No

Date

PENGANTAR MATEMATIKA KOMPUTASI

LKP 3

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1) Buktikan fungsi berikut konvergen atau divergen

$$\sum_{n=1}^{\infty} \left(\frac{3}{n^2+5n+6} - 9^{-n+2} \, 4^{n+1} \right)$$

Jawab

$$\left(\sum_{n=1}^{\infty} \frac{3}{n^2+5n+6} - \sum_{n=1}^{\infty} 9^{-n+2} \, 4^{n+1} \right)$$

$$= \left(\sum_{n=1}^{\infty} \frac{3}{(n+2)(n+3)} \right) - \left(\sum_{n=1}^{\infty} 9^{-n} \cdot 9^2 \cdot 4^n \cdot 4 \right)$$

$$= 3 \sum_{n=1}^{\infty} \frac{1}{(n+2)(n+3)} - 324 \sum_{n=1}^{\infty} \frac{4^n}{9^n}$$

$$\sum_{n=1}^{\infty} \frac{1}{(n+2)(n+3)} = \frac{A}{n+2} + \frac{B}{n+3}$$

$$= \frac{A(n+3) + B(n+2)}{(n+2)(n+3)}$$

$A(n+3) + B(n+2) = 1$

$n = -3 \longrightarrow -B = 1$

$B = -1$

$n = -2 \longrightarrow A = 1$

$$\Rightarrow 3 \sum_{n=1}^{\infty} \frac{1}{n+2} - \frac{1}{n+3}$$

$$= \left(\frac{1}{3} - \frac{1}{4}\right) + \left(\frac{1}{4} - \frac{1}{5}\right) + \left(\frac{1}{5} - \frac{1}{6}\right) + \dots + \left(\frac{1}{n+2} - \frac{1}{n+3}\right)$$

$$= \frac{1}{3} - \frac{1}{k+3}$$

$$S_n = 3 \sum_{n=1}^{\infty} \frac{1}{n+2} - \frac{1}{n+3}$$

$$= 3 \cdot \left(\frac{1}{3} - \frac{1}{k+3}\right) = 1 - \frac{3}{k+3}$$

$$= \frac{k+3-3}{k+3} = \frac{k}{k+3}$$

$$\lim_{k \to \infty} \frac{k}{k+3} = \lim_{k \to \infty} \frac{k/k}{k/k + 3/k} = 1$$

karena, deret $\sum_{n=1}^{\infty} \frac{1}{(n+2)(n+3)}$ merupakan deret kolaps, maka $3 \sum_{n=1}^{\infty} \frac{1}{(n+2)(n+3)}$ merupakan deret konvergen.

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$$\Rightarrow 324 \sum_{n=1}^{\infty} \left(\frac{4}{9}\right)^{n} \Rightarrow \quad a = 4/9 \quad u_2 = 4^2/9^2$$

$$r = \frac{4^2/9^2}{4/9}$$

$$= 324 \ \frac{a}{1-r} \qquad r = 4/9$$

$$= 324 \cdot \frac{4/9}{1 - 4/9}$$

$$= 324 \cdot 4/5$$

$$= \frac{1296}{5}$$, sehingga konvergen

Karena $\sum_{n=1}^{\infty} \left(\frac{3}{n^2+5n+6}\right)$ dan $\sum_{n=1}^{\infty} 9^{-n+2} \, 4^{n+1}$ konvergen, maka

$\sum_{n=1}^{\infty} \left(\frac{3}{n^2+5n+6} - 9^{-n+1} + 4^{n+1}\right)$ juga konvergen.

2) Buktikan fungsi berikut konvergen atau divergen

$$\sum_{n=1}^{\infty} \frac{n^2+2}{n^4+5}$$

Jawab:

menggunakan uji banding limit

jika $\sum_{n=1}^{\infty}$ an konvergen maka $\lim_{n \to \infty}$ an $= 0$

$$\lim_{n \to \infty} \frac{n^2+2}{n^4+5}$$

$$\lim_{n \to \infty} \frac{n^2/n^4 + 2/n^4}{n^4/n^4 + 5/n^4} = \frac{0+0}{1+0} = 0$$

Karena nilai $\lim_{n \to \infty}$ an $= 0$, maka $\sum_{n=1}^{\infty}$ terbukti konvergen.