RBE502 - Homework Set 2

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Due date: September 8, 2021

Problem 1

We are presented with two figures representing two figures, seen below:

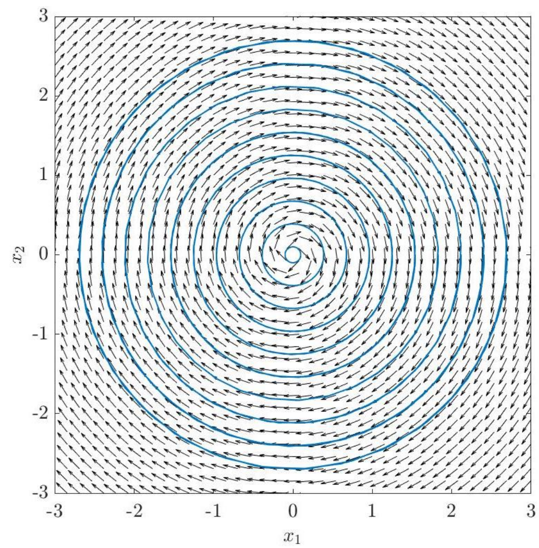

Figure 1: A simple harmonic oscillator $\dot{x} = -kx$, where $k > 0$

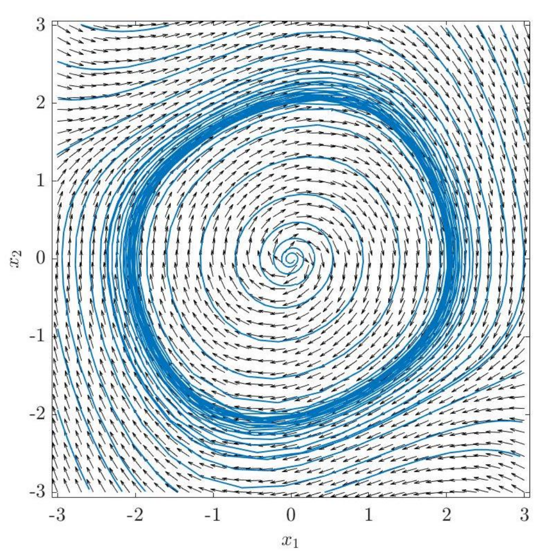

Figure 2: A Van der Pol system $\dot{x} + 0.2(x^2 - 1)\dot{x} + x = 0$

The state space form for the simple harmonic oscillator state vectors is $x_1 = x, x_2 = \dot{x}$, for $\dot{x}_1 = x_2$ and $\dot{x}_2 = -kx_1$. The state space form for the Van der Pol system is $x_1 = x$, $x_2 = \dot{x}$, for $\dot{x}_1 = x_2$ and $\dot{x}_2 = 0.2(1-x_1^2)x_2 - x_1$.

The key differences between a simple sustained oscillation (such as in our first system) and limit cycle (such as in our second system) are twofold. First, the amplitude of a limit cycle is independent of its initial conditions. Once the limit cycle is reached, irregardless of where it started, it will continue through the self-sustaining limit cycle loop. In a simple oscillation, initial conditions are what determines which of the oscillations it can reached.

The second difference is that simple oscillations are extremely sensitive to system parameter changes. A small change can either cause the system to converge or become unstable. A limit cycle, however, is less susceptible to initial conditions heavily affecting the behaviour of the system.

Problem 2

In this problem, we are presented with a pendulum system defined by the equation $ml^2\ddot{q} = mgl\sin q + u$.

Part A

Here, we let:

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} q \\ \dot{q} \end{bmatrix}$$

We then find the state space formulation of the system (assuming $u(t) = 0$) by definining $\dot{x}_1$ and $\dot{x}_2$:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} x_2 \\ \frac{mgl\sin x_1}{ml^2} \end{bmatrix}$$

.

Part B

Assuming the $m = 0.1$ kg , $l = 1$ m, and $g = 9.81$ $\frac{m}{s^2}$, we utilize MATLAB to draw a phsae portrait of the system provided. We make use of provided ranges, $-3\pi \leq x_1 \leq 3\pi$ and $-4 \leq x_2 \leq 4$. The phase portrait generated is below:

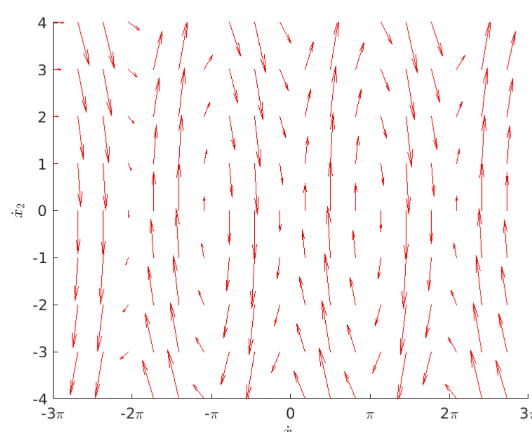

Figure 3: Phase portrait of our system pendulum system

Part C

In this problem, we are tasked to utiliez the phase portrait in from Part B and identify the equilibrium points of the system, classifying them as stable or unstable groups.

Using the phase portrait from before, we solved the ordinary differential equation for different initial conditions to better demonstrate the system's diverging and converging behaviours. We then highlighted key points that we believe are equilibrium points.

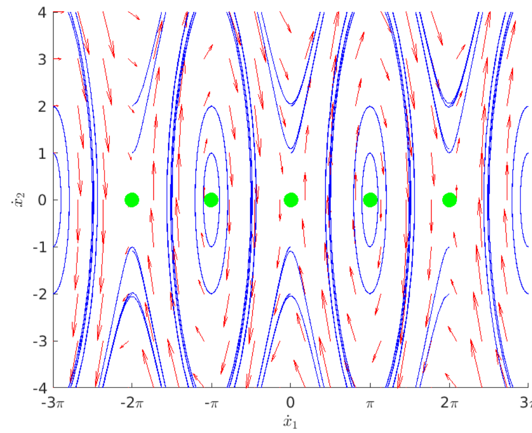

Figure 4: Phase portrait of our pendulum system with equilibrium points

When $\dot{x}_2 = 0$, we have an equilibrium point. When the equilibrium point is an odd integer multiple of π - ie $-\pi$ or π - the system converges towards the equilibrium point and is thus **stable**. At even integer multiples of π - ie -2π or 2π - the system diverges away from the equilibrium point, and is thus **unstable**.