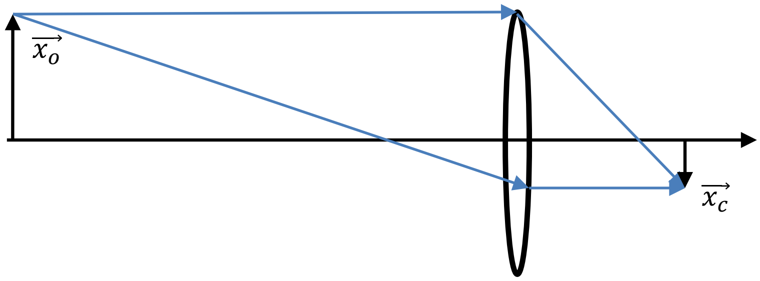
CS/RBE 549 Computer Vision Name:______________________________
Fall 2022

HW #1

1. Prove that for a thin lens, the image is in focus when

$$\frac{1}{-z_o} + \frac{1}{z_c} = \frac{1}{f}$$



Reason as follows: A ray leaving the object at $\overrightarrow{x_o} = (y_o, z_o)$ parallel to the z axis passes through the lens, then bends to pass through focal point $(0, f)$ before hitting the image plane at $\overrightarrow{x_c} = (y_c, z_c)$. If the image is in focus, then similarly, a ray leaving the object at $\overrightarrow{x_o}$ passing through the negative focal point $(0, -f)$ will be bent parallel to the z axis and hit the image plane at the same point $\overrightarrow{x_c}$.

Hint: As discussed in class, consider similar triangles from the lens to the focal point and the focal point to the image plane. There are 2 pairs of similar triangles, one for the positive and negative focal point. Then show that

$$-z_c f + z_o f = z_c z_o$$

2. A typical human eyeball is 2.4 cm in diameter and contains roughly 150,000,000 receptors. Ignoring the fovea, assume that the receptors are uniformly distributed across a hemisphere (it is actually closer to 160°).
   a. How many receptors are there per mm^2?
   b. Mars has a diameter of 8,000 km and an average distance from Earth of 225,000,000 km. Using a value of f equal to the eye's diameter, on how many receptors does the image of Mars fall?

3. An image has object and background pixels whose brightness values are distributed according to the Gaussian (Normal) distribution with the same mean μ but different variances σ_o and σ_b with $\sigma_o < \sigma_b$.

$$P_o(x) = \frac{1}{\sqrt{2\pi}\sigma_o} e^{-\frac{1}{2}\frac{x^2}{\sigma_o{}^2}} \text{ and } P_b(x) = \frac{1}{\sqrt{2\pi}\sigma_b} e^{-\frac{1}{2}\frac{x^2}{\sigma_b{}^2}}$$

It is desired to segment the image into object and background. Show that the decision rule that maximizes the probability of a correct decision is to label each pixel with brightness x as *object* if

$$|x - \mu| < \sigma_o \sigma_b \sqrt{2 \frac{\ln \sigma_b - \ln \sigma_o}{\sigma_b^2 - \sigma_o^2}}$$

and *background* otherwise.

4. In heterogeneous coordinates, a rotation followed by a translation is represented by

$$\begin{bmatrix} \boldsymbol{R} & \vdots & \vec{T} \\ \cdots & & \cdots \\ 0 & \vdots & 1 \end{bmatrix}$$

What is the inverse operation? Hint: It is not $\begin{bmatrix} \boldsymbol{R}^{-1} & \vdots & -\vec{T} \\ \cdots & & \cdots \\ 0 & \vdots & 1 \end{bmatrix}$.

5. Not for extra credit: Binarize your selfie from HW0 in 2 ways:

   a. Threshold the image such that ~20% of the pixels are black and ~80% are white.
   b. Some other method of your choosing. Explain your method.