CS/RBE 549 Computer Vision (Online) Name:______________________________

Fall 2022

HW #8

1. **Binary Image Matching (30%):** Let I_1 and I_2 be binary images. Show that
$$|I_1 - I_2|^2 = \sum \text{\# of pixels where } I_1 \neq I_2$$
Where $|I|^2 = \sum i_{jk}^2$ is the sum of all (pixels squared) in *I*.

2. **Bayes classifier for 3D patterns (40%):** Let's explore the mechanics of the preceding development. We assume that the patterns are samples from two Gaussian populations, and that classes c_1 and c_2 occurs equally likely, c_1 has sample pattern vectors (1,2,3), (2,2,4), (2,2,3), and (2,3,3), and c_2 has sample pattern vectors (1,2,4), (1,3,4), (2,3,4), (1,3,3)
   a. what is m_1 and m_2 when applying $m_j = \frac{1}{n_j}\sum_{x \in c_j} x$ to the sample pattern vectors? Note that n_j is the number of sample pattern vectors from class c_j, and the summation is taken over these vectors.
   b. from $\mathrm{C}_j = \frac{1}{n_j}\sum_{\mathrm{x} \in c_j} \mathrm{x}\,\mathrm{x}^T - m_j m_j^T$, what does $\mathrm{C}_1 = \mathrm{C}_2 =$?, and the inverse of this matrix?
   c. what are the decision functions? Assuming classes are equally likely as $d_j(\mathrm{x}) = \mathrm{x}^T\mathrm{C}^{-1}\mathrm{m}_j - \frac{1}{2}\mathrm{m}_j^T\mathrm{C}^{-1}\mathrm{m}_j$.
   d. The decision boundary separating the two classes is $d_1(\mathrm{x}) - d_2(\mathrm{x})$

3. PCA analysis (30%): For six multispectral satellite images corresponding to six spectral bands,
   a. please organize the images that leads to the formation of a six-element vector x from each set of corresponding pixels in the images (256×256), so the population consisted of $256^2 = 65536$ vectors from which the **mean vector** $\mathrm{m_x} = E\{\mathrm{x}\}$, **covariance matrix** $\mathrm{C_x} = E\{(\mathrm{x} - \mathrm{m_x})(\mathrm{x} - \mathrm{m_x})^T\}$, and corresponding **eigenvalues and eigenvectors** can be computed.
   b. using eigenvectors as rows of matrix A, compute a set of **y vectors** according to $\mathrm{y} = \mathrm{A}(\mathrm{x} - \mathrm{m_x})$. Similarly compute $\mathbf{C_y} = \mathrm{AC_xA^T}$ which is a matrix with diagonal elements as eigenvalues of $\mathrm{C_x}$.
   c. generate a set of six principal component images using the y vectors, notice the significant portion of the contrast detail is contained in the first two images and decreases rapidly from there.