CS/RBE 549 Computer Vision — Name: **Sol U. Tion**

Fall 2020 — Thu 10 Dec 3:30 pm - Fri 11 Dec 11:59 pm

Exam #2 Solutions

1. **Optical Flow (30 pts):** Suppose the image brightness is given by

$$I(x,y,t) = I_0 + \ln\big((x - c_1 t)(y - c_2 t)\big)$$

Ignore any singularity when $\ln(\ldots) = 0$

   a. (10 pts) What are I_x, I_y, and I_t?

$$\boldsymbol{I_x = \frac{1}{(x - c_1 t)}, I_y = \frac{1}{(y - c_2 t)}, I_t = -\frac{c_1}{(x - c_1 t)} - \frac{c_2}{(y - c_2 t)}}$$

   b. (10 pts) Express the Optical Flow Constraint Equation $I_x u + I_y v + I_t = 0$ in the simplest terms possible for this image sequence, that is, with no fractions.

$$\boldsymbol{\frac{1}{(x - c_1 t)} u + \frac{1}{(y - c_2 t)} v - \frac{c_1}{(x - c_1 t)} - \frac{c_2}{(y - c_2 t)} = 0}$$

$$\boldsymbol{(y - c_2 t)(u - c_1) + (x - c_1 t)(v - c_2) = 0}$$

   c. (10 pts) The equation from b. must hold for all *x*, *y*, and *t*. Find a constant solution for *u* and *v* that makes this true, that is, such that *u* and *v* do not depend on *x*, *y*, and *t*.

   Only true when $\boldsymbol{(u - c_1) = 0, (v - c_2) = 0 \Rightarrow u = c_1, v = c_2}$

2. **Moving Camera (25 pts):** Suppose that the viewer (camera) is moving. We can model this in the imaging equations

$$\vec{X}^C = R\vec{X}^W + \vec{T} \text{ and } \vec{X}^I = \frac{\left|\vec{f}\right|^2 \vec{X}^C}{\vec{f} \cdot \vec{X}^C}$$

   by letting R and $\vec{T}$ depend on time, $R = R(t), \vec{T} = \vec{T}(t)$. Assume that the camera is translating with velocity $\vec{V}^C \equiv \frac{d}{dt}\vec{T}(t)$ and that there is no rotation, $\frac{d}{dt}R(t) = 0$. As we know from experience, points in the image will seem to move.

   a. (15 pts) Show that image point $\vec{X}^I$ will appear to move with velocity

$$\vec{V}^I \equiv \frac{d}{dt}\vec{X}^I(t) = \frac{1}{\vec{f} \cdot \vec{X}^C}\big((\vec{X}^I \times \vec{V}^C) \times \vec{f}\big)$$

Hint: Use the vector triple product formula $(\vec{a}\times\vec{b})\times\vec{c} = (\vec{c}\cdot\vec{a})\vec{b} - (\vec{b}\cdot\vec{c})\vec{a}$.

$$\vec{\boldsymbol{V}}^{\boldsymbol{I}} \equiv \frac{\boldsymbol{d}}{\boldsymbol{dt}}\vec{\boldsymbol{X}}^{\boldsymbol{I}}(\boldsymbol{t}) = \frac{\boldsymbol{d}}{\boldsymbol{dt}}\frac{|\vec{\boldsymbol{f}}|^2\vec{\boldsymbol{X}}^{\boldsymbol{C}}}{\vec{\boldsymbol{f}}\cdot\vec{\boldsymbol{X}}^{\boldsymbol{C}}} = \frac{(\vec{\boldsymbol{f}}\cdot\vec{\boldsymbol{X}}^{\boldsymbol{C}})\frac{\boldsymbol{d}}{\boldsymbol{dt}}\left(|\vec{\boldsymbol{f}}|^2\vec{\boldsymbol{X}}^{\boldsymbol{C}}\right) - |\vec{\boldsymbol{f}}|^2\vec{\boldsymbol{X}}^{\boldsymbol{C}}\frac{\boldsymbol{d}}{\boldsymbol{dt}}(\vec{\boldsymbol{f}}\cdot\vec{\boldsymbol{X}}^{\boldsymbol{C}})}{(\vec{\boldsymbol{f}}\cdot\vec{\boldsymbol{X}}^{\boldsymbol{C}})^2}$$

$$= \frac{(\vec{\boldsymbol{f}}\cdot\vec{\boldsymbol{X}}^{\boldsymbol{C}})|\vec{\boldsymbol{f}}|^2\vec{\boldsymbol{V}}^{\boldsymbol{C}} - |\vec{\boldsymbol{f}}|^2\vec{\boldsymbol{X}}^{\boldsymbol{C}}(\vec{\boldsymbol{f}}\cdot\vec{\boldsymbol{V}}^{\boldsymbol{C}})}{(\vec{\boldsymbol{f}}\cdot\vec{\boldsymbol{X}}^{\boldsymbol{C}})^2}$$

$$= \frac{\mathbf{1}}{(\vec{\boldsymbol{f}}\cdot\vec{\boldsymbol{X}}^{\boldsymbol{C}})^2}\left((\vec{\boldsymbol{f}}\cdot\vec{\boldsymbol{X}}^{\boldsymbol{C}})(\vec{\boldsymbol{f}}\cdot\vec{\boldsymbol{f}})\vec{\boldsymbol{V}}^{\boldsymbol{C}} - (\vec{\boldsymbol{V}}^{\boldsymbol{C}}\cdot\vec{\boldsymbol{f}})(\vec{\boldsymbol{f}}\cdot\vec{\boldsymbol{X}}^{\boldsymbol{C}})\frac{|\vec{\boldsymbol{f}}|^2}{(\vec{\boldsymbol{f}}\cdot\vec{\boldsymbol{X}}^{\boldsymbol{C}})}\vec{\boldsymbol{X}}^{\boldsymbol{c}}\right)$$

$$= \frac{(\vec{\boldsymbol{f}}\cdot\vec{\boldsymbol{X}}^{\boldsymbol{C}})}{(\vec{\boldsymbol{f}}\cdot\vec{\boldsymbol{X}}^{\boldsymbol{C}})^2}\left((\vec{\boldsymbol{f}}\cdot\vec{\boldsymbol{f}})\vec{\boldsymbol{V}}^{\boldsymbol{C}} - (\vec{\boldsymbol{V}}^{\boldsymbol{C}}\cdot\vec{\boldsymbol{f}})\frac{|\vec{\boldsymbol{f}}|^2}{(\vec{\boldsymbol{f}}\cdot\vec{\boldsymbol{X}}^{\boldsymbol{C}})}\vec{\boldsymbol{X}}^{\boldsymbol{c}}\right)$$

$$= \frac{\mathbf{1}}{\vec{\boldsymbol{f}}\cdot\vec{\boldsymbol{X}}^{\boldsymbol{C}}}\left((\vec{\boldsymbol{f}}\cdot\vec{\boldsymbol{X}}^{\boldsymbol{I}})\vec{\boldsymbol{V}}^{\boldsymbol{C}} - (\vec{\boldsymbol{V}}^{\boldsymbol{C}}\cdot\vec{\boldsymbol{f}})\vec{\boldsymbol{X}}^{\boldsymbol{I}}\right)$$

$$= \frac{\mathbf{1}}{\vec{\boldsymbol{f}}\cdot\vec{\boldsymbol{X}}^{\boldsymbol{C}}}\left((\vec{\boldsymbol{X}}^{\boldsymbol{I}}\times\vec{\boldsymbol{V}}^{\boldsymbol{C}})\times\vec{\boldsymbol{f}}\right)$$

b. (10 pts) The Focus of Expansion is the point in the image toward which the camera appears to be moving, given by the projection of $\vec{V}^C$ into the image. That is,

$$\vec{X}^I{}_{\mathrm{FOE}} = \frac{|\vec{f}|^2\vec{V}^C}{\vec{f}\cdot\vec{V}^C}$$

Points in the image will seem to move away from the Focus of Expansion. Show that a point at the Focus of Expansion does not appear to move.

$$\vec{\boldsymbol{V}}^{\boldsymbol{I}}{}_{\mathbf{FOE}} = \frac{\mathbf{1}}{\vec{\boldsymbol{f}}\cdot\vec{\boldsymbol{X}}^{\boldsymbol{C}}}\left((\vec{\boldsymbol{X}}^{\boldsymbol{I}}{}_{\mathbf{FOE}}\times\vec{\boldsymbol{V}}^{\boldsymbol{C}})\times\vec{\boldsymbol{f}}\right) = \frac{\mathbf{1}}{\vec{\boldsymbol{f}}\cdot\vec{\boldsymbol{X}}^{\boldsymbol{C}}}\left(\left(\frac{|\vec{\boldsymbol{f}}|^2}{\vec{\boldsymbol{f}}\cdot\vec{\boldsymbol{V}}^{\boldsymbol{C}}}\vec{\boldsymbol{V}}^{\boldsymbol{C}}\times\vec{\boldsymbol{V}}^{\boldsymbol{C}}\right)\times\vec{\boldsymbol{f}}\right)$$

$$= \frac{\mathbf{1}}{\vec{\boldsymbol{f}}\cdot\vec{\boldsymbol{X}}^{\boldsymbol{C}}}\left((\vec{\mathbf{0}}\times\vec{\boldsymbol{V}}^{\boldsymbol{C}})\times\vec{\boldsymbol{f}}\right) = \vec{\mathbf{0}}$$

3. **Time to Impact – Volume (10 pts):**
We saw in class that time-to-impact of a ball, with radius $R(t)$ in the image, heading towards a viewer (ignoring gravity, wind resistance, etc.) is given by

$$T_{\text{Impact}} = \frac{R(t)}{\frac{d}{dt}R(t)}$$

Using his X-ray vision to see in 3 dimensions, Superman can use measure the volume $V(t)$ of a ball heading towards him. What is T_{impact} in terms of $V(t)$ and its derivative with respect to time $\frac{d}{dt}V(t)$?

You could derive this from scratch, following the same approach as in the class notes. Or you could use $\boldsymbol{V = \frac{4}{3}\pi R^3, \frac{dV}{dt} = 4\pi R^2 \frac{dR}{dt}}$ to derive

$$\boldsymbol{T_{\text{Impact}} = \frac{R(t)}{\frac{d}{dt}R(t)} = \frac{\frac{4}{3}\pi R^3(t)}{\frac{4}{3}\pi R^2 \frac{d}{dt}R(t)} = 3\frac{V(t)}{\frac{d}{dt}V(t)}}$$

4. **Stereo Velocity (15 pts):** We saw that one way to formulate the stereo problem yields

$$\vec{X}^W = \vec{X}_{AVG}\frac{|\vec{B}|^2}{\vec{B}\cdot\vec{\Delta}}$$

as the solution to finding $\vec{X}^W$ from a stereo pair. If the world point is moving with velocity $\vec{V}^W$, then image points $\vec{X}_L$ and $\vec{X}_R$ will appear to move also. Express $\vec{V}^W$ in terms of $\vec{X}_L$ and $\vec{X}_R$ and their time derivatives $\vec{V}_L$ and $\vec{V}_R$

$$\boldsymbol{\vec{V}^W = \frac{d}{dt}\left(\vec{X}_{AVG}\frac{|\vec{B}|^2}{\vec{B}\cdot\vec{\Delta}}\right) = \frac{(\vec{B}\cdot\vec{\Delta})\frac{d}{dt}\left(\vec{X}_{AVG}|\vec{B}|^2\right) - \left(\vec{X}_{AVG}|\vec{B}|^2\right)\frac{d}{dt}(\vec{B}\cdot\vec{\Delta})}{(\vec{B}\cdot\vec{\Delta})^2}}$$

$$\boldsymbol{= \frac{(\vec{B}\cdot\vec{\Delta})\vec{V}_{AVG}|\vec{B}|^2 - \left(\vec{X}_{AVG}|\vec{B}|^2\right)\left(\vec{B}\cdot\frac{d}{dt}\vec{\Delta}\right)}{(\vec{B}\cdot\vec{\Delta})^2}}$$

$$\boldsymbol{= \frac{\vec{V}_{AVG}|\vec{B}|^2}{\vec{B}\cdot\vec{\Delta}} - \frac{\left(\vec{X}_{AVG}|\vec{B}|^2\right)\vec{B}\cdot\left(\frac{d}{dt}\vec{\Delta}\right)}{(\vec{B}\cdot\vec{\Delta})^2}}$$

$$\boldsymbol{= \frac{|\vec{B}|^2}{2}\left(\frac{(\vec{V}_L+\vec{V}_R)}{\vec{B}\cdot(\vec{X}_L-\vec{X}_R)} - \frac{(\vec{X}_L+\vec{X}_R)\vec{B}\cdot(\vec{V}_L-\vec{V}_R)}{(\vec{B}\cdot(\vec{X}_L-\vec{X}_R))^2}\right)}$$

5. **Sonar Vision (10 pts):** In the movie *Batman Forever*, Batman (Bruce Wayne) wears a "Sonar Suit," which displays a sonar image of his environment. Although the physics of

sound vision are never quite explained, presumably, the suit emits sound waves, which are then reflected back to him. Although sound waves can be focused, this is difficult to do. More commonly, sound waves are detected by a phased array of receivers. However they are detected, objects tend to lack distinct surface markings, because the reflected sound depends only on distance, surface orientation, and material reflectivity. *Briefly* discuss some of the challenges imaging the world using Sonar Vision.

a. **Cannot discern surface markings**
b. **Need a lot of processing to reconstruct images**
c. **Some materials, such as clothing, absorb ultrasound well**
d. **Multiple echoes (reflections)**
e. **Noise / interference**
f. **Lack of resolution might or might not be a challenge. The speed of sound through air is 343 m/s, so ultrasound at 20 kHz and above has a wavelength of no more than 1.7 cm.**
g. **Unlike light, sound can be easily refracted around objects, which might be an advantage**
h. **Batman's sonar could confuse any real bats in the vicinity**

6. **Object Detection (10 pts):** Based on your experience in Computer Vision, you are hired to design a pedestrian detection system for self-driving cars. List 4 types of knowledge / sources of information that you could use.

Shape: Elongated rectangle
Orientation: Vertical
Speed: 0-4mph, possibly as high as 12 mph when running
Location: Often on side of road or in crosswalk, but could be anywhere
Color / brightness: Any. Not usually a light source, unless carrying a small source, such as a flashlight or beacon
Texture: Non-uniform due to clothing, body parts, etc.
Nearby objects: Possibly other pedestrians, baby strollers, shopping carts, skateboard, etc.
Other?

How would you convince a governmental regulatory agency that the system is effective?
Show high selectivity / recall / accuracy / AUC on testing

Exhaustive or extensive testing

Drive for 1B miles with a human supervisor on board with no pedestrian collisions

Confusion matrix = identity

You can't

Note: Bribery is **not an acceptable answer**

7. **Teamwork (1 pt):** On a scale of 1 to 5, with 1 being the lowest, 3 is neutral and 5 being the highest, rate how well your project team is working together.

5 – Ashay and Haoxiang are great to work with.