ECE535: Digital Signal Processing

Matlab Project 2

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1 Deterministic spectral analysis

Part 1a)

The first task is to write in MATLAB a function to compute the sum of two cosines as follow:

$$s_c(t) = A_0 cos(2\pi f_0 t) + A_1 cos(2\pi f_1 t)$$

Fig. 1 presents the tests to verify the function in MATLAB is working properly. The first and second plots show only one cosine at a time. In the first plot, the amplitude is 1, and 10 cycles can be observed which verifies a frequency of 10 Hz. Similarly, 5 cycles of a cosine with an amplitude 2 can be seen on the second plot. The third plot is the sum of the first and second plots. The maximum and minimum amplitudes for the third plot is 3 and -1 respectively. The locations of maxima and minima agrees with the sum of the peaks of the first and the second plots. The function is verified to behave as expected.

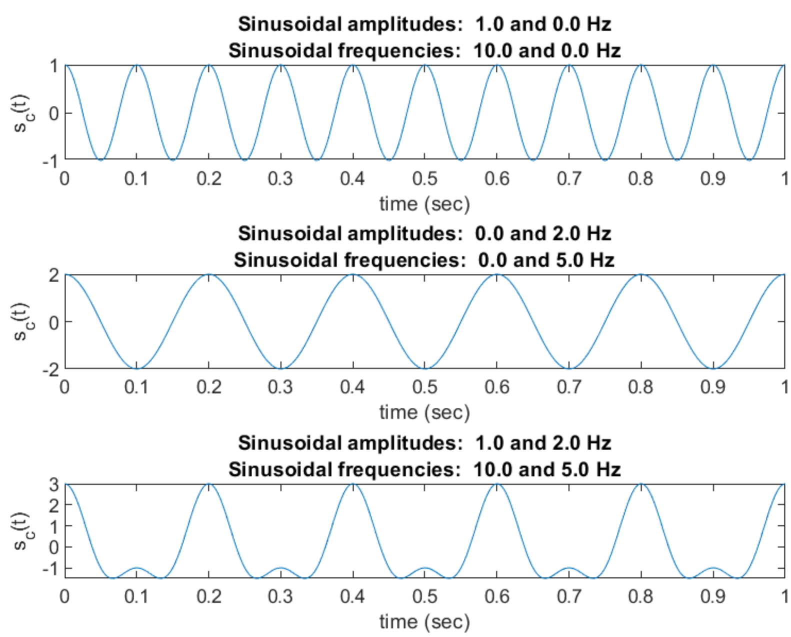


Figure 1: Testing the sum of two consines function

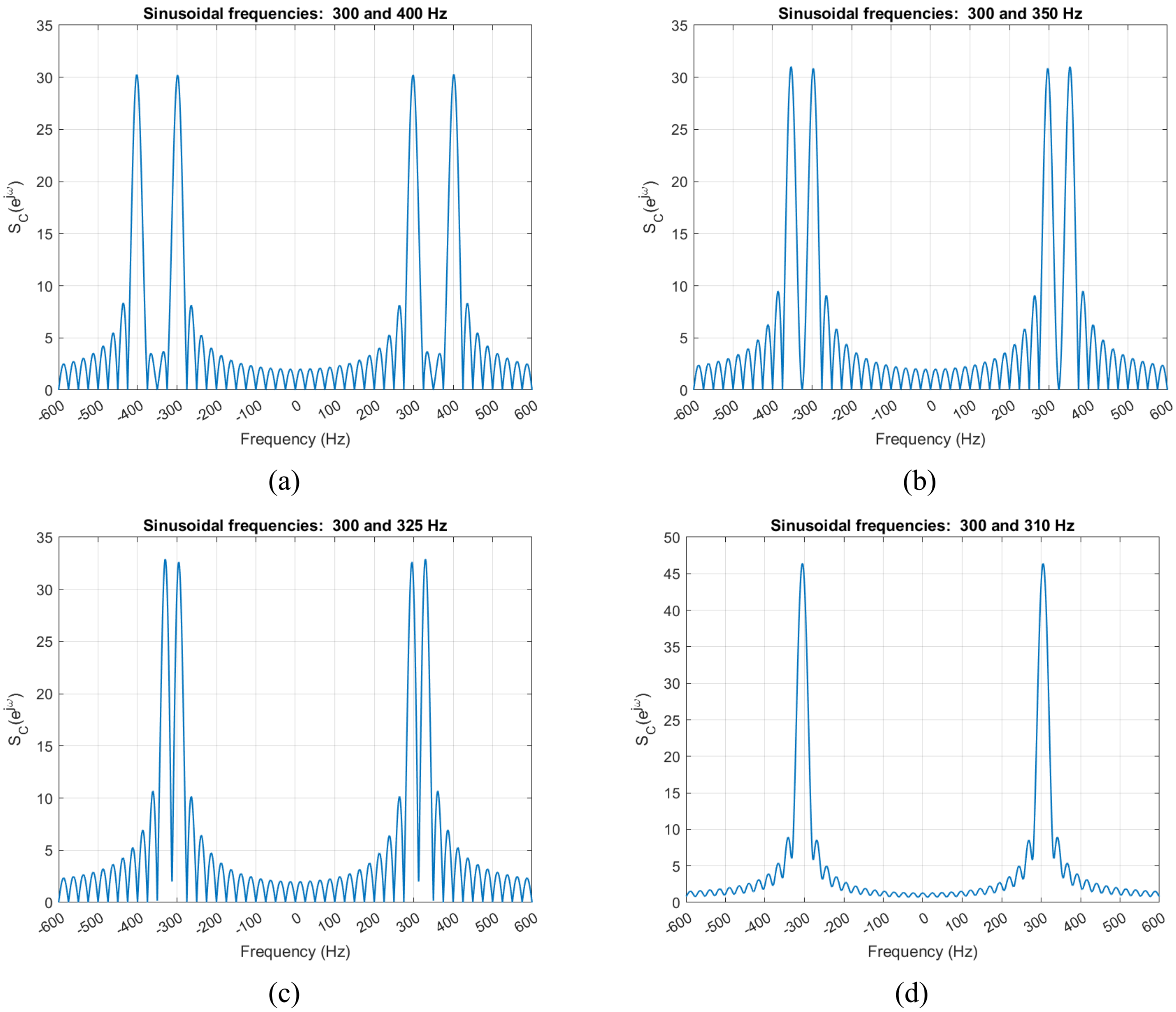


Figure 2: Decreasing f_1 until the two frequencies are indistinguishable. 8192-pt DFT and window length $L = 60$.

Part 1b)

Experiment with different value for f_1

The task is to investigate how close f_1 can be to f_0. A fixed rectangular window of length $L = 60$ is chosen for this test. The main lobe (ML) width of a rectangular is $\varDelta\varOmega_{ML} = \frac{4\pi}{L} f_s$. Equivalently, the ML width is $\varDelta f_{ML} = \frac{2}{L} f_s = \frac{2}{60}(1500) = 50\,\text{Hz}$. Since f_0 is at $300\,\text{Hz}$, for any f_2 that is between $250\,\text{Hz}$ and $350\,\text{Hz}$, overlaps and leakage are expected between the two spectra. Fig. 2a shows two distinct spectra at the same peak values. Fig. 2a is at the "edge" of $50\,\text{Hz}$ resolution of the spectral analyzer, and the two MLs are right next to each other. Fig. 2c show leakages from f_0 to f_1 and vice versa; the peak is now higher than of Fig. 2a. Fig. 2d shows that the frequencies content at f_0 and f_1 are no longer distinguishable.

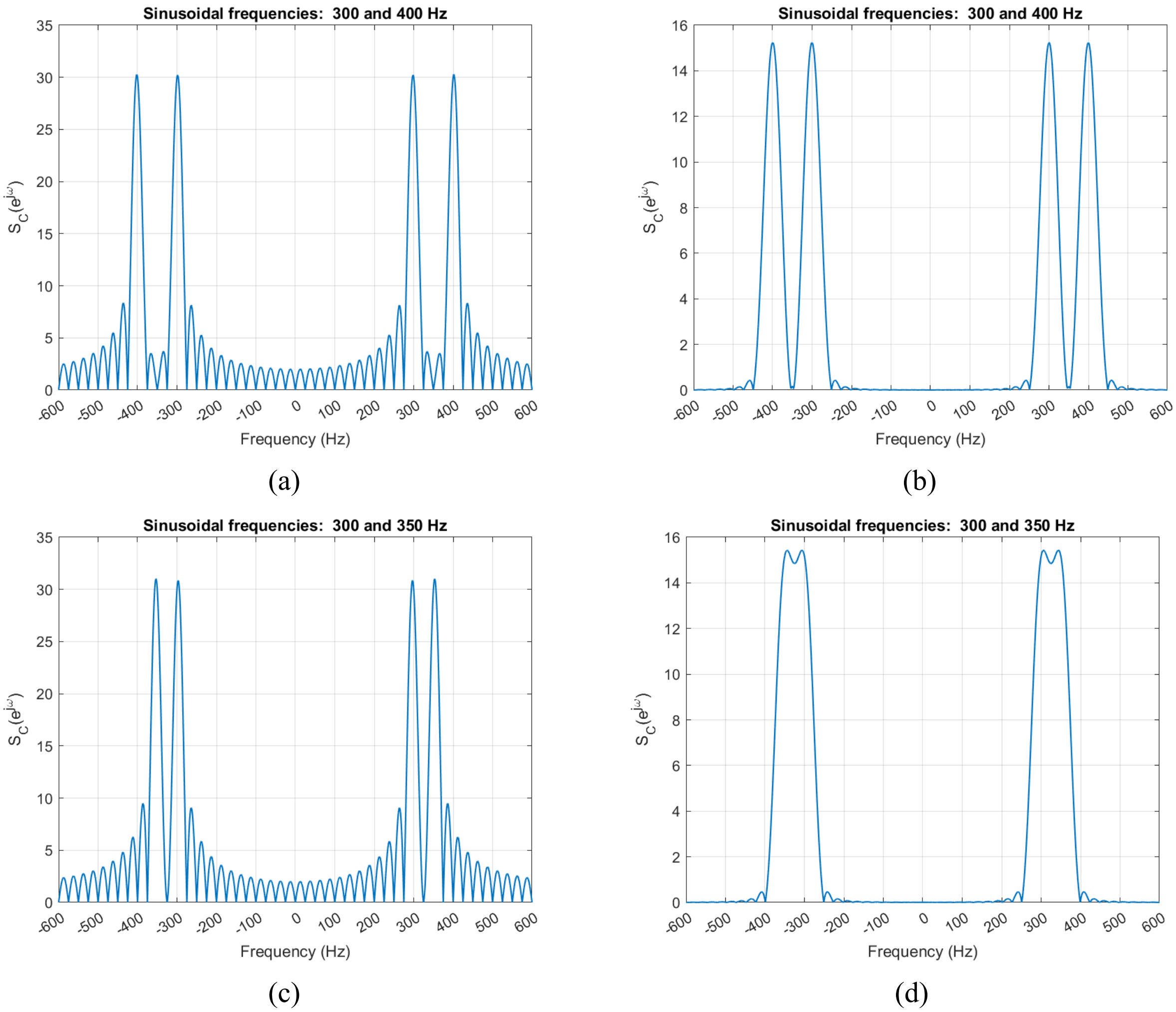


Figure 3: Rectangular versus Hanning Window. Figures (a) and (c) are rectangular windows. Figures (b) and (d) are Hanning windows. 8192-pt DFT and window length $L = 60$.

Experiment with rectangular and Hanning windows

The ML width of a Hanning window is $\Delta\Omega_{ML} = \frac{8\pi}{L} f_s$. Equivalently, the ML width is $\Delta f_{ML} = \frac{4}{L} f_s = \frac{4}{60}(1500) = 100\,\text{Hz}$. Therefore, the ML of the Hanning window is twice of the rectangular window. Between Fig. 3a and Fig. 3b, Hanning window attenuates the side lobes more than the rectangular window. However, the two spectra Fig. 3b are at the ”edges” of the resolution of the spectral analyzer before significant leakage occurs. In Fig. 3c, the two frequencies content f_0 and f_1 are still distinguishable due to the smaller side lobes of the rectangular window. Comparing to Fig. 3d, the leakage is significant because of the wider ML of the Hanning window, and the two frequencies contents are no longer distinguishable except for their peaks.

Experiment with FFT size

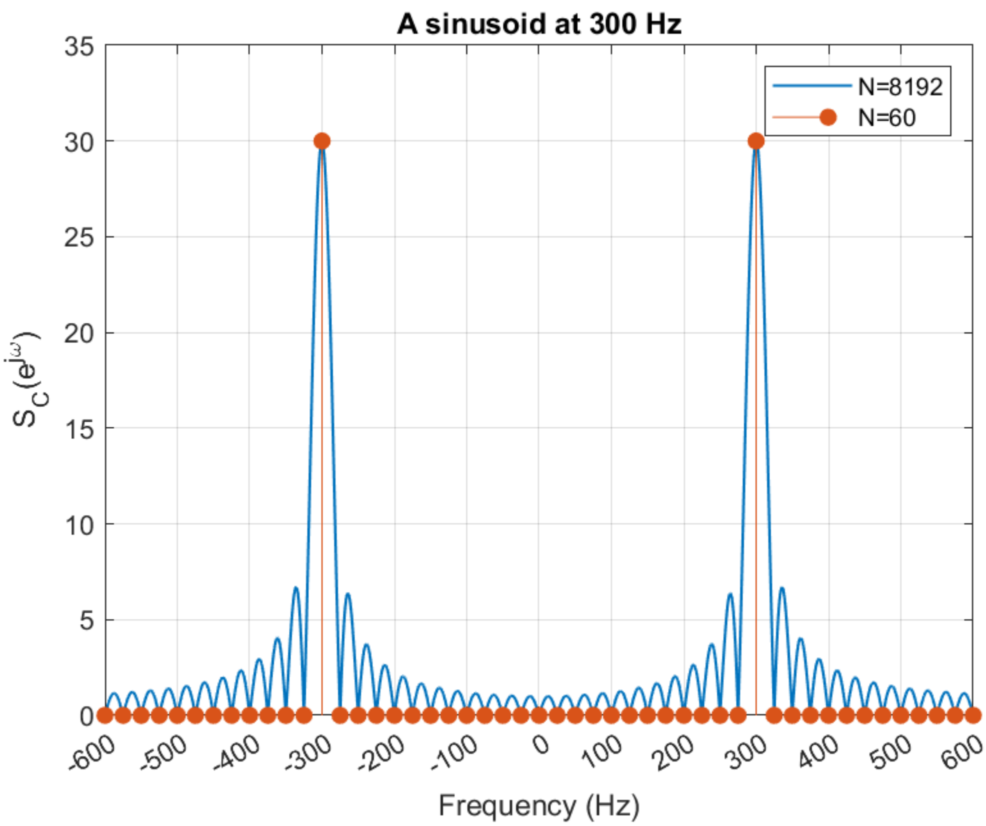


Figure 4: Spectrum of a 300 Hz cosine computed with different FFT sizes

The FFT size must be at least the same length to the length of the signal. In Fig. 4, the FFT size is the same to the window length $L = 60$. The frequency samples in red produces an illusion of a clean spectrum where the rectangular window attenuates all other frequency contents perfectly except at the frequency of interest $f_0 = 300$ Hz. The blue curve is a more densely sampled spectrum where linear interpolation produces a similar spectrum to the continuously frequency ω. Clearly, with only the red samples, we may have got the wrong impression about the spectrum in blue.

Part 1c)

The task in this part is to change the the amplitude $A_0 = 1,\ A_1 = 0.25$ and repeat the experiments in part 1b.

Experiment with different value for f_1

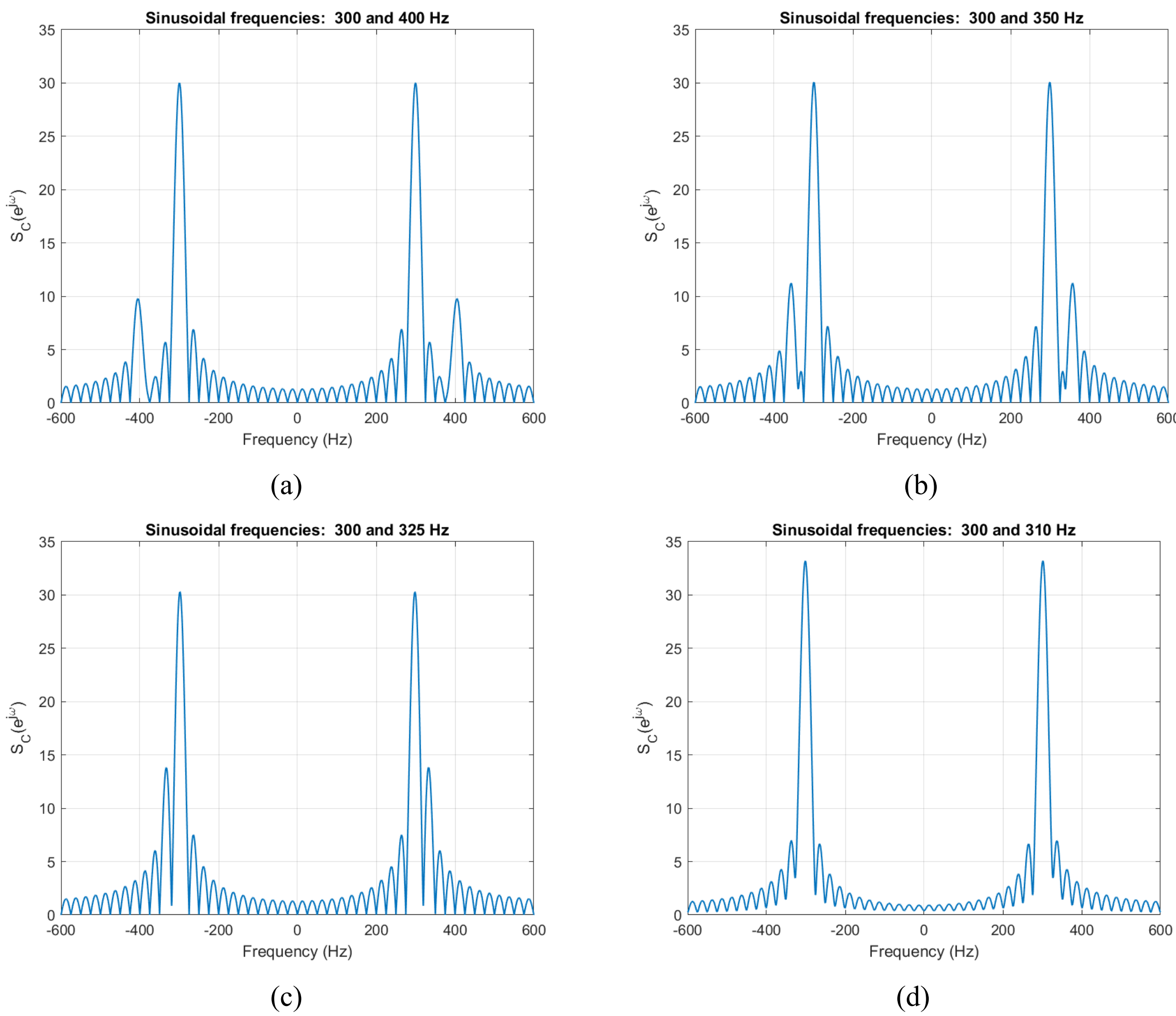


Figure 5: Modified amplitudes: $A_0 = 1,\ A_1 = 0.25$. Decreasing f_1 until the two frequencies are indistinguishable. 8192-pt DFT and window length $L = 60$.

Fig. 5a shows the relative spectral height at $300\,\text{Hz}$ to be 300, and the spectral height at $400\,\text{Hz}$ to be 10. Recall that in Fig. 2c, the two spectral components are distinguishable at the edge of resolution $25\,\text{Hz}$. However,Fig. 5a show a significant leakage from $300\,\text{Hz}$, with $A_0 = 1$, to spectral content at $325\,\text{Hz}$, with $A_1 = 0.25$, where the height at f_1 is now 15 instead of 10.

Experiment with rectangular and Hanning windows

The ML width of a Hanning window is two times wider than of a rectangular window. The two spectral contents are distinguishable in Fig. 6c with rectangular window. However, with Hanning window in Fig. 6d, the leakage is too significant that the two spectral contents are no longer distinguishable.

Experiment with FFT size

Fig. 7 shows that $N = 60$ the frequencies are sampled at the zero crossings giving an illusion of a clean spectrum. Moreover, the peak of spectral content at $350\,\text{Hz}$ falls in between the equidistant samples. That is, with only the red samples, we might have read the wrong frequency for $350\,\text{Hz}$.

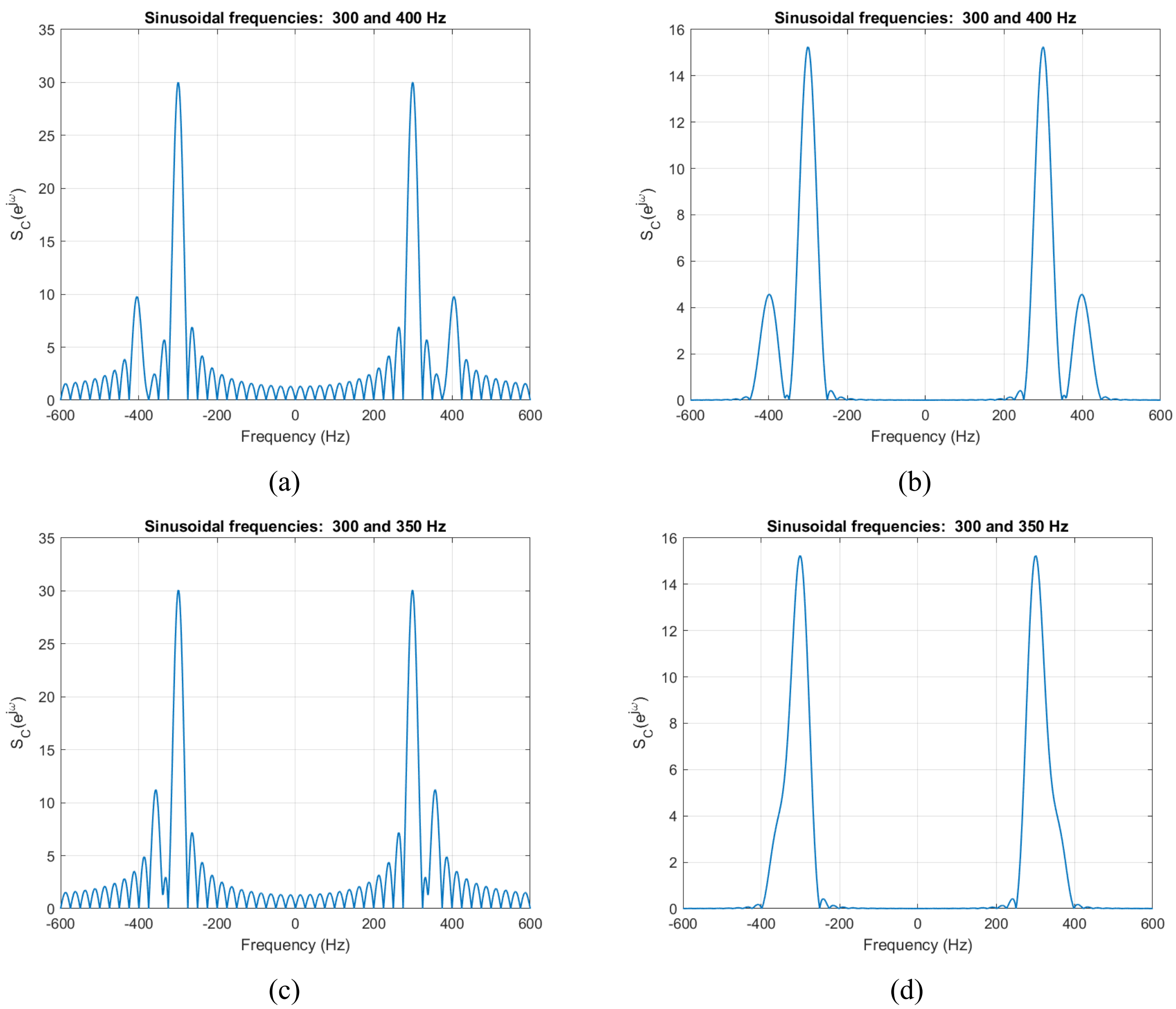


Figure 6: Rectangular versus Hanning Window. Figures (a) and (c) are rectangular windows. Figures (b) and (d) are Hanning windows. 8192-pt DFT and window length $L = 60$.

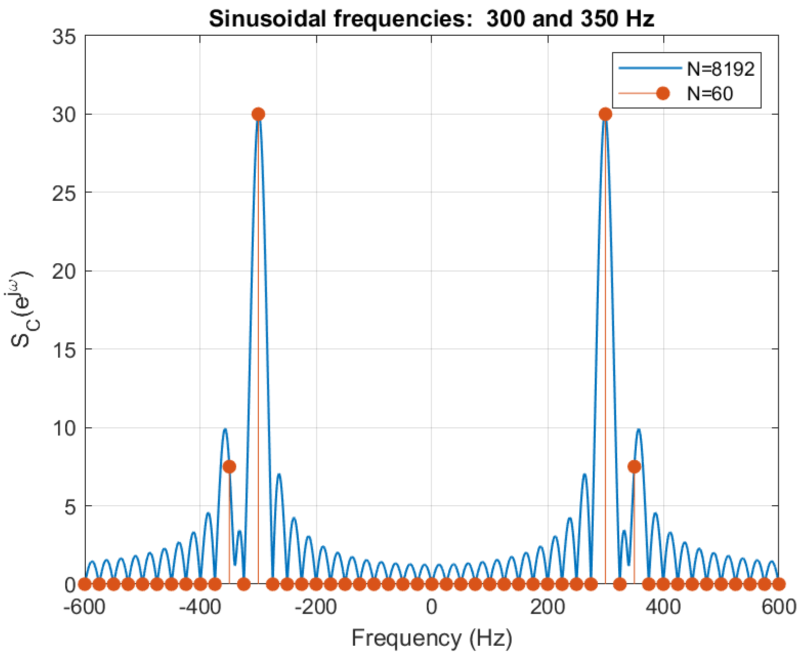
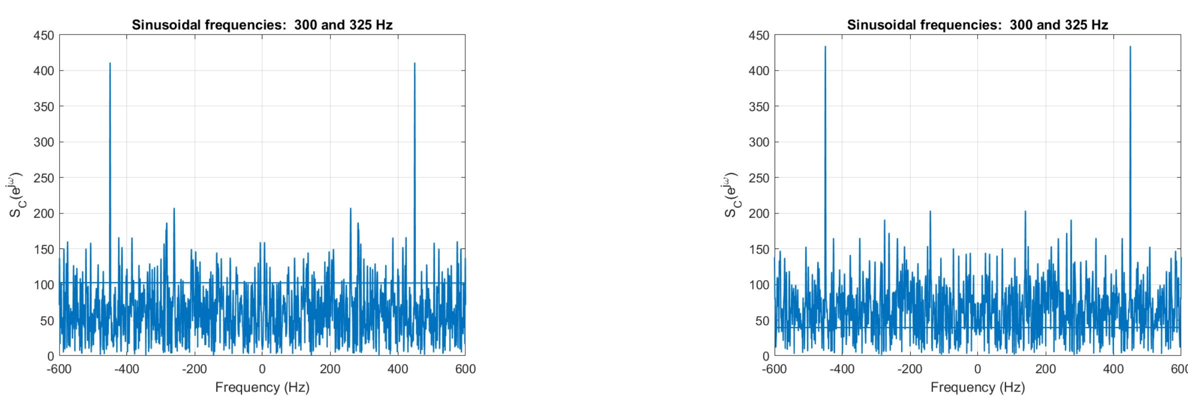


Figure 7: Testing the sum of two consines function

2 Averaging over multiple segments to reduce variance

Part 2a)

Fig. 8 shows that the spectral content is no longer clear if contains sine waves for 300 Hz and 325 Hz. The effect of their amplitude $A_0 = 1,\ A_1 = 0.25$ is not readable from the power spectrum.



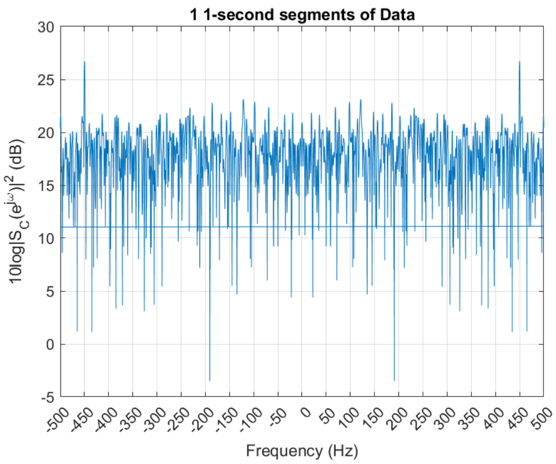
(a)

(b)

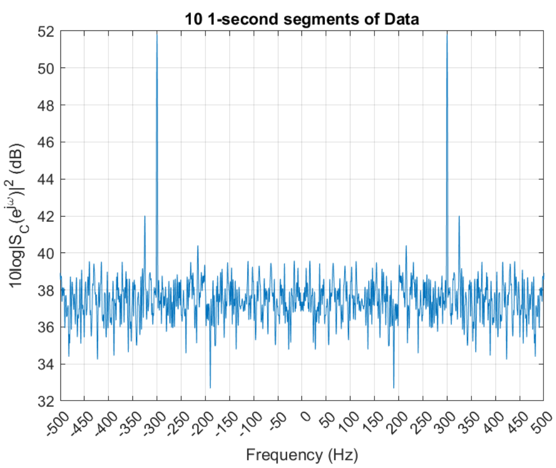
Figure 8: Sum of two cosines with Gaussian random noise. Experiments are run two times.

Part 2c) and d)

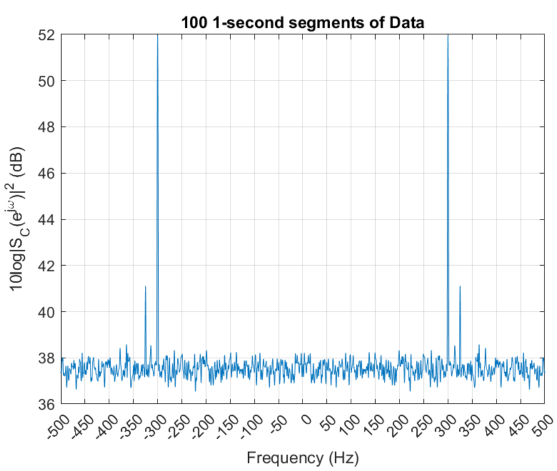
The more segments in averaging, the lower the variance of the noise. Without knowing ahead that the sinusoidal waves contain $300\,\mathrm{Hz}$ and $325\,\mathrm{Hz}$, Fig. 9c is the preferred power spectrum to look at to determine the frequencies as the noise are flat and the frequencies content of the sinusoids are impulse like. The effect of their amplitudes $A_0 = 1,\ A_1 = 0.25$ is observable on Fig. 9c.



(a)



(b)



(c)

Figure 9: Figures (a)-(c) are organized in an increasing order of the number of segments in averaging the power spectra. Applied a 50 percent overlapping Hanning windows of length 1 second and $f_s = 1500\,\text{Hz}$ (or 1500-sample window segment)

3 Analysis of simulated data

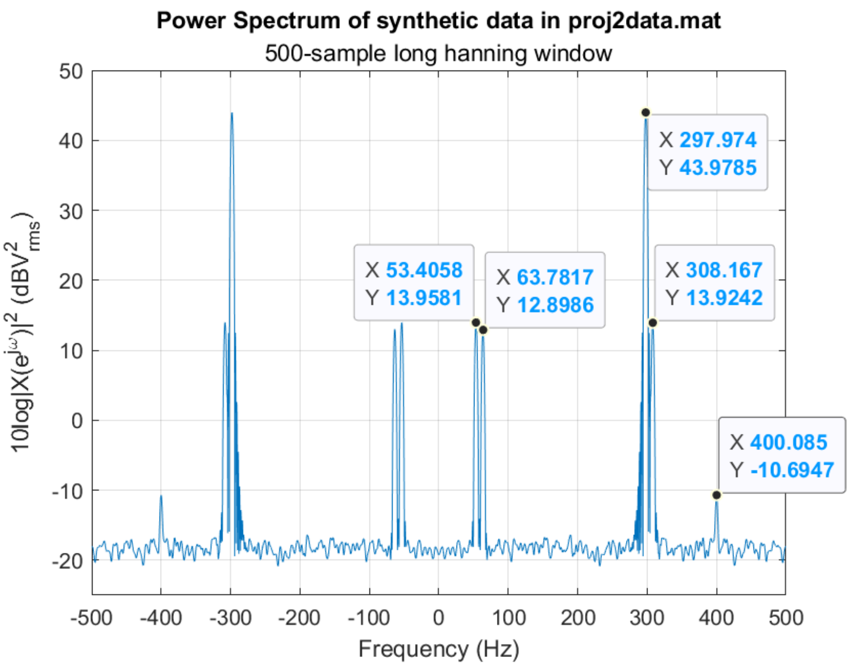


Figure 10: Analyze synthetic data file with a resolution of 10 Hz and resolve sinusoids with powers that differ by 30dB

Table 1: Estimate RMS voltage of each sinusoids

Frequency (Hz)	**Vrms** (dBV_{rms}^2)	**Vrms** (V)
53	14	5.01
64	13	4.47
298	44	158.49
308	14	5.01
100	-10	0.32

Let A be the power spectral heights reading from Fig. 10, and organized in the second column of 1. The RMS voltage is calculated and stored in the third column of 1 as follow:

$$V_{RMS} = 10^{A/20}$$

Hanning widow is chosen for the spectral analyzer because the window type has sidelodes of -31dB relative to the mainlobe. The spectral analyzer uses a 500-sample window. The ML width of a Hanning window is $\Delta\Omega_{ML} = \frac{8\pi}{L} f_s$. Equivalently, the ML width is $\Delta f_{ML} = \frac{4}{L} f_s =$

$\frac{4}{500}(1000) = 2\,\text{Hz}$, which satisfies the $10\,\text{Hz}$ resolution requirement. Experiment is carried out again with 700-sample Blackman window in Fig. 11.

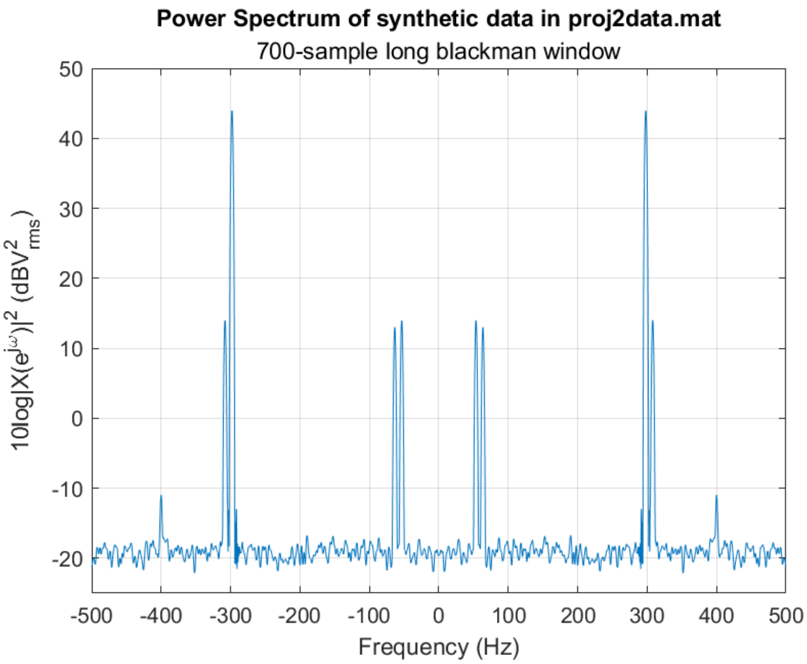


Figure 11: Another set of parameter used to verified the solution is correct

4 Analysis of real data

Fig. 12 shows the spectral analysis results for real data SWellEx-96 experiment. From the SWellEx website, the spectral analyzer must have at least a 2 Hz resolution. Choosing a 4000-sample window yields a $\Delta f_{ML} = \frac{2}{L} f_s = \frac{2}{4000}(1500) = 0.75\,\text{Hz}$ resolution. The choice window is rectangular because this type of window has the smallest ML, which prevents leakage from large spectral content to smaller neighboring ones. Using a 16384-pt FFT is a enough samples to have a clear pictures of spectrum.

With the given window length, 50 percent overlap, and the data set is known to have 1 350 000 samples, the number blocks is calculated to be 769 using the algorithm provided in avgspec function.

I learned how to justified for parameters used in function avgspec, or spectrogram in MATLAB. The window type and length shall affect the resolution of the spectral analyzer, while the leakage is mainly affected by the window type. I find the website SWellEx to be helpful when the expected frequencies are given, so that I have an idea what frequencies to look for and what is the required resolution.

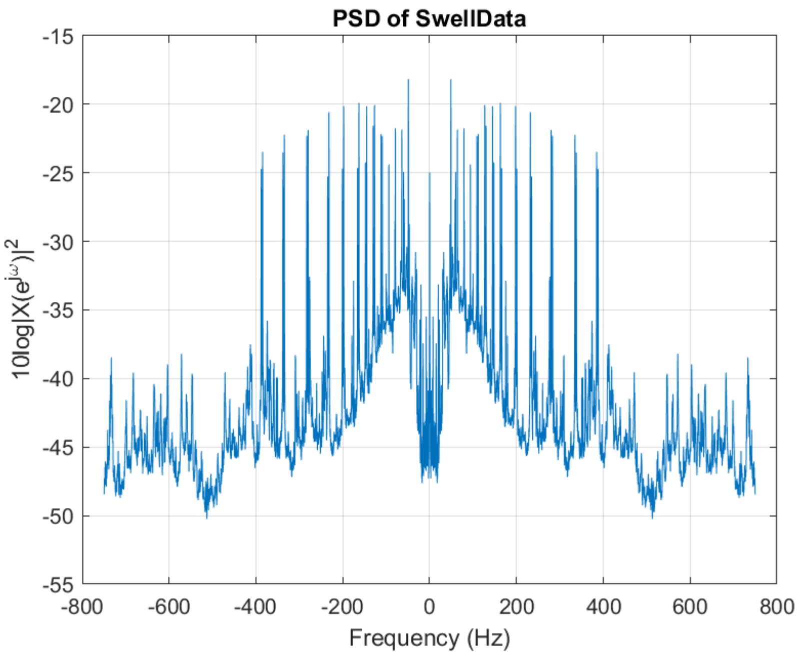


Figure 12: Spectral analyzing SWellEx-96 experiment using 4000-sample rectangular window, and 16384-point FFT

Fig. 13 to Fig. 16 are zoom-in version of Fig. 12 to read the frequencies contained in SWellEx data. Here is the list of the frequencies read, in Hertz: 49, 64, 79, 94, 109, 112, 127, 130, 145, 148, 163, 166, 198, 201, 232, 235, 280, 283, 335, 338, 385, 388.

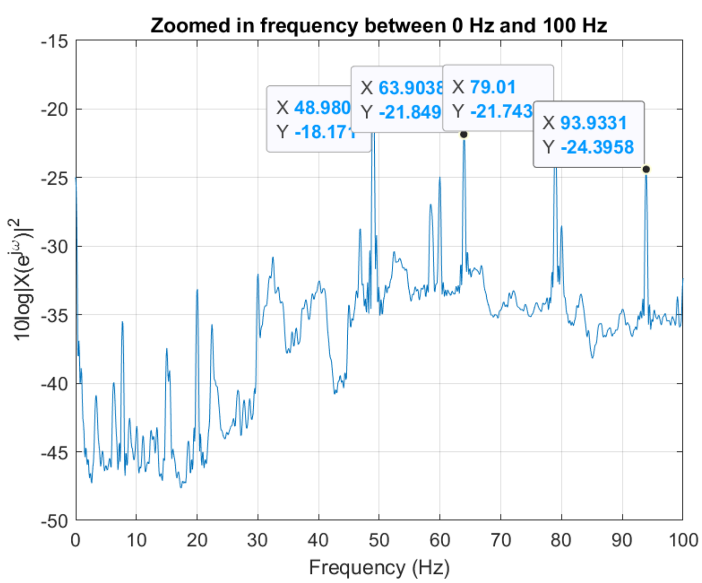


Figure 13: Zoom in to read spectral content

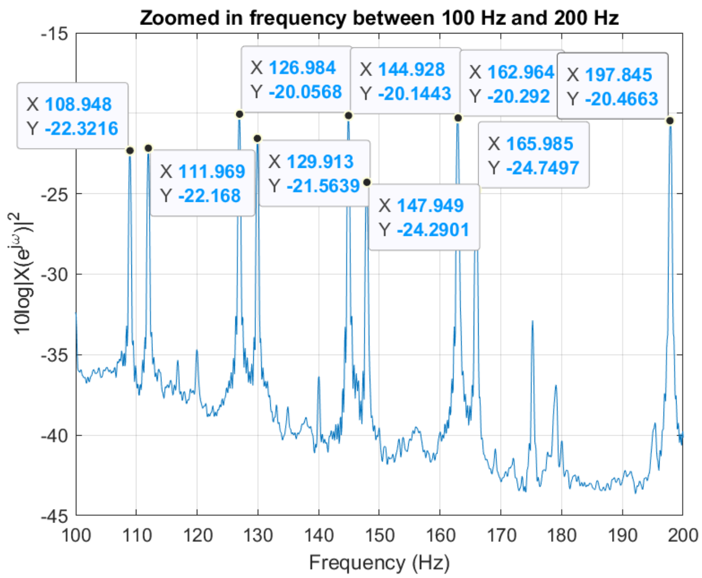


Figure 14: Zoom in to read spectral content

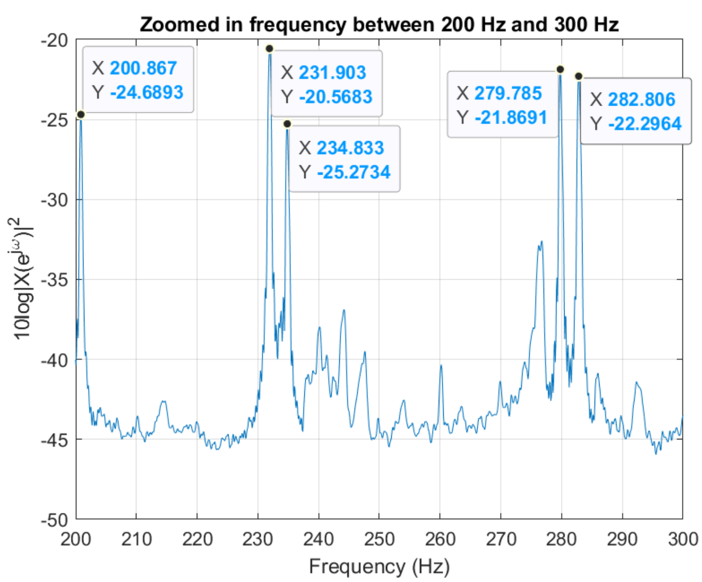


Figure 15: Zoom in to read spectral content

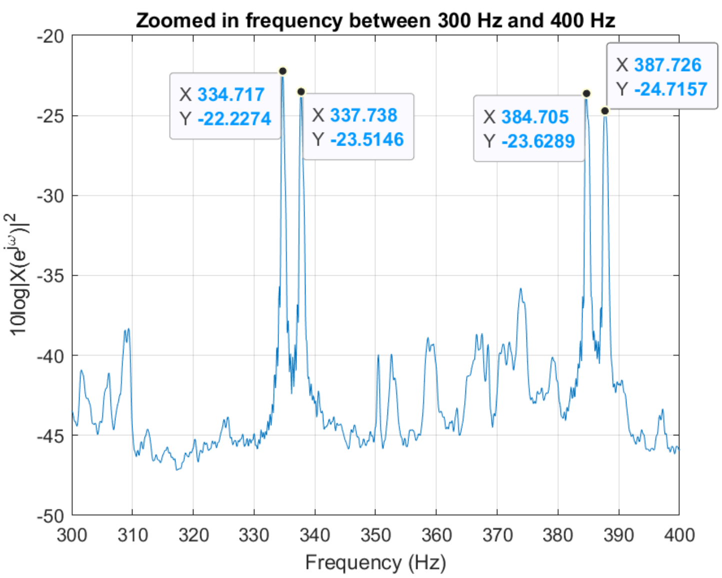


Figure 16: Zoom in to read spectral content