Design of Sliding mode and Backstepping Controllers for a Quadcopter

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***Abstract*—The quadcopter is a great platform for control systems research as it is highly nonlinear and under-actuated system. The nonlinear dynamic model of the quadcopter is formulated using the Newton-Euler method. The motion of the quadcopter can be divided into two subsystems; a rotational subsystem (attitude and heading) and a translational subsystem (altitude, x and y motion). In this paper two Non-linear Control strategies, sliding mode control (SMC) and Backstepping control (BSC) have been proposed. A SMC is a type of Variable Structure Control. It uses a high speed switching control law to force the state trajectories to follow a specified user defined surface in the states space and to maintain the state trajectories on this surface. BSC is a recursive control algorithm that works by designing intermediate control laws for some of the state variables. These state variables are called "virtual controls" for the system. These Controllers have been implemented on the Quadcopter through simulations using MATLAB/Simulink. The results have been compared with traditional PID controller.**

***Keywords*—Quadcopter, UAV, Nonlinear Control, Sliding Mode, Backstepping, PID.**

I. INTRODUCTION

Unmanned aerial vehicles (UAVs) are very useful when it has to perform a desired task in a dangerous and/or inaccessible environment where high human risk is involved. In recent days, UAVs are becoming very popular among the researcher and control engineers. The rotorcraft UAVs has several advantages over fixed wing UAVs, such as hovering at a particular place, vertical takeoff and landing and aggressive maneuvering. Due to this features rotorcraft UAVs are being used in several applications such as search and rescue operations, traffic monitoring, Wild fire suppression, Border surveillance, pipeline inspection and crop spraying in agriculture etc. Under the category of rotorcraft UAVs, Quadcopter have acquired much attention among researcher. Quadcopter is a multi-copter that is lifted and propelled by four rotors, each mounted in one end of a cross-like structure as shown in Figure 1. Each rotor consists of a propeller fitted to a separately powered Brushless DC motor. Quadcopter has 6 degree of freedom (three translational and three rotational) and only four actuators [1]. Hence quadcopter is an underactuated system and highly nonlinear in nature. Unlike a conventional helicopter, a quadcopter's rotor blade pitch angle need not be varied, which makes the quadcopter manufacturing and maintenance easier. Moreover, quadcopters have capacity to carry large payload due to the presence of four motors providing higher thrust. One drawback of quadcopter is more energy consumption due to presence of four motors which restrict the flying time of quadcopter. The main contribution of this work is to develop a Sliding mode and Backstepping controller to stabilize and regulate quadcopter orientation as well as its altitude and positions. Finally designed controller performance is compared with conventional PID controller using MATLAB simulations.

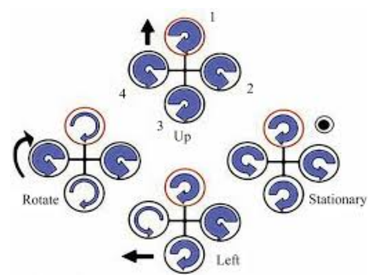


Fig. 1: Quadcopter motion description

A. Quadcopter Configuration

The quadcopter is a highly non-linear, six degree-of-freedom, multi-input-multi-output and under-actuated system. Quadcopter motion can be controlled by changing the speed of the rotors. A quadcopter has two sets of counter-rotating propellers, therefore neutralizing the effective aerodynamic drag. Vertical movement is controlled by simultaneously increasing or decreasing the thrust of all rotors. Yaw moment is created by proportionally varying the speeds of counter-rotating parts. Roll and pitch moments can be controlled by applying differential thrust forces on opposite rotors of the quadcopter. propellers (1,3) are rotating clockwise and propellers (2,4) are rotating in counter clockwise direction.Roll rotation coupled with lateral motion is achieved by Changing the 2 and 4 propeller's speed conversely. Pitch rotation and the corresponding lateral motion is achieved by changing 1 and 3 propeller's speed conversely. Yaw rotation is achieved by the difference in the counter-torque between each pair of propellers. The motions along x axis is achieved by pitch

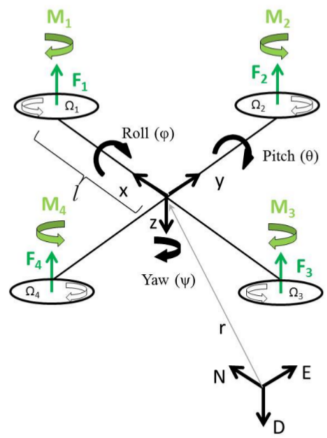


Fig. 2: Quadcopter frame system with a vehicle frame(x,y,z) and inertial frame(N,E,D)

rotation and motion along y axis is obtained by roll rotation of a quadcopter as shown in figure 2. This roll and pitch angles are known as tilt angle. Therefore control of the tilt angles and the motion of the quadcopter are closely related and estimation of the orientation (roll and pitch) is critical. Quadcopter motions are highly coupled. Reduction in one rotor speed results not only variations in the tilting angles but also a change in total yaw moment and thrust. Due to the coupled nature of the quadcopter, to create any motion all of the rotor speeds should be properly controlled. In [2] a PID based attitude and position controller was developed for quadcopter to stabilize its position and orientation angles. A simple path-following LQR controller was applied on a full dynamic model of the quadcopter in [3]. In [4] the backstepping based attitude controller for a quadcopter was developed to stabilize its attitude. A robust attitude controller was developed based on linear control and robust compensation in [5] and the sliding mode controller was developed to stabilize cascaded under actuated system in [6].

II. Quadcopter dynamic modelling

To design a good controller, an appropriate mathematical model, which includes the kinematics and dynamic equation of the quadcopter is required. The mathematical model of a quadcopter will be developed based on a Newton-Euler formalism with some assumptions listed below [7]:

- The quadcopter structure is rigid and symmetrical.
- The center of gravity of the quadcopter coincides with the vehicle frame origin.
- The propellers are rigid.
- Thrust force and drag torque are proportional to the square of propeller's speed.

To obtain the appropriate model of the quadcopter, one need to define the coordinate frames associated with the system. Figure 2 shows the Earth reference frame or inertial frame with N, E and D axis and the vehicle frame with x, y and z axis. The inertial frame is fixed on a specific place at ground level and the vehicle frame is at the centre of quadcopter body and axis parallel to inertial frame. Besides these two frames, there are three more coordinate frames for obtaining the adequate model i.e. yaw adjusted frame, pitch adjusted frame and body frame. Vehicle frame's yaw is adjusted to match the quadrotor's yaw to get the yaw-adjusted frame which is then pitch adjusted to get pitch-adjusted frame. Finally body frame is obtained by adjusting the roll of the pitch adjusted frame. the inertial to vehicle frame is just a simple translation. The transformation from vehicle to body frame is given by the following rotation matrix:

$$\begin{aligned} R_v^b(\phi,\theta,\psi) &= R_p^b(\phi)R_y^p(\theta)R_v^y(\psi) \\ &= \begin{pmatrix} c\theta c\psi & c\theta s\psi & -s\theta \\ s\phi s\theta c\psi - c\phi s\psi & s\phi s\theta s\psi + c\phi c\psi & s\phi c\theta \\ c\phi s\theta c\psi + s\phi s\psi & c\phi s\theta s\psi - s\phi c\psi & c\phi c\theta \end{pmatrix} \end{aligned} \tag{1}$$

where $s \triangleq \sin$ and $c \triangleq \cos$.

R_p^b is the transformation from the pitch-adjusted frame to body frame, R_y^p is the transformation from the yaw-adjusted frame to pitch-adjusted frame, R_v^y is the transformation from the vehicle frame to yaw-adjusted frame. Since, each of the quadcopter parameters; roll, pitch and yaw are measured relative to different frames, these transformations help us in interplay of the variables. (u, ϕ, p) are the linear velocity along the roll-axis direction, angle rotated and the angular velocity about the roll-axis. (v, θ, q) are the linear velocity along the pitch-axis direction, angle rotated and the angular velocity about the pitch-axis. (w, ψ, r) are the linear velocity along the yaw-axis direction, angle rotated and the angular velocity about the yaw-axis. (x, y, z) are the position of the quadcopter along N,E and D axis respectively in inertial frame.

A. Kinematics

The position derivative $(\dot{x}, \dot{y}, \dot{z})$ are inertial frame quantities and velocities (u, v, w) are the body frame quantities. They can be related through the transformation matrix as follows:

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{pmatrix} = \begin{pmatrix} c\theta c\psi & s\phi s\theta c\psi - c\phi s\psi & c\phi s\theta c\psi + s\phi s\psi \\ c\theta s\psi & s\phi s\theta s\psi + c\phi c\psi & c\phi s\theta s\psi - s\phi c\psi \\ -s\theta & s\phi c\theta & c\phi c\theta \end{pmatrix} \begin{pmatrix} u \\ v \\ w \end{pmatrix} \tag{2}$$

angular velocities (p, q, r) and angular rates $(\dot{\phi}, \dot{\theta}, \dot{\psi})$ are related as follows:

$$\begin{pmatrix} p \\ q \\ r \end{pmatrix} = \begin{pmatrix} 1 & 0 & -s\theta \\ 0 & c\phi & s\phi c\theta \\ 0 & -s\phi & c\phi c\theta \end{pmatrix} \begin{pmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{pmatrix} \tag{3}$$

B. Dynamics

By applying the Newton's second law to the translational motion, we get:

$$\mathbf{f} = m\frac{d\mathbf{v}}{dt} = m\left(\frac{d\mathbf{v}}{dt} + \omega_{\mathbf{b}} \times \mathbf{v}\right) \tag{4}$$

where $\mathbf{v} = (u, v, w)^T$ and $\omega_{\mathbf{b}} = (p, q, r)^T$.

Consider coriolis equation to evaluate time derivative of velocities in body frame. Substituting the relevant equations we get:

$$\begin{pmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{pmatrix} = \begin{pmatrix} rv - qw \\ pw - ru \\ qu - pv \end{pmatrix} + \frac{1}{m} \begin{pmatrix} f_x \\ f_y \\ f_z \end{pmatrix} \tag{5}$$

Assuming external disturbances are absent, we have;

$$\begin{pmatrix} f_x \\ f_y \\ f_z \end{pmatrix} = R_v^b \begin{pmatrix} 0 \\ 0 \\ mg \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ -f \end{pmatrix} \tag{6}$$

where f is the total upward thrust and mg is the weight of the quadcopter. Now applying Newton's law to rotational motion, we get;

$$\tau = \frac{d\mathbf{L}}{dt} = (\frac{d\mathbf{L}}{dt} + \omega_{\mathbf{b}} \times \mathbf{L}) \tag{7}$$

where $\mathbf{L} = \mathbf{J}\omega_{\mathbf{b}}$ is the angular momentum and τ is the applied torque. by using coriolis equation to evaluate time derivative of angular momentum in the body frame, equation can be simplified as;

$$\begin{pmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{pmatrix} = \begin{pmatrix} qr\frac{J_y - J_z}{J_x} \\ pr\frac{J_z - J_x}{J_y} \\ pq\frac{J_x - J_y}{J_z} \end{pmatrix} + \begin{pmatrix} \frac{\tau_\phi}{J_x} \\ \frac{\tau_\theta}{J_y} \\ \frac{\tau_\psi}{J_z} \end{pmatrix} \tag{8}$$

where, $\tau = (\tau_\phi, \tau_\theta, \tau_\psi)^T$ and J_x, J_y, J_z are the diagonal elements of the inertial matrix J.

Equations 2-8 now describe the 6DOF model of the quadcopter. [7] describe some approximations to make these set of equations more suitable for control design. The coriolis terms and time derivative of the transformation matrix are neglected to get these much simpler set of equations;

$$\begin{aligned} \ddot{\phi} &= \dot{\theta}\dot{\psi}(\frac{J_y - J_z}{J_x}) + \frac{l}{J_x}U_2 \\ \ddot{\theta} &= \dot{\phi}\dot{\psi}(\frac{J_z - J_x}{J_y}) + \frac{l}{J_y}U_3 \\ \ddot{\psi} &= \dot{\phi}\dot{\theta}(\frac{J_x - J_y}{J_z}) + \frac{U_4}{J_z} \\ \ddot{z} &= \frac{U_1}{m}(\cos\phi\cos\theta) - g \\ \ddot{x} &= \frac{U_1}{m}(\cos\phi\sin\theta\cos\psi + \sin\phi\sin\psi) \\ \ddot{y} &= \frac{U_1}{m}(\cos\phi\sin\theta\sin\psi - \sin\phi\cos\psi) \end{aligned} \tag{9}$$

where, the control inputs $U_1 - U_4$ are given as;

$$\begin{aligned} U_1 &= b(\Omega_1^2 + \Omega_2^2 + \Omega_3^2 + \Omega_4^2) \\ U_2 &= b(\Omega_4^2 - \Omega_2^2) \\ U_3 &= b(\Omega_3^2 - \Omega_1^2) \\ U_4 &= d(\Omega_4^2 + \Omega_2^2 - \Omega_1^2 - \Omega_3^2) \end{aligned} \tag{10}$$

where, b and d are thrust and drag factor respectively.

III. Control Design

The mathematical model developed in section II can be represented in a state-space form $\dot{X} = f(X, U)$ by considering $X = (x_1 ... x_{12})^T \in R^{12}$ as state vector of the system.

$$X = (x_1, x_2, ... x_{12})^T = \begin{pmatrix} x_1 = \phi \\ x_2 = \dot{\phi} \\ x_3 = \theta \\ x_4 = \dot{\theta} \\ x_5 = \psi \\ x_6 = \dot{\psi} \\ x_7 = z \\ x_8 = \dot{z} \\ x_9 = x \\ x_{10} = \dot{x} \\ x_{11} = y \\ x_{12} = \dot{y} \end{pmatrix} \tag{11}$$

$$\dot{X} = f(X, U) = \begin{pmatrix} x_2 \\ x_4 x_6 a_1 + U_2 b_1 \\ x_4 \\ x_2 x_6 a_2 + U_3 b_2 \\ x_6 \\ x_2 x_4 a_3 + U_4 b_3 \\ x_8 \\ \frac{U_1}{m}(\cos x_1 \cos x_3) - g \\ x_{10} \\ \frac{U_1}{m} U_x \\ x_{12} \\ \frac{U_1}{m} U_y \end{pmatrix} \tag{12}$$

where,

$$\left.\begin{aligned} U_x &= \cos x_1 \sin x_3 \cos x_5 + \sin x_1 \sin x_5 \\ U_y &= \cos x_1 \sin x_3 \sin x_5 - \sin x_1 \cos x_5 \\ a_1 &= \frac{J_y - J_z}{J_x}, b_1 = \frac{l}{J_x} \\ a_2 &= \frac{J_z - J_x}{J_y}, b_2 = \frac{l}{J_y} \\ a_3 &= \frac{J_x - J_y}{J_z}, b_3 = \frac{1}{J_z} \end{aligned}\right\} \tag{13}$$

The control strategy is implemented such that the altitude and position of the quadcopter is controlled by using the vertical thrust input U_1 . Based on altitude and position, desired roll and pitch angles are supplied by position controller to the attitude controller. The attitude and heading controllers are then stabilizing the quadcopter near quasi stationary conditions with control inputs U_2, U_3 and U_4 respectively. This control structure [8] is shown in figure 3.

A. *Sliding Mode Control*

The main objective of this section is to provide a detailed explanation of designing control law based on SMC for the attitude, heading, altitude and x,y position control of the quadcopter [9] [10].

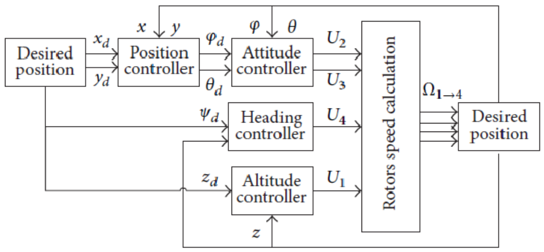


Fig. 3: Quadcopter control structure

1) Roll control: Let the tracking error in roll angle is defined as:

$$e_1 = \phi_d - \phi \tag{14}$$

The sliding surface for roll control is defined as,

$$s = \dot{e_1} + c_1 e_1; \tag{15}$$

Derivative of sliding surface will therefore be:

$$\dot{s} = c_1(\dot{\phi_d} - \dot{\phi}) + \ddot{\phi_d} - \ddot{\phi} \tag{16}$$

Let us define a Lyapunov function as:

$$V(e_1, s) = \frac{1}{2}(e_1^2 + s^2) \tag{17}$$

Corresponding to above defined Lyapunov function, an exponential reaching law is proposed for the SMC as follows:

$$\dot{s} = -k_1 sgn(s) - k_2 s \tag{18}$$

To satisfy the sliding mode condition $s\dot{s} < 0$, the value of k_1 and k_2 is chosen such that $k_1 > 0$ and $k_2 > 0$.
By equating the proposed reaching law Eq.(18) to the derivative of the sliding surface in Eq.(16) and substituting $\ddot{\phi}$ from Eq.(9), the control input U_2 can be extracted as:

$$U_2 = \frac{1}{b_1}[k_1 sgn(s) + k_2 s - a_1\dot{\theta}\dot{\psi} + \ddot{\phi_d} + c_1(\dot{\phi_d} - \dot{\phi})] \tag{19}$$

2) Pitch control: Similarly the control input U_3 responsible for generating the pitch rotation θ can be calculated as:

$$U_3 = \frac{1}{b_2}[k_3 sgn(s) + k_4 s - a_2\dot{\phi}\dot{\psi} + \ddot{\theta_d} + c_2(\dot{\theta_d} - \dot{\theta})] \tag{20}$$

3) Yaw or heading control: Similarly the control input U_4 responsible for generating the yaw rotation ψ can be calculated as:

$$U_4 = \frac{1}{b_3}[k_5 sgn(s) + k_6 s - a_3\dot{\phi}\dot{\theta} + \ddot{\psi_d} + c_3(\dot{\psi_d} - \dot{\psi})] \tag{21}$$

4) Altitude control: Similarly the control input U_1 responsible for generating the vertical motion z can be calculated as:

$$U_1 = \frac{m}{\cos\phi\cos\theta}[k_7 sgn(s) + k_8 s + g + \ddot{z_d} + c_4(\dot{z_d} - \dot{z})] \tag{22}$$

5) x and y Motion Control: Consider U_x and U_y are the orientations of U_1 which is responsible for the x and y motion respectively, can be extracted in a similar way as:

$$U_x = \frac{m}{U_1}[k_9 sgn(s) + k_{10} s + \ddot{x_d} + c_5(\dot{x_d} - \dot{x})] \tag{23}$$

$$U_y = \frac{m}{U_1}[k_{11} sgn(s) + k_{12} s + \ddot{y_d} + c_6(\dot{y_d} - \dot{y})] \tag{24}$$

B. Backstepping Control

The main objective of this section is to provide a detailed explanation of designing control law based on BSC for the attitude, heading, altitude and x,y position control of the quadcopter [11] [12].

1) Roll control: Let tracking error in roll is defined as:

$$e_1 = x_{1d} - x_1 \tag{25}$$

Consider positive definite Lyapunov function

$$V_1 = \frac{1}{2}e_1^2 \tag{26}$$

Taking differentiation of Lyapunov function with respect to time we get:

$$\dot{V_1} = e_1(\dot{x_{1d}} - x_2) \tag{27}$$

To make system asymptotically stable, time derivative of a Lyapunov function must be negative semi-definite. To satisfy this, let us choose a positive definite bounding function $W_1(e) = c_1 e_1^2$ to bound $\dot{V_1}$ as in Eq.(27).

$$\dot{V_1} = e_1(\dot{x_{1d}} - x_2) \leq -c_1 e_1^2, with : c_1 > 0 \tag{28}$$

To satisfy above inequality, let us choose the virtual control input as:

$$x_{2d} = \dot{x_{1d}} + c_1 e_1 \tag{29}$$

Defining a new error variable e_2, which is the difference between x_2 and x_{2d}

$$e_2 = x_2 - \dot{x_{1d}} - c_1 e_1 \tag{30}$$

Rewriting Lyapunov's function time derivative $\dot{V_1}$ as:

$$\dot{V_1} = -e_1 e_2 - c_1 e_1^2 \tag{31}$$

The next step is to add V_1 with a quadratic term in the second error variable e_2 to get a positive definite V_2,

$$V_2 = V_1 + \frac{1}{2}e_2^2 \tag{32}$$

with time derivative

$$\dot{V_2} = -e_1 e_2 - c_1 e_1^2 + e_2(\dot{x_2} - \ddot{x_{1d}} - c_1\dot{e_1}) \tag{33}$$

substituting the value of $\dot{x_2}$ from equation (12) we get:

$$\dot{V_2} = -e_1 e_2 - c_1 e_1^2 + e_2(a_1 x_4 x_6 + b_1 U_2 - \ddot{x_{1d}} - c_1\dot{e_1}) \tag{34}$$

The control input U_2 can be extracted, satisfying $\dot{V_2} = -c_1 e_1^2 - c_2 e_2^2 < 0$:

$$U_2 = \frac{1}{b_1}[e_1 - a_1 x_4 x_6 + \ddot{x_{1d}} - c_2 e_2 + c_1(\dot{x_{1d}} - x_2)] \tag{35}$$

2) Pitch control: Similarly Control input U_3 for generating pitch rotation can be extracted as:

$$U_3 = \frac{1}{b_2}[e_3 - a_2 x_2 x_6 + \ddot{x}_{3d} - c_4 e_4 + c_3(\dot{x}_{3d} - x_4)] \quad (36)$$

3) Yaw control: Similarly the control input U_4 for the yaw rotation can be calculated as:

$$U_4 = \frac{1}{b_3}[e_5 - a_3 x_2 x_4 + \ddot{x}_{5d} - c_6 e_6 + c_5(\dot{x}_{5d} - x_6)] \quad (37)$$

4) Altitude control: The control input U_1 for altitude can be extracted as:

$$U_1 = \frac{m}{\cos x_1 \cos x_3}[g + e_7 + \ddot{x}_{7d} - c_8 e_8 + c_7(\dot{x}_{7d} - x_8)] \quad (38)$$

5) x and y Motion Control: For x and y motion control, U_x and U_y can be extracted as:

$$U_x = \frac{m}{U_1}[e_9 + \ddot{x}_{9d} - c_{10} e_{10} + c_9(\dot{x}_{9d} - x_{10})] \quad (39)$$

$$U_y = \frac{m}{U_1}[e_{11} + \ddot{x}_{11d} - c_{12} e_{12} + c_{11}(\dot{x}_{11d} - x_{12})] \quad (40)$$

with:

$$\left.\begin{array}{c} e_1 = x_{1d} - x_1, e_2 = x_2 - \dot{x}_{1d} - c_1 e_1 \\ e_3 = x_{3d} - x_3, e_4 = x_4 - \dot{x}_{3d} - c_3 e_3 \\ e_5 = x_{5d} - x_5, e_6 = x_6 - \dot{x}_{5d} - c_5 e_5 \\ e_7 = x_{7d} - x_7, e_8 = x_8 - \dot{x}_{7d} - c_7 e_7 \\ e_9 = x_{9d} - x_9, e_{10} = x_{10} - \dot{x}_{9d} - c_9 e_9 \\ e_{11} = x_{11d} - x_{11}, e_{12} = x_{12} - \dot{x}_{11d} - c_{11} e_{11} \end{array}\right\} \quad (41)$$

C. PID Control

As a baseline method [8], each of the control inputs U_i are modelled as $U_i = K_p e_i + K_d \dot{e_i} + K_i \int e_i$ Where, $e_1 = \phi_d - \phi, e_2 = \theta_d - \theta, e_3 = \psi_d - \psi, e_4 = z_d - z, e_5 = x_d - x$ and $e_6 = y_d - y.(\phi_d, \theta_d, \psi_d, z_d, x_d, y_d)$ are the desired value for $(\phi, \theta, \psi, z, x, y)$. The parameters K_p, K_i and K_d are Proportional , Integral and Derivative gain respectively.

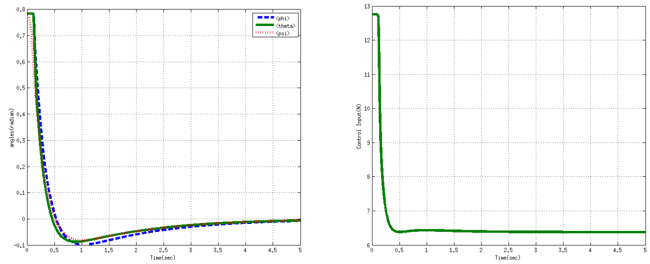
IV. SIMULATION RESULTS

This section provides simulation results of the quadcopter using the PID, BSC and SMC. The proposed control strategy is tested by using MATLAB/Simulink in order to test the performance acquired for the control of attitude, altitude and linear position of the quadcopter. The values of the model parameters used for simulations are listed in table 1.

The attitude controller's task is basically to stabilize the roll, pitch and yaw motion. Similarly altitude controller will stabilize the height and position controller will regulate x and y motion. For this simulations, dynamic model Eq.(9) is used. The attitude control goal is to reach and stay at zero from some initial values. The initial roll, pitch and yaw angle is $\frac{\pi}{4}$ rad. The attitude and heading control response using PID, BSC and SMC is shown in figure 4,5 and 6 respectively.

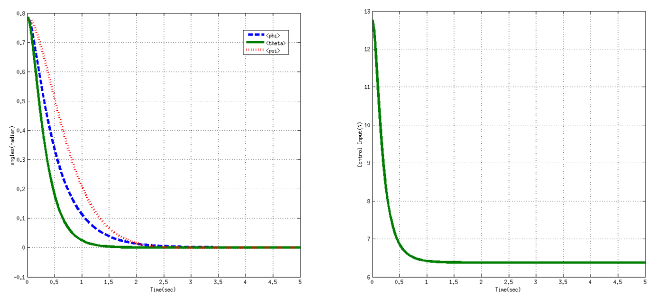
TABLE I

Parameter	Value	Parameter	Value
m	0.65 kg	J_y	7.5x10^{-3} $kg.m^2$
g	9.81 m/s^2	J_z	1.3x10^{-2} $kg.m^2$
l	0.23 m	b	3.13x10^{-5} $kg.m$
J_x	7.5x10^{-3} $kg.m^2$	d	7.5x10^{-7} $kg.m^2$



(a) PID control to stabilize the orientation angles. (b) PID control input U_1.

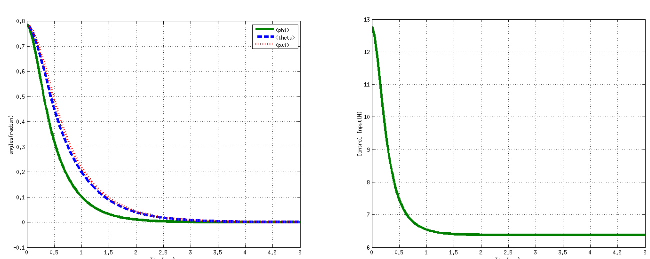
Fig. 4: PID based roll, pitch and yaw control



(a) Backstepping control to stabilize the orientation angles. (b) Backstepping control input U_1.

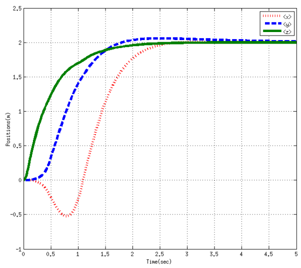
Fig. 5: Backstepping based roll, pitch and yaw control

From the response it can be seen that overshoot appears and also require more time to reach zero values (settling time is large) in case of PID Control compare to BSC and SMC. The position and altitude response (x,y and z motion) using PID, BSC and SMC is shown in figure 7, 8 and 9 respectively. The control task is to reach the position $x_d = y_d = z_d = 2$ m from zero.Altitude response shows better performance and tracking (steady state error is zero) having no overshoot in

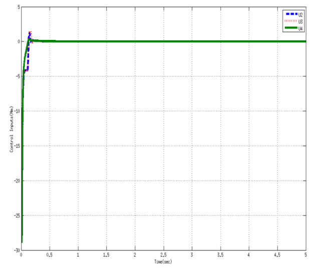


(a) Sliding mode control to stabilize the orientation angles. (b) Sliding mode control input U_1.

Fig. 6: Sliding mode based roll, pitch and yaw control

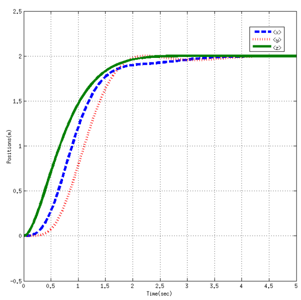


(a) PID control to stabilize the orientation angles.

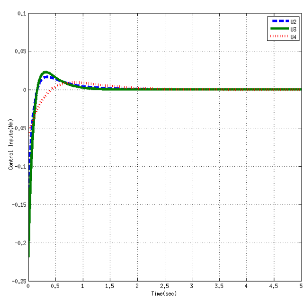


(b) PID control inputs U_2, U_3, U_4 during attitude control mode.

Fig. 7: PID based position and altitude control



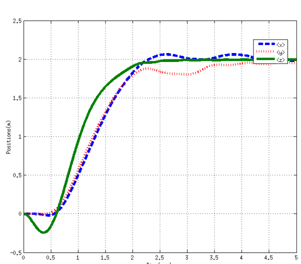
(a) Backstepping control to stabilize the x,y and z motion.



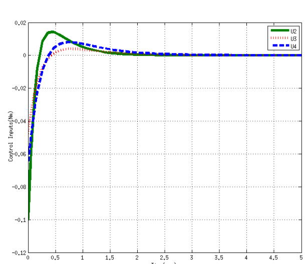
(b) Backstepping control inputs U_2, U_3, U_4 during attitude control mode.

Fig. 8: Backstepping based position and altitude control

case of BSC and SMC. Position response is quite satisfactory and fast, as settling time is low and slightly better tracking performance (steady state error is almost zero) in case of BSC as compared to PID and SMC in linear region (hovering mode). BSC provide good control over SMC. Chattering is found in case of SMC. Chattering can be minimized with a continuous approximation of a pre-determined "sign" function [9]. When the controllers is operating outside the linear region (away from hover), the PID controller is not able to stabilize the system. The SMC and the BSC are nonlinear control approach, their controller parameter tuning is not much affected by the operating region.



(a) Sliding mode control to stabilize the x,y and z motion.



(b) Sliding mode control inputs U_2, U_3, U_4 during attitude control mode.

Fig. 9: Sliding mode based position and altitude control

V. CONCLUSION

The six degree of freedom mathematical model of the quadcopter is developed using Newton-Euler method. Attitude, altitude and position controller based on Backstepping and Sliding mode approach is developed. The performance and tracking ability of the controllers is examined and compared with traditional PID Controller through MATLAB simulation and stability is guaranteed using Lyapunov global stability theorem. The attitude and heading controller using SMC and BSC approach provide better control of orientation angles compare to PID and altitude controller also able to stabilize z motion with no overshoot and steady state error. Position controller introduced using the SMC approach provides average results compare to PID and BSC approach as chattering is asssociated with the SMC due to discontinuous control law. Each controller approves the ability to control the linear translations, althought not quite as good. Therefore further research should be carried out on the robust control design and focus on extending the quadcopter dynamics and to implement this designed controller in real environment. the effect of disturbances and uncertainty is not taken in to account in this work so future work is to consider all these aspect and design a disturbance observer based control to estimate and reject the effect of disturbances and uncertainty.

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