Coevolutionary Opinion Dynamics with Sparse Interactions in Open-ended Societies

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Abstract

Opinion dynamics is a crucial topic in complex social systems. However, existing models rarely study limited information accessibility, sparse interactions, and the coevolution of opinion in an open-ended structure. In this paper, we propose the **S**parse **CO**evolutionary **O**pen-**E**nded (**SCOOE**) model. We address the sparse interaction limitation through extrinsic collective interaction and intrinsic observation based on incomplete neighborhood information. We also consider the coevolution of opinion and open-ended structure by studying structure-opinion co-dynamics when dissidents are leaving and when newcomers with novel opinions are joining. From an opinion dynamics perspective, we find that the proposed mechanisms effectively form lean and fast decision strategies to reduce conflicts under uncertainty. The model is robust in boosting and enhancing a global consensus with only small odds of extreme results. The structure evolves toward a small-world network. We find that an emergent dialectic relationship exists between community segregation and community cohesion viewed from a structural dynamics perspective. We also study the influence of agent heterogeneity under different cognitive ability distributions.

Keywords: Opinion Dynamics, Sparse Interactions, Collective Decision-making, Open-endedness, Coevolution.

1. Introduction

The study of opinion dynamics, i.e., the study of the formation and dynamics of public opinions, is a crucial research topic in complex systems and social networks. The topic has been widely explored for several decades in theoretical models and in real-world applications among different disciplines, including social science, control engineering, statistical physics, and computer science. Elucidation of the mechanisms behind macro-level opinion dynamics is vital for

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the understanding of social interactions/dynamics, complexity, distributed control, and decision-making. It also holds valuable lessons to apply to real-world empirical studies and applications like marketing and social media [1].

An agent-based model is a type of computational model focusing on the bottom-level (micro) interactions and their effects on the holistic (macro) system. It is a powerful tool to study evolutionary phenomena in both the natural and social sciences. Many classic agent-based models have been explored under various assumptions to study opinion dynamics from different perspectives. For example, the Hegselmann-Krause model [2] studies opinion polarization with the bounded confidence assumption, i.e., agents interact only if their opinions are sufficiently close to each other by falling within a confidence interval. The Sznajd model and its variations [3] study the evolution of consensus in a closed society through majority voting. In that model, a focal agent polls its complete neighborhood (i.e., the group of agents sharing connections with the focal agent in the social network) and selects the opinion of the majority. However, some assumptions in existing models, e.g., polling the complete neighborhood, seem to be no longer suitable, notably when people with bounded rationality only have a partial view and cannot access the full neighborhood information in their social networks. Meanwhile, when we interact with neighbors, the literature from psychology suggests that we are mainly concerned with the overall opinion of neighbors (e.g., a joint opinion through collective decision-making), and we adjust our own opinions according to this feedback [4]. Some other work uses the bounded-confidence assumption [5] applying dense interactions and serial opinion updates through interacting with all selected neighbors. While existing models thoroughly describe the dynamics of opinions and interactions, they ignore the built-in structural dynamics caused by opinion dynamics and open-endedness, e.g., through the effects of newcomers, leavers, and their impact on structure-opinion coevolution.

The intent of this paper is to develop the **S**parse **CO**evolutionary **O**pen-**E**nded (**SCOOE**) model, then use this model to answer two questions: **How do opinions evolve under conditions of sparse interactions and incomplete information, and relatedly how do opinion dynamics guide the coevolution of an open-ended society?** For the first question, the focal agent in our model has a limited view of a partial neighborhood. It first only considers the interactions with the joint opinion formed by the collective decision-making based on incomplete neighborhood information, rather than interacting with all neighbors or polling the entire neighborhood. We then introduce an observation mechanism by which the focal agent adjusts its opinion by observing the incomplete neighborhood environment without direct interactions with neighbors. The two mechanisms play roles in influencing opinion evolution at different stages. For the second question, interactions and opinions in our open-ended society coevolve in the model: The opinion dynamics affects which agent will become a dissident, exiting the society and the associated structural dynamics. Meanwhile, a joiner with possibly novel opinions affects its neighbors and its opinion might cascade through the system via interactions. We carry out experiments to study the structural dynamics, opinion dynamics in search for answers to these two questions.

The rest of the paper is organized as follows: We critically review the related literature and comment on the current limitations in Section 2. Our SCOOE model is then introduced in Section 3, including agent design/model initialization (Section 3.1) and model dynamics (Section 3.2). Experimental results are reported in Section 4. In Section 5, we present a comprehensive discussion of the model. We conclude by pointing out future research directions in Section 6.

2. Previous Work

Research in opinion dynamics models is mainly conducted in two directions. If the focal entity (agent) chooses opinions from a discrete set of opinions, the model is called a discrete-opinion model [6], whereas if the focal agent's opinions are represented by a continuous interval, the model is called a continuous-opinion model [2, 7]. In the general framework of an opinion dynamics model, agents update their opinions according to interactions with randomly chosen others or with connected neighbors in a networked structure. Stable patterns of opinions will evolve over time, e.g., group agreement, polarization, or multiple local opinions distributed in different communities [8].

Recent work has revised the following three perspectives to improve the two basic models: (i) the representation of opinions, (ii) the opinion fusion methods, and (iii) the heterogeneity of agents. Some work has explored novel representations of opinions with multiple topics [9]. Liu et al. [10] propose a fuzzy set-based representation of opinions reflecting the attitudes of tolerance and stubbornness of humans. Some work has introduced different mechanisms for opinion fusion, e.g., the multi-level bounded confidence model [11], where a society is divided into multiple subgroups with different levels of confidence intervals. There, agents prefer to only interact with others whose opinions are falling into their corresponding confidence intervals. Some work has introduced different types of heterogeneity, including antagonistic agents [12], leader-follower relationships [13], and stubborn agents [14].

Although this previous work has offered important insights, it also involves two important limitations. First, a widely adopted assumption in existing models is complete information, i.e., rational agents have perfect information to poll their neighborhoods or conduct interaction preference (e.g., selecting neighbors), for an update of their opinions. However, this assumption is unrealistic, notably when the complete information of the neighborhood is inaccessible. We can indeed be affected by many people. However, the literature from psychology suggests that agents are primarily influenced by taking the collective opinions of others (e.g., neighbors in the social network) into account, rather than updating their opinions serially and densely by interacting with all their neighbors or certain selected neighbors with similar opinions [4, 15]. Sparse interactions with incomplete information have rarely been studied previously [16, 17].

Second, the majority of previous model extensions are carried out in closed systems. Insufficient attention is paid to the built-in system dynamics. Nevertheless, it is valuable to consider an open-ended evolutionary system where new agents can join and dissidents can leave, where agent opinions are updated continuously (opinion evolution), and where the structure coevolves simultaneously with opinion evolution (the coevolution of structure and opinion).

We address these limitations in the proposed SCOOE model, which takes into account the bounded rationality (e.g., incomplete information) of agents and the coevolution of structure and opinion in an open-ended society.

3. Sparse Coevolutionary Open-ended (SCOOE) Model

In this section we describe the two main aspects of the proposed model, model initialization and model dynamics.

3.1. Agent Design and Model Initialization

First we describe the basic building blocks of the SCOOE model, heterogeneous agents and opinion representation.

3.1.1. Heterogeneous Agents

The majority of previous work has aimed to assign agents different role-based or function-based heterogeneity (e.g., leader-follower) to empower the model describing different real-world behaviors. In this paper, we ask the following question: What factors primarily influence the integration of a new opinion with an original opinion? Literature from psychology suggests that the most critical factor in opinion fusion is the built-in cognitive heterogeneity of humans. For instance, the Big Five personality traits model asserts that people's personality exists on a spectrum with multiple dimensions, e.g., inventive/curious v.s. consistent/cautious [18]. Cognitive dissonance theory describes the psychological stress when people are exposed to contradictory opinions and reconcile them to be consistent [8]. For agent i being exposed to a new opinion, we assume that agent i has a built-in probability to stick to its opinion, i.e., a stubbornness probability w_i. It describes the degree to which an agent relies on its original opinion. In contrast, the complement of stubbornness, i.e., an openness probability $1 - w_i$, quantifies the degree to which agent i is willing to adopt a new opinion derived from the interaction with other agents. Heterogeneity is produced when agents hold different built-in cognitive features represented by different stubbornness (or openness) probabilities. We assume that stubbornness w_i follows a probability distribution in the population, like a Poisson or Gaussian distribution. In the experimental section below, we report on the influence of different stubbornness distributions.

3.1.2. Opinions of Agents

The opinion O_i of an agent i is represented by a real number in the continuous interval [0,1]. It describes the degree to which an agent believes the propagated information, e.g., news or rumors. A higher value of opinion O_i means that agent i believes the propagated information more strongly. Initially, each agent is assigned a random opinion, i.e., a random number in the range [0,1].

Note that the opinion represented by O_i is independent of the stubbornness probability. The former refers to the attitude of an agent toward news items. It will be expressed and updated by taking the opinions of others into account. The latter describes the personality and cognitive ability of an agent. It is built into an agent, only refers to the agent itself, and will not be expressed or changed. In the real world, people (e.g., agents i) could initially believe a piece of news by selectively collecting information themselves (confirmation bias, i.e., people collect information trying to follow their original beliefs or biases, e.g., original opinions). However, they might easily be persuaded to change their opinions through some new evidence or short-term interactions with others. In such a case, the initial opinion of an agent i is $O_i \approx 1$, while its stubbornness is $w_i \approx 0$. In contrast, agent i might disagree with the news and could be difficult to convince. In such a case, $O_i \approx 0$ while $w_i \approx 1$. It is common to find that stubbornness and belief-based opinions are not clearly differentiated in existing work.

3.2. Model Dynamics

Here we describe the mechanisms of the SCOOE model dynamics, i.e., the sparse interaction protocol of opinion dynamics and the coevolution of opinion and open-ended structure. An overall picture of the SCOOE model is shown in Figure 1.

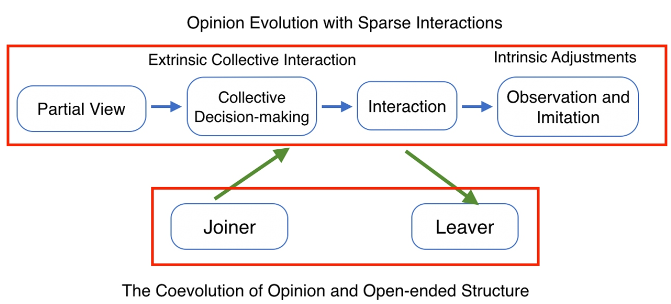


Figure 1: The whole picture of the SCOOE model dynamics. Opinion dynamics with two components of sparse interactions (extrinsic and intrinsic forms): The focal agent with a limited view can only access a partial neighborhood. It aggregates a joint opinion of the incomplete neighborhood by collective decision-making. Then the focal agent only takes this joint opinion from the extrinsic incomplete neighborhood into account by the interaction with the joint opinion, rather than by dense interactions with all neighbors or certain neighbors with similar opinions selected by polling the neighborhood. Imitation is also introduced to drive opinion intrinsic adjustments based on observation of the incomplete neighborhood environment without direct interactions with neighbors. The coevolution of open-ended structure and opinion: The opinion dynamics affect the leaver exiting society and associated structural dynamics. A joiner with a random opinion joins. It changes the structural features and the neighborhood settings, and the neighborhood settings in turn affect the opinion dynamics.

3.2.1. Opinion Dynamics with Sparse Interactions

We first discuss the opinions dynamics. The critical theme of opinion dynamics is sparse interaction and incomplete information. As we pointed out in the introduction and related work section, people do not serially poll the neighborhood in their social networks to update the opinion, but are mainly influenced through considering the joint opinion of others as their feedback [4]. This contrasts with most agent-based models which are formulated with such complete information assumptions, e.g., polling the entire neighborhood to select neighbors and conduct dense interactions serially to update opinions [4, 15].

The literature from psychology suggests two types of motivations for humans to change behavior, extrinsic motivation (people are assimilated into extrinsic environments) and intrinsic motivation (people are motivated by internal desire) [19]. We take inspiration from this and assume two types of actions forming the sparse interaction, extrinsic collective interactions and an intrinsic observation mechanism. The interplay between these two actions enhances group opinion evolution. However, they play different roles in various stages of the model dynamics reported in the experimental section.

Extrinsic Collective Interaction with Incomplete Information. Here, we introduce a collective decision-making approach to incorporate the sparse joint opinion formation and interaction based on a limited neighborhood (i.e., incomplete information). Though several collective decision-making approaches have been proposed in discrete-opinion models, e.g., majority voting [20], this approach in continuous-opinion models has not been fully developed so far.

Suppose a focal agent i with a connection degree d_i in its social network is able to only access a random subset of neighbors i_1, i_2,..., i_j, where j is randomly chosen and satisfies $1 \leq j \leq d_i$. This assumption means incomplete information by a limited view and only partial access to neighbors, and it allows more dynamic interactions, e.g., an agent will not interact with its entire neighborhood. Agent i generates a joint opinion O_i^{Joint} of its random partial neighborhood, rather than by interacting with all its neighbors or certain neighbors with similar opinions serially. An intuitive way to generate a joint opinion is by taking the weighted average of the selected neighbors' opinions [21]. The weights assigned to different neighbors are proportional to their relative connection degree strength, as shown in Equation 1, where d_{i_k} is the degree of neighbor i_k, $k \in [1, j]$.

$$O_i^{Joint} = \sum_{k=1}^{j} (d_{i_k} / \sum_{k=1}^{j} d_{i_k}) \times O_{i_k} \tag{1}$$

Thus, the more a neighbor is connected in the local network (measured by its relative connection strength), the greater its weight and influence on the joint opinion in the collective decision-making process.

Another critical factor in designing an interaction protocol is confirmation bias. That is, people collect and interpret information selectively by trying to follow their original bias (e.g., their original opinions) [22]. The most widely adopted interaction protocol with confirmation bias is a bounded confidence model where rational agents owning the perfect information poll their entire neighborhoods and select others to interact only if their opinions fall within a confidence interval [7, 23, 5]. Here we take inspiration from game theory and model this as an opinion interaction game with confirmation bias among bounded-rational agents with limited information. So, after generating a weighted joint opinion based on limited neighborhood information, agent i with opinion O_i receives a payoff R_i represented by Equation 2.

$$R_i = 1 - |O_i - O_i^{Joint}| \tag{2}$$

Equation 2 means that if the opinion O_i of agent i is very different from the joint opinion in its selected neighborhood (the local environment), it receives a low payoff. Neighborhoods with more similar opinions are considered more trustworthy, thus, resulting in a higher payoff.

After considering the collective interaction by the game-playing and interaction with the joint opinion, the focal agent i adapts to the neighborhood. Suppose the stubbornness of i is ω_i and its openness is $1-\omega_i$, then the adapted opinion $O_i^{Adapted}$ of agent i is calculated by Equation 3. It represents a combination of relying on its original opinion and accepting a new opinion [21, 17].

$$O_i^{Adapted} = O_i \times \omega_i + O_i^{Joint} \times (1 - \omega_i) \tag{3}$$

Intrinsic Self-adjustments. We have now seen how agents take advantage of extrinsic collective information within their incomplete neighborhoods. Agents also observe the local environment to adjust their opinions to seek a higher payoff. This is driven by intrinsic motivation. People sometimes engage in an activity just because they are drawn to do it [19, 24].

We apply the imitation rule here that does not need direct interactions but transforms information in the population through observation and self-adjustment [25, 26]. Again, a focal agents i only accesses a random partial neighborhood as its observation environment[1]. For these random neighbors, the focal agent i holds a probability W_{i,i_r} to imitate the local best-performing neighbor i_r (i.e., the neighbor with the highest cumulative payoff) by adopting i_r's opinion as its own opinion. The imitation probability W_{i,i_r} is expressed by Equation 4 [26].

$$W_{i,i_r} = \frac{1}{1 + exp[(E_i - E_{i_r})/\mu]} \tag{4}$$

E_i and E_{i_r} are cumulative payoffs of agent i and the local best-performing neighbor i_r. They reflect the long-term adaptation and assimilation into the society as stubborn agents with fairly different opinions will ultimately have a

[1]These random partial neighbors for observation are different from the interaction neighborhoods as we allow more randomness. People also do not always consult with the same group within their social networks.

low cumulative payoff. μ is a noise parameter modeling irrational choices, and we set μ to $\mu = 1.5$ [26]. [2] μ allows the possibility to imitate opinions of agents with a lower cumulative payoff due to making an irrational choice.

In summary, we introduce (i) sparse opinion updates by taking incomplete information-based collective decision-making into account, and (ii) observation and self-adjustment of opinions without direct interaction with neighbors. Sparse interactions are achieved.

3.2.2. Open-ended Structural Dynamics

This section presents the open-ended structural dynamics with leaving and joining agents and the opinion-structure coevolution.

Leavers: At each time step, the agent with the lowest cumulative payoff leaves the society, which models an intention to exit a society where most individuals have fairly different positions (e.g., opinions). The stubbornness and openness of a leaver are recorded. All adjacent edges of this agent are removed from the society upon leaving. As society evolves, leaver-driven structural dynamics will demonstrate the confirmation bias more strongly because stubborn agents with opinions fairly different from others will have a low payoff leading to their removel from the model. Opinion dynamics affects the cumulative payoff, influences which agents become leavers, and thus drives the structural coevolution of the system.

Joiners: At each time step, a newcomer v will join. As society evolves, the community structure constantly changes. Agent v has incomplete information about different communities. It detects the real-time community structure and attempts to join a random $community_v$ by connecting to randomly selected nodes within $community_v$. We assign a random opinion O_v to v and the recorded stubbornness/openness of the leaver (see above) to v in order to keep the cognitive ability distribution stable within the society. Note that the cumulative payoff E_v of the incoming agent v is not comparable to that of existing agents when calculating the imitation probability and removing leavers, especially for long-term experiments (see Equation 4). We accordingly assume that given v joining at time step t_v, agent v's initialized cumulative payoff E_v is adjusted by the corresponding payoff $R_v^{t_v}$ at time t_v multiplied by the number of completed interactions t_v. After initialization, the cumulative payoff E_v is calculated by regularly adding the corresponding payoff R_v^t at each time step t until v is removed or the system terminates globally.

After joining a community, the newcomer v chooses and connects to a node u in another community. We apply the preferential attachment principle (i.e., nodes with a higher connection degree have a stronger ability to attract new nodes added to the network) because "the rich getting richer" phenomenon is widely observed in real-world societies [27]. Thus, the probability $p_{v,u}$ for v

[2] When conducting long-term interactions, the cumulative payoff could reach several hundred. Thus, the value of noise is chosen to be slightly greater than the range [0,1] in previous work.

Algorithm 1: The SCOOE Model

Initialize society, opinions, stubbornness, openness, payoffs, and neighbors;
for *each time step* $t(t = 1, \cdots, T)$ **do**
  //Opinion Dynamics with Sparse Interactions;
  for *each agent* i *in the society* **do**
    Agent i accesses partial random neighbors and generates a joint opinion O_i^{Joint} (see Equation 1);
    Agent i receives the corresponding payoff R_i and updates the cumulative payoff E_i by O_i^{Joint} (see Equation 2);
    Agent i is assimilated into the neighborhood with the adapted opinion $O_i^{Adapted}$ (see Equation 3);
    Agent i chooses a local best-performing neighbor i_r to imitate with an imitation probability W_{i,i_r} (see Equation 4);
  end
  //The Coevolution of Open-ended Structure and Opinions;
  The agent with the lowest cumulative payoff leaves the society;
  Assign a random opinion O_v to a newcomer v;
  Assign the stubbornness/openness of the leaver to v;
  Adjust the cumulative payoff E_v of v;
  v detects the real-time community structure;
  if *multiple communities* (≥ 2) *can be found in the society* **then**
    v joins a random community by connecting to partial random nodes within this community;
    An edge is generated connecting v to a node u in another community, following the preferential attachment (see Equation 5);
  else
    v connects to a node u in the society following the preferential attachment (see Equation 5);
  end
end

choosing u to connect to follows Equation 5.

$$\forall u \in (G - community_v) : p_{u,v} \propto d_u / \sum d_u \tag{5}$$

$G-community_v$ represents all of the other communities except for $community_v$, which the new node v joins. d_u represents the degree of node u. If only one community exists as the society evolves, the new node joins by connecting to only one node following preferential attachment. An algorithmic view of the SCOOE model is shown in Algorithm 1.

4. Experiments

In this section, we describe our experiments and their results.

4.1. Experimental Settings

We create two small-world networks holding 500 nodes each to model two physically separated groups of people interacting to a certain degree. So, randomly-chosen edges connect the two small-world networks. We apply the Watts–Strogatz model to generate an individual small-world network [28]. Each node is connected to four nearest neighbors. The rewiring probability is set to 0.05. This structure constitutes the agent society, with each node representing an agent. A focal agent will only consider those agents connected by edges as neighbors and conduct sparse interactions based on the neighborhood. For initialization, the stubbornness distribution follows a Gaussian distribution with a mean of 0.5 and a standard deviation of 0.25. These parameters are chosen so that most values lie between 0 and 1. In addition we apply a cut-off so that generated random number can only lie between 0 and 1, i.e., we constrain stubbornness to the interval between 0 to 1, as shown in Figure 2 (a). We apply the modularity-based method for real-time community detection [29].[3] Imitation noise is set to $\mu = 1.5$. The simulation is run for 450 Monte Carlo time steps.

4.2. Experimental Results and Analysis

Here we report our experimental results by answering the following questions. We will explain and discuss the phenomena found in the discussion section.

4.2.1. How Do Group Opinions Evolve with Sparse Interactions?

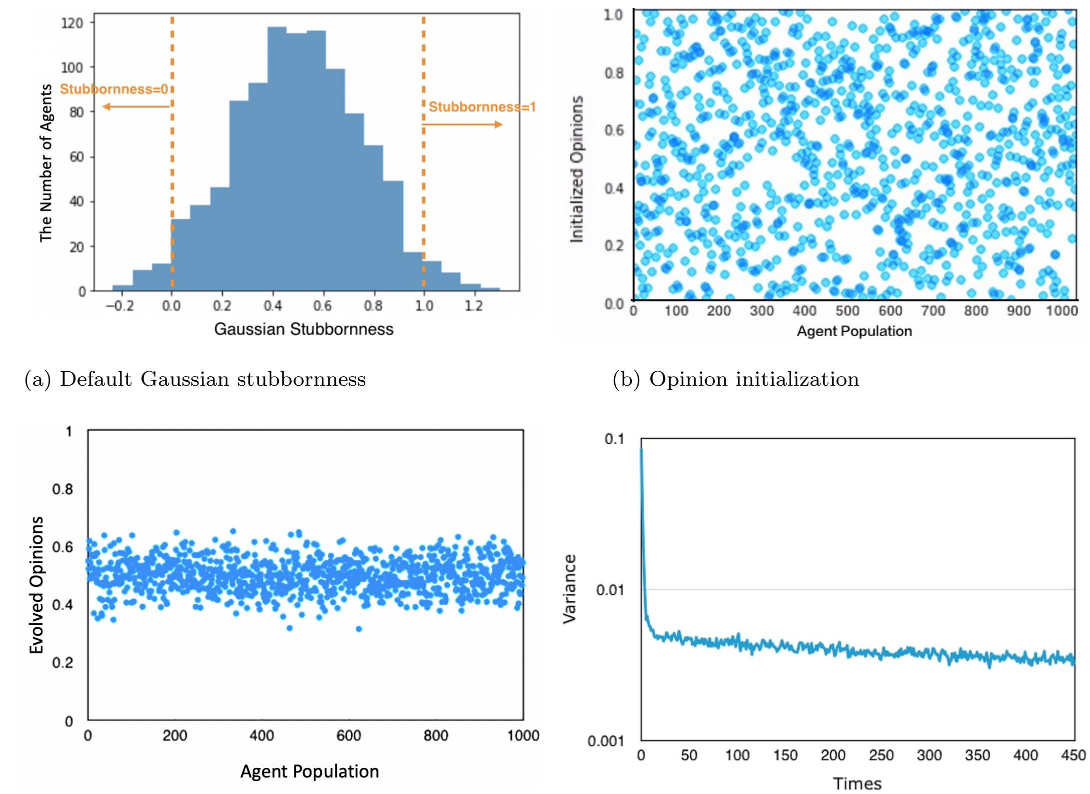
We study how far the group opinions evolve away from their initial states, measured by the variance dynamics shown in Figure 2. The opinions of agents are reasonably different at the start because agents are assigned random opinions initially. As society evolves, we find two stages of evolution: a fast-decay phase (i.e., the variance of group opinions dramatically decreases from 0.084 to 0.005) and a slow-decrease phase (i.e., the variance slowly continues dropping to 0.003 at the end of the simulation). It is interesting to find that the final opinions are in a relatively narrow band and less polarized without firmly believing or strongly unbelieving the rumors among the agent population, even with some agents never changing their opinions (stubbornness =1) but being removed by the model. The Gaussian stubbornness distribution is also evenly distributed. The majority of the population keeps a balance between maintaining their original opinions and accepting a new opinion. Mirroring reality, we find that agents are more likely to stay open-minded to propagated news/rumors during *long-term* interactions in an *open-ended* society.

4.2.2. How Do the Opinion Dynamics Shape the Structural Co-dynamics?

For this question, we primarily focus on the dynamics of the clustering coefficient, average path length, degree distribution, and community structure.

The society coevolves to be a holistically dense small-world structure with a heavy-tailed degree distribution. As shown in Figures 3 and 4, we find an increase in the clustering coefficient and average degree, as well as a decrease in average path length and the number of communities. We initialize the society as two interconnected small-world networks. The random edges between them change the initialized small-world features by randomizing them to a certain degree.

[3] We have compared several community detection methods. We find they do not strongly affect the results.



(a) Default Gaussian stubbornness

(b) Opinion initialization

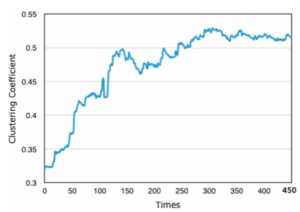
(c) Evolved opinions under Gaussian stubbornness distribution

(d) Variance dynamics

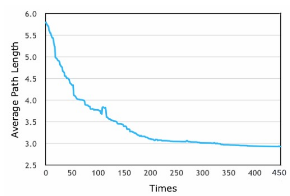
Figure 2: Opinion dynamics in the population with Gaussian stubbornness.

Thus, we find a chaotic society initially with 26 detected tiny communities and a coevolved society with nine segregated communities by the modularity-based method [29]. We also observe that the coevolved society has a small-world feature with a high clustering coefficient. It coevolves to be a more tightly knit group with dense connection degrees, high information transmission efficiency, and a low average path length due to network homophily. That is, the final opinions of the population are relatively consistent, leading to an increase in payoff and a decrease in conflicts (e.g., confirmation bias for fairly different opinions) in the game-playing upon interactions. This coevolutionary trend of the structure in turn boosts the evolution of a global opinion [30].

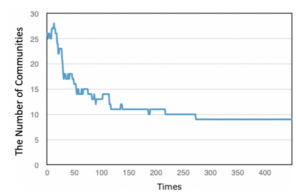
Although some small-world generation models, e.g., the Kleinberg model [31], do not generate heavy-tailed degree distributions, it is not surprising to find a heavy-tailed degree distribution appearing in the SCOOE model. The advantages of “the rich” become significant eventually because of preferentially added joiners. Specifically, we calculate the proportion $P(k)$ of nodes with connection degree k. We find that the relationships between node proportion $P(k)$ and node degree k can be approximated by a linear relationship $\log[P(k)] \propto (-\gamma) \times \log(k)$ with a negative slope $-\gamma \approx -2.758$. Note that the data points in Figure 3 (f) represent the average degree and the node proportion in different degree ranges. We only study the nodes in these degree ranges because they fill most of the network. These nodes are enough to illustrate a linear relationship.



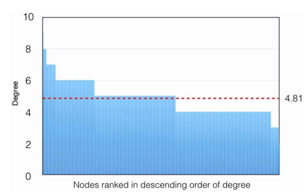
(a) Clustering coefficient



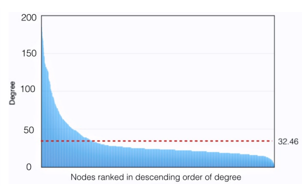
(b) Average path length



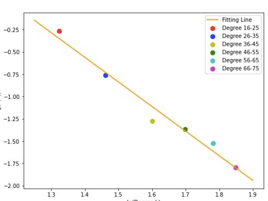
(c) The Number of communities



(d) Initialized degree distribution



(e) Coevolved degree distribution



(f) The relationships between node degree and node proportion

Figure 3: Structural Dynamics.

An emergent dialectic relationship between community segregation and cohesion. Cohesion is a concept of togetherness and connectedness among nodes within a network. There is no unified definition of cohesion because it depends on the context. Previous literature has referred to it as cliques/communities, clusters, or average degree [32].[4] Figure 4 shows our assessment of the community segregation and cohesion. The initialized society is desegregated and chaotic with a low level of cohesion (i.e., with a low average degree and clustering coefficient). As society coevolves, we find it has a clear pattern of fewer segregated communities that become densely clustered (i.e., with a high average degree and clustering coefficient). Agents have disconnected social networks initially but highly cohesive social clusters eventually.

It is interesting to note that society becomes segregated but dense spontaneously and simultaneously with a global consensus and cohesion, but without multiple local-opinion "barycenters" that might emerge aligned with segregated communities [23]. Mirroring reality, as Neal et al. [34] suggest, a widely observed example in the real world is policy-making to reduce detrimental residential segregation. A widely adopted approach to introduce desegregated neighborhoods and reduce residential segregation is to improve cohesion, e.g., dense connections. However, a paradox exists between community segregation and cohesion. The society evolves to be dense with segregated communities, whereas a desegregated society is not as cohesive as we would expect.

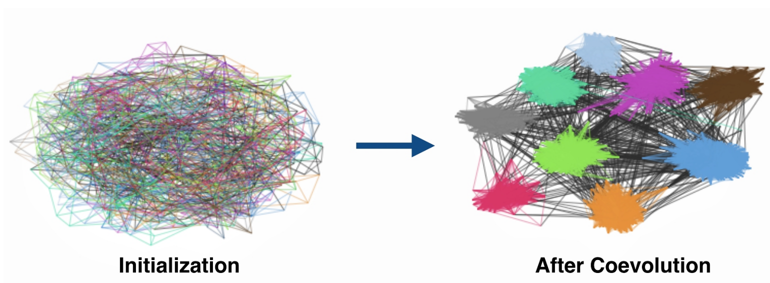


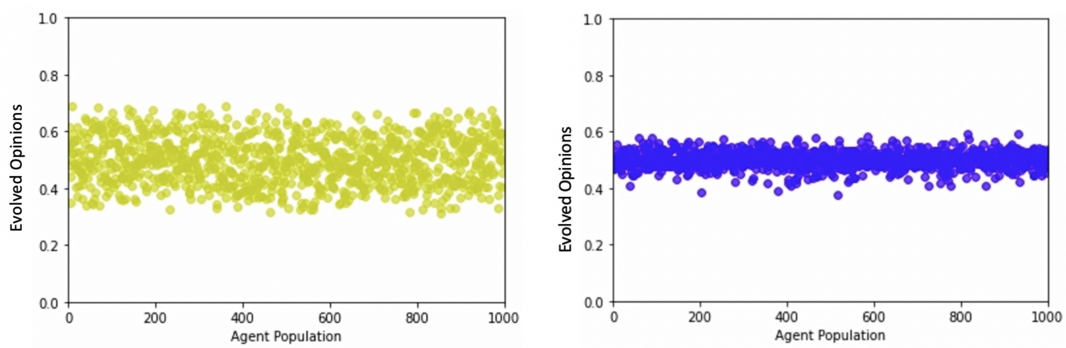
Figure 4: The coevolved society after 450 time steps. The nodes and edges within a community are set with the same color. The color of external edges connecting two communities is set to black.

4.2.3. Do Different Stubbornness Distributions Matter?

To study the influence of stubbornness distributions, we also test a Beta distribution and a Poisson distribution. We initialize the Beta distribution with two positive shape parameters $\alpha = 7$ and $\beta = 1$, and the Poisson distribution with the expected rate of occurrences $\lambda = 1$. We normalize the two generated distributions with the maximum value representing stubbornness=1. The evolved opinions in these two cases are shown in Figure 5. The variance developments and their comparison to the baseline Gaussian stubbornness distribution

[4] Some work has also applied the k-component for measuring network cohesion. The k-component of a network is the maximum sub-graph in which we need to remove at least k nodes to break this sub-graph into more components. We do not study the k-component here because the "giant component" that fills most of the network is always found in an undirected network, while the rest of the network is divided into many scattered small components [33]. This does not help us understand the cohesion and segregation but works for connectivity.

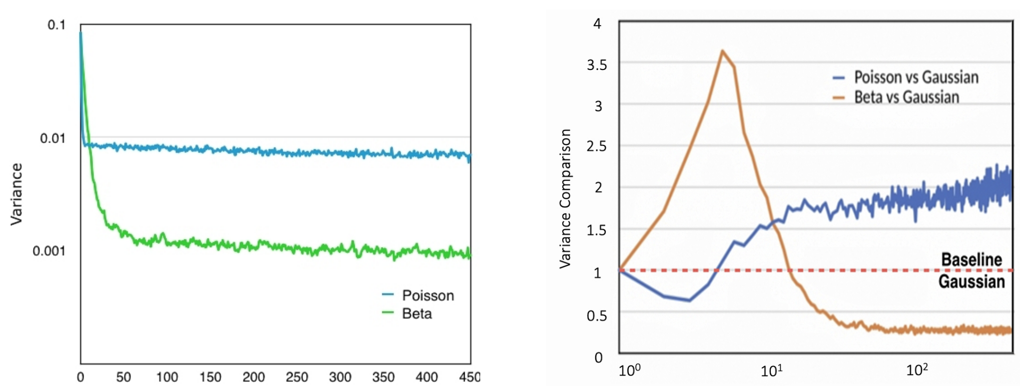
are shown in Figure 6. The variance comparison is defined as the ratio of the opinion variance for the Poisson/Beta stubbornness distributions to that for the Gaussian stubbornness distribution at each time step t, $t \in [0, 450]$.



(a) Poisson stubbornness distribution

(b) Beta stubbornness distribution

Figure 5: The evolved opinions with different stubbornness distributions.



(a) Variance

(b) Variance compared with Gaussian stubbornness distribution

Figure 6: The dynamics of the variance of opinions with different stubbornness distributions.

Stubbornness is generally small in a population with Poisson stubbornness. Agents are very flexible to become followers of the propagated news/rumor. As a result, it will be easier to pass the fast-decay phase. Variance rapidly drops from 0.084 to 0.009, and we observe an initial lower variance than the baseline shown in Figure 6 (b). Because of the flexibility in updating opinions, evolved opinions are still inconsistent at the end of 450 time steps, and the final variance is relatively large (0.007). In contrast, the agent population generally has much higher Beta stubbornness. So, it takes many steps (more than 80 time steps shown in Figure 6 (a)) to reach the slow-decrease phase of variance dynamics. Accordingly, we find an initial increase in the variance ratio to pass the fast-decay phase shown in Figure 6 (b). Because of the high stubbornness, final opinions are stable with few changes, and only a small final variance (0.001) can be observed.

It is challenging to drive the global opinion evolution among a stubborn population, e.g., the initially weak emergence of the global opinion in the population with high Beta stubbornness. However, it is interesting to find the most unified global consensus in such a society with many agents only weakly changing opinions. This unusual phenomenon is due to the open-endedness of the society. The most stubborn agents will be considered maladapted to the environment and removed as the society evolves. Agents will be assimilated by agents who surround them. No matter the initial opinions they hold in stubborn crowds, they will finally have a relatively unified group consensus after the *long-term* interactions and the slow assimilation of opinions crowding out dissidents in an *open-ended* society. We can say that these high stubbornness values serve as a "wall" — newcomers with similar opinions will be accepted, while newcomers with opinions out of this range will be removed quickly.

5. Discussion

It is crucial to design simple but practical agent-based models linked to the phenomena of interest. This section revisits and discusses the proposed mechanisms by focusing on their effects on the consensus evolution within groups.

5.1. Enhancement by Sparse Interactions

Our results validate several earlier findings with different mechanisms and remarkably boost the evolution even in a stubborn population [35, 36, 37]. A widely studied contagion phenomenon in social networks is that the chance to adopt a contested "innovation" (e.g., firmly believing a news/rumor) will be smaller for an individual with more neighbors [17, 16]. When a focal agent aggregates the joint opinion by collective decision-making, extreme opinions (e.g., a strong endorsement) of selected neighbors are neutralized by weighted averaging. This effect will be more significant for high-degree nodes, given the larger share of their neighbors. On the other hand, high-degree nodes with fewer likelihood of being extreme have a more substantial impact on the weighted aggregation method and a more extensive influence range. At the same time, collective interactions decrease the probability of interacting directly with extreme agents and being affected by them. So, the extrinsic collective interaction mechanism boosts the emergence of a global consensus. It plays fewer roles when the population rapidly reaches a pre-consensus (the start of the slow-decrease phase in variance dynamics), given the constantly adapted local interaction environment with the randomness to select neighbors, the joiners/leavers, and a constant injection of new opinions. The intrinsic adjustment mechanism continues to further the emergence of a global consensus and weakens conflicts by direct imitation. Extrinsic collective interactions primarily play a role in the fast-decay phase of the variance dynamics, whereas intrinsic adjustments mainly play a role in the slow-decrease phase. Their interplay works to enhance the evolution of a global opinion, as shown in Figures 2 and 6.

5.2. Lean and Fast Decision Strategies with Incomplete Information

A broad assumption in the widely cited bounded confidence model is that rational agents owning the perfect information poll their neighborhood and select neighbors to interact only if their opinions are sufficiently close to their own. This assumption facilitates polarization and global conflicts [23]. It has been widely recognized that it is difficult to evolve a global consensus for large population sizes [11, 38, 39], because multiple local consensus might be distributed in a society. As a result, such a system needs more bottom-level interactions to pass the formation of these local consensus. Unlike some bounded confidence models, e.g., [23, 7], here we start by assuming that bounded-rational agents only access a partial neighborhood (incomplete information) to aggregate a joint opinion. Confirmation bias is represented by the stipulation that adopting more similar opinions will bring a higher payoff. We find that conflicts among bounded-rational agents are weakened globally and rapidly. Bounded rationality with incomplete information forms lean and fast decision strategies to reduce conflicts under uncertainties, whereas complete information weakens group coordination, as suggested by some literature from psychology [40].

5.3. Open-endedness Enables Few Extreme Opinions and Permanently Novel Opinions

The echo chamber effect in social media studies describes a situation where local opinions are reinforced by repetition inside a closed society and insulated from rebuttal or different opinions (confirmation bias). The SCOOE model weakens the echo chamber effect of local opinions by the open-endedness feature [41, 42]. When we examine previous models based on a closed structure, some work has shown global/local polarization and extreme opinions [43, 2, 44]. The open-endedness feature in the SCOOE model with its constant injection of new opinions helps a population to defend against the echo chamber effect and to stay open-minded. It reduces the chances of extreme results because extreme agents are likely to be removed from the society. It also mirrors the findings of a global consensus formation in a population with high Beta stubbornness.

On the other hand, we find that eventually evolved opinions are wholly unified in some closed-society models [36]. The continuous addition and removal of agents and the structure/opinion dynamics they bring with them influence neighbors and neighbors' surroundings in a cascading fashion. Though the designed mechanisms strongly facilitate the evolution, it is impossible to reach a highly unified global consensus. One can only approach it, as the slow-decrease phase in variance dynamics shown in Figures 2 and 6 depict. It can also be said that the SCOOE model is robust to boost and enhance the evolution of a global opinion as it successfully defends against the interference of a constant injection of novel opinions.

6. Conclusions

In this paper, we propose the SCOOE model, a coevolutionary opinion dynamics model with sparse interactions in an open-ended society. Two-phase evolution shows that the extrinsic collective interaction mechanism boosts the evolution; the intrinsic adjustment mechanism slowly reduces conflicts. Their interplay facilitates the effective and robust formation of a global opinion. The model also provides a new direction for small-world network generation from social complexity and interaction perspectives. As the model evolves, agents tend to be connected closer to others. The agent society shows a small-world characteristic with a heavy-tailed degree distribution and emerges to become more segregated and cohesive. Different emergent trends can be found in flexible, normal, and stubborn populations, with a link to a potential explanation in cognitive science and psychology. We expect to apply the SCOOE model in a broader field, e.g., media and communication studies and decentralized control of asynchronous systems.

Though the proposed mechanisms show promise in experiments, there are some open questions. While we successfully introduced a limited view of focal agents, payoff information is accessible to all agents. We are interested in a model with even less information. Is it possible to design an incomplete information model with hidden payoff information when agents adjust the opinions according to the environmental feedback? Additionally, the mechanisms we create are fully interconnected and coevolved. Opinion dynamics with sparse interactions provide citeria (i.e., cumulative payoff) for nodes to leave society. Then agents actively behave to drive structural evolution and opinion evolution. We can study the different influence of structural and opinion interaction mechanisms on the formation of global consensus. An empirical study based on real-world data to calibrate the model and a further relaxation of assumptions to make the model even simpler are also valuable directions in the future.

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