LKP PERTEMUAN 3
PMK

1. Buktikan fungsi berikut konvergen atau divergen

$$\sum_{n=1}^{\infty} \left(\frac{3}{n^2+5n+6} - 9^{-n+2}4^{n+1}\right)$$

$\sum_{n=1}^{\infty} \left(\frac{3}{n^2+5n+6} - 9^{2-n} \cdot 4^{n+1}\right)$ merupakan deret divergen/konvergen?

Suatu deret $\sum_{n=1}^{\infty} X_k$ dikatakan konvergen jika barisan jumlah parsial (S_n) konvergen, dengan kata lain, $\lim_{n\to\infty} S_n = l$

→ Deret diatas dipecah menjadi $\sum_{n=1}^{\infty} \frac{3}{n^2+5n+6}$ dan $\sum_{n=1}^{\infty} 9^{2-2} \cdot 4^{n+1}$

→ $\sum_{n=1}^{\infty} \frac{3}{n^2+5n+6} = 3 \sum_{n=1}^{\infty} \frac{1}{(n+2)(n+3)}$

$= 3 \cdot \sum_{n=1}^{\infty} \left(\frac{1}{n+2} - \frac{1}{n+3}\right)$

$= 3 \left(\left(\frac{1}{3} - \cancel{\frac{1}{4}}\right) + \left(\cancel{\frac{1}{4}} - \frac{1}{5}\right) + \ldots + \left(\frac{1}{n+2} - \frac{1}{n+3}\right)\right)$

$= 3 \left(\frac{1}{3} - \frac{1}{(n+3)}\right)$

$= \lim_{n\to\infty} \left(3 \cdot \left(\frac{1}{3} - \frac{1}{n+3}\right)\right) = 1$ (konvergen)

→ $\sum_{n=1}^{\infty} 9^{2-n} \cdot 4^{n+1} = \sum_{n=1}^{\infty} 9^{-(n-2)} \cdot 4^{n+1} = \sum_{n=1}^{\infty} \frac{4^{n+1}}{9^{n-2}}$

$= \sum_{n=1}^{\infty} \frac{4^{n-1} \cdot 4^2}{9^{n-1} \cdot 9^{-1}}$

$= \sum_{n=1}^{\infty} 16 \cdot 9 \cdot \frac{4^{n-1}}{9^{n-1}}$

$= \sum_{n=1}^{\infty} 144 \left(\frac{4}{9}\right)^{n-1}$

$= \frac{144}{1 - 4/9} = \frac{1296}{5}$ (konvergen)

Jumlah deret

$$\sum_{n=1}^{\infty}\left(\frac{3}{n^2+5n+6} - 9^{2-n}\cdot 4^{n+1}\right) = 1 - \frac{1296}{5} = -\frac{1291}{5}$$

karen barisan jumlah parsial (sn) konvergen dan memiliki nilai l

maka deret tsb dikatakan konvergen.

2. Buktikan fungsi berikut konvergen atau divergen

$$\sum_{n=1}^{\infty} \frac{n^2+2}{n^4+5}$$

$\sum_{n=1}^{\infty} \left(\frac{n^2+2}{n^4+5} \right)$ divergen/ konvergen ? (uji banding limit)

Misalkan $a_n \geq 0$, $b_n > 0$ dan $\lim_{n \to \infty} \left(\frac{a_n}{b_n} \right) = L$

(i) jika $0 < L < \infty$, maka $\sum_{n=1}^{\infty} a_n$ dan $\sum_{n=1}^{\infty} b_n$ samas konvergen atau divergen

(ii) jika $L = 0$ dan $\sum_{n=1}^{\infty} b_n$ konvergen maka a_n konvergen.

Dari deret diatas misal diterapkan

$a_n = \frac{n^2+2}{n^4+5} > 0$ dan $b_n = \frac{n^2+2}{n^4} > 0$

$\rightarrow \lim_{n \to \infty} \left(\frac{a_n}{b_n} \right) = L$

$$\lim_{n \to \infty} \left(\frac{\cancel{n^2+2}}{n^4+5} \cdot \frac{n^4}{\cancel{n^2+2}} \right) = \lim_{n \to \infty} \left(\frac{n^4}{n^4+5} \right)$$

$$= \lim_{n \to \infty} \left(\frac{\cancel{n^4}}{\cancel{n^4} \cdot \left(1 + \frac{5}{n^4} \right)} \right)$$

$$= \lim_{n \to \infty} \left(\frac{1}{1 + \frac{5}{n^4}} \right)$$

$$= \frac{1}{1+5 \times 0} = 1 \longrightarrow 0 < L < \infty$$

samas konvergen / divergen

karena $b_n = \frac{n^2+2}{n^4} \longrightarrow$ konvergen, maka a_n juga <u>konvergen</u>