CS/RBE 549 Computer Vision Name:______________________________
Fall 2022

HW #12

1. **Inexact Match (60%):** In stereo imaging, if the rays defined by $\vec{X}_L$ and $\vec{X}_R$ do not intersect, we can find $\vec{X}^W$ anyway by minimizing an error measure. One way to do this is to project $\vec{X}^W$ into the left and right image planes to give $\vec{X}_L{}'$ and $\vec{X}_R{}'$. The error *E* is defined as the squared difference between the observed image points $\vec{X}_L$ and $\vec{X}_R$ and the projected image points $\vec{X}_L{}'$ and $\vec{X}_R{}'$.
   a. Show that for the simplified parallel optical axis camera geometry used in class where $\vec{f} \cdot \vec{B} = 0, R = I, \vec{T} = \pm\frac{\vec{B}}{2}$

   $$E \equiv \left|\vec{X}_L' - \vec{X}_L\right|^2 + \left|\vec{X}_R' - \vec{X}_R\right|^2$$
   $$= \left(\frac{f}{z^W}\left(x^W + \frac{b}{2}\right) - x_L\right)^2 + \left(\frac{f}{z^W}y^W - y_L\right)^2 + \left(\frac{f}{z^W}\left(x^W - \frac{b}{2}\right) - x_R\right)^2 + \left(\frac{f}{z^W}y^W - y_R\right)^2$$

   b. By differentiating *E* with respect to x^W and y^W, show that

   $$x^W = \frac{x_L + x_R}{2}\frac{z^W}{f} \text{ and } y^W = \frac{y_L + y_R}{2}\frac{z^W}{f}$$

   c. By differentiating *E* with respect to z^W and using the results of b., show that

   $$z^W = \frac{fb}{\Delta_x}$$

   and conclude that

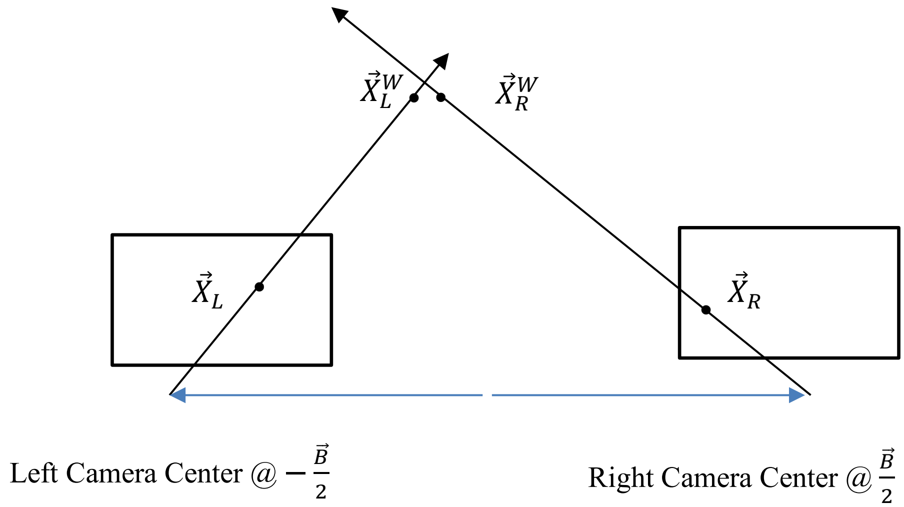
   $$\vec{X}^W = \vec{X}_{AVG}\frac{\left|\vec{B}\right|^2}{\vec{B} \cdot \vec{\Delta}}$$

2. **Inexact Match, Alternative Version (40%):** Another way to determine distance when $\vec{X}_L$ and $\vec{X}_R$ do not intersect is to find the point $\vec{X}^W$ closest to the rays defined by $\vec{X}_L$ and $\vec{X}_R$. The left image ray is defined by

   $$\vec{X}_L^W = -\frac{\vec{B}}{2} + l\vec{X}_L, 0 \le l$$

   Similarly, the right image ray is defined by

   $$\vec{X}_R^W = \frac{\vec{B}}{2} + r\vec{X}_R, 0 \le r$$



The goal is to find points $\vec{X}_L^W$ and $\vec{X}_R^W$ on the left and right image rays that are closest to each other; the point equidistant between them is the desired $\vec{X}^W$. We can find these points by minimizing the following:

$$\min_{l,r} \left|\vec{X}_L^W - \vec{X}_R^W\right|^2$$

Show that the solution is given by

$$\vec{X}^W = \frac{1}{2}\frac{\left(\left|\vec{X}_R\right|^2(\vec{X}_L\cdot\vec{B}) - (\vec{X}_L\cdot\vec{X}_R)(\vec{X}_R\cdot\vec{B})\right)\vec{X}_L + \left((\vec{X}_L\cdot\vec{X}_R)(\vec{X}_L\cdot\vec{B}) - \left|\vec{X}_L\right|^2(\vec{X}_R\cdot\vec{B})\right)\vec{X}_R}{\left|\vec{X}_L\right|^2\left|\vec{X}_R\right|^2 - (\vec{X}_L\cdot\vec{X}_R)^2}$$