Ma323 Lab 02

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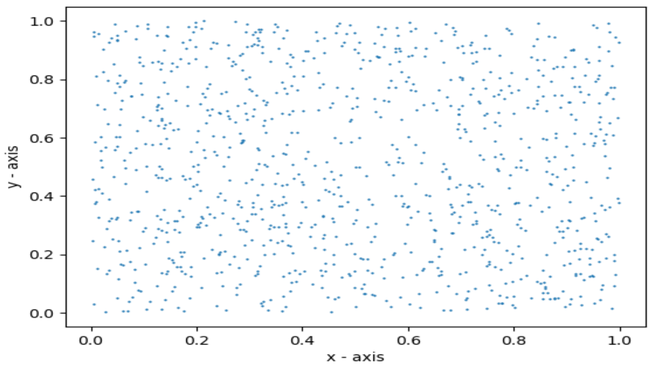
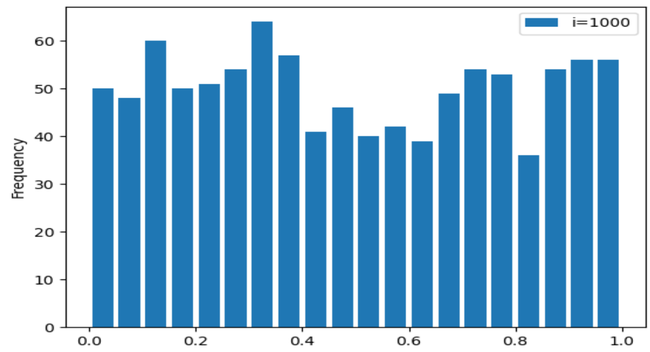
Question 1

The parameters used for generating the first 17 values of U_i, using a linear congruence generator, are : $a = 1597, b = 5, m = 244944, x_0 = 17$.

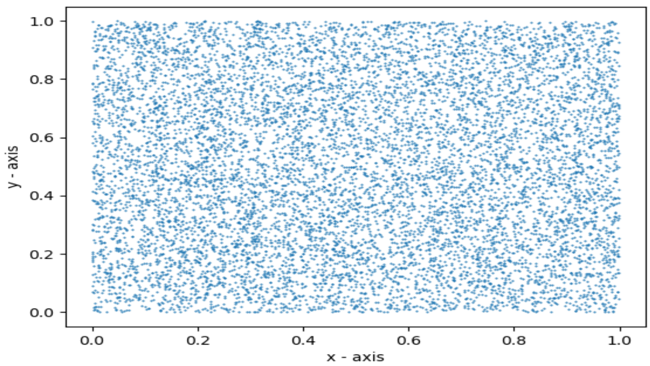
The scattered plot of (U_i, U_{i+1}) and the corresponding frequency bar diagram for different number of values generated:



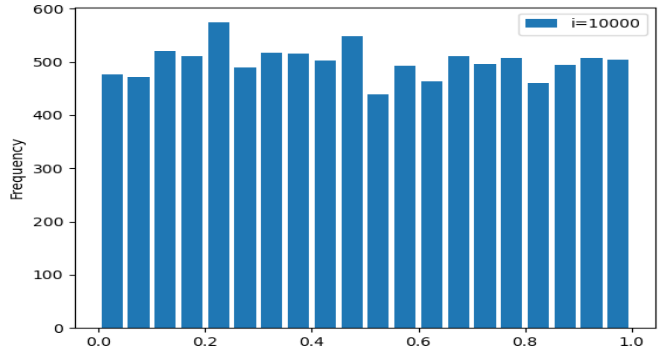
(a) $i = 1000$



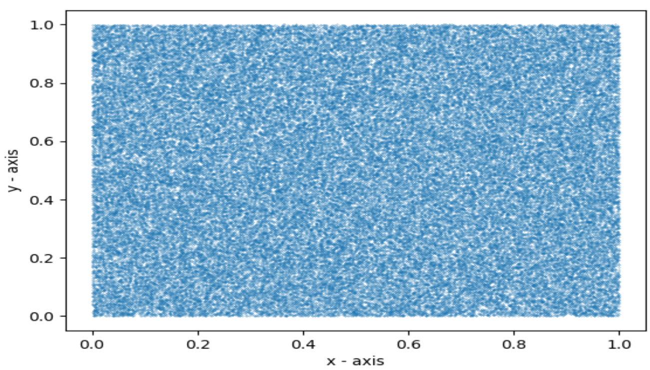
(b) $i = 1000$



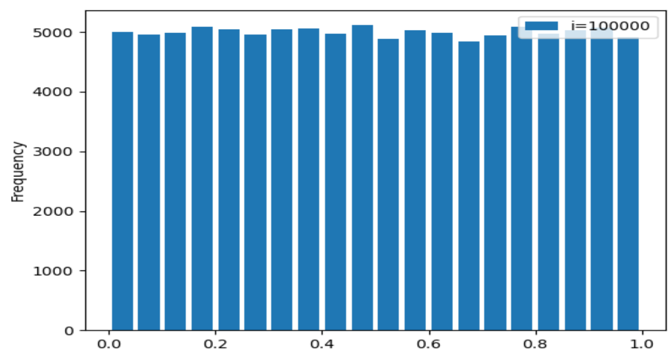
(a) $i = 10000$



(b) $i = 10000$



(a) $i = 100000$



(b) $i = 100000$

Observations:

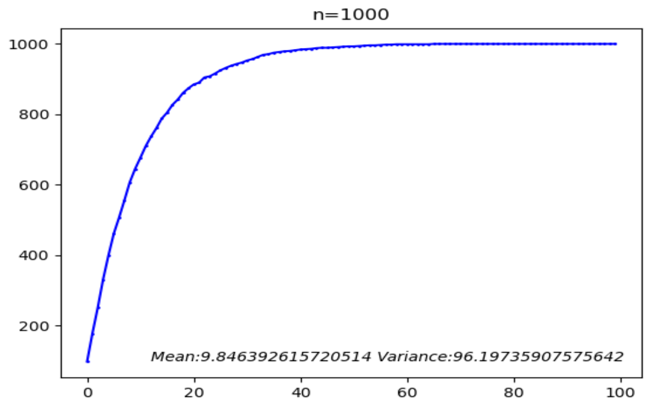
- There is no apparent pattern in the (U_i, U_{i+1}) plot which indicates that U_i is not directly dependent on U_{i+1} (as suggested by the formula of lagged Fibonacci Generator), points are uniformly scattered in 1x1 square. From this we can say that the generated values(points) are uniformly distributed.
- As the sample size is increased, the uniformity of the generated values increases which can be seen in the bar diagrams and also as the sample size increases density of the plots also increases.

Question 2

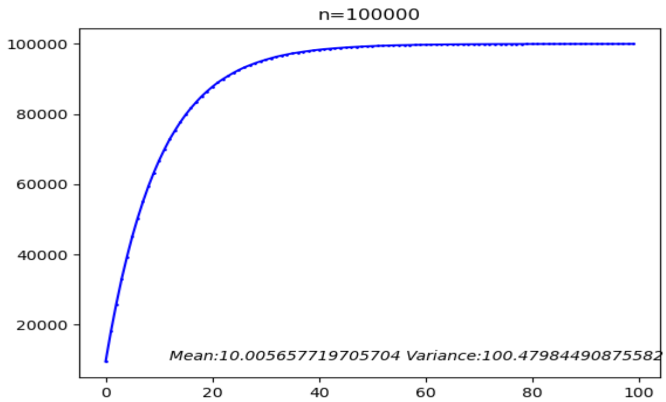
The given exponential distribution is:

$$F(x) = 1 - e^{-x/\theta}, x \geqslant 0$$

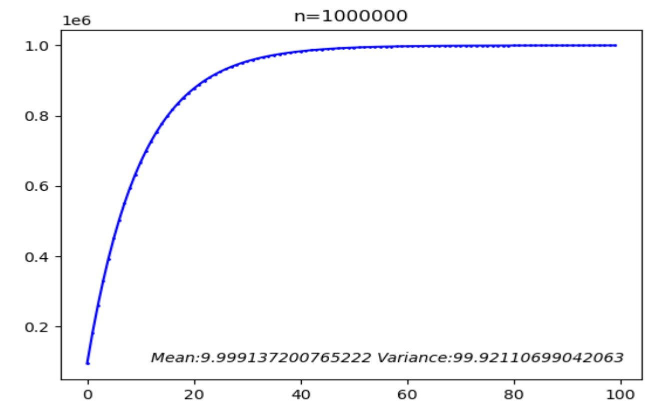
The plot of the cumulative distribution function for the values generated using the inverse transform method, for various sample size (θ taken as 10, Number of intervals = 100) with parameters a=1597, m=244944, b=5 and x_0=17 are given below:



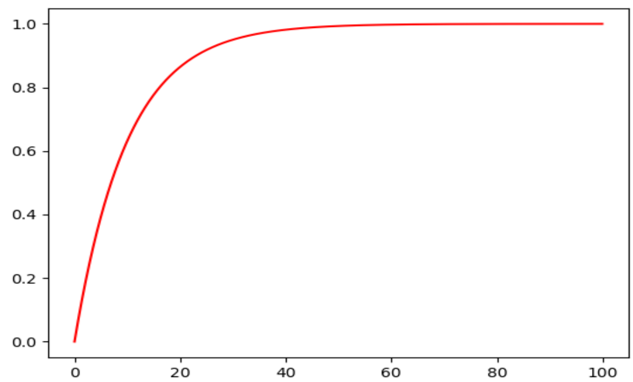
(a) n = 1000



(b) n = 100000



(c) n = 1000000



(d) Actual Distribution Function

The corresponding values of the sample mean and variance:

1. For n = 1000, Mean = 9.846392615720514, Variance = 96.19735907575642.
2. For n = 100000, Mean = 10.005657719705704, Variance = 100.47984490875582.

3. For n = 1000000, Mean = 9.999137200765222, Variance = 99.92110699042063.

Theoretical Mean = $\theta = 10$
Theoretical Variance = $\theta^2 = 10^2 = 100$

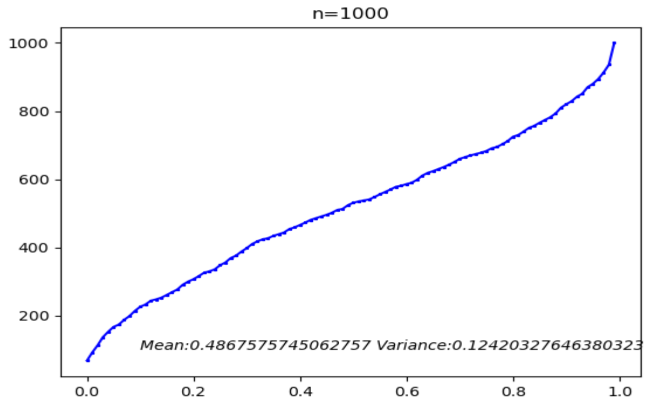
It is evident from above figures that as the sample size is increased, the sample mean and variance go closer to the theoretical mean and variance and hence will eventually **converge** to the theoretical mean and variance. It can be seen that the CDF of random samples approaches the CDF of exponential distribution with θ=10.

Question 3

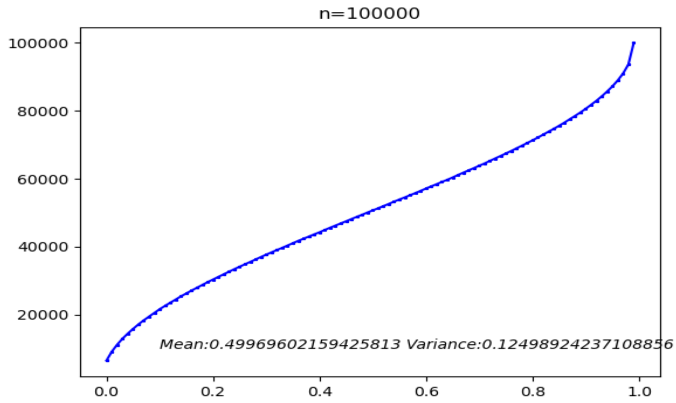
The given distribution is:

$$F(x) = \frac{2}{\pi} \arcsin \sqrt{x}, 0 \leqslant x \leqslant 1$$

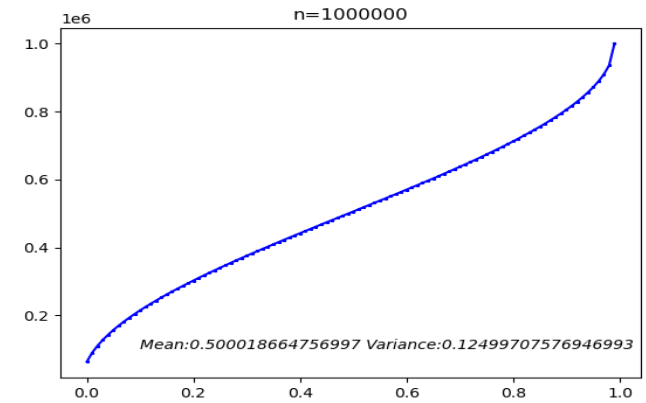
The plot of the cumulative distribution function for the values generated using the inverse transform method, for various sample size (Number of intervals = 100) with parameters a=1597, m=244944, b=5 and x_0=19 are given below:



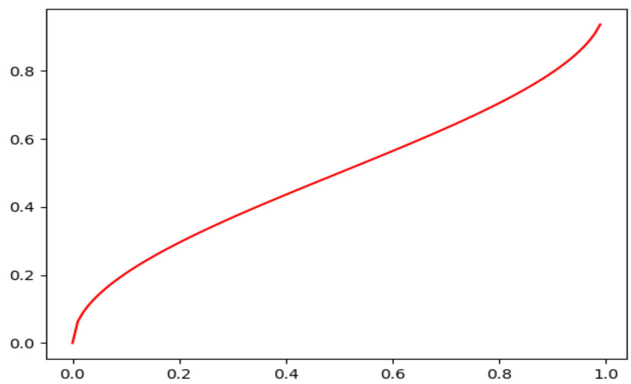
(a) n = 1000



(b) n = 100000



(c) n = 1000000



(d) Actual Distribution Function

The corresponding values of the sample mean and variance:

1. For n = 1000, Mean = 0.4867575745062757, Variance = 0.12420327646380323.
2. For n = 100000, Mean = 0.49969602159425813, Variance = 0.12498924237108856.
3. For n = 1000000, Mean = 0.500018664756997, Variance = 0.12499707576946993.