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No
Date

4.38) if $E[X]=1$ and $Var(X)=5$

q: a) $E[(2+X)^2]$

b) $Var(4+3X)$

a: a) $E[(2+x)^2] = E[(4+4x+x^2)]$

$= 4+4E[x]+E[x^2]$

$= 4+4\cdot1+(var(x)+\mu^2)$

$= 4+4+(5+1^2)$

$= 8+6$

$= 14$

b) $var(4+3x) = 3^2 \cdot var(x)$

$= 9\cdot5$

$= 45$

5.7) the density function of X is given by

$$f(x)\begin{cases} a+bx^2 & 0\le x\le 1 \\ 0 & \text{otherwise}\end{cases}$$

if $E[x]=3/5$

q: a and b?

a: $\int_{-\infty}^{\infty} f(x)\,dx = \int_{-\infty}^{0} f(x)\,dx + \int_0^1 f(x)\,dx + \int_1^{\infty} f(x)\,dx = 1$

$= 0 + \int_0^1 a+bx^2\,dx + 0 = 1$

$= \int_0^1 a+bx^2\,dx = 1$

$= ax+\frac{b}{3}x^3\Big]_0^1 = 1$

$= a+b/3 = 1$

$E[x]=3/5 = \int_{-\infty}^{\infty} x f(x)\,dx$

$= \int_{-\infty}^{0} xf(x)\,dx + \int_0^1 xf(x)\,dx + \int_1^{\infty} xf(x)\,dx$

$= 0 + \int_0^1 x(a+bx^2)\,dx + 0$

$= a/2\,x^2 + b/4\,x^4\Big]_0^1$

$= a/2+b/4 = 3/5$

$a + \frac{b}{3} = 1 \Rightarrow 3a + b = 3$

$\frac{a}{2} + \frac{b}{4} = \frac{3}{5} \Rightarrow 10a + 5b = 12$

$15a + 5b = 15$

$10a + 5b = 12$

$5a = 3$

$a = 3/5,\ b = 6/5$ //

5.4) The probability density function of X, the lifetime of a certain type of electronic device (measured in hours), is given by

$$f(x) \begin{cases} \frac{10}{x^2} & x > 10 \\ 0 & x \leq 10 \end{cases}$$

q: a) find $P\{X > 20\}$

b) what is the cumulative distribution function of X?

c) what is the probability that, of 6 such types of devices, at least 3 will function for at least 15 hours? what assumptions

q: a) $\int_{-\infty}^{10} f(x)\,dx + \int_{10}^{20} f(x)\,dx + \int_{20}^{\infty} f(x)\,dx = 1$

$0 + \int_{10}^{20} 10/x^2\,dx + \int_{20}^{\infty} f(x)\,dx = 1$

$\int_{20}^{\infty} f(x)\,dx = 1 - \left(-10/x\right]_{10}^{20})$

$= 1 - ((-1/2) - (-1))$

$= 1 - (1/2)$

$\int_{20}^{\infty} f(x)\,dx = 1/2$

b) $\int_{-\infty}^{10} f(x)\,dx + \int_{10}^{x} f(x)\,dx$

$= 0 + \int_{10}^{x} 10/x^2\,dx$

$= \left(-10/x\right]_{10}^{x}$

$= \left(-\frac{10}{x}\right) + 1$ //

c) $P(x \geq 3) = \binom{6}{3}\left(\frac{2}{3}\right)^3\left(\frac{1}{3}\right)^3 + \binom{6}{4}\left(\frac{2}{3}\right)^4\left(\frac{1}{3}\right)^2 + \binom{6}{5}\left(\frac{2}{3}\right)^5\left(\frac{1}{3}\right)^1 + \binom{6}{6}\left(\frac{2}{3}\right)^6\left(\frac{1}{3}\right)^0$

$= \frac{656}{724} \approx 0.9$ //

6.9) successful attempts

$P(A) : 0,3$ $P(B) : 0,2$

Failure attempts

$P'(A) : 0,7$ $P'(B) : 0,8$

q: a) verify the probability that the series ends in the 2nd round is 23.56%.

b) find the probability distribution of x = number of rounds played to end the series

c) find the expected number of rounds to be played in the series.

a: a) finish 1st round ⇒ A win B loss $= P(A) \times P'(B) = 0.3 \times 0.8 = 0.24$

A loss B win $= P'(A) \times P(B) = 0.2 \times 0.7 = 0.14$

Finish 2nd round ⇒ A win B win $= P(A) \times P(B) = 0.3 \times 0.2 = 0.06$

A loss B loss $= P'(A) \times P'(B) = 0.7 \times 0.8 = 0.56$

chance to 2nd round ⇒ $0.06 + 0.56 = 0.62$ ⇒ $P(A) \cdot P'(B) = 0.62 \times 0.24 = 0.1488$

$P'(A) \cdot P(B) = 0.62 \times 0.14 = 0.0868$

probability series end in 2nd round

$= 0.1488 + 0.0868$

$= 0.2356 \times 100\% = 23.56\%$ //

b) $P(x=1) = 0.24 + 0.14 = 0.38 \times 100\% = 38\%$

$P(x=2) = 23,56\%$

$P(x=3) = 1 - 38\% - 23.56\% = 38.44\%$

c) $E[x] = \Sigma x f(x) = 1(38\%) + 2(23,56\%) + 3.(38.44\%)$

$= 2.0044\%$

6.10) Probability distribution of X = Ideal Number of Children

$P(x)$ Females $\Rightarrow$ $P(0) = 0.01$ $\quad$ $P(x)$ Males $\Rightarrow$ $P(0) = 0.02$

$P(1) = 0.03$ $\quad$ $P(1) = 0.03$

$P(2) = 0.55$ $\quad$ $P(2) = 0.60$

$P(3) = 0.31$ $\quad$ $P(3) = 0.28$

$P(4) = 0.11$ $\quad$ $P(4) = 0.08$

q: a) show that the means are similar : 2.50 for females & 2.39 for males

b) stdev females = 0.770 & stdev males = 0.758

explain why a practical implication of the values of standard dev is that males hold slightly more consistent views than females about the ideal family size

a: a) means for females : $(0 \times 0.01) + (1 \times 0.03) + (2 \times 0.55) + (3 \times 0.31) + (4 \times 0.11)$

$= 2.5$

means for males $= (0 \times 0.02) + (1 \times 0.03) + (2 \times 0.60) + (3 \times 0.28) + (4 \times 0.08)$

$= 2.39$

b) standar deviasi menyatakan persebaran nilai, di mana semakin kecil nilai standar deviasi maka semakin konsisten.

Oleh karena itu males lebih konsisten dari females karena stdev males < stdev females.

3.153) Find the distributions of random variables that have each of the following moment-generating function:

a:

q: a) $m(t) = [(1/3)e^t + (2/3)]^5$ $\Rightarrow$ sebaran binomial ; $p = 1/3$, $n = 5$

b) $m(t) = \dfrac{e^t}{2 - e^t}$ $\Rightarrow$ sebaran geometrik ; $p = 1/2$

c) $m(t) = e^{2(e^t - 1)}$ $\Rightarrow$ sebaran poisson ; $\lambda = 2$

2. a: a) $m(t) = [(1/3)e^t + (2/3)]^5$ $\Rightarrow$ $[pe^t + (1-p)]^5$

maka $p = 1/3$ dan $n = 5$

sehingga $f(x) = \binom{n}{x} p^x (1-p)^{n-x} = \binom{5}{x}\left(\frac{1}{3}\right)^x \left(\frac{2}{3}\right)^{5-x}$

b) $m(t) = \dfrac{e^t}{2 - e^t}$

$E(x) = m'(0) = n \cdot p = \dfrac{2 \cdot e^0}{(2 - e^0)^2} = \dfrac{2}{1^2} = 2$

$E(x^2) = m''(0) = n \cdot p(1-p) + np = \dfrac{2 \cdot e^0 (e^0 + 2)}{(2 - e^0)^3} = \dfrac{2 \cdot 3}{1^3} = 6$

$\Rightarrow np = 2$, $n(n-1)p + np = 6$ $\Rightarrow$ $p = -1$, $n = -2$ $\Rightarrow$ $f(x) = \binom{-2}{x} (-1)^x (2)^{-2-x}$

c) $m(t) = e^{2(e^t - 1)}$

$E(x) = n \cdot p = m'(0) = 2 \cdot e^{0 + 2 \cdot e^0 - 2} = 2$

$E(x^0)$: $np(1-p) + np = m''(0) = e^{0 + 2 \cdot e^0 - 2}(4 \cdot e^0 + 2) = 6$

$2(1-p) + 2 = 6$

$1 - p = 2$ $\Rightarrow$ $p = -1$, $n = -2$

$f(x) = \binom{n}{x} p^x (1-p)^{n-x}$

$f(x) = \binom{-2}{x} (-1)^x (2)^{-2-x}$

3.156) y is random variable with $m(t)$.

q: a) $m(0)$?

b) if $W = 3Y$, show that moment-generating function of W is $m(3t)$

c) if $X = Y - 2$, show that moment-generating function of X is $e^{-2t} m(t)$

a: a) $m(t) = E(e^{ty})$

$m(0) = E(e^{t(0)})$

$= E(1)$

$= 1$

b) $m(w) = p(3y)$

$= 3 \cdot p(y)$

$= 3 \cdot E(e^{ty})$

$= 3 \cdot m(t)$

$= m(3t$

c) $m(x) = p(Y-2)$

$= e^{-2} \cdot p(Y)$

$= e^{-2+y} \, p(Y)$

$= e^{-2t} \, m(t)$