

PERTEMUAN 13

TEKNIK NUMERIK UNTUK PENYELESAIAN INTEGRASI DAN TURUNAN NUMERIK

TUJUAN PRAKTIKU

Mahasiswa mampu menerapkan teknik-teknik penyelesaian integrasi dan turunan numerik menggunakan Program R.

TUGAS PRAKTIKUM

Lakukan penghitungan manual dan program R.

1. Hitunglah

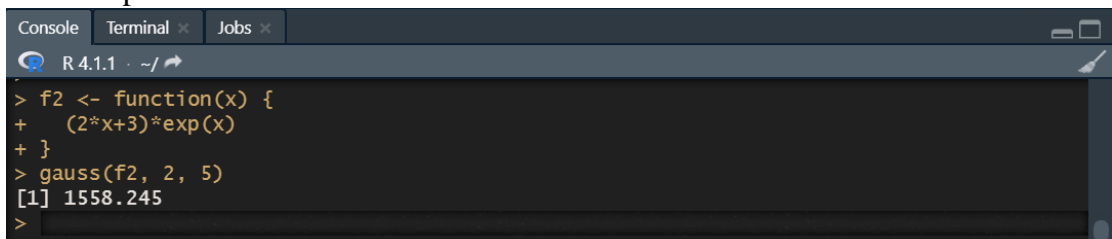
$$\int_2^5 (2x + 3) e^x$$

Gunakan kaidah Gauss-Legendre 2-titik untuk menghitung nilai integral diatas.

Syntax program R:

```
gauss <- function(f, a, b) {  
  x <- function(u) {  
    ((u+1) * (b-a) + 2*a) / 2  
  }  
  t1 <- x(1/sqrt(3))  
  t2 <- x(-1/sqrt(3))  
  hasil <- ((b-a) / 2) * (f(t1) + f(t2))  
  return(hasil)  
}  
  
f2 <- function(x) {  
  (2*x+3)*exp(x)  
}  
  
gauss(f2, 2, 5)
```

Hasil output:



```
R 4.1.1 ~/  
> f2 <- function(x) {  
+   (2*x+3)*exp(x)  
+ }  
> gauss(f2, 2, 5)  
[1] 1558.245  
>
```

Perhitungan manual:

hitung integral $\int_2^5 (2x+3) e^x$ menggunakan kaidah Gauss-Legendre 2 titik

► Mengubah peubah x menjadi t

$$x = \frac{(a+b) + (b-a)t}{2} = \frac{(5+2) + (5-2)t}{2} = 3.5 + 1.5t$$

► Diferensial dx menjadi dt

$$dx = \frac{b-a}{2} dt = \frac{5-2}{2} dt = 1.5 dt$$

► fungsi integral

$$\begin{aligned} \int_2^5 (2x+3) e^x dx &= \int_{-1}^1 (2(3.5+1.5t) + 3) e^{(3.5+1.5t)} 1.5 dt \\ &= 1.5 \int_{-1}^1 (2(3.5+1.5t) + 3) e^{(3.5+1.5t)} dt \end{aligned}$$

$$f(t) = (2(3.5+1.5t) + 3) e^{(3.5+1.5t)}$$

$$f(1/\sqrt{3}) = (2(3.5 + 1.5(1/\sqrt{3})) + 3) e^{(3.5+1.5 \cdot \frac{1}{\sqrt{3}})} = 923,6654$$

$$f(-1/\sqrt{3}) = (2(3.5 + 1.5(-1/\sqrt{3})) + 3) e^{(3.5+1.5 \cdot \frac{-1}{\sqrt{3}})} = 115,16445$$

sehingga

$$\begin{aligned} \int_2^5 (2x+3) e^x dx &= 1.5 \int_{-1}^1 (2(3.5+1.5t) + 3) e^{(3.5+1.5t)} dt \\ &= 1.5 [f(1/\sqrt{3}) + f(-1/\sqrt{3})] \\ &= 1.5 (923,6654 + 115,16445) \\ &= 1152,244175 \end{aligned}$$

2. Misalkan $f(x) = \ln(x^3 - x + 1)$

n	x
-1	0.99
0	1.00
1	1.01

Hitunglah hampiran $f'(1)$ dengan menggunakan rumus selisih maju, rumus selisih mundur, dan rumus selisih bedda pusat untuk nilai-nilai $h = 0.1, 0.01, 0.001, 0.0001$. Bandingkan galat untuk tiap rumusnya.

Syntax Program R:

```
turunan <- function(f, x, h) {  
  mb = NULL  
  mc = NULL  
  md = NULL  
  me = NULL  
  mf = NULL  
  mg = NULL  
  mh = NULL  
  
  x0 = x-h  
  x1 = x  
  x2 = x+h  
  
  dx <- genD(func = f, x = x)$D[1]  
  
  maju <- (f(x2)-f(x1))/h  
  pusat <- (f(x2)-f(x0))/2*h  
  mundur <- (f(x1)-f(x0))/h  
  
  galat1 <- abs (maju-dx)  
  galat2 <- abs (pusat-dx)  
  galat3 <- abs (mundur-dx)  
  
  mb[1] = h  
  mc[1] = maju  
  md[1] = pusat  
  me[1] = mundur  
  mf[1] = galat1  
  mg[1] = galat2  
  mh[1] = galat3  
  
  matriks <- matrix(c(mb, mc, md, me, mf, mg, mh), ncol=7,  
                    dimnames = list(NULL, c("h", "selisih  
maju", "selisih pusat", "selisih mundur", "galat maju",  
"galat pusat", "galat mundur" )))  
  return(matriks)  
}  
  
f3 <- function(x) {  
  log(x^3-x+1)  
}  
  
turunan(f3, 1, 0.1)  
turunan(f3, 1, 0.01)  
turunan(f3, 1, 0.001)  
turunan(f3, 1, 0.0001)
```

Hasil output:

```

Console Terminal Jobs
R 4.1.1 ~ /
> turunan(f3, 1, 0.1)
h selisih maju selisih pusat selisih mundur galat maju galat pusat galat mundur
[1,] 0.1 2.078268 0.0197681 1.875351 0.07826847 1.980232 0.1246488
> turunan(f3, 1, 0.01)
h selisih maju selisih pusat selisih mundur galat maju galat pusat galat mundur
[1,] 0.01 2.009768 0.0001999767 1.989765 0.00976818 1.9998 0.01023482
> turunan(f3, 1, 0.001)
h selisih maju selisih pusat selisih mundur galat maju galat pusat galat mundur
[1,] 0.001 2.000998 1.999998e-06 1.998998 0.0009976682 1.999998 0.001002335
> turunan(f3, 1, 0.0001)
h selisih maju selisih pusat selisih mundur galat maju galat pusat galat mundur
[1,] 1e-04 2.0001 2e-08 1.9999 9.997668e-05 2 0.0001000233
>

```

Perhitungan manual:

► $h = 0.1$

n	x	f(x)
-1	0.9	-0.18753
0	1	0
1	1.1	0.20783

a) hampiran selisih maju

$$f'(1) = \frac{f(1.1) - f(1)}{h} = \frac{0.20783 - 0}{0.1} = 2.0783$$

b) hampiran selisih pusat

$$f'(1) = \frac{f(1.1) - f(0.9)}{2h} = \frac{0.20783 - (-0.18753)}{2(0.1)} = 1.9768$$

c) hampiran selisih mundur

$$f'(1) = \frac{f(1) - f(0.9)}{h} = \frac{0 - (-0.18753)}{0.1} = 1.8753$$

► $h = 0.01$

n	x	f(x)
-1	0.99	-0.01980
0	1	0
1	1.01	0.020098

a) hampiran selisih maju

$$f'(1) = \frac{f(1.01) - f(1)}{h} = \frac{0.020098 - 0}{0.01} = 2.0098$$

b) hampiran selisih pusat

$$f'(1) = \frac{f(1.01) - f(0.99)}{2h} = \frac{0.020098 - (-0.01980)}{2(0.01)} = 1.995$$

c) hampiran selisih mundur

$$f'(1) = \frac{f(1) - f(0,99)}{h} = \frac{0 - (-0,01990)}{0,01} = 1,990$$

► $h = 0,001$

n	x	f(x)
-1	0,999	-0,001999
0	1	0
1	1,001	0,002001

a) hampiran selisih maju

$$f'(1) = \frac{f(1,001) - f(1)}{h} = \frac{0,002001 - 0}{0,001} = 2,001$$

b) hampiran selisih pusat

$$f'(1) = \frac{f(1,001) - f(0,999)}{2h} = \frac{0,002001 - (-0,001999)}{2(0,001)} = 2$$

c) hampiran selisih mundur

$$f'(1) = \frac{f(1) - f(0,999)}{h} = \frac{0 - (-0,001999)}{0,001} = 1,999$$

► $h = 0,0001$

n	x	f(x)
-1	0,9999	-0,0001999
0	1	0
1	1,0001	0,00020001

a) hampiran selisih maju

$$f'(1) = \frac{f(1,0001) - f(1)}{h} = \frac{0,00020001 - 0}{0,0001} = 2,0001$$

b) hampiran selisih pusat

$$f'(1) = \frac{f(1,0001) - f(0,9999)}{2h} = \frac{0,00020001 - (-0,0001999)}{2(0,0001)} = 1,99955$$

DATE :

c) hampiran selisih mundur

$$f'(1) = \frac{f(1) - f(0,9999)}{h} = \frac{0 - (-0,0001999)}{0,0001} = 1,999$$

Perbandingan nilai galat

h	Selisih maju	Selisih pusat	Selisih mundur
0.1	.078268	.02319	.124649
0.01	.009768	.000233	.010235
0.001	.000998	.000002	.001002
0.0001	.0001	0	.0001

* Galat = Nilai turunan asli – Nilai selisih