

Exam #1

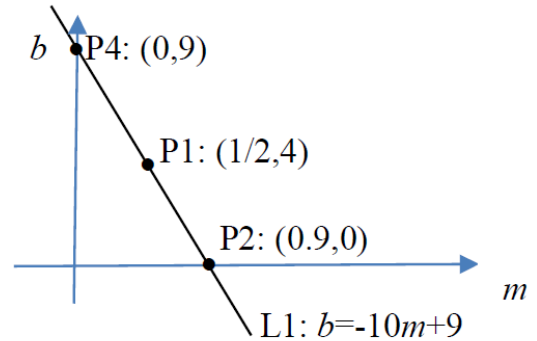
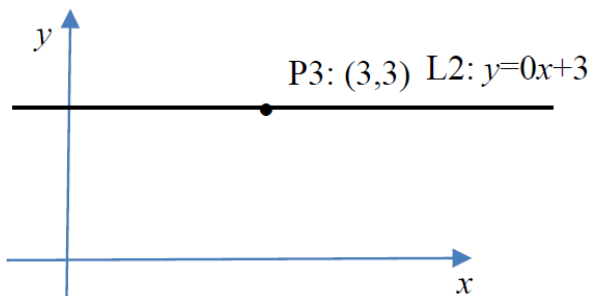
This examination is open book and open notes. Please complete individually.

Write all your answers on the examination in the space provided except Problem 5. Partial credit will be given, but you must justify your work.

The examination will permit submission within 24 hours once it begins. Good luck.

Problem 1: /20
Problem 2: /20
Problem 3: /20
Problem 4: /20
Problem 5: /20
Total: /100

1. **Hough Transform (20 pts).** In this problem x , y , b , and m may be positive or negative, integers or fractions. 2 points in (m, b) space are given by P1: $m=0.5$, $b=4$; P2: $m=0.9$, $b=0$. Hint: It might help to construct the (x, y) and (m, b) spaces.



- a) (5 pts) What line $L1$ in (m, b) space passes through points $P1$ and $P2$?
- b) (10 pts) What point $P3$ in (x, y) space corresponds to line $L1$?
- c) (10 pts) Horizontal line $L2$ in (x, y) space passes through $P3$. What is its corresponding point $P4$ in (m, b) space?

2. **Morphology (20 pts).** Robot SeeWee dilates binary image II (left) with an unknown 3 by 3 structuring element SE to yield output image OI (right)

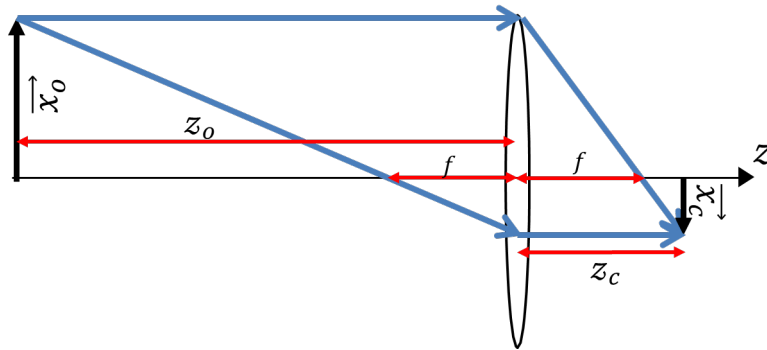
0	0	0	0	0
0	1	0	1	0
0	0	1	0	0
0	1	0	1	0
0	0	0	0	0

1	0	1	0	1
0	1	0	1	0
1	0	1	0	1
0	1	0	1	0
1	0	1	0	1

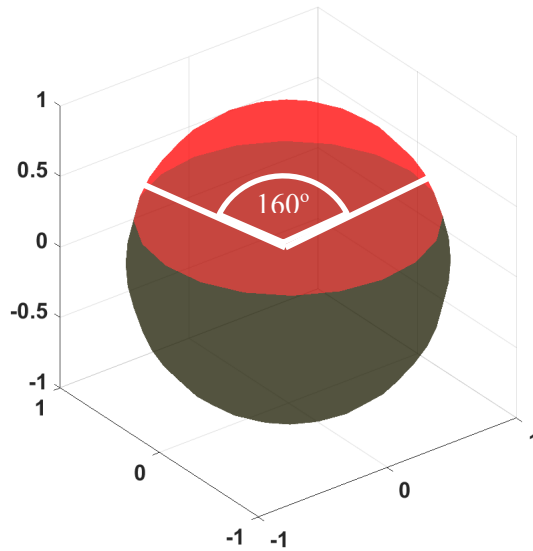
- (10 pts) What is a 3 by 3 structuring element SE such that $II \oplus SE = OI$?
- (10 pts) Find another 3 by 3 structuring element SE_2 that is different from the one in a) that also satisfies $II \oplus SE_2 = OI$.

3. **Focus (20 pts).**

- a) (5 pts) Prove that for a thin lens, the image is in focus when $\frac{1}{-z_O} + \frac{1}{z_c} = \frac{1}{f}$



- b) (15 pts) For a typical human eyeball of 12mm in radius and contains roughly 1.5×10^8 ($1.5E8$) receptors. Assuming that the receptors are uniformly distributed across a 160° partial sphere (see red zone out of brown sphere, please note it is not a perfect hemisphere by definition). For Planet Mars of 4×10^3 km in radius with an average distance to Earth of 2.25×10^8 km. Using a value of focal length f equal to the eye's diameter, on how many receptors does the image of Mars fall? *Please show calculation steps for partial credits.*

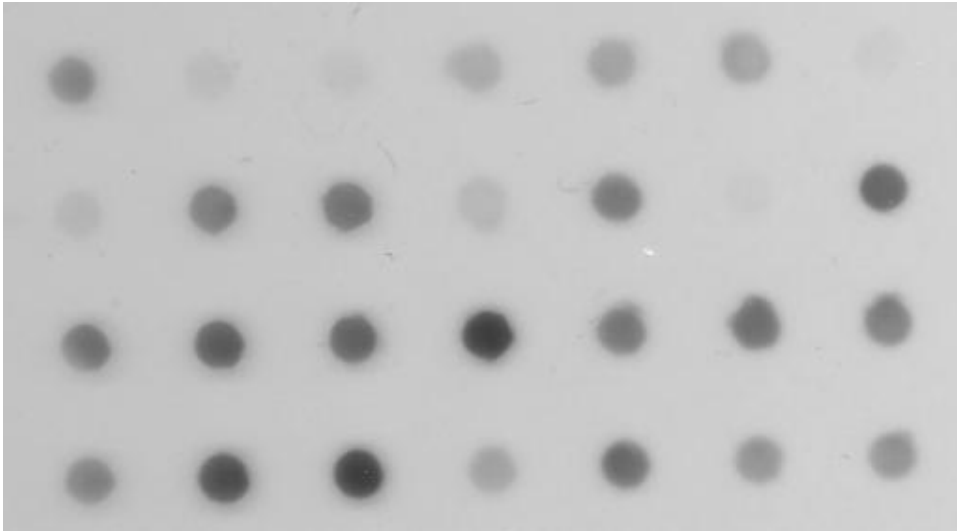


4. **Segmentation (20 pts).** An image has two objects and background pixels whose brightness values are distributed according to the Rayleigh distribution with parameters σ_b , σ_{o1} and σ_{o2} with $0 < \sigma_b < \sigma_{o1} < \sigma_{o2}$. The probability of a pixel having brightness k is given by

$$P_b(k) = \frac{k}{\sigma_b^2} e^{-k^2/2\sigma_b^2}, P_{o1}(k) = \frac{k}{\sigma_{o1}^2} e^{-k^2/2\sigma_{o1}^2}, P_{o2}(k) = \frac{k}{\sigma_{o2}^2} e^{-k^2/2\sigma_{o2}^2}$$

Robot SeeWee wants to segment the image into object o1, object o2 and background. Assuming that background and object pixels are equally likely, find the decision rule that maximizes the probability of a correct decision. Hint: more than one thresholds are needed.

5. **Coding (20 pts).** Please use the [image](#) (download by clicking) to perform following operations. Hint: we only need one channel for grayscale image, instead of RGB channels.



- d) (5 pts) Compute the Sobel edges of the grayscale image dot-blot.jpg
- e) (10 pts) Compute the Marr-Hildreth edges (zero-crossings of $\nabla^2 g_\sigma * f$) for various values of σ . Try 1, 2, 4, 8, 16. Do you get closed contours?
- f) (5 pts) Design a 2D filter (e.g., low-pass, band-pass, or high-pass) on Fourier domain to perform the function of edge detection. Hint: Try to avoid sharp edges of the filter to prevent Gibb's ringing effect. Figure below is an example, see if you can achieve similarly if not better.

