Understanding Normal, Gamma, and Beta Distributions

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In the study of statistics and probability, understanding different types of distributions is essential for analyzing data and making predictions. Among these, the normal, gamma, and beta distributions are fundamental, and we unfortunately did not have enough time to go over these concepts in class. Our class was tasked with the job of understanding these concepts and writing a paper to share what we’ve learned. This essay aims to explain these concepts in plain English, providing a clear understanding of their characteristics and applications.

The normal distribution, often referred to as the Gaussian distribution, is one of the most prominent probability distributions in statistics, characterized by its symmetrical, bell shaped curve when graphed. This symmetry means that the distribution is centered around its mean. In a perfectly normal distribution, the mean, median, and mode are equal and lie at the top of the curve. The normal distribution is especially useful because of the Central Limit Theorem, which states that “the sampling distribution of the mean will always be normally distributed, as long as the sample size is large enough. Regardless of whether the population has a normal, Poisson, binomial, or any other distribution, the sampling distribution of the mean will be normal.” This theorem is powerful because it allows us to draw conclusions about population parameters even when the population distribution is unknown, as long as the sample size is sufficiently large.

In the real world, the normal distribution is used across various fields. In finance, it is used to model asset prices, despite the reality that financial returns often exhibit heavier tails than the normal distribution predicts. In quality control, the normal distribution is instrumental in setting up control charts and establishing acceptable ranges of variation. The normal distribution is also applied to understand and interpret test scores, where most individuals score around the average while fewer score extremely high or low. With that said, it is important to note that not all data is normally distributed. In the real world. It is not uncommon for data to sometimes be lopsided or have more extreme highs and lows than the usual bell-shaped curve. The unusual outliers are sometimes referred to as skewness.

Moving on to the gamma distribution, this distribution is a way to predict scenarios relating to time. For example, the time a person spends at a bus stop waiting for the next bus. The time a factory has to wait for a machine to finish a task. Or the response time an IT department has for their tickets. Statistical models can be used to manage and reduce these waiting times. Unlike the normal distribution, which looks like a perfectly symmetrical bell when graphed, the gamma distribution is lopsided. Picture a hill that slightly slopes up on one side and then drops off more steeply on the other side.

In what situation is a gamma distribution useful? Well, it is most useful in situations where things are not perfectly balanced. Circling back to the previously mentioned IT department example, an IT manager may clock in for work and have one or two open tickets which he will take him and hour to work on and close. After that the IT manager may not receive another ticket for a few hours, then suddenly he receives 4 or 5 tickets in the next hour. Of course, each ticket will have a different response time as the amount of tickets he receives each day will be different and the time it takes to close each ticket is not always the same. The gamma distribution helps to describe this more random waiting time. It will tell you things like the most common wait time, the average wait time, and the chances of having to wait a really long or really short time. There are other wait times that we don’t often think about as “wait” times because the wait is extended over a much longer period of time. For example, a brand new car, or a new computer. These things will eventually die on you, and however long they last is the “wait” time. Given the wait times, the gamma distribution can give you an idea on the how long a new car, or laptop will last before it needs to be replaced. This can come in handy for businesses that need to plan for repairs or replacements and the cash requirements that come along with replacing these things. In reliability engineering, for example, engineers use the gamma distribution to estimate the lifespan of machinery and parts. It's based on how similar machines have lasted in the past. It helps companies to plan ahead and avoid sudden breakdowns that can cause delays and cost money. The gamma distribution can even be used to model the amount of time between earthquakes. While it’s not perfect, this is a powerful tool statics has provided the world with.

The beta distribution is a tool for understanding chances and odds when things are uncertain. Imagine you have a coin, but it's not a regular coin. It's a weird coin that doesn't always have a 50/50 chance of landing heads or tails. Sometimes, maybe because of how it's weighted or shaped, it might land on heads more often than tails or vice versa. The beta distribution helps us figure out the odds of getting heads or tails when we're not dealing with a fair coin. In the real world, lots of things are like that weird coin. They don't always happen in a way we can predict with simple odds. For example, the chance of rain. Weather experts don't just say there's a 50% chance of rain all the time. They look at clouds, temperature, wind, and a bunch of other stuff to say, "Okay, today, there's a 70% chance of rain," and "Tomorrow, there's only a 10% chance." The beta distribution is perfect for this because it deals with probabilities that change. It's not just about two outcomes like rain or no rain. This distribution can handle a whole range of outcomes, like the chance of a team winning a season based on their past wins and losses, or how likely it is that people will like a new flavor of ice cream based on taste tests. companies use the Beta distribution to model things like project completion times. Let’s say a project could finish super fast if everything goes right, or it could drag on if things go wrong. The beta distribution helps managers understand the most likely outcome so he or she can plan accordingly. From what I understand it seems beta distribution is the most flexible. It can be stretched and squished to model different levels of certainty and uncertainty. Which comes in handy when figuring the odds of pretty much anything that can happen between "never ever" and "absolutely for sure."

In summary, the normal, gamma, and beta distributions are powerful statistical tools, each with unique properties and applications. The normal distribution, with its characteristic bell curve, is pivotal for representing symmetric data. The gamma distribution is key in modeling skewed, continuous data, especially for time to event analysis. Lastly, the beta distribution's ability to handle probabilities makes it indispensable in fields such as meteorology and Bayesian statistics. Understanding these distributions enhances our ability to analyze various types of data and make informed predictions.

Works Cited

Chen, James. “Normal Distribution: What It Is, Properties, Uses, and Formula.” *Investopedia*, Investopedia, www.investopedia.com/terms/n/normaldistribution.asp#:~:text=The%20assumption%20of%20a%20normal,is%20being%20over%20or%20undervalued. Accessed 5 Dec. 2023.

*Gamma Distribution Definition, Formula & Examples - Study.Com*, study.com/academy/lesson/gamma-distribution-definition-equations-examples.html. Accessed 5 Dec. 2023.

*Projectmanagement.Com - 7 Key Concepts for Controlling Quality*, www.projectmanagement.com/blog-post/19598/7-Key-Concepts-For-Controlling-Quality. Accessed 5 Dec. 2023.

*Quora*, www.quora.com/What-are-some-real-life-examples-of-beta-distribution. Accessed 5 Dec. 2023.

Turney, Shaun. “Central Limit Theorem: Formula, Definition & Examples.” *Scribbr*, 22 June 2023, www.scribbr.com/statistics/central-limit-theorem/#:~:text=The%20central%20limit%20theorem%20says,the%20mean%20will%20be%20normal.

“What Is the Gamma Distribution?” *Built In*, builtin.com/data-science/gamma-distribution. Accessed 5 Dec. 2023.

Wackerly, Dennis, Mendenhallm William, and Scheaffer, Richard. *Mathematical Statistics with Applications* 7th Edition, Thomson Brooks/Cole, 2008