

Alinkom 4

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$$1) F: \mathbb{R}^2 \rightarrow \mathbb{R}^3 \text{ dengan } F((x, y)^T) = (x, y+1)^T$$

$$F(\vec{u}) = (u_1, u_2+1)^T$$

$$F(\vec{v}) = (v_1, v_2+1)^T$$

$$F(\vec{u} + \vec{v}) = (u_1 + v_1, u_2 + v_2 + 1)^T$$

$$F(\vec{u}) + F(\vec{v}) = (u_1 + v_1, u_2 + v_2 + 2)^T$$

$$F(\vec{u} + \vec{v}) \neq F(\vec{u}) + F(\vec{v})$$

maka bukan transformasi linier

$$2) F: \mathbb{R}^3 \rightarrow \mathbb{R}^2 \text{ dengan } F((x, y, z)^T) = (x+y+z, z-y-x)^T$$

$$F(\vec{u}) = (u_1 + u_2 + u_3, u_3 - u_2 - u_1)^T$$

$$F(\vec{v}) = (v_1 + v_2 + v_3, v_3 - v_2 - v_1)^T$$

$$F(\vec{u} + \vec{v}) = ((u_1 + v_1) + (u_2 + v_2) + (u_3 + v_3), (u_3 + v_3) - (u_2 + v_2) - (u_1 + v_1))^T$$

$$= F(\vec{u}) + F(\vec{v})$$

aksioma 1 ✓

$$F(k\vec{u}) = ((ku_1 + ku_2 + ku_3, ku_3 - ku_2 - ku_1))^T$$

$$= k \cdot F(\vec{u})$$

aksioma 2 ✓

maka terbukti
transformasi linier

$$3) F: \mathbb{R}^2 \rightarrow \mathbb{R}^3 \text{ dengan } F(\langle x, y \rangle) = \langle x-y, y-x, y \rangle$$

$$F(\vec{u}) = \langle u_1 - u_2, u_2 - u_1, u_2 \rangle$$

$$F(\vec{v}) = \langle v_1 - v_2, v_2 - v_1, v_2 \rangle$$

$$F(\vec{u} + \vec{v}) = \langle u_1 + v_1 - (u_2 + v_2), (u_2 + v_2) - (u_1 + v_1), u_2 + v_2 \rangle$$

$$= F(\vec{u}) + F(\vec{v})$$

aksioma 1 ✓

$$F(k \cdot \vec{u}) = \langle ku_1 - ku_2, ku_2 - ku_1, ku_2 \rangle$$

$$= k \cdot F(\vec{u})$$

aksioma 2 ✓

maka terbukti
transformasi linier

$$4) F: \mathbb{R}^2 \rightarrow \mathbb{R}^2 \text{ dengan } F((x, y)^T) = \langle 5, x+ty \rangle$$

$$F(\vec{u}) = \langle 5, u_1 + tu_2 \rangle$$

$$F(\vec{v}) = \langle 5, v_1 + tv_2 \rangle$$

$$F(\vec{u} + \vec{v}) = \langle 5, (u_1 + v_1) + t(u_2 + v_2) \rangle$$

$$F(\vec{u}) + F(\vec{v}) = \langle 10, (u_1 + u_2) + t(v_1 + v_2) \rangle$$

$$F(\vec{u} + \vec{v}) \neq F(\vec{u}) + F(\vec{v})$$

maka bukan transformasi linier

5) $F: M_{2 \times 2} \rightarrow R$ dengan $F(A) = \det(A)$

$$A = \begin{pmatrix} a_1 & a_2 \\ a_3 & a_4 \end{pmatrix} \quad F(A) = a_1 a_4 - a_2 a_3$$

$$B = \begin{pmatrix} b_1 & b_2 \\ b_3 & b_4 \end{pmatrix} \quad F(B) = b_1 b_4 - b_2 b_3$$

$$F(A) + F(B) = a_1 a_4 + b_1 b_4 - (a_2 a_3 + b_2 b_3)$$

$$A+B = \begin{pmatrix} a_1+b_1 & a_2+b_2 \\ a_3+b_3 & a_4+b_4 \end{pmatrix}$$

$$F(A+B) = (a_1+b_1)(a_4+b_4) - (a_3+b_3)(a_2+b_2)$$

$$F(A+B) \neq F(A) + F(B)$$

makanya bukan
transformasi
linier

6) $F: M_{2 \times 2} \rightarrow M_{2 \times 2}$ dengan $F(A) = A + A^T$

$$A = \begin{pmatrix} a_1 & a_2 \\ a_3 & a_4 \end{pmatrix} \quad F(A) = \begin{pmatrix} a_1 & a_2 \\ a_3 & a_4 \end{pmatrix} + \begin{pmatrix} a_1 & a_3 \\ a_2 & a_4 \end{pmatrix} = \begin{pmatrix} 2a_1 & a_2+a_3 \\ a_2+a_3 & 2a_4 \end{pmatrix}$$

$$B = \begin{pmatrix} b_1 & b_2 \\ b_3 & b_4 \end{pmatrix} \quad F(B) = \begin{pmatrix} 2b_1 & b_2+b_3 \\ b_2+b_3 & 2b_4 \end{pmatrix}$$

$$F(A+B) = \begin{pmatrix} 2(a_1+b_1) & (a_2+b_2)+(a_3+b_3) \\ (a_2+b_2)+(a_3+b_3) & 2(a_4+b_4) \end{pmatrix}$$

$$= F(A) + F(B)$$

aksioma 1 ✓

$$F(k \cdot A) = \begin{pmatrix} k \cdot 2a_1 & k(a_2+a_3) \\ k(a_2+a_3) & k \cdot 2a_4 \end{pmatrix}$$

$$= k \cdot F(A)$$

aksioma 2 ✓

maka terbukti

transformasi
linier

7) $F: P_3 \rightarrow P_2$ dengan $F(a_0 + a_1x + a_2x^2 + a_3x^3) = 5a_0 + a_3x^2$

$$F(b_0 + b_1x + b_2x^2 + b_3x^3) = 5b_0 + b_3x^2$$

$$F(a+b) = 5(a_0+b_0) + (a_3+b_3)x^2$$

$$= F(a) + F(b)$$

aksioma 1 ✓

$$F(k \cdot a) = k \cdot 5a_0 + k \cdot a_3x^2$$

$$= k \cdot F(a)$$

aksioma 2 ✓

maka terbukti

transformasi linier

yang bukan = {1, 4, 5}