Ma323 Lab 01

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Question 1

The below table shows the sequence of numbers x_i generated for $a = 6, b = 0, m = 11$ and value of x_0 ranging from 0 to 10.

$x_0 = 0$	$x_0 = 1$	$x_0 = 2$	$x_0 = 3$	$x_0 = 4$	$x_0 = 5$	$x_0 = 6$	$x_0 = 7$	$x_0 = 8$	$x_0 = 9$	$x_0 = 10$
0	1	2	3	4	5	6	7	8	9	10
0	6	1	7	2	8	3	9	4	10	5
0	3	6	9	1	4	7	10	2	5	8
0	7	3	10	6	2	9	5	1	8	4
0	9	7	5	3	1	10	8	6	4	2
0	10	9	8	7	6	5	4	3	2	1
0	5	10	4	9	3	8	2	7	1	6
0	8	5	2	10	7	4	1	9	6	3
0	4	8	1	5	9	2	6	10	3	7
0	2	4	6	8	10	1	3	5	7	9
0	1	2	3	4	5	6	7	8	9	10
0	6	1	7	2	8	3	9	4	10	5

The below table shows the sequence of numbers x_i generated for $a = 3, b = 0, m = 11$ and value of x_0 ranging from 0 to 10.

$x_0 = 0$	$x_0 = 1$	$x_0 = 2$	$x_0 = 3$	$x_0 = 4$	$x_0 = 5$	$x_0 = 6$	$x_0 = 7$	$x_0 = 8$	$x_0 = 9$	$x_0 = 10$
0	1	2	3	4	5	6	7	8	9	10
0	3	6	9	1	4	7	10	2	5	8
0	9	7	5	3	1	10	8	6	4	2
0	5	10	4	9	3	8	2	7	1	6
0	4	8	1	5	9	2	6	10	3	7
0	1	2	3	4	5	6	7	8	9	10
0	3	6	9	1	4	7	10	2	5	8
0	9	7	5	3	1	10	8	6	4	2
0	5	10	4	9	3	8	2	7	1	6
0	4	8	1	5	9	2	6	10	3	7
0	1	2	3	4	5	6	7	8	9	10
0	3	6	9	1	4	7	10	2	5	8

For $a = 6$, 10 distinct values appear before repetition for all x_0 ranging from 1 to 10 (except for $x_0 = 0$).

For $a = 3$, only 5 distinct values appear before repetition for x_0 ranging from 1 to 10 (except for $x_0 = 0$).

The **first option** ($a = 6, b = 0, m = 11$) is a better choice because it has a **full period** i.e., 10, compared to the second option which as a period length of only 5.

When $b = 0$ then the sequence x_i becomes ax_0, ax_0^2, ax_0^3...(mod m) and if $x_0 = 0$ then the sequence becomes constant which is easily predictable.

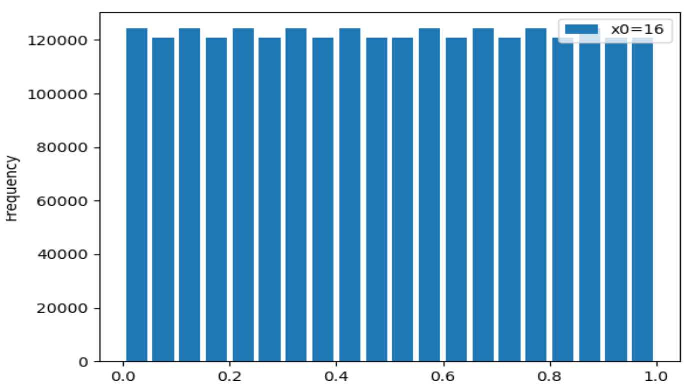
So, the best combination in above 2 cases is $a = 6, b = 0, m = 11$ with any x_0 other than 0. With all other conditions being equal, generators with larger period should be preferred because it makes the sequence less discernible.

Question 2

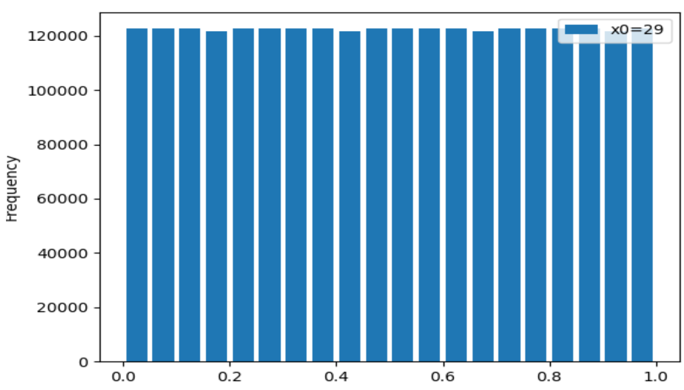
The frequency table for $a = 1597, b = 0, m = 244944$ is shown below;

Frequency	$x_0 = 16$	$x_0 = 29$	$x_0 = 331$	$x_0 = 867$	$x_0 = 1000$
0.0-0.05	124320	122640	121800	123480	120960
0.05-0.1	120960	122640	122640	120960	120960
0.1-0.15	124320	122640	122640	123480	124320
0.15-0.2	120960	121800	122640	120960	120960
0.2-0.25	124320	122640	122640	123480	124320
0.25-0.3	120960	122640	121800	123480	120960
0.3-0.35	124320	122640	122640	120960	124320
0.35-0.4	120960	122640	122640	123480	120960
0.4-0.45	124320	121800	122640	120960	124320
0.45-0.5	120960	122640	122640	123480	120960
0.5-0.55	120960	122640	121800	123480	120960
0.55-0.6	124320	122640	122640	120960	124320
0.6-0.65	120960	122640	122640	123480	120960
0.65-0.7	124320	121800	122640	120960	124320
0.7-0.75	120960	122640	122640	123480	120960
0.75-0.8	124320	122640	121800	123480	124320
0.8-0.85	120960	122640	122640	120960	120960
0.85-0.9	124320	122640	122640	123480	124320
0.9-0.95	120960	121800	122640	120960	120960
0.95-1.0	120960	122640	122640	123480	124320

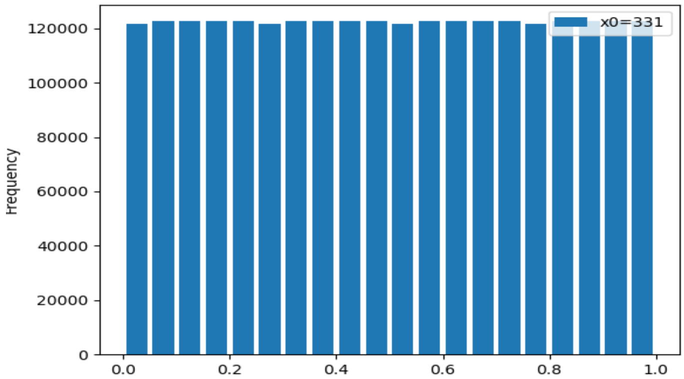
The histograms for 5 different values of x_0 and $a = 1597, b = 0, m = 244944$ are shown below:



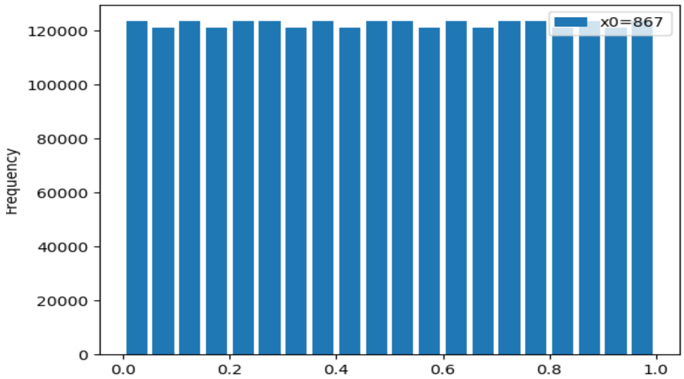
(a) $a = 1597, b = 0, m = 244944$ and $x_0 = 16$



(b) $a = 1597, b = 0, m = 244944$ and $x_0 = 29$



(a) $a = 1597, b = 0, m = 244944$ and $x_0 = 331$



(b) $a = 1597, b = 0, m = 244944$ and $x_0 = 867$

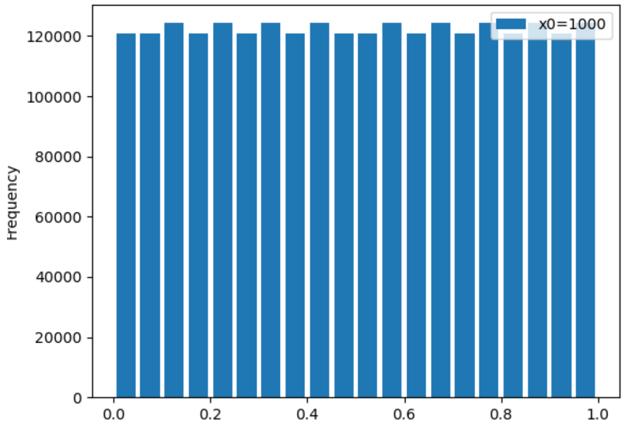
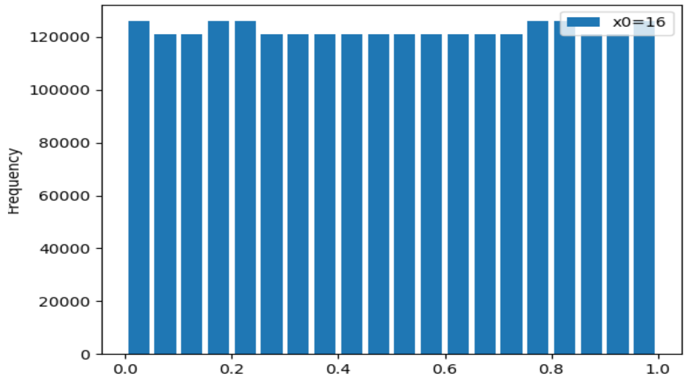


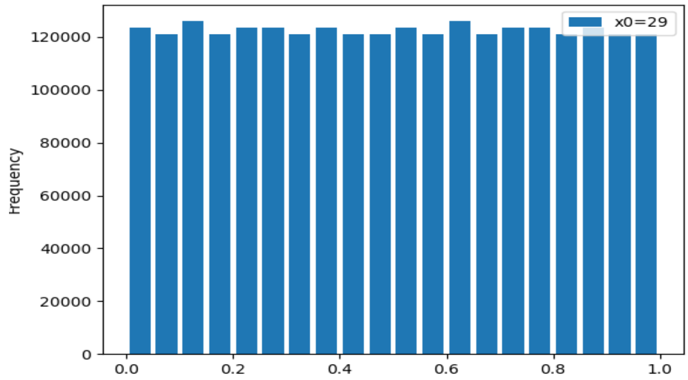
Figure 3: $a = 1597, b = 0, m = 244944$ and $x_0 = 1000$

The frequency table for $a = 51749, b = 0, m = 244944$ is shown below;

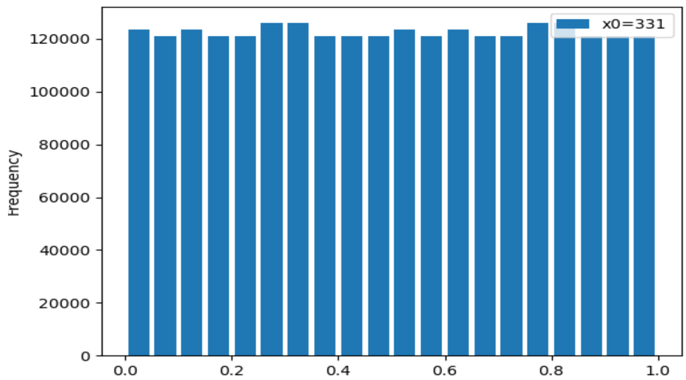
Frequency	$x_0 = 16$	$x_0 = 29$	$x_0 = 331$	$x_0 = 867$	$x_0 = 1000$
0.0-0.05	126000	123480	123480	136080	120960
0.05-0.1	120960	120960	120960	113400	120960
0.1-0.15	120960	126000	123480	120960	120960
0.15-0.2	126000	120960	120960	128520	120960
0.2-0.25	126000	123480	120960	113400	126000
0.25-0.3	120960	123480	126000	136080	126000
0.3-0.35	120960	120960	126000	113400	126000
0.35-0.4	120960	123480	120960	128520	120960
0.4-0.45	120960	120960	120960	120960	120960
0.45-0.5	120960	120960	120960	113400	120960
0.5-0.55	120960	123480	123480	136080	120960
0.55-0.6	120960	120960	120960	113400	120960
0.6-0.65	120960	126000	123480	120960	120960
0.65-0.7	120960	120960	120960	128520	126000
0.7-0.75	120960	123480	120960	113400	126000
0.75-0.8	126000	123480	126000	136080	126000
0.8-0.85	126000	120960	126000	113400	120960
0.85-0.9	120960	123480	120960	128520	120960
0.9-0.95	120960	120960	120960	120960	120960
0.95-1.0	126000	120960	120960	113400	120960



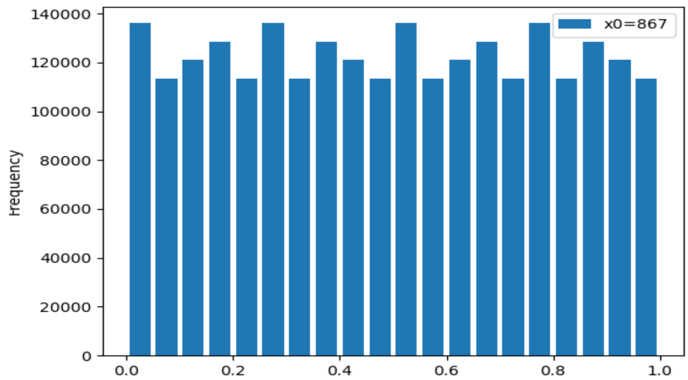
(a) $a = 51749, b = 0, m = 244944$ and $x_0 = 16$



(b) $a = 51749, b = 0, m = 244944$ and $x_0 = 29$



(a) $a = 51749, b = 0, m = 244944$ and $x_0 = 331$



(b) $a = 51749, b = 0, m = 244944$ and $x_0 = 867$

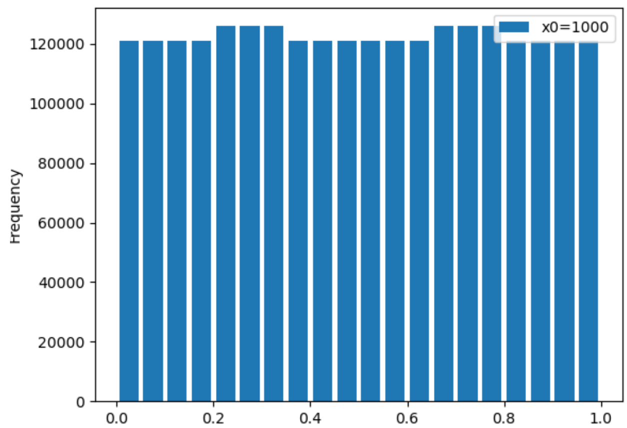


Figure 6: $a = 51749, b = 0, m = 244944$ and $x_0 = 1000$

Observations:

1. From above two cases it can be observed that the longer the period length higher is the uniformity.

2. If the parameters follow the knuth condition then the histogram/ distribution is most uniform because the generator attains full period length(this can be verified by taking a=1597,b=5 and m=244944).

3. For above condition we will have the exactly same histogram/distribution irrespective of x_0 because every value possible (0 to m-1) will be included in the sequence x_i.

4. Moreover, from the above histograms and the frequency tables, it can be observed that frequency of values for each bin (0-0.05,0.05-0.1,..) is approximately equal, indicating that the values generated by the linear congruence generator is uniformly distributed between 0 and 1.

Question 3

For $a = 1229, b = 1, m = 2048$ & $x_0 = 10$, the plot of (u_{i-1}, u_i) is:

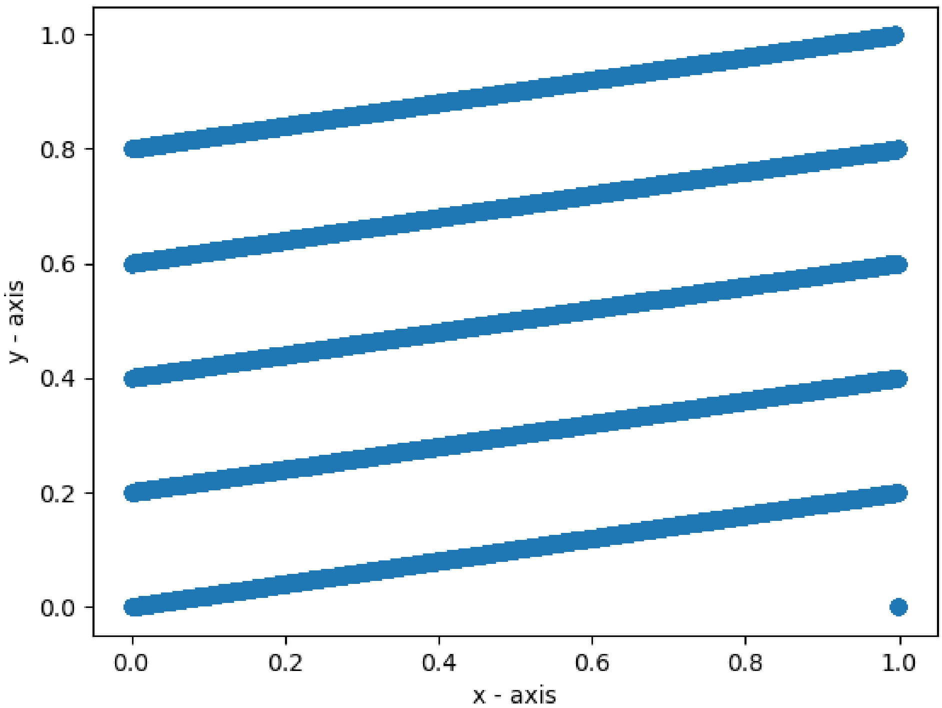


Figure 7: *Plot*

A direct consequence of knuth condition is that if b is odd, m is a power of 2 and a=4n+1 for some n then the generator has a full period. All these conditions are satisfied by the given parameters (b=1,m=2^{11} and a=1229(4*307+1). Hence, the generator has a full period which can be easily verified by the plot.