Automating Modeling and Simulation of Dynamic and Control Systems Using the Bond Graph Method

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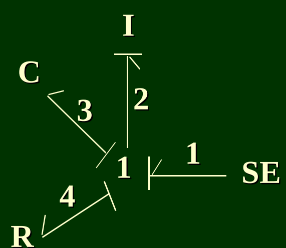
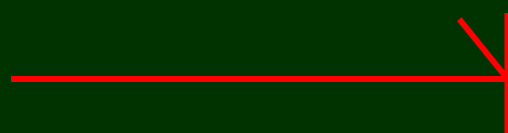
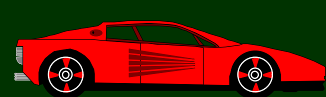


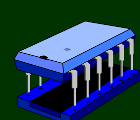


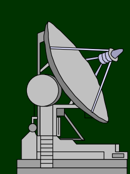
Dynamic Systems

- Electrical
- Mechanical
- Hydraulic
- Thermal

- Examples:
  - Moving car
  - Electric circuits
  - Telescope positioning system







Computer Aided Modeling and Design of Dynamic Systems
Basic Concepts

Step 1. Develop an engineering model
Step 2. Write differential equations
Step 3. Determine a solution
Step 4. Write a program

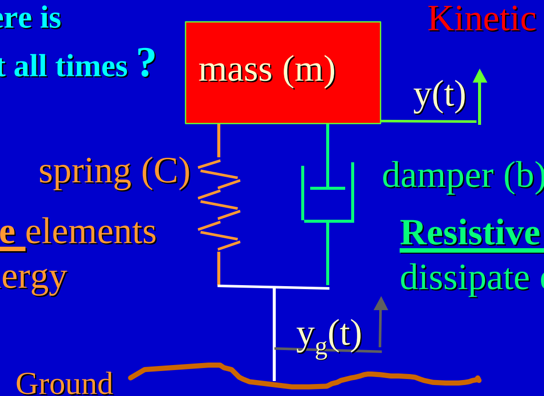
Physical System → **Schematic Model** → **Differential Equations** → **Block Diagram** or **Bond Graph** → **Simulation Language** → **Output Data tables & Graphs**

E.L.W.

1. Develop an engineering model

Where is the mass at all times ?

Inertia elements
Kinetic energy

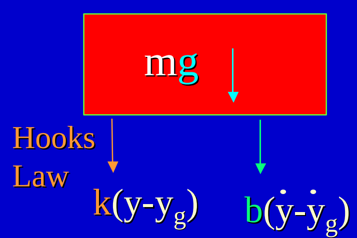


Compliance elements
Potential energy

Resistive elements
dissipate energy

$$-k(y-y_g)-b(\dot{y}-\dot{y}_g)-mg=m\ddot{y}$$

2. Write differential equations



$$\Sigma F_y = m\ddot{y} = m\frac{d^2y}{dt^2}$$

$$\Sigma F_y = \frac{d}{dt}\left(m\frac{dy}{dt}\right)$$

$$-ky - b(\dot{y}-\dot{y}_g) - mg = m\ddot{y}$$

$$m\ddot{y} + b(\dot{y}-\dot{y}_g) + k(y-y_g) = mg$$

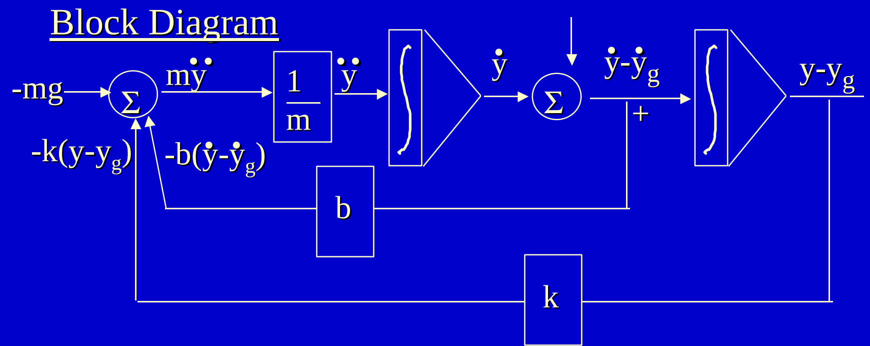
(mass)	(damper)	(spring)	External forces
Inertia	**Resistive**	**Compliance**	
Kinetic energy	dissipate energy	Potential energy	

3. Determine the solution

Options

- **Analytical**
- **Block diagram**
- **Bond graph model**
- **Write a program**
- **Use simulation tools**
- **Frequency domain (Laplace Transforms)**

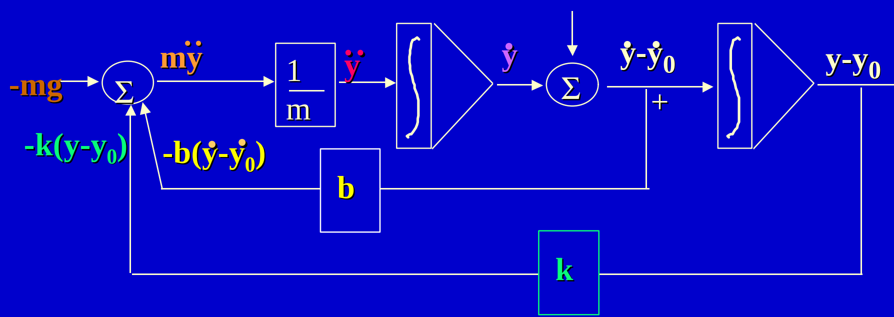
Block Diagram



A block diagram represents the dynamics of the system and describes program statements in single instructions.

4. Write a Program

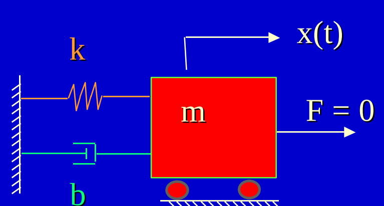
Options
- Your own
- Simulation Language Input



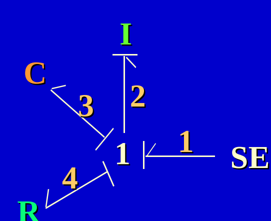
MYDD = - M*Y - K*Y - B*(YD-Y0)
YD = (1/M)* INTEG(MYDD,MYDDIN)
DIFF = YD - Y0D
YMY0 = INTEG (DIFF, DIFFIN)
Y = YMY0 + Y0

} For Simulation Language (no logical sort)

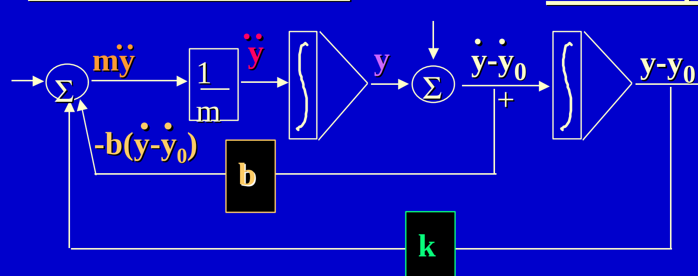
EQUIVALENT REPRESENTATIONS



Physical representation

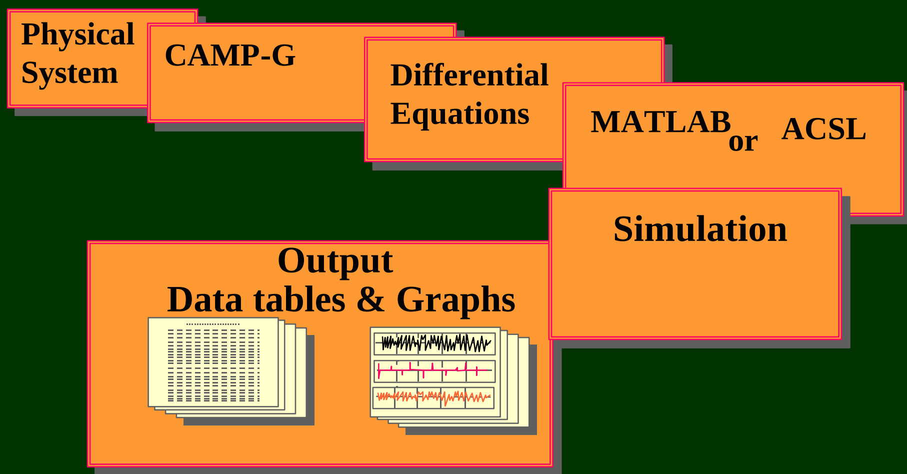


Bond Graph



Block Diagram

Bond Graphs and Simulation



E.L.W.

Basic Bond Graph Concepts

Word Bond Graphs, basic elements single and multiport devices

BOND GRAPHS and PHYSICAL VARIABLES

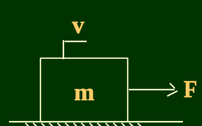
Power Flow Concept

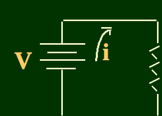
Causality Concept

A imposes effort on B,
B responds with a flow

B imposes effort on A,
A responds with a flow

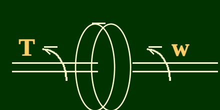
System variables

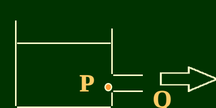




Power = F x v

Power = V x i





Power = T x w

Power =P x Q

Power = effort x flow

Effort & flow are power variables

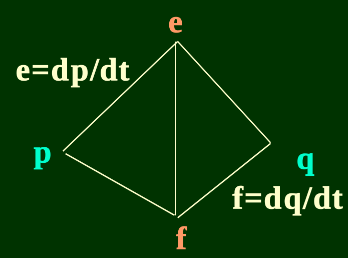
- **Efforts ....**
  - **Force (F)** **Newtons**
  - **Voltage (V)** **Volts V**
  - **Torque (T)** **N-meters**
  - **Pressure (P)** **N /m^2**

- **Flows....**
  - **Velocity (V)** **m/s**
  - **Current (i)** **Amps**
  - **Ang. velocity(w)** **rad/s**
  - **Volume flow (Q)** **m^3**

- ***power = effort (e) x flow (f)***

System variables

- Generalized power and energy variables have the following relations:
- **f = dq / dt** **q is a generalized displacement**
- **e = dp / dt** **p is a generalized momentum.**
- A state tetrahedron explains these relations

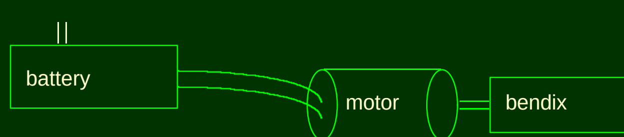


Physical Systems Variable Types

VARIABLE	MECHANICAL TRANSLATION	MECHANICAL ROTATION	ELECTRICAL	HYDRAULIC
Effort	Force (F) (Newtons N)	Torque (T) (N-m)	Voltage (Volts V)	Pressure (P) (N/m^2)
Flow	Velocity (v) (m/s)	Angular velocity (w) (rad/s)	Current (i) (Amperes A)	Volume flow (Q) (m^3/s)
Displacement	Displacement (x) (m)	Angle (rad)	Charge (q) (A-s)	Volume (m^3)
Momentum	Momentum (N-s)	Angular momentum (N-m-s)	Flux linkage (V-s)	Pressure momentum ($N\text{-}s/m^2$)

Word bond graphs.

- **Word bond graph** : Simple representation of physical system; using words to imply a system component.
- Example: Car Starter/Solenoid

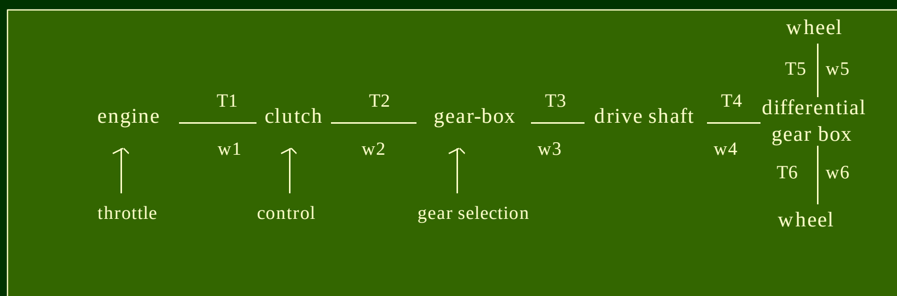


The word bond graph can be drawn as

battery $\underset{\text{current}}{\overset{\text{voltage}}{\rule{3em}{0.4pt}}}$ motor $\underset{\text{angular velocity}}{\overset{\text{torque}}{\rule{3em}{0.4pt}}}$ bendix

Word bond graphs

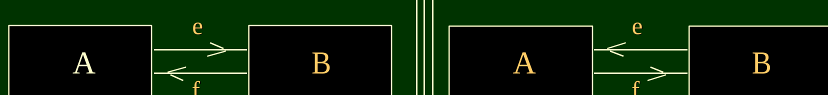
- **Car Model**: - Power from engine is fed to clutch
  - Transmitted to gear box. Selects gear
  - Power flows to wheels via a differential gear box and drive shaft.



Efforts: Torques (T1,T2,T3,T4,T5,T6)
Flows: Velocities (w1,w2,w3,w4,w5,w6)

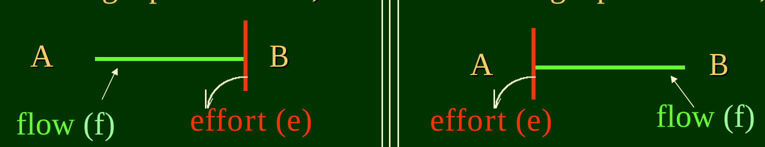
Concept of causality

- **Causality** : Indicates **WHO** causes **WHAT** to **WHOM**



in bond graph notation,

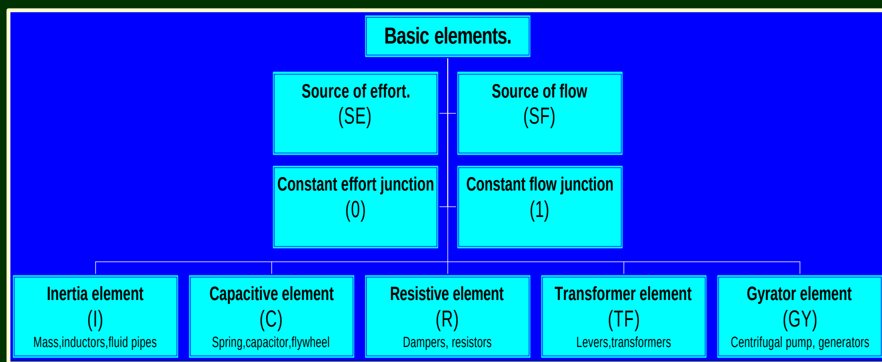
in bond graph notation,



If element **A** imposes an **effort** on element **B,** then element **B** **responds back with** a **flow** or vice-versa,

Basic elements

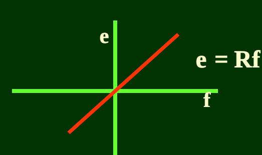
To convert a word bond graph to “complete bond” graph we need some basic elements.



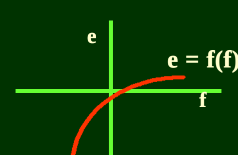
With these elements, bond graph models of dynamic systems can be created in any energy domain.

A resistive element (R).

- There is a static **relation** between **effort & flow.**
- Resistive elements are **idealization** of devices like, dampers, resistors, fluid carrying pipes.



Linear R



Non linear R

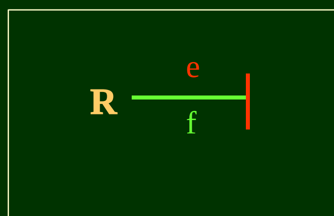
Units of R

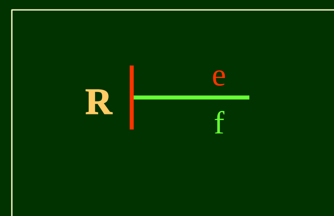
Mech. translation	Mech rotation	Electrical	Hydraulic
N-s/m	N-m-s	V/A (Ohms)	N-s/ m^2

Resistive element (R)

Causality considerations :

- A resistive element takes either form





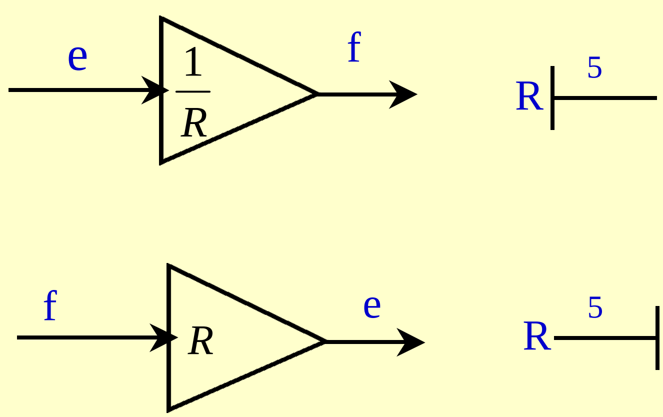
Relation :e=g (f)

Relation :f=g^{-1}(e)

$$\boxed{e = Rf}$$

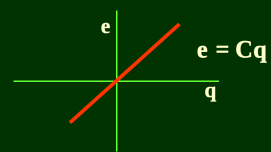
$$\boxed{f = \frac{1}{R}e}$$

R element

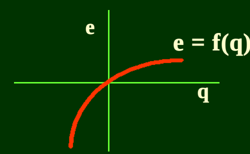


Capacitive element (C).

- In a capacitive element a static relation exists between effort & displacement.
- These devices store or dissipate energy without loss.
- Capacitive elements are idealization of devices like, springs, capacitors, accumulators.



Linear C



Non linear C

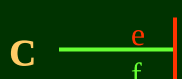
Units of C

Mech. translation	Mech rotation	Electrical	Hydraulic
N / m	N-m / rad	farads	N / m^5

Capacitive element (C)

Causality considerations :

- Integral causality

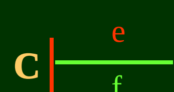


$$e = \frac{1}{C}\int f \; dt = \frac{1}{C} q$$

$$\therefore \frac{dq}{dt} = f$$

Preferred form for computational purposes

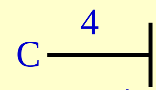
- Derivative causality



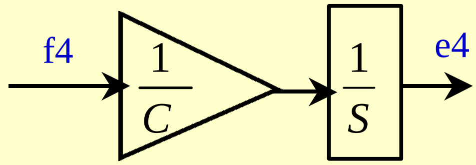
$$f = C\frac{de}{dt} = \frac{dq}{dt}$$

$$\therefore \frac{dq}{dt} = C\frac{de}{dt}$$

This is not preferred form for computational purposes

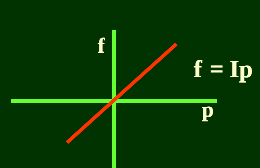


C element

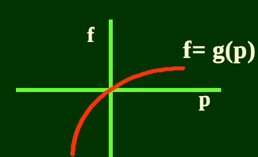


Inertia element (I).

- There is a static **relation** between **flow & momentum.**
- These devices store kinetic energy
- Inertia elements are to model inductance effects in electrical circuits, mass & inertia effects in mechanical & hydraulic systems.



Linear I



Non linear I

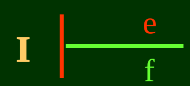
Units of I

Mech. translation	Mech rotation	Electrical	Hydraulic
N-s^2/m	N-m-s^2	V-s/A (Henrys)	N-s^2/m^5

Inertia element (I)

Causality considerations :

- Integral causality



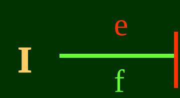
$$f = \frac{1}{I}\int e\,dt = \frac{1}{I}p$$

$$\therefore \frac{dp}{dt} = e$$

Impulse Momentum form

Preferred form for computational purposes.

- Derivative causality



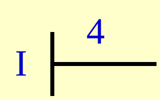
$$e = I\frac{df}{dt} = \frac{dp}{dt}$$

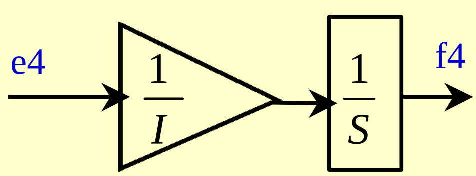
$$\therefore \frac{dp}{dt} = I\frac{df}{dt}$$

Newton's law form

This is not preferred form of causality for I element.

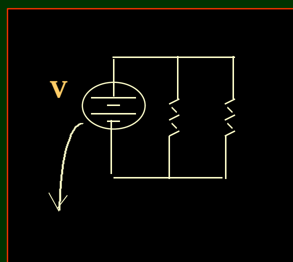
I element

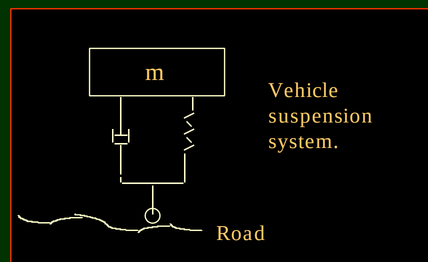




The source elements SE & SF)

- **An effort source** : System/element which maintains an input **effort.** SE's are voltage sources, forces, pressure.
- **A flow source** : System/device which maintains a an input **flow.** SF's are velocity sources, current, flow sources



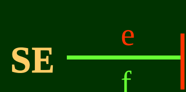


Effort Source **Flow source**

Source elements (SE & SF)

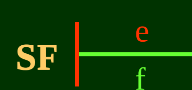
Causality considerations :

- Effort Source



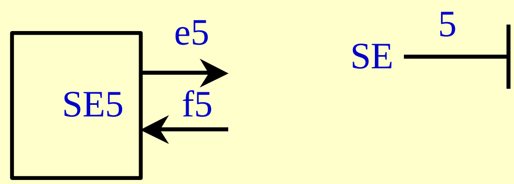
The effort source imposes an effort on the connected junction or element

- Flow source

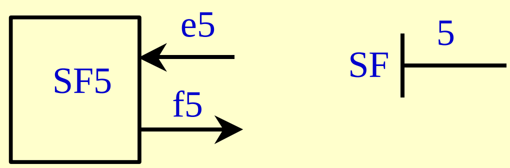


Flow source imposes a flow onto the system, connected junction or element.

Source SE element

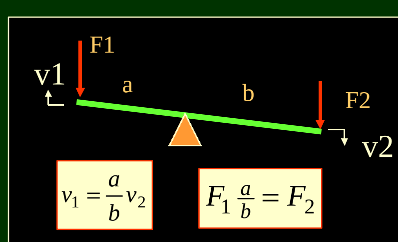


Source SF element

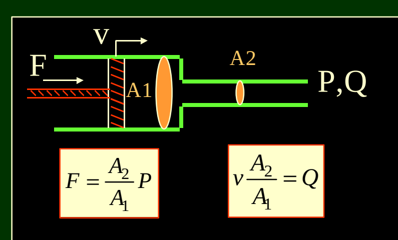


Transformer element (TF)

- Two port elements **altering magnitude** of either **flow or effort** are **transformer** elements.
- Transformers have static relation between input flow/effort & output flow/effort by means of a **transformer modulus.**



A lever

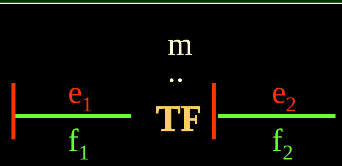


Hydraulic Ram

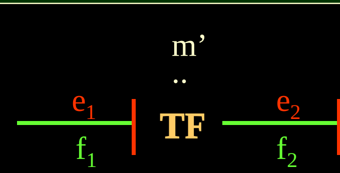
Ratio **(a/b)** & **(A2/A1)** is **transformer modulus.**
Other examples: gear set, electrical transformer, pulleys.

Transformer element (TF)

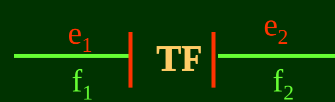
Causality considerations :



$$e_1 = m\, e_2$$
$$f_1\, m = f_2$$

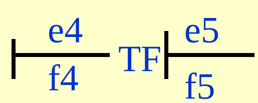


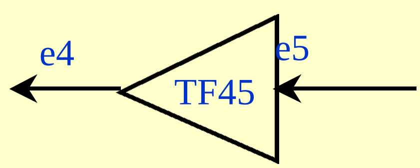
$$e_1\, m' = e_2$$
$$f_1 = m'\, f_2$$

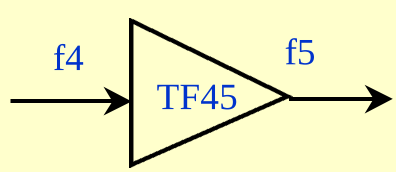


Incorrect causal form, not possible

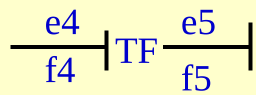
TF element

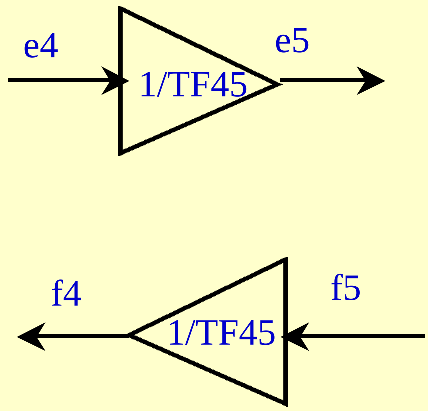






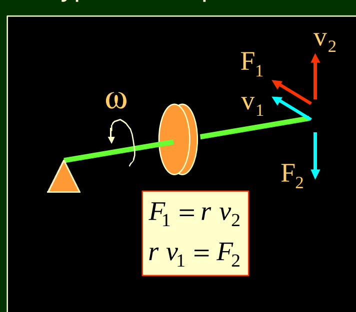
TF element





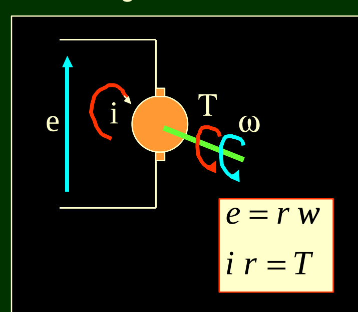
Gyrator element (GY)

- **Gyrators** : Two port elements which relate input effort to output flow or viceversa by means of a modulus.
- Typical examples: voice coil, electric motor, generator.



Gyro-scope

If the rotor spins rapidly, & a small F1 will yield a proportional velocity v2, & vice-versa

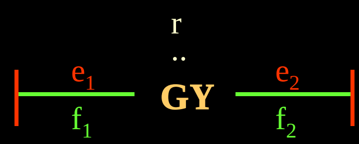


Motor

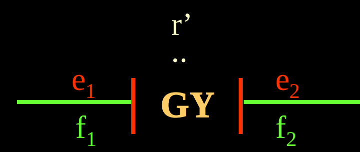
Angular velocity output is proportional to applied voltage e

Gyrator element (GY)

Causality considerations :



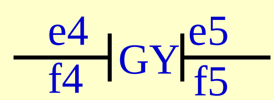
$$e_1 = r\ f_2$$
$$f_1\ r = e_2$$

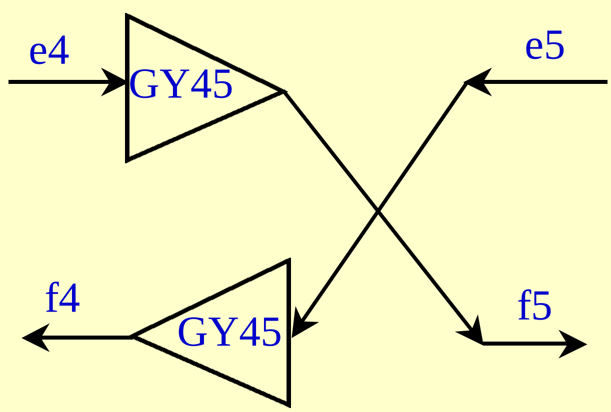


$$e_1\ r' = f_2$$
$$f_1 = r'\ e_2$$

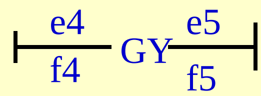
Incorrect causal form, not possible

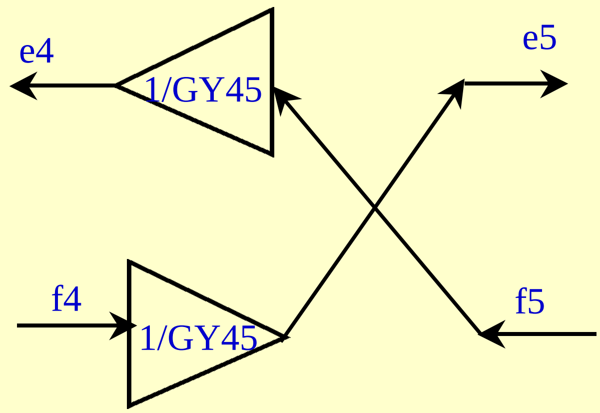
GY element



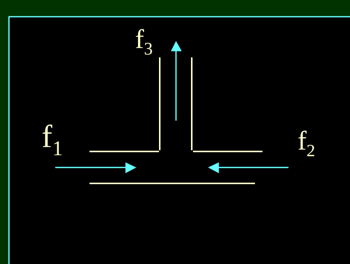


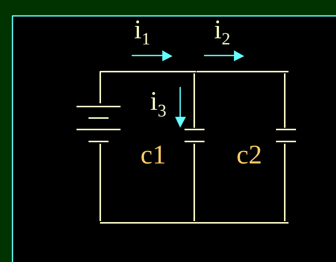
GY element





The (0) junction element





f1+f2=f3

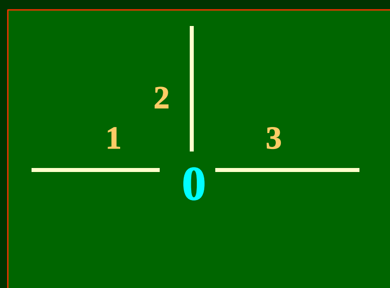
P1=P2=P3

i_1-i_2=i_3

e1=e2=e3

The (0) junction cont.

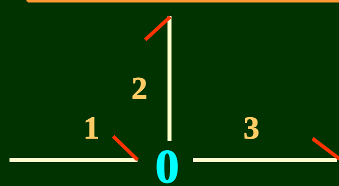
- (**0 junction**) : Is a **common effort** junction.
  - All efforts are equal
  - The sum of the flows equal zero.



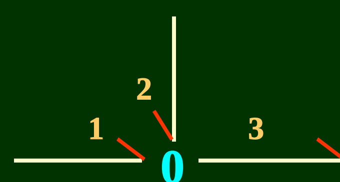
$$e_1 = e_2 = e_3$$
$$f_1 + f_2 + f_3 = 0$$

Summation signs will be determined by Power Flow.

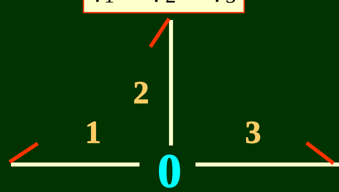
Power flow and the (0) Junction



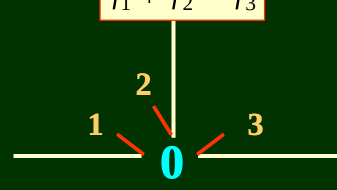
$$e_1 = e_2 = e_3$$
$$f_1 = f_2 + f_3$$



$$e_1 = e_2 = e_3$$
$$f_1 + f_2 = f_3$$

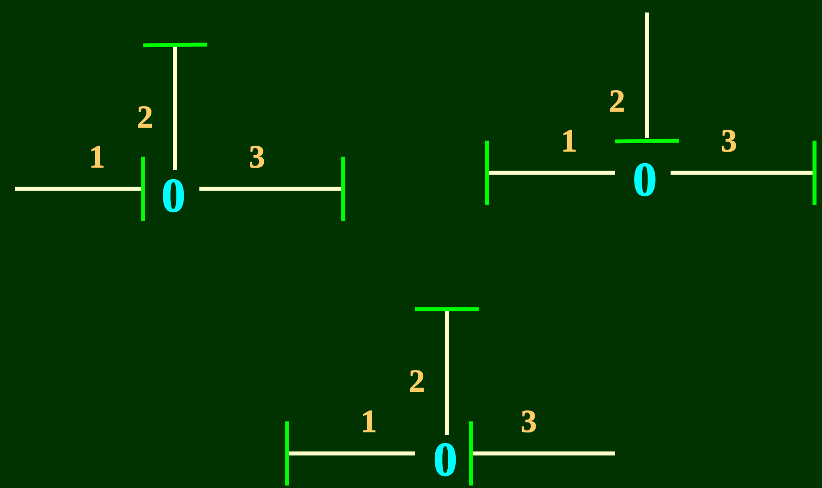


$$e_1 = e_2 = e_3$$
$$-f_1 - f_2 - f_3 = 0$$

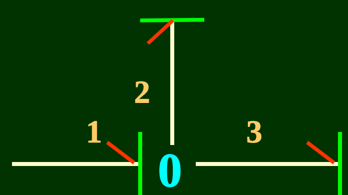


$$e_1 = e_2 = e_3$$
$$f_1 + f_2 + f_3 = 0$$

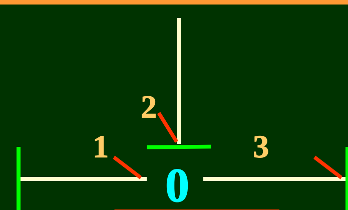
Causality and the (0) Junction



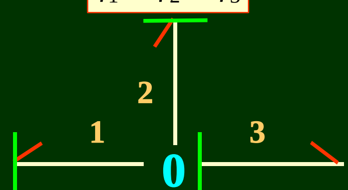
Power flow and Causality (0) Junction



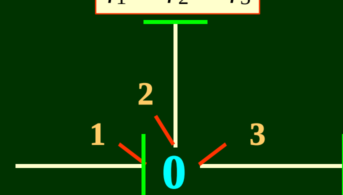
$$e_1 = e_2 = e_3$$
$$f_1 = f_2 + f_3$$



$$e_1 = e_2 = e_3$$
$$f_1 + f_2 = f_3$$



$$e_1 = e_2 = e_3$$
$$-f_1 - f_2 - f_3 = 0$$

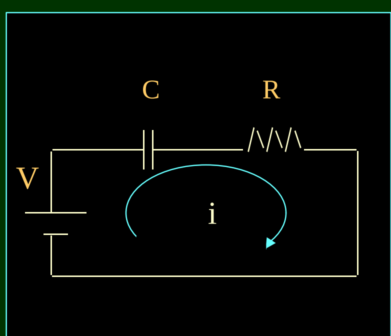


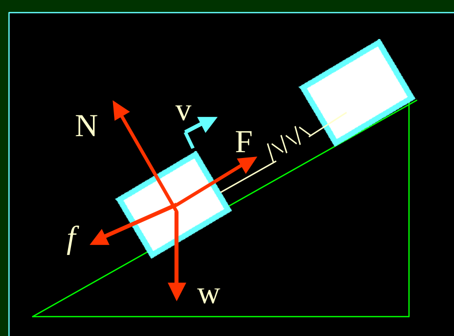
$$e_1 = e_2 = e_3$$
$$f_1 + f_2 + f_3 = 0$$

(0) Junction properties

- It is a **common effort** junction for all bonds attached
- All efforts are equal
- The sum of the flows equal zero.
- Power conserving, power in equals power out
- Only one causal mark determines the input effort and thus all other efforts will be outputs
- There can only be one bond and only one bond that sets the effort input
- Power flow half arrows determine how the flows will sum

The (1) junction element



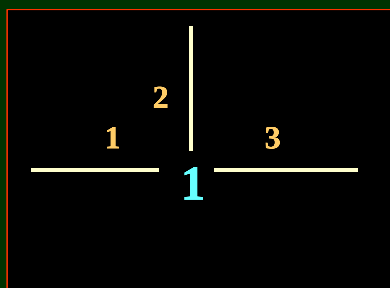


Current through C and R is the same. Summation of voltages

Velocity is common but sumation of forces must follow Newton's law

The (1) junction cont.

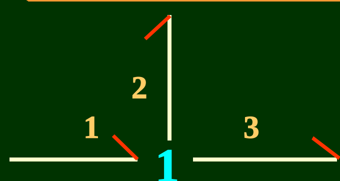
- (**1 junction**) : Is a **common flow** junction.
  - All flows are equal
  - The sum of the efforts equal zero.



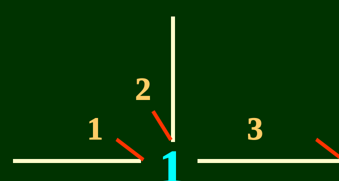
$$f_1 = f_2 = f_3$$
$$e_1 + e_2 + e_3 = 0$$

Summation determined by Power Flow.

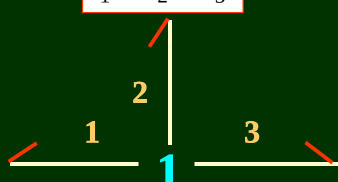
Power flow and the (1) Junction



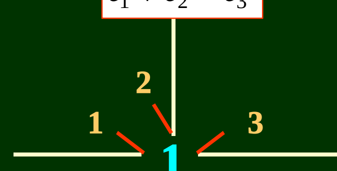
$$f_1 = f_2 = f_3$$
$$e_1 = e_2 + e_3$$



$$f_1 = f_2 = f_3$$
$$e_1 + e_2 = e_3$$

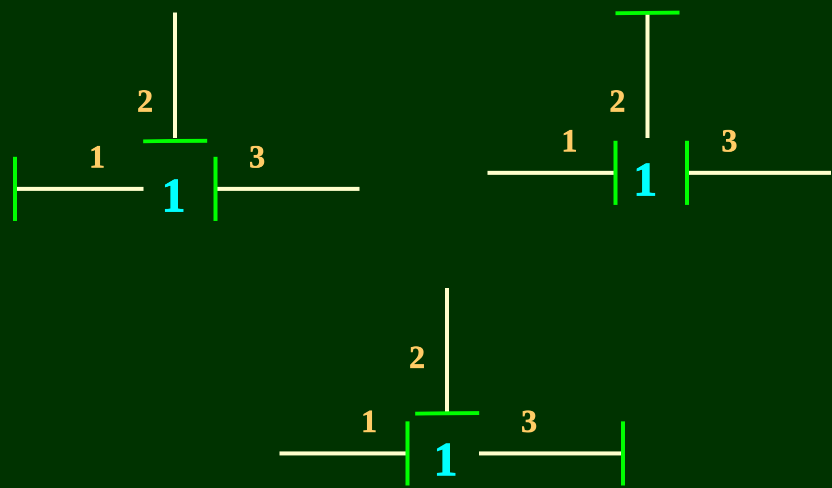


$$f_1 = f_2 = f_3$$
$$-e_1 - e_2 - e_3 = 0$$

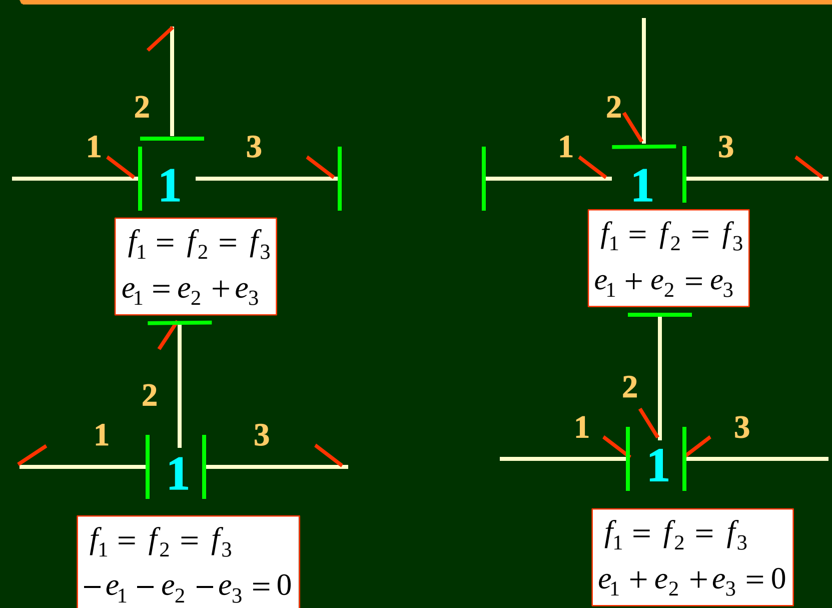


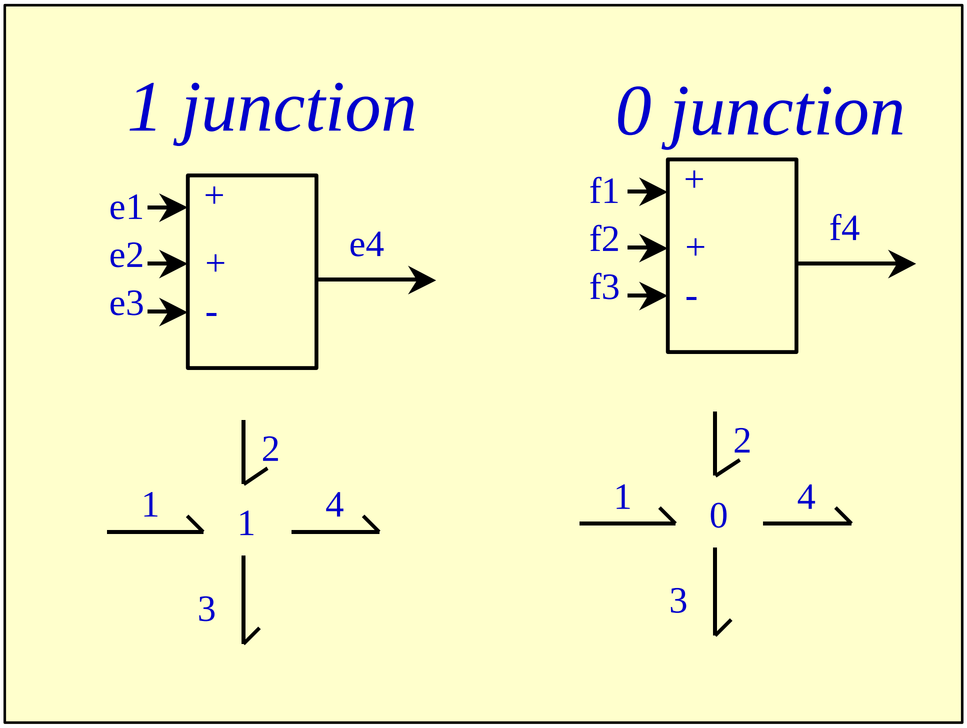
$$f_1 = f_2 = f_3$$
$$e_1 + e_2 + e_3 = 0$$

Causality and the (1) Junction



Power flow and Causality (1) Junction

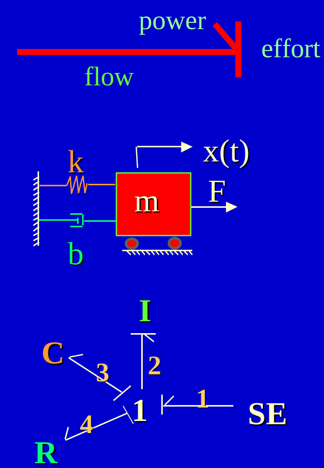




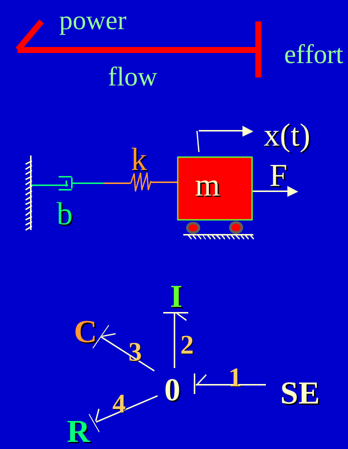
(1) Junction properties

- It is a **common flow** junction for all bonds attached
- All **flows** are equal
- The sum of the efforts equal zero.
- Power conserving, power in equals power out
- Only one causal mark determines the input flow and thus all other flows will be outputs
- There can only be one bond and only one bond that sets the flow input
- Power flow half arrows determine how the efforts will sum

FUNDAMENTALS OF BOND GRAPH MODELING

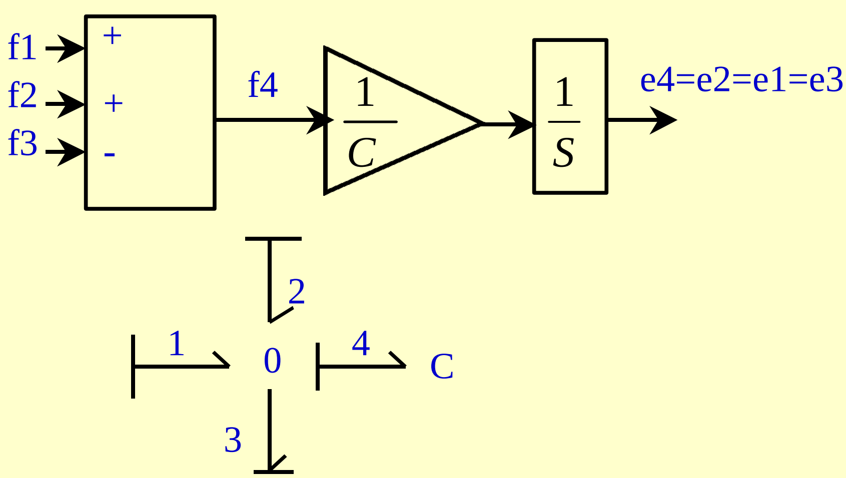


To Sum Efforts

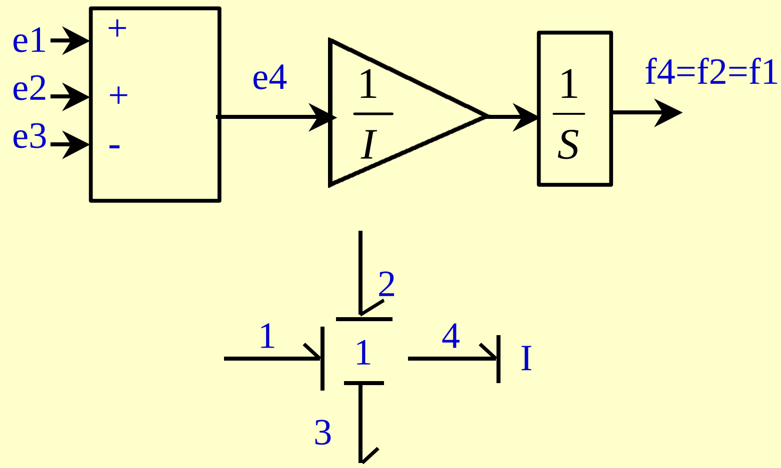


To Sum Flows

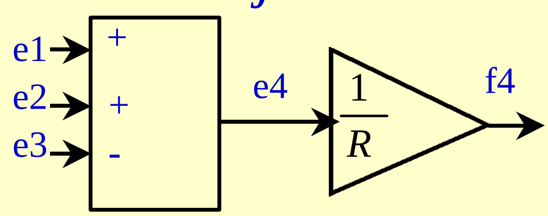
Causal bond(4)
0 junction-C element



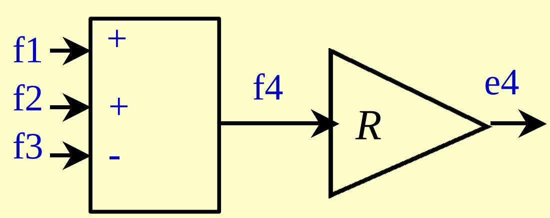
Causal bond 1 junction-I element



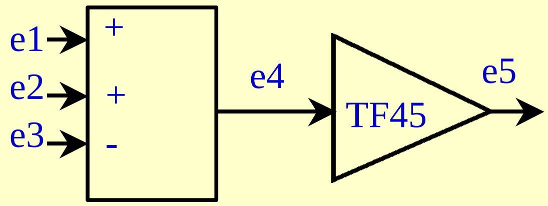
Causal bond 1 junction R element



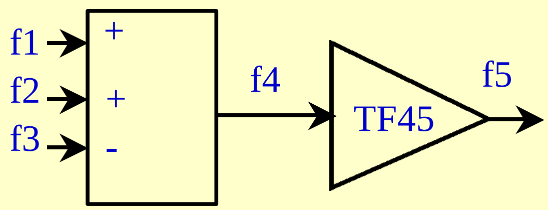
Causal bond 0 junction R element



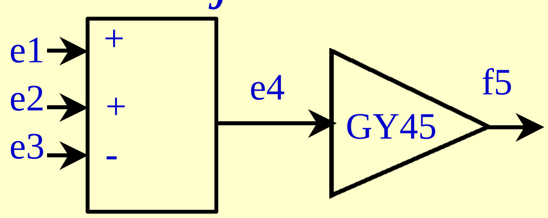
Causal bond 1 junction TF element



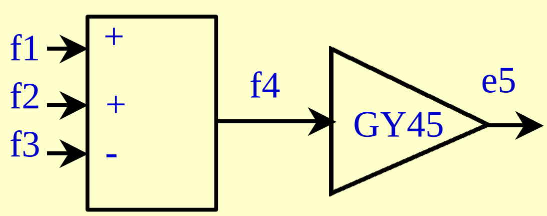
Causal bond 0 junction TF element



Causal bond 1 junction GY element



Causal bond 0 junction GY element



Causal forms.

Symbol	Implied meaning & variable relation.
C	Preferred integral causal form.... f= dq/dt
I	Preferred integral causal form.... e= dp/dt
C	Derivative causal form....q= integ (f)... not preferred
I	Derivative causal form....p= integ (e)... not preferred
1 TF 2	Preferred form for transformer element. (or opposite) f1=f2/m & e1=e2/m OR e1=me2 & f1=mf2
GY	Preferred form for gyrator element. (or opposite)