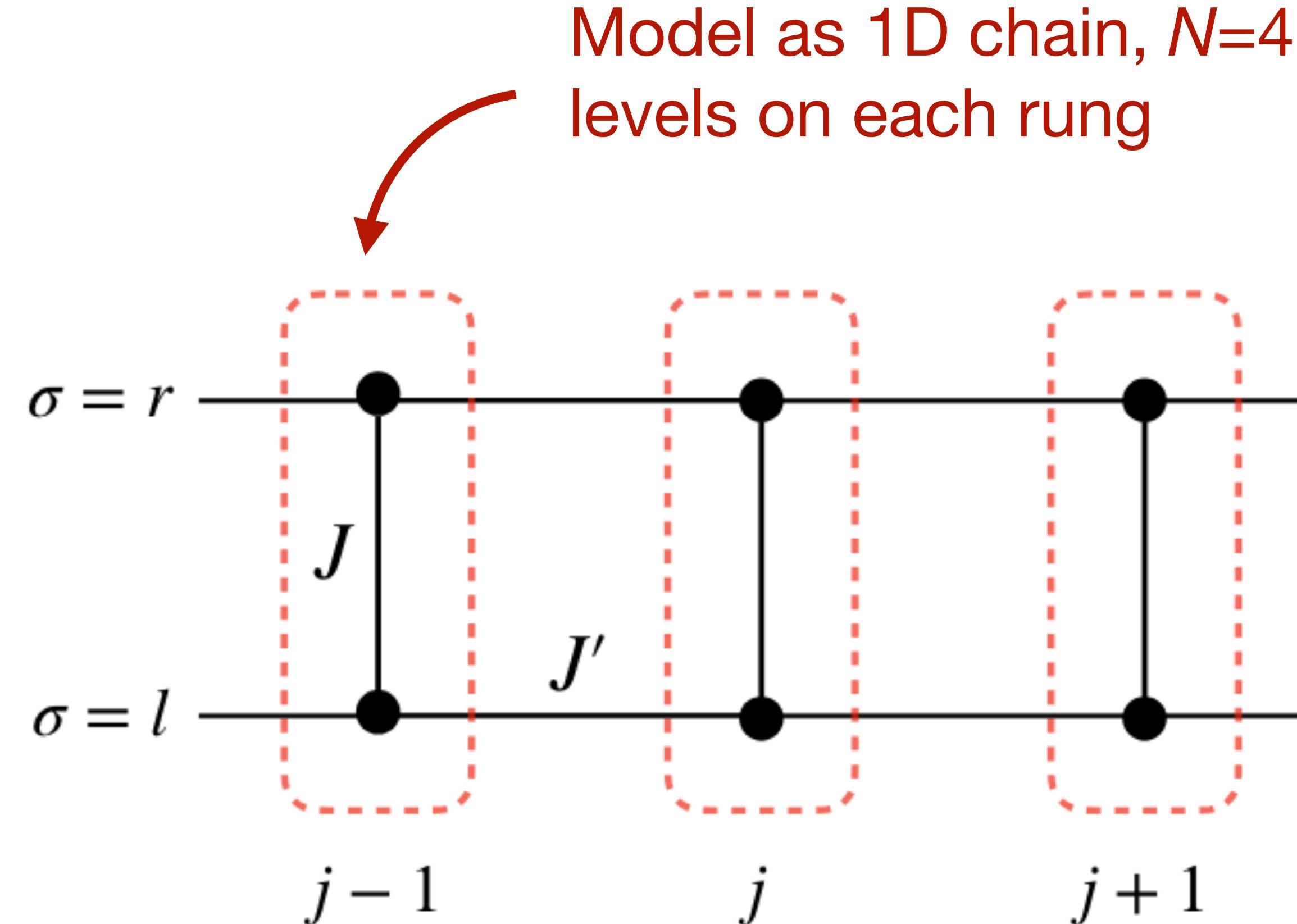
## Sublimation



# Feature preview: Entangled units



D. Dahlbom et al, Phys. Rev. B **110**, 104403 (2024)

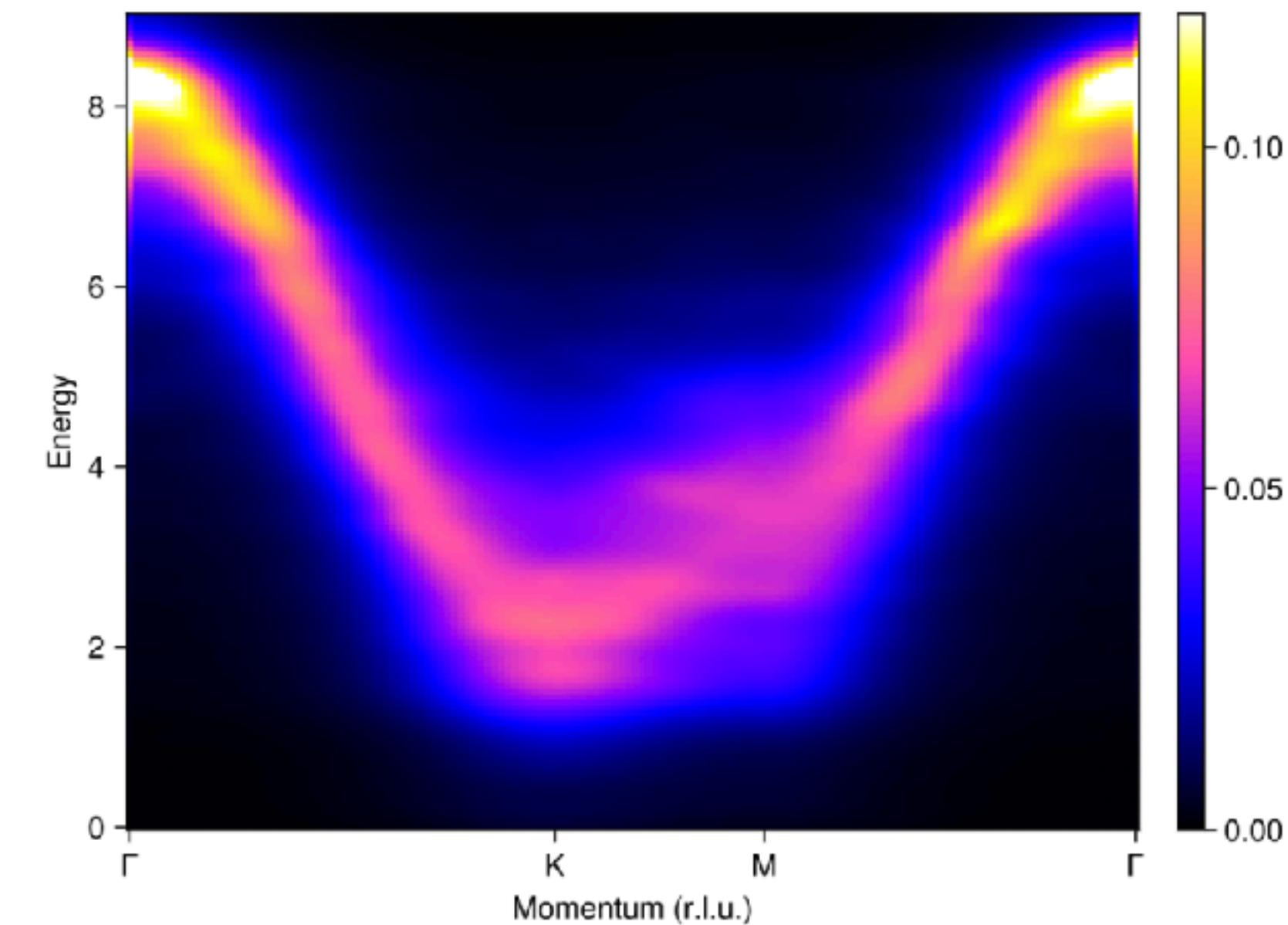
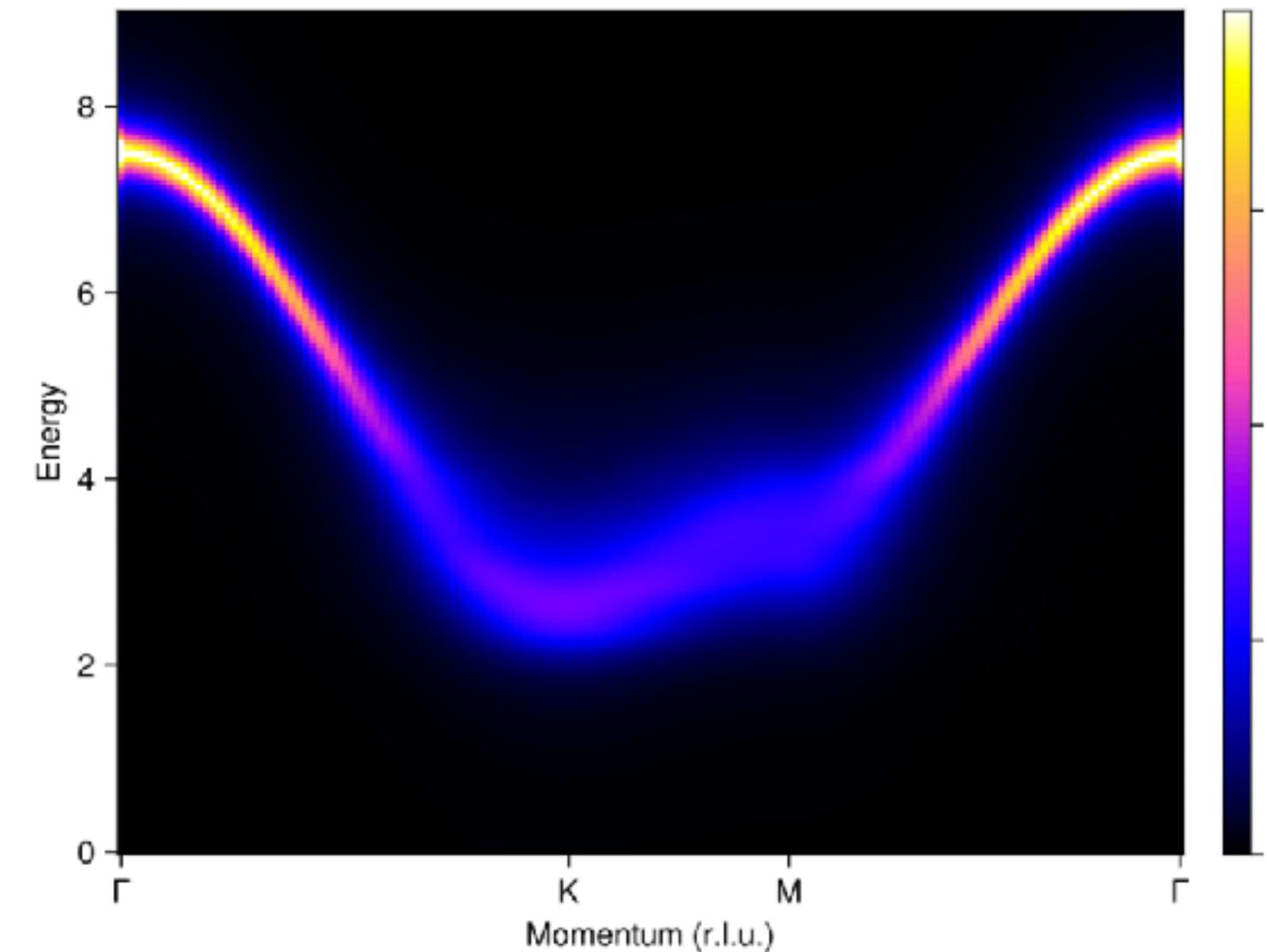
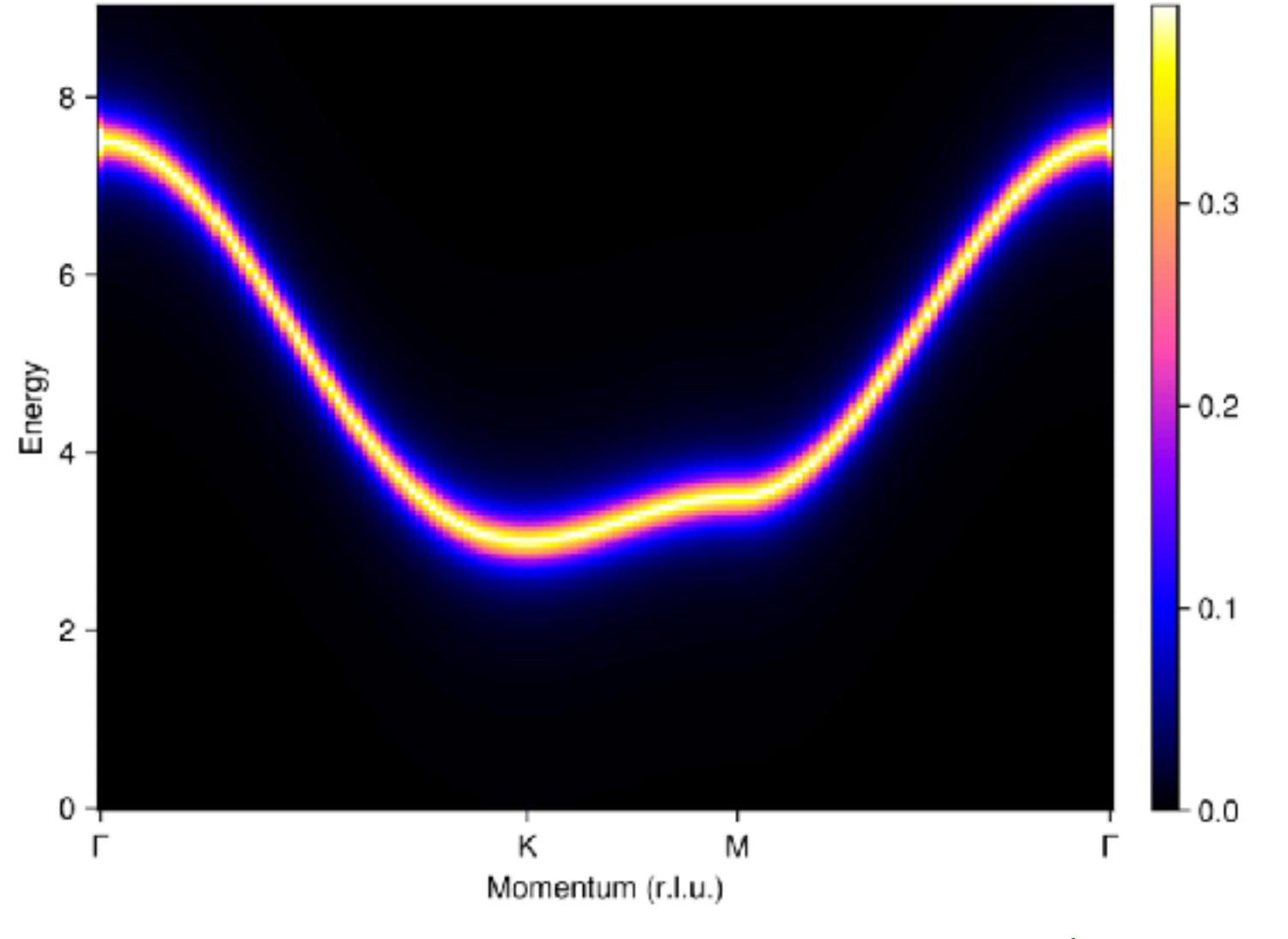


# Disordered systems with KPM



H. Lane et al, SciPost Phys. **17**, 145 (2024)

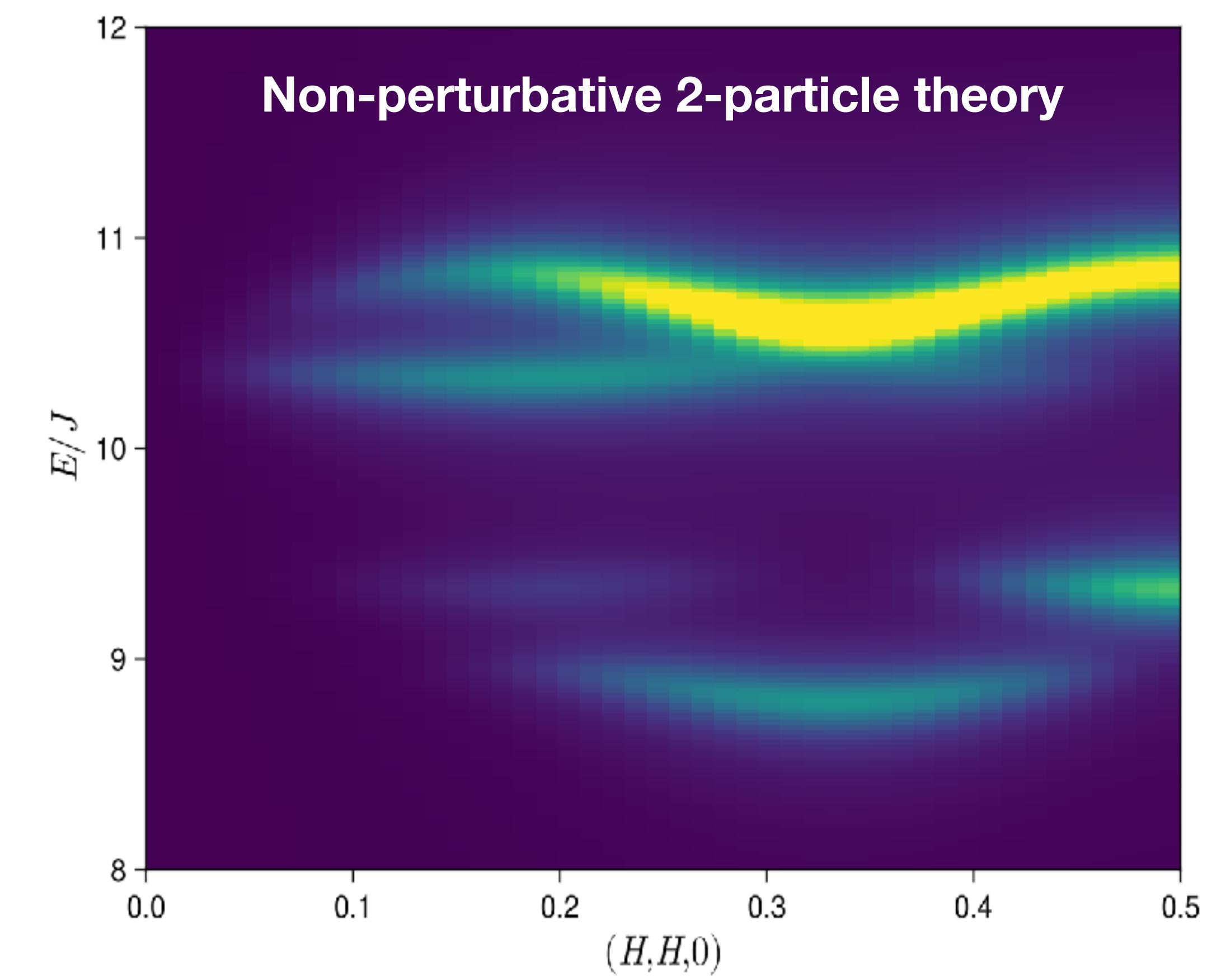
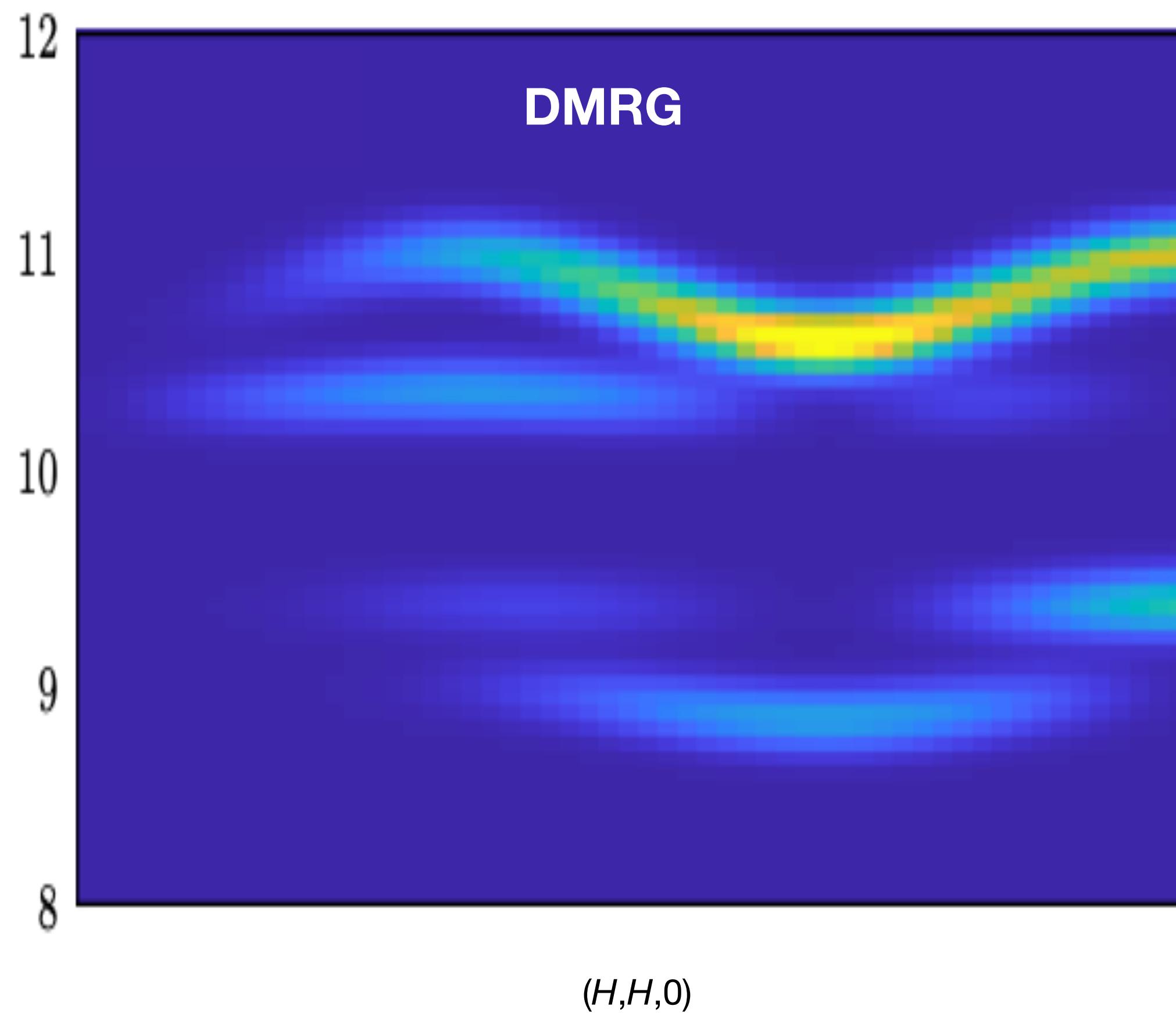
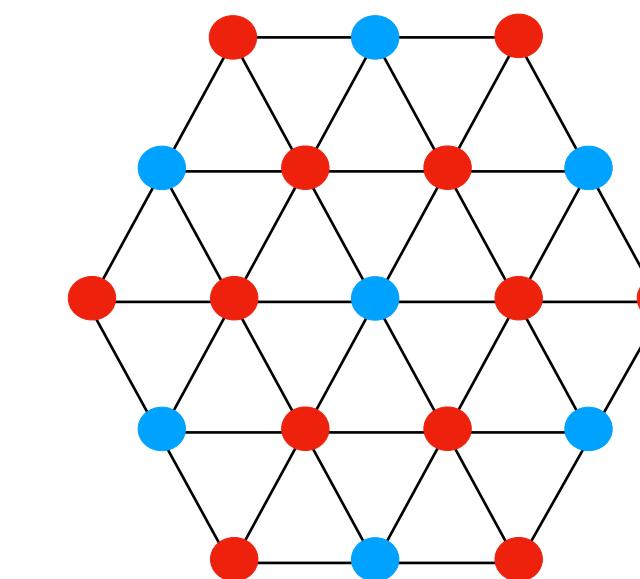
H. Lane et al. [arXiv: 2501.03980]



# Feature preview: Corrections to Spin Wave Theory

## Spin bound states, Hao Zhang

$$\mathcal{H} = J \sum_{\langle i,j \rangle} [S_i^x S_j^x + S_i^y S_j^y + \Delta S_i^z S_j^z] - H \sum_i S_i^z$$

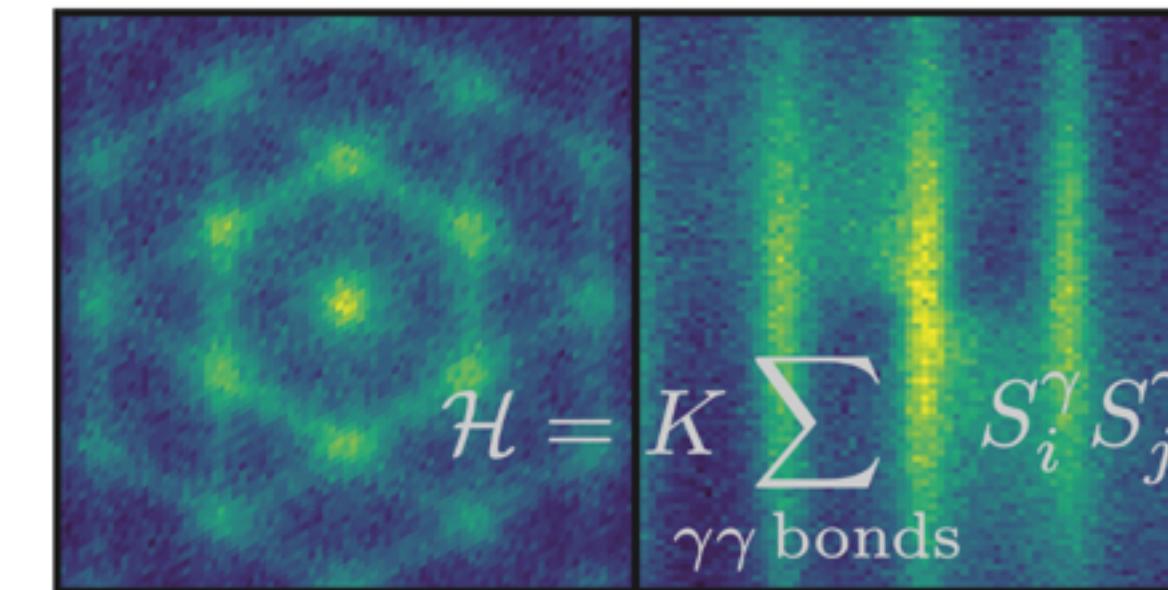


# Feature preview: Self-consistent Gaussian approximation

(Harry Lane)

$$P[S] \sim \exp[-\beta S_{-\mathbf{q}}(J_{\mathbf{q}} + \lambda)S_{\mathbf{q}}]$$

$$\int \langle S_{-\mathbf{q}}S_{\mathbf{q}} \rangle d^3\mathbf{q} = \text{const.}$$



Spinteract

<https://github.com/SunnySuite/Sunny.jl/pull/355>

# Thanks!

Kipton Barros - [kbarros@lanl.gov](mailto:kbarros@lanl.gov)



- D. Dahlbom, H. Zhang, C. Miles, S. Quinn, A. Niraula, B. Thipe, M. Wilson, S. Matin, H. Mankad, S. Hahn, D. Pajerowski, S. Johnston, Z. Wang, H. Lane, Y. W. Li, X. Bai, M. Mourigal, C. D. Batista, K. Barros, *Sunny.jl: A Julia Package for Spin Dynamics* [[arXiv:2501.13095](https://arxiv.org/abs/2501.13095)].
- H. Zhang, C. D. Batista, *Classical spin dynamics based on  $SU(N)$  coherent states*, Phys. Rev. B **104**, 104409 (2021) [[arXiv:2106.14125](https://arxiv.org/abs/2106.14125)].
- D. Dahlbom, H. Zhang, C. Miles, X. Bai, C. D. Batista, K. Barros, *Geometric integration of classical spin dynamics via a mean-field Schrödinger equation*, Phys. Rev. B **106**, 054423 (2022) [[arXiv:2204.07563](https://arxiv.org/abs/2204.07563)].
- D. Dahlbom, C. Miles, H. Zhang, C. D. Batista, K. Barros, *Langevin dynamics of generalized spins as  $SU(N)$  coherent states*, Phys. Rev. B **106**, 235154 (2022) [[arXiv:2209.01265](https://arxiv.org/abs/2209.01265)].
- D. Dahlbom, H. Zhang, Z. Laraib, D. M. Pajerowski, K. Barros, C. D. Batista, *Renormalized Classical Theory of Quantum Magnets*, Submitted, [[arXiv:2304.03874](https://arxiv.org/abs/2304.03874)].
- D. Dahlbom, D. Brooks, M. S. Wilson, S. Chi, A. I. Kolesnikov, M. B. Stone, H. Cao, Y.-W. Li, K. Barros, M. Mourigal, C.D. Batista, X. Bai, *Quantum to classical crossover in generalized spin systems -- the temperature-dependent spin dynamics of FeI<sub>2</sub>*, Phys. Rev. B **109**, 014427 [[arXiv:2310.19905](https://arxiv.org/abs/2310.19905)].
- H. Lane, H. Zhang, D. Dahlbom, S. Quinn, R. D. Somma, M. Martin, C. D. Batista, K. Barros, *Kernel Polynomial Method for Linear Spin Wave Theory*, SciPost Phys. **17**, 145 (2024) [[arXiv:2312.08349](https://arxiv.org/abs/2312.08349)].