Fall 2022

HW #7

1. **Object Signatures (30%):** Please draw and describe the function to represent $r(\theta)$ for the following objects

a. Ellipse (10%)	A A
b. Dome (10%)	R
c. Rectangle (10%)	W

- 2. Classification(30%): Suppose we have a 2-class classification problem with class means μA and μB . Assuming that both classes are equally likely, show that the Nearest Mean classifier decision boundary is the hyper-plane perpendicular to, and midway along, the line segment connecting μA to μB . No need to assume any particular distribution for classes A and B.
- 3. Object Representation (40%): We can represent an object by its boundary $(x(s), y(s)), 0 \le s \le S$ where S is the length of the object's boundary and s is distance along that boundary from some arbitrary starting point. Combine x and y into a single complex function z(s) = x(s) + jy(s). The Discrete Fourier Transform (DFT) of z is

$$Z(k) = \sum_{s=0}^{S-1} e^{-2\pi j \frac{ks}{S}} z(s), 0 \le k \le S - 1$$

We can use the coefficients Z(k) to represent the object boundary. The limit on s is S-1 because for a closed contour z(S) = z(0). The Inverse Discrete Fourier Transform is

$$z(s) = \frac{1}{S} \sum_{k=0}^{S-1} e^{+2\pi j \frac{ks}{S}} Z(k), 0 \le s \le S - 1$$

- a. (15%) Suppose that the object is translated by $(\Delta x, \Delta y)$, that is, $z'(s) = z(s) + \Delta x + j\Delta y$. How is z''s DFT Z'(k) related to Z(k)?
- b. (10%) What object has $z(s) = R \cos \frac{2\pi s}{s} + jR \sin \frac{4\pi s}{s}$? Sketch it. Hint: This is infinitely easy!
- c. (15%) What is Z(k) corresponding to z(s) from Part b? Hint: Most coefficients are 0.