CS/RBE 549 Computer Vision Name: **Sol U. Tion**
Fall 2018 Monday 03 Dec 2018

Exam #2 Solutions

1. **Optical Flow (40 pts):** Suppose the image brightness is given by

$$I(x,y,t) = I_0 + \frac{1}{2}[(x - c_1 t)^2 + (y - c_2 t)^2]$$

   a. (15 pts) What are I_x, I_y, and I_t? Hint: You should find that these derivatives have a simple form.

$$\boldsymbol{I_x = (x - c_1 t), I_y = (y - c_2 t), I_t = -c_1(x - c_1 t) - c_2(y - c_2 t)}$$

   b. (15 pts) Express the Optical Flow Constraint Equation $I_x u + I_y v + I_t = 0$ in the simplest terms possible for this image sequence.

$$\boldsymbol{I_x u + I_y v + I_t = (x - c_1 t)u + (y - c_2 t)v - c_1(x - c_1 t) - c_2(y - c_2 t)}$$
$$\boldsymbol{= (x - c_1 t)(u - c_1) + (y - c_2 t)(v - c_2) = 0}$$

   c. (10 pts) The equation from b. must hold for all *x*, *y*, and *t*. Find a constant solution for *u* and *v* that makes this true, that is, such that *u* and *v* do not depend on *x*, *y*, and *t*.

$$\boldsymbol{u = c_1, v = c_2}$$

2. **Stereo Motion (35 pts):** In stereo imaging we can compute a point's world coordinates from left and right images as

$$\vec{X}^W = \vec{X}_{AVG} \frac{\left|\vec{B}\right|^2}{\vec{B} \cdot \vec{\Delta}}$$

   a. (20 pts) Show that if the world point is in motion, we can compute its velocity as

$$\vec{V}^W = \vec{V}_{AVG} \frac{\left|\vec{B}\right|^2}{\vec{B} \cdot \vec{\Delta}} - \vec{X}_W \frac{\vec{B} \cdot \frac{d\vec{\Delta}}{dt}}{\vec{B} \cdot \vec{\Delta}}$$

$$\boldsymbol{\vec{V}^W = \frac{d\vec{X}^W}{dt} = \frac{\left(\vec{B} \cdot \vec{\Delta}\right)\left|\vec{B}\right|^2 \frac{d\vec{X}_{AVG}}{dt} - \vec{X}_{AVG}\left|\vec{B}\right|^2 \left(\vec{B} \cdot \frac{d\vec{\Delta}}{dt}\right)}{\left(\vec{B} \cdot \vec{\Delta}\right)^2}}$$

$$\boldsymbol{= \frac{\left|\vec{B}\right|^2 \frac{d\vec{X}_{AVG}}{dt}}{\left(\vec{B} \cdot \vec{\Delta}\right)} - \vec{X}_{AVG} \frac{\left|\vec{B}\right|^2}{\vec{B} \cdot \vec{\Delta}} \frac{\left(\vec{B} \cdot \frac{d\vec{\Delta}}{dt}\right)}{\left(\vec{B} \cdot \vec{\Delta}\right)} = \vec{V}_{AVG} \frac{\left|\vec{B}\right|^2}{\vec{B} \cdot \vec{\Delta}} - \vec{X}_W \frac{\vec{B} \cdot \frac{d\vec{\Delta}}{dt}}{\vec{B} \cdot \vec{\Delta}}}$$

   b. (15 pts) Suppose that a moving world point is imaged as

$$\vec{X}_L^I = \begin{bmatrix} \frac{c}{2t} \\ 0 \\ f \end{bmatrix}, \vec{X}_R^I = \begin{bmatrix} \frac{-c}{2t} \\ 0 \\ f \end{bmatrix}$$

With the usual imaging geometry of $\vec{B} = [b \quad 0 \quad 0]^T, \vec{F} = [0 \quad 0 \quad f]^T$, what is $\vec{V}^W$? Express $\vec{V}^W$ in the simplest terms.

$$\vec{X}_{AVG} = \begin{bmatrix} 0 \\ 0 \\ f \end{bmatrix}, \vec{V}_{AVG} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \vec{\Delta} = \begin{bmatrix} \frac{c}{t} \\ 0 \\ 0 \end{bmatrix}, \frac{d\vec{\Delta}}{dt} = \begin{bmatrix} -\frac{c}{t^2} \\ 0 \\ 0 \end{bmatrix}, \vec{X}^W = \begin{bmatrix} 0 \\ 0 \\ \frac{fbt}{c} \end{bmatrix},$$

$$\vec{V}^W = \vec{0} - \begin{bmatrix} 0 \\ 0 \\ \frac{fbt}{c} \end{bmatrix} \frac{\left(-\frac{bc}{t^2}\right)}{\frac{bc}{t}} = \begin{bmatrix} 0 \\ 0 \\ \frac{fb}{c} \end{bmatrix}$$

3. **Image Knowledge (15 pts):** Describe 3 sources of knowledge about *your* assigned object that could be exploited by a computer vision system. How robust is each of these knowledge sources?
   Varies by object. Could include color, subparts, shape (represented how?), contours, edges, features, …

4. **X-Ray Vision (10 pts):** As is well-known, the comic book hero Superman has X-Ray vision. Although the physics of x-ray vision are never quite explained, presumably, Superman's eyes are sensitive to photons in the x-ray spectrum, which can penetrate most objects. *Briefly* discuss some of the challenges imaging the world using x-rays.
   Many possible good answers. Here are a few:
   Imaging – How to build a detector that can reliably detect x-rays, which can pass through most objects. Forming an image with x-rays is difficult, although not impossible.
   Illumination – Most objects do not emit x-rays naturally, so Superman might have to emit x-rays to provide enough illumination. Doing this without damaging himself is a challenge.
   Surfaces – x-rays are presumably emitted from bulk material, not just the surface. Therefore, Superman cannot assume that he is seeing object surfaces. Instead, he sees objects at all distances unless he can focus on a particular depth.
   Color – Although x-rays span many wavelengths, they do not correspond to our usual sense of color.
   Etc.

5. **Teamwork (1 pt):** On a scale of 1 to 5, with 1 being the lowest, 3 is neutral and 5 being the highest, rate how well your project team is working together.
   No correct answer.