LKP 3 Analgor

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1) a) Buktikan bahwa :

i) $T(n) = 5n^4 - 37n^3 + 13n - 4 \in O(n^4)$

ii) $T(n) = n^3 + 20n + 1 \notin O(n^2)$

iii) $T(n) = n^3 + 20n + 1 \in O(n^3)$

iv) $T(n) = n^3 + 20n + 1 \in O(n^4)$

b) tentukan Big-O

v) $3n + n^3 + 4$

vi) $1 + 2 + 3 + \ldots + n + 3n^2$

Jawab :

i) $5n^4 - 37n^3 + 13n - 4 \leq cn^4$

$5 - \frac{37}{n} + \frac{13}{n^3} - \frac{4}{n^4} \leq c$ untuk $n = -1$, $c = 35$

so, for $5n^4 - 37n^3 + 13n - 4 \leq 35n^4$ $\boxed{\therefore \text{terbukti} \in O(n^4)}$

ii) $n^3 + 20n + 1 \leq cn^2$

$n + \frac{20}{n} + \frac{1}{n^2} \leq c$

$n \leq c \rightarrow$ kontradiktif $\boxed{\therefore \text{terbukti} \notin O(n^2) \therefore}$

iii) $n^3 + 20n + 1 \leq cn^3$

$1 + \frac{20}{n^2} + \frac{1}{n^3} \leq c$ untuk $n = 1$ $c = 22$

$n^3 + 20n + 1 \leq 22n^3$ $\boxed{\therefore \text{terbukti} \in O(n^3)}$

iv) $n^3 + 20n + 1 \leq cn^4$

$\frac{1}{n} + \frac{20}{n^3} + \frac{1}{n^4} \leq c$ untuk $n = 1$ $c = 22$

$n^3 + 20n + 1 \leq 22n^4$ $\boxed{\therefore \text{terbukti} \in O(n^4)}$

v) $n^3 + 3n + 4 \leq cn^3$ untuk $O(n^3)$

$1 + \frac{3}{n^2} + \frac{4}{n^3} \leq c$ untuk $n = 1$, $c = 8$

$n^3 + 3n + 4 \leq 8n^3$ $\boxed{\therefore \text{terbukti untuk } n \geq 1 \text{ Big } O(n^3)}$

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vi) $\underbrace{1+2+3+\dots+n}$ $+ 3n^2$

$\left(\frac{1}{2} n (n+1)\right) + 3n^2$

$\frac{1}{2} n^2 + \frac{1}{2} n + 3n^2$

$\Rightarrow \frac{7}{2} n^2 + \frac{1}{2} n \leq cn^2$ untuk $O(n^2)$

$7/2 + \frac{1/2}{n} \leq c$ untuk $n=1$, $c=4$

$7/2\, n^2 + 1/2\, n \leq 4n^2$ $\boxed{\therefore \text{terbukti untuk } n \geq 1 \text{ Big } O(n^2)}$

2) dik: $T(N) = cn^2$ dit: $T(N)$ untuk $N=5000$?

untuk $N=100 \rightarrow T(N) = 1\,ms$

jawab: $T(N) = cn^2$

$c = \frac{T(N)}{n^2}$

$c = \frac{1\,ms}{100^2}$

$c = 10^{-4}\,ms$

$T(N) = 10^{-4} n^2\,ms$

$T(5000) = 10^{-4} \cdot (5 \cdot 10^3)^2\,ms$

$\boxed{T(5000) = 25 \times 10^2\,ms}$

3) dik: data: 10^{12} record

$T_A(n) = 0{,}1 \cdot n \cdot \log_2 n$ mikrodetik

$T_B(n) = 5 \cdot n$ mikrodetik

dit: a) mana yang lebih baik kinerjanya?

b) kapan A lebih baik dari B?

c) kapan B lebih baik dari A?

jawab: pertama, tentukan titik potong kedua persamaan

$T_A(n) = T_B(n)$

$0{,}1 \cdot \cancel{n} \cdot \log_2 n = 5 \cdot \cancel{n}$

$\log_2 n = 50$

$n = 2^{50}$ → titik potong

⇒ Uji coba daerah $n < 2^{50}$

co: $n = 2$

$T_A(n) = 0{,}1 \cdot 2 \cdot \log_2 2$

$= 0{,}2$

$T_B(n) = 5 \cdot 2$

$= 10$

⇒ uji coba daerah $n > 2^{50}$

co: $n = 2^{60}$

$T_A(n) = 0{,}1 \cdot 2^{60} \cdot \log_2 2^{60}$

$= 6 \cdot 2^{60}$

$T_B(n) = 5 \cdot 2^{60}$

A < B   A > B

2^{50}

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a) dapat terlihat bahwa untuk $n < 2^{50}$ algoritma $T_A(n)$ lebih baik dari $T_B(n)$ karena waktu pemrosesannya lebih cepat

b) $T_A(n)$ lebih baik dari $T_B(n)$ saat $n < 2^{50}$

c) $T_B(n)$ lebih baik dari $T_A(n)$ saat $n > 2^{50}$

4) dik: $T_A(n) : O(n^2) = an^2$ dit: berapa waktu yang dihabiskan setiap pemrosesan untuk 10.000 item?

$T_B(n) : O(n^{1.5}) = bn^{1.5}$

$T_C(n) : O(n \log n) = cn \log n$

ketiganya menghabiskan 10 detik untuk 100 item data

jawab: cari masing² a, b, dan c

$T_A(100) = a \cdot 100^2 = 10\,s \longrightarrow a = 10^{-3}\,s$

$T_B(100) = b \cdot 100^{1.5} = 10\,s \longrightarrow b = 10^{-2}\,s$

$T_C(100) = c \cdot 100 \cdot \log 100 = 10\,s \longrightarrow c = 5 \cdot 10^{-2}\,s$

sehingga, waktu yang dihabiskan masing² pemroses untuk 10.000 item data yaitu

$T_A(n) = 10^{-3} \times n^2 = 10^{-3} \times (10^4)^2\,s = \boxed{10^5\,s}$

$T_B(n) = 10^{-2} \times n^{1.5} = 10^{-2} \times (10^4)^{1.5}\,s = \boxed{10^4\,s}$

$T_C(n) = 5 \cdot 10^{-2} \cdot n \cdot \log n = 5 \cdot 10^{-2} \cdot (10^4) \cdot \log(10^4)\,s = 20 \cdot 10^2\,s = \boxed{2 \times 10^3\,s}$

5) dit: hitung T(n) & kompleksitas dan

```
i <- n;
while (i>1){
    j = i;
    while (j<n) {
        k <- 0;
        while (k<n) {
            k = k+2;
        }
        j <- j*2;
    }
    i <- i/2;
}
```

jawab:

loop	operasi	times
1	$i \leftarrow n$	1
1	$i > 1$	$\log_2(n)+1$
2	$j = i$	$\log_2(n)$
2	$j < n$	$(\log_2 n + 1) \times (\frac{1}{2}\log_2 n)$
3	$k \leftarrow 0$	$(\log_2(n)) \times (\frac{1}{2}(\log_2 n - 1))$
3	$k < n$	$(\log_2 n) \times (\frac{1}{2}(\log_2 n - 1)) \times (\frac{n+1}{2} + 1)$
3	$k = k+2$	$(\log_2 n) \times (\frac{1}{2}(\log_2 n - 1)) \times (\frac{n+1}{2})$
2	$j \leftarrow j*2$	$(\log_2(n)) \times (\frac{1}{2}\log_2 n - 1)$
1	$i \leftarrow i/2$	$\log_2(n)$

```
for (i=n; i>1; i/=2) {
    for(j=i; j<n; j*=2) {
        for (k=0; k<n; k+=2) {
        }
    }
}
```

Fungsi sama seperti ini

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$\sum$ operasi loop 1 $= 1 + \log_2 n + 1 + \log_2 n$
$= 2\log_2 n + 2$

$\sum$ operasi loop 2 $= \log_2 n + \left(\frac{1}{2}\log_2 (n)^2 + \frac{1}{2}\cancel{\log_2 n}\right) + \left(\frac{1}{2}\log_2 (n)^2 - \frac{1}{2}\cancel{\log_2 n}\right)$
$= \log_2 (n)^2 + \log_2 (n)$

$\sum$ operasi loop 3 $= \left(\frac{1}{2}\log_2 (n)^2 - \frac{1}{2}\log_2 n\right) + \left(\left(\frac{n+3}{2}\right)\times\frac{1}{2}\log_2 (n)^2 - \left(\frac{n+3}{2}\right)\times\frac{1}{2}\log_2 n\right).$
$\left(\left(\frac{n+1}{2}\right)\times\frac{1}{2}\log_2 (n)^2 - \left(\frac{n+1}{2}\right)\times\frac{1}{2}\log_2 (n)\right)$
$= \left(\frac{1}{2}\log_2 (n)^2 - \frac{1}{2}\log_2 n\right) + \left(\left(\frac{2n+4}{2}\right)\times\frac{1}{2}\log_2 (n)^2 - \left(\frac{2n+4}{2}\right)\times\frac{1}{2}\log_2 n\right)$
$= \left(\frac{2n+5}{2}\right)\log_2 (n)^2 - \left(\left(\frac{2n+5}{2}\right)\times\log_2 n\right)$

$T(n) = (2\log_2 n + 2) + (\log_2 (n)^2 + \log_2 n) + \left(\frac{2n+5}{2}\log_2 (n)^2 - \frac{2n+5}{2}\log_2 n\right)$

$$\boxed{T(n) = \frac{2n+7}{2}\log_2 (n)^2 + \frac{-2n+1}{2}\log_2 (n) + 2}$$

$\boxed{\text{kompleksitas} = O(n \cdot \log_2 (n)^2)}$

6) Hitung T(n) dan complexity

a)
```
int a=0, b=0
for(i=0; i<N; i++) {
    a = a + rand();
}
For(j=0; j<M; j++) {
    b = b + rand();
}
```

loop	operasi	times versi N M	times
	a=0	1	1
	b=0	1	1
1	i=0	1	1
	i<N	N+1	n+1
	i++	N	n
	a+=rand()	N	n
2	j=0	1	1
	j<M	M+1	n+1
	j++	M	n
	b+=rand()	M	n

$T(n) = 1+1+1+(n+1)+n+n+1+(n+1)+n+n$

$\boxed{T(n) = 6n+6}$
$\boxed{\text{kompleksitan} = O(n)}$

b)
```
int a=0;
for(i=0; i<N; i++) {
    for(j=N; j>i; j--) {
        a = a+i+j;
    }
}
```

loop	operasi	times
	a=0	1
1	i=0	1
	i<N	n+1
	i++	n
2	j=N	n
	j>i	$\left(\frac{1}{2}(n+1)(n+2)\right)-1$
	j--	$\frac{1}{2}n(n+1)$
	a=a+i+j	$\frac{1}{2}n(n+1)$

$\sum$ operasi loop 1
$= 1+1+n+1+n$
$= 2n+3$

$\sum$ operasi loop 2
$= n + \left(\frac{1}{2}n^2 + \frac{3}{2}n\right) + \left(2\times\frac{1}{2}(n^2+n)\right)$
$= \frac{3}{2}n^2 + \frac{7}{2}n$

$T(n) = 2n+3+\frac{3}{2}n^2+\frac{7}{2}n$

$\boxed{T(n) = \frac{3}{2}n^2 + \frac{11}{2}n + 3}$
$\boxed{\text{kompleksitas} = O(n^2)}$

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c)
```
int i,j,k=0
for( i=n/2; i<=n; i++){
    for (j=2; j<=n; j=j*2){
        k = k+n/2;
    }
}
```

loop	operasi	times
1	int i	1
	int j	1
	int k=0	1
	i=n/2	1
	i≤n	$\left(\frac{n+1}{2}\right)+2$
	i++	$\left(\frac{n+1}{2}\right)+1$
2	j=2	$\left(\frac{n+1}{2}\right)+1$
	j≤n	$\left(\left(\frac{n+1}{2}\right)+1\right)\times(\log_2 n+1)$
	j*=2	$\left(\left(\frac{n+1}{2}\right)+1\right)\times(\log_2 n)$
	k+=n/2	$\left(\left(\frac{n+1}{2}\right)+1\right)\times(\log_2 n)$

$\sum$operasi loop1 $= 1+1+1+1+\left(\frac{n+1}{2}+2\right)+\left(\frac{n+1}{2}+1\right)$

$= 7+n+1$

$= n+8$

$\sum$operasi loop2 =

$= \left(\frac{n+3}{2}\right)+\left(\left(\frac{n+1}{2}\right)\cdot\log_2 n+\left(\frac{n+1}{2}\right)+\log_2 n+1\right)$

$+\left(\left(\frac{n+1}{2}\right)\log_2 n+\log_2 n\right)\times 2$

$= \frac{2n+4}{2}+1+\left(\left(\frac{n+1}{2}+1+(n+1)+2\right)\log_2 n\right)$

$= n+3+\left(\frac{3n+9}{2}\log_2 n\right)$

$T(n) = 2n+11+\frac{3n+9}{2}\log_2 n$

Kompleksitas: $O(n\log_2 n)$

d)
```
min=0, max=0
while(n!=0) {
    scanf("%d",&num)
    if (num >= max)
        max =num
    else if (num <= min)
        min=num
    printf("%d %d\n", min, max)
    n=n-1
}
```

operasi	times
min=0	1
max=0	1
n!=0	n+1
scanf	n
num≥max	n
max=num	n
num≤min	n
min=num	n
printf	n
n=n-1	n

$T(n) = 1+1+(n+1)+n+n+n+n+n+n+n$

$T(n) = 8n+3$

Kompleksitas $= O(n)$

e)
```
for (int i=0; i<n; i++) {
    for(int j=0; j<m; j++){
        printf("%d", a[i][j]);
        if(j==m-1)
            printf("\n");
        else
            Printf(" ");
    }
}
```

operasi	times
i=0	1
i<n	n+1
i++	n
j=0	n
j<m	n·(n+1)
j++	n·n
Printf	n·n
if-else	n·n

$T(n) = 1+(n+1)+n+n+(n^2+n)+n^2+n^2+n^2$

$T(n) = 4n^2+4n+2$

Kompleksitas $= O(n^2)$

CORET-CORETAN :)

NOMOR 5

$i=n$	8	4	2	1 +1	$i=n$	64	$i=32$	$i=16$	
$i>1$	$8>1$	$4>1$	$2>1$	$1>1$ +1	$i>1$	$64>1$	$32>1$	$16>1$	
$i/=2$	4	2	1	$\log_2$	$j=i$	$j=64$	$j=32$	$j=16$	
					$j<n$	x	$32<64$	$16<n$	$32<n$
					$k=0$	x	$k=0$	$k=2$	✓ $k=64$
					$k<n$	x	$0<64$	$2<64$	✓ x
					$k+=2$	x	$k=2$	$k=4$	✓ x
					$j*=2$	x	$j=64$	$j=32$	64
					$i/=2$	$i=32$	$i=16$		

$j=i \equiv i/=2$

$k=0 \equiv j*=2$

2^8

64 32 16 8 4 2
64 32 16 8 4
64 32 16 8
64 32 16
64 32
64

16 8 4 2
16 8 4
16 8
16

$i=n$	16	~~8~~	4			2			1	
$i>1$	$16>1$	$8>1$	$4>1$			$2>1$			$1>1$	
$j=i$	$j=16$	$j=8$ ~~j=16~~	$j=4$	~~j=8~~	~~j=16~~	$j=2$	~~j=4~~	~~j=8~~	~~j=16~~	
$j<n$	$16<16$	$8<16$ $16<16$	$4<16$	$8<16$	$16<16$	$2<16$	$4<16$	$8<16$	$16<16$	
$j*=2$	x	$j=16$ x	$j=8$	$j=16$	x	$j=4$	$j=8$	$j=16$	x	
$i/=2$	8	4				2			1	

~~$\log_2 n+1$~~ 1

$\log_2 n+1$

$\log_2 n$

$\frac{1}{2}\log_2 n \cdot (\log_2 n+1)$

$\frac{1}{2}(\log_2 n-1)(\log_2 n-1+1)$

$\log_2 n$

$j<n$	$16<16$ $8<16$	$16<16$ $4<16$ $8<16$ $16<16$ $2<16$ $4<16$ $8<16$ $16<16$
$k=0$	0	0 0 0 0 0
$k<n$	$0,2,4,6,\dots,64$	$\frac{n+1}{2}+1$
$k+=2$	$0,2,\dots,32$	$\frac{n+1}{2}$

$(\log_2 n-1)=x$ $\quad(\log_2 n)=y$

$1+2+3=\frac{1}{2}x(x+1)$ dengan $x=3$

$1+2+3+4=$ $\quad y=4$

$\frac{1}{2}\cdot 4 \cdot 5 \quad 10$

$\frac{1}{2}\cdot 3\cdot 4$

$k=0 \quad \log_2 n\cdot(\frac{1}{2}\log_2 n-1)$

0, 2, 4, 6, 8

0, 2, 4, 6, 8, 10, 12

$(\frac{1}{2}(\log_2 x))\times(\log_2 x+1)$

$(\frac{1}{2}(\log_2 x-1)\times(\log_2 x-1+1)=\frac{1}{2}(\log_2 x-1)\times\log_2 x$

$G = U\Sigma^{\alpha}$ $\quad \alpha \begin{cases} =1 & \text{objek} \\ =0 & \text{peubah} \\ =0.5 & \text{objek \& peubah} \end{cases}$

$H^{T} = \Sigma^{1-\alpha} A$

$\boxed{\text{NOMOR 6C}}$

n=16 k=0

i=n/2 i=8

i≤n 8≤16 9≤16 10≤16 16≤16 i≤17

j=2 j=2 j=2 j=2

j≤n 2≤16 4≤16 8≤16 16≤16 32≤16

k=k+n/2 k=8 k= k k same

j*=2 j=4 j=8 j=16 j=32

i++ i=9 i=10 i=11 ...... i=16 i=17

9 10 11 12 13 14 15 16 17

i≤n 8, 9, 10, 11, 12, 13, 14, 15, 16, 17 n=16 10 i++

7, 8, 9, 10, 11, 12, 13, 14, 15, 16 n=15

4, 5, 6, 7, 8, 9 n=8 6 $\frac{n+1}{2} + 2$

3, 4, 5, 6, 7, 8 n=7

4, 5, 6, 7, 8, 9, 10 n=9 7

n=16 2, 4, 8, 16, 32

n=15 2, 4, 8, 16 16≤15 $\log_2 n$

n=32 2, 4, 8, 16, 32

n=30 2, 4, 8, 16, 32

$\boxed{\text{no 6d}}$ n=8 8, 7, 6, 5, 4, 3, 2, 1, 0

num=5

num>max 5>0 ✓

max=num max=5

NUMOR 6B1

a=0	N=6											
i=0	i=0											
i<N	0<6							1<6			5<6	6<6
j=N	j=6							j=6			j=6	
j>i	6>0	5>0	4>0	3>0	2>0	1>0	0>0	6>1	5>1		6>5	
a=a+i+j	a=6	a=11	a=	a	a	a						
j--	5	4	3	2	1	0	X					
i++	i=1									i=5	i=6	

$\frac{1}{2}n(n+1)$

i=1 6>0 5>0 4>0 3>0 2>0 1>0 0>0

i=2 6>1 5>1 4>1 3>1 2>1 1>1

i=3 6>2 5>2 4>2 3>2 2>2

i=4 6>3 5>3 4>3 3>3

i=5 6>4 5>4 4>4

i=6 6>5 5>5

~~6>6~~

$\frac{1}{2}n(n+1) \rightarrow a=a+i+j$

$\frac{1}{2}(n+1)(n+2)-1 \rightarrow j<i$