No

Date

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1. $F: \mathbb{R}^2 \to \mathbb{R}^2$ dengan $F((x,y)^T) = (x, y+1)^T$

$\bar{u} = (u_1, u_2)^T \quad F(\bar{u}) = (u_1, u_2+1)^T$

$\bar{v} = (v_1, v_2)^T \quad F(\bar{v}) = (v_1, v_2+1)^T$

- $F(\bar{u}+\bar{v}) = F(u_1+v_1, u_2+v_2)$
  $= (u_1+v_1, u_2+v_2+1)^T$
- $F(\bar{u})+F(\bar{v}) = (u_1+v_1, u_2+v_2+2)^T$

maka $F(\bar{u}+\bar{v}) \neq F(\bar{u})+F(\bar{v})$ pada $F((x,y)^T) = (x, y+1)^T$

Jadi bukan merupakan transformasi linear

2. $F: \mathbb{R}^3 \to \mathbb{R}^2$ dengan $F((x,y,z)^T) = (x+y+z, z-y-x)^T$

$\bar{a} = (a_1, a_2, a_3)^T \to F(\bar{a}) = (a_1+a_2+a_3, a_3-a_2-a_1)^T$

$\bar{b} = (b_1, b_2, b_3)^T \to F(\bar{b}) = (b_1+b_2+b_3, b_3-b_2-b_1)^T$

$F(\bar{a}+\bar{b}) = F(a_1+b_1, a_2+b_2, a_3+b_3)$

$= ((a_1+b_1)+(a_2+b_2)+(a_3+b_3), (a_3+b_3)-(a_2+b_2)-(a_1+b_1))^T$

$= (a_1+a_2+a_3+b_1+b_2+b_3, a_3-a_2-a_1+b_3-b_2-b_1))^T$

$= (a_1+a_2+a_3, a_3-a_2-a_1)^T + (b_1+b_2+b_3, b_3-b_2-b_1))^T$

$= F(\bar{a}) + F(\bar{b})$

terbukti aksioma 1 ✓

$F(k\bar{a}) = F((ka_1, ka_2, ka_3)^T)$

$= (ka_1+ka_2+a_3, ka_3-ka_2-ka_1)^T$

$= k(a_1+a_2+a_3, a_3-a_2-a_1)^T$

$= k \cdot F(\bar{a})$, terbukti untuk aksioma 2

maka $F((x,y,z)^T) = (x+y+z, z-y-x)^T$ merupakan transformasi linear

3. $F: \mathbb{R}^2 \to \mathbb{R}^3$ dengan $F((x,y)) = (x-y, y-x, y)$

$\bar{a} = (a_1, a_2) \to F(\bar{a}) = (a_1-a_2, a_2-a_1, a_2)$

$\bar{b} = (b_1, b_2) \to F(\bar{b}) = (b_1-b_2, b_2-b_1, b_2)$

$F(\bar{a}+\bar{b}) = F(a_1+b_1, a_2+b_2)$

$= ((a_1+b_1)-(a_2+b_2), (a_2+b_2)-(a_1+b_1), a_2+b_2)$

$= (a_1-a_2+b_1-b_2, a_2-a_1+b_2-b_1, a_2+b_2)$

$= (a_1-a_2, a_2-a_1, a_2) + (b_1-b_2, b_2-b_1, b_2)$

$= F(\bar{a}) + F(\bar{b})$ terbukti aksioma 1

$F(k\bar{a}) = F(ka_1, ka_2)$

$= (ka_1-ka_2, ka_2-ka_1, ka_2)$

$= (k(a_1-a_2), k(a_2-a_1), ka_2)$

$= k \cdot F(\bar{a})$ maka aksioma 2 terbukti dan merupakan transformasi linear

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4. $F : R^2 \to R^2$ dengan $F((x,y)^T) = (5, x+y)$

$\bar{a} = (a_1, a_2)^T \quad \to F(\bar{a}) = (5, a_1 + a_2)$

$\bar{b} = (b_1, b_2)^T \quad \to F(\bar{b}) = (5, b_1 + b_2)$

$F(\bar{a}+\bar{b}) = F(a_1+b_1, a_2+b_2)$

$= (5, a_1 + b_1 + a_2 + b_2) \quad \dots (1)$

$F(\bar{a}) + F(\bar{b}) = (10, a_1+b_1+a_2+b_2) \quad \dots (2)$

persamaan (1) dan (2) $\neq$ maka aksioma 1 tidak terbukti dan bukan transformasi linear.

5. $F: M_{2\times2} \to R$ dengan $F(A) = \det(A)$

$A = \begin{pmatrix} a_1 & a_2 \\ a_3 & a_4 \end{pmatrix} \quad F(A) = a_1 a_4 - a_2 a_3$

$B = \begin{pmatrix} b_1 & b_2 \\ b_3 & b_4 \end{pmatrix} \quad F(B) = b_1 b_4 - b_2 b_3$

$F(A) + F(B) = a_1 + a_4 - a_2 a_3 + b_1 b_4 - b_2 b_3$

$F(A+B) = F\begin{pmatrix} a_1+b_1 & a_2+b_2 \\ a_3+b_3 & a_4+b_4 \end{pmatrix}$

$= (a_1+b_1)(a_4+b_4) - ((a_2+b_2)(a_3+b_3))$

$= (a_1 a_4 + a_1 b_4 + a_4 b_1 + b_1 b_4) - (a_2 a_3 + a_2 b_3 + a_3 b_2 + b_2 b_3)$

$= (a_1 a_4 - a_2 a_3) + (b_1 b_4 - b_2 b_3) + a_1 b_4 + a_4 b_1 + a_2 b_3 + a_3 b_2$

$= F(\bar{a}) + F(\bar{b}) + a_1 b_4 + a_4 b_1 + a_2 b_3 + a_3 b_2$

aksioma 1 tidak terpenuhi maka, bukan transformasi linear.

6. $F: M_{2\times2} \to M_{2\times2}$ dengan $F(A) = A + A^T$

$A = \begin{pmatrix} a_1 & a_2 \\ a_3 & a_4 \end{pmatrix} \quad \to F(A) = \cancel{\begin{pmatrix} 2a_1 & a_2+a_3 \\ a_2+a_3 & 2a_4 \end{pmatrix}} \begin{pmatrix} 2a_1 & a_2+a_3 \\ a_2+a_3 & 2a_4 \end{pmatrix}$

$B = \begin{pmatrix} b_1 & b_2 \\ b_3 & b_4 \end{pmatrix} \quad \to F(B) = \cancel{\begin{pmatrix} 2b_1 & b_2+b_3 \\ b_2+b_3 & 2b_4 \end{pmatrix}} \begin{pmatrix} 2b_1 & b_2+b_3 \\ b_2+b_3 & 2b_4 \end{pmatrix}$

$F(A+B) = F\begin{pmatrix} a_1+b_1 & a_2+b_2 \\ a_3+b_3 & a_4+b_4 \end{pmatrix}$

$= \begin{pmatrix} 2a_1 + 2b_1 & a_2+b_2+a_3+b_3 \\ a_2+a_3+b_2+b_3 & 2a_4+2b_4 \end{pmatrix}$

$= \begin{pmatrix} 2a_1 & a_2+a_3 \\ a_2+a_3 & 2a_4 \end{pmatrix} + \begin{pmatrix} 2b_1 & b_2+b_3 \\ b_2+b_3 & 2b_4 \end{pmatrix}$

$= F(A) + F(B)$ aksioma 1 terbukti

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lanjutan no 6 :

$$F(kA) = F\begin{pmatrix} ka_1 & ka_2 \\ ka_3 & ka_4 \end{pmatrix}$$

$$= \begin{pmatrix} k2a_1 & ka_2+ka_3 \\ ka_3+ka_2 & k2a_4 \end{pmatrix}$$

$$= k.\begin{pmatrix} 2a_1 & a_2+a_3 \\ a_2+a_3 & 2a_4 \end{pmatrix}$$

$= k \cdot F(A)$ aksioma 2 terbukti

maka 6 merupakan transformasi linear.

7 $F: P_3 \to P_2$ dengan $F(a_0+a_1x\, a_2x^2+a_3x^3) = 5a_0+a_2x^2$

$a = a_0+a_1x+a_2x^2+a_3x^3 \Rightarrow F(a) = 5a_0+a_2x^2$

$b = b_0+b_1x+b_2x^2+b_3x^3 \quad F(b) = 5b_0+b_2x^2$

$$F(a+b) = F(a_0+b_0+a_1x+b_1x+a_2x^2+b_2x^2+a_3x^3+b_3x^3)$$

$$= 5(a_0+b_0)+(a_2x^2+b_2x^2)$$

$$= (5a_0+a_2x^2)+(5b_0+b_2x^2)$$

$= F(a)+F(b)$ aksioma 1 terbukti

$$F(ka) = F(ka_0+ka_1x+ka_2x^2+ka_3x^3)$$

$$= 5ka_0+ka_2x^2$$

$$= k(5a_0+a_2x^2)$$

$= k \cdot F(a)$ aksioma 2 terbukti

maka no 7 merupakan transformasi linear