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Kelas : Pararel 1

4.38 If $E[X] = 1$ and $Var(X) = 5$, find

(a) $E[(2+X)^2]$;

(b) $Var(4+3X)$.

Jawab :

(a) $E[(2+X)^2] = E[4 + 4X + X^2]$

$= 4 + 4E[X] + E[X^2]$

$= 4 + 4 + (Var(X) + \mu^2)$

$= 8 + (5 + 1^2)$

$= 14$

(b) $Var(4+3X) = 9\,Var(X)$

$= 9 \cdot 5$

$= 45$

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5.7) The density function of x is given by

$$f(x) \begin{cases} a+bx^2 & 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$$

If $E[x] = \frac{3}{5}$, find a and b?

Jawab.

↳ ■ $\int_{-\infty}^{\infty} f(x)\,dx = \int_{-\infty}^{0} f(x)\,dx + \int_{0}^{1} f(x)\,dx + \int_{1}^{\infty} f(x)\,dx = 1$

$= 0 + \int_0^1 a+bx^2\,dx + 0 = 1$

$= \int_0^1 a+bx^2\,dx = 1$

$= ax + \frac{b}{3}x^3 \Big|_0^1 = 1$

$= a + \frac{b}{3} = 1$

■ $E[x] = \frac{3}{5} = \int_{-\infty}^{\infty} x\,f(x)\,dx$

$= \int_{-\infty}^{0} x\,f(x)\,dx + \int_0^1 x\,f(x)\,dx + \int_1^{\infty} x\,f(x)\,dx$

$= 0 + \int_0^1 x(a+bx^2)\,dx + 0$

$= \frac{a}{2}x^2 + \frac{b}{4}x^4 \Big|_0^1$

$= \frac{a}{2} + \frac{b}{4}$

$= \frac{3}{5} \rightarrow 5(2a+b) = 12 \rightarrow a = \frac{3}{5}\ ;\ b = \frac{6}{5}$

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5.4 The probability density function of X, the lifetime of a certain type of electronic device (measured in hours), is given by

$$f(x) = \begin{cases} \frac{10}{x^2} & x > 10 \\ 0 & x \leq 10 \end{cases}$$

(a) Find $P\{X > 20\}$
(b) What is the cumulative distribution function of X ?
(c) What is the probability that, of 6 such types of devices, at least 3 will function for at least 15 hours ? What assumptions are you making ?

Jawab :

↳ (a) Find $P\{x > 20\}$

$$P(x > 20) = 1 - \int_{10}^{20} \frac{10}{x^2} \, dx$$

$$= 1 - 10 \int_{10}^{20} \frac{1}{x^2} \, dx$$

$$= 1 + 10 \cdot \frac{1}{x} \Big|_{10}^{20}$$

$$= 1 + 10 \left(\frac{1}{20} - \frac{1}{10} \right)$$

$$= 1 - 10 \cdot \frac{1}{20}$$

$$= \frac{1}{2}$$

(b) Cumulative distribution function of x

$$F_X(x) = P(X \leq x)$$

$$= \int_{10}^{x} \frac{10}{a^2} \, da$$

$$= -10 \cdot \frac{1}{a} \Big|_{10}^{x}$$

$$= -10 \cdot \left(\frac{1}{x} - \frac{1}{10} \right)$$

$$= 1 - \frac{10}{x}$$

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(c) probability 6 tyres of device

$P(A \geq 3) = 1 - A \leq 2$

$= 1 - P(A = 0) - A$

$= 1 - P(A = 2)$

$P(A = 0) = \binom{6}{0} \cdot P(x < 15)^6$

$= \dfrac{1}{3^6}$

$P(A = 1) = \binom{6}{1} \cdot \dfrac{2}{3} \cdot \dfrac{1}{3^5}$

6.4 Basketball shots To win a basketball game, two competitors play three rounds of one three-point shot each. The series ends if one them scores in a round but the other misses his shot or if both get the same result in each of the three rounds. Assume competitors A and B have 30 % and 20 % of successful attempts, respectively, in three-point shots and that the outcomes of the shots are independent events

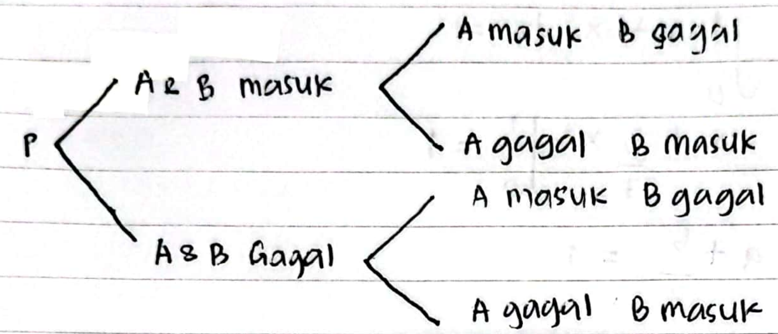
a. Verify the probability that the series ends in the second round is 23,56 %.

b. Find the probability distribution of X = number of rounds played to end the series.

c. Find the expected number of rounds to be played in the series

Jawab :

a)



∴ Ronde 1

$P(A, B)$ masuk $= 0{,}3 \times 0{,}2$

$= 0{,}06$

$P(A, B)$ gagal $= 0{,}7 \times 0{,}8$

$= 0{,}56$

∴ Ronde 2

$P(A \text{ masuk}, B \text{ gagal}) = 0{,}3 \times 0{,}8$

$= 0{,}24$

$P(A \text{ gagal}, B \text{ masuk}) = 0{,}7 \times 0{,}2$

$= 0{,}14$

∴ peluang selesai ronde dua

$P = (0{,}06 + 0{,}56)\,0{,}24 + (0{,}06 + 0{,}56)\,0{,}14$

$= (0{,}62)\,0{,}24 + (0{,}62)(0{,}14)$

$= 0{,}1488 + 0{,}0868$

$= 0{,}2356$

$= 23{,}56\ \%$

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(b) $P(X=1) = 0{,}24 + 0{,}14$
$= 0{,}38$
$= 38\%$

$P(X=3) = 1 - 38\% - 23{,}56\%$
$= 38{,}44\%$

$P(X=2) = 0{,}2356$
$= 23{,}56\%$

(c) $E[X] = \sum_{1}^{3} x f(x) = 1 \times 38\% + 2 \times 23{,}56\% + 3 \times 38{,}44\%$
$= 2{,}0044$

6.10 Ideal number of children Let X denote the response of a randomly selected person to the question, "what is the ideal number of children for a family to have?" The probability distribution of X in the United states is approximately as shown in the table, according to the gender of the person asked the question.

Probability Distribution of X = Ideal Number of Children

X	P(X) Females	P(X) Males
0	0,01	0,02
1	0,03	0,03
2	0,55	0,60
3	0,31	0,28
4	0,11	0,08

Note that the probabilities do not sum to exactly 1 due to rounding error.

a. shows that the means are similar, 2,50 for females and 2,39 for males.
b. The standard deviation for the females is 0,770 and 0,758 for the males. Explain why a practical implication of the values for the standard deviations is that males hold slightly more consistent views than females about the ideal family size.

Jawab :
a. nilai tengah : $E[X] = \sum x f(x)$
$E[X]_F : 0 \times 0{,}01 + 1 \times 0{,}03 + 2 \times 0{,}55 + 3 \times 0{,}31 + 4 \times 0{,}11$
$= 2{,}5$
$E[X]_m : 0 \times 0{,}02 + 1 \times 0{,}03 + 2 \times 0{,}60 + 3 \times 0{,}28 + 4 \times 0{,}08$
$= 2{,}39$

b. Keragaman laki² lebih sedikit dari perempuan. Oleh karena itu, standar deviasi laki² lebih konsisten dibanding perempuan.

3.153) Find the distributions of the random variables that have each of the following moment-generating functions:

a. $m(t) = [(1/3)e^t + (2/3)]^5$

b. $m(t) = \dfrac{e^t}{2 - e^t}$

c. $m(t) = e^{2(e^t - 1)}$.

Jawab:

a. $m(t) = [(1/3)e^t + (2/3)]^5$ $n = 5$, $p = 1/3$ → sebaran binomial

$= [pe^t + (1-p)]^n$

$f(x) = \binom{n}{x} p^x (1-p)^{n-x}$

$= \binom{5}{x} (1/3)^x (2/3)^{5-x}$

b. $m(t) = \dfrac{e^t}{2 - e^t}$ $E[X] = m'(0) = n \cdot p$ → sebaran geometrik

$= \dfrac{2 \cdot e^0}{(2 - e^0)^2}$

$= \dfrac{2}{(1)2} = 2$

$E(X^2) = m''(0)$

$= np(1-p) + np$

$= \dfrac{2 \cdot e^0 (e^0 + 2)}{(2 - e^0)^3}$

$= \dfrac{2(3)}{(1)^3}$

$= 6$

$np = 2$ $2(1-p) + 2 = 6$ $n(-1) = 2$

$n(n-1)p + np = 6$ $p = -1$ $n = -2$

$f(x) = \binom{n}{x} p^x (1-p)^{n-x}$

$= \binom{-2}{x} (-1)^x (2)^{-2-x}$

c. $m(t) = e^{2(e^t - 1)}$

$E[X] = n \cdot p = m'(0) = 2 \cdot e^{0 + 2 \cdot e^0} = 2$ → sebaran poisson

$E[X^2] = np(1-p) + np = m''(0) = e^{0 + 2e^0 - 2}(4 \cdot e^0 + 2) = 6$

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$np = 2$ $\quad$ $2(1-p)+2 = 6$ $\quad$ $n(-1) = 2$

$np(1-p)+np = 6$ $\quad$ $p = -1$ $\quad$ $n = -2$

$f(x) = \binom{n}{x} p^x (1-p)^{n-x}$

$= \binom{-2}{x} (-1)^x (2)^{-2-x}$

3.156 Suppose that Y is a random variable with moment-generating function $m(t)$

a. What is $m(0)$?

b. If $W = 3Y$, show that the moment-generating function of W is $m(3t)$.

c. If $X = Y-2$, show that the moment-generating function of X is $e^{-2t}m(t)$.

Jawab :

a) $m(t) = E(e^{ty})$

$m(0) = E(e^{t(0)})$

$m(0) = E(1)$

$m(0) = 1$

b) $m(W) = E(e^{tW})$

$m(W) = E(e^{t.3Y})$

$m(W) = E(e^{3tY})$

$m(W) = m(3t)$

c) $m(X) = E(e^{tX})$

$m(X) = E(e^{t(Y-2)})$

$m(X) = E(e^{tY}.e^{-2t})$

$m(X) = e^{-2t}.E(e^{tY})$

$m(X) = e^{-2t}.m(t)$