RBE549 - Homework 10

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Problem 1

In this problem ,we are asked to consider two surfaces:

$$z_1 = \frac{1}{2}\ln(x^2+y^2) \tag{1}$$

...and...

$$z_2 = \tan^- 1(\frac{x}{y}) \tag{2}$$

A

We are asked to find $p(x,y)$ and $q(x,y)$ for both surfaces. $p(x,y) = \frac{\partial z_n}{\partial x}$ and $q(x,y) = \frac{\partial z_n}{d_y}$. So we solve for each:

$$p_1 = \frac{\partial z_1}{\partial x} = \frac{\partial}{\partial x}\frac{1}{2}\ln(x^2+y^2) = \frac{x}{x^2+y^2} \tag{3}$$

$$q_1 = \frac{\partial z_1}{\partial y} = \frac{\partial}{\partial y}\frac{1}{2}\ln(x^2+y^2) = \frac{y}{x^2+y^2} \tag{4}$$

$$p_2 = \frac{\partial z_2}{\partial x} = \frac{\partial}{\partial x}\tan^- 1(\frac{x}{y}) = \frac{y}{x^2+y^2} \tag{5}$$

$$q_2 = \frac{\partial z_2}{\partial y} = \frac{\partial}{\partial y}\tan^- 1(\frac{x}{y}) = -\frac{x}{x^2+y^2} \tag{6}$$

B

Here, we are tasked in showing that z_1 and z_2 gives rise to the same shading when a rotationally symmetric reflectance map applies; $R(p,q) = R(p^2+q^2)$.

Here, we can show this by demonstrating that $p_1^2 + q_1^2 = p_2^2 + q^2$.

$$p_1^2 + q_1^2 = p_2^2 + q^2 \tag{7}$$

$$\frac{x}{x^2+y^2}^2 + \frac{y}{x^2+y^2}^2 = \frac{y}{x^2+y^2}^2 + -\frac{x}{x^2+y^2}^2 \tag{8}$$

$$\frac{x^2}{x^2+y^2} + \frac{y^2}{(x^2+y^2)^2} = \frac{y^2}{(x^2+y^2)^2} + \frac{x^2}{x^2+y^2} \tag{9}$$

$$\frac{1}{x^2+y^2} = \frac{1}{x^2+y^2} \tag{10}$$

1 Problem 2

In this problem, we're looking at a surface normal $\hat{n} = \begin{bmatrix} n_x & n_y & n_z \end{bmatrix}^T$, where $n_x^2 + n_y^2 + n_z^2 = 1$, and can be represented by surface orientation $p, q . p = \frac{n_x}{\sqrt{1 - n_x^2 - n_y^2}}$, etc. In addition to the original origin, $\begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T$, we can use another origin from the point $\begin{bmatrix} 0 & 0 & -1 \end{bmatrix}^T$.

A

Here, we are tasked with solving for f and g. We can do this by solving for the line of the vector from our new origin point of $\begin{bmatrix} 0 & 0 & -1 \end{bmatrix}^T$.

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \end{bmatrix} + k \begin{bmatrix} l \\ m \\ n \end{bmatrix} \tag{11}$$

Note that the slopes represented by our variables can be further defined as:

$$\begin{bmatrix} l \\ m \\ n \end{bmatrix} = \begin{bmatrix} x_2 - x_1 \\ y_2 - y_1 \\ z_2 - z_1 \end{bmatrix} \tag{12}$$

Our x and our y are the values we're seeking to solve for, f and g, respectively. Now we fill in our known values to solve for our line:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \end{bmatrix} + k \begin{bmatrix} x_2 - x_1 \\ y_2 - y_1 \\ z_2 - z_1 \end{bmatrix} \tag{13}$$

$$\begin{bmatrix} f \\ g \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} + k \begin{bmatrix} n_x - 0 \\ n_y - 0 \\ n_z + 1 \end{bmatrix} \tag{14}$$

We can then break this down into a system of equations. First, we solve for t using our z value as we have an additional known here.

$$1 = -1 + k(n_z + 1) \tag{15}$$

$$k = \frac{2}{n_z + 1} \tag{16}$$

...and now we can solve for f and g:

$$f = 0 + k(n_x - 0) = \frac{2n_x}{n_z + 1} \tag{17}$$

$$g = 0 + k(n_y - 0) = \frac{2n}{n_z + 1} \tag{18}$$

Resulting in a final f, g, z =

$$\begin{bmatrix} f \\ g \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{2n_x}{n_z+1} \\ \\ \frac{2n_y}{n_z+1} \\ \\ 1 \end{bmatrix} \tag{19}$$

B

Now we are taskedwith showing that when $n_z = 0$, f and g are finite and lie on a circle of radius 2.

$$f = \frac{2n_x}{n_z + 1} = \frac{2n_x}{0 + 1} = 2n_x \tag{20}$$

$$g = \frac{2n}{n_z + 1} = \frac{2n_y}{0 + 1} = 2n_y \tag{21}$$

Given these values, we can confirm the statement that f and g are finite as they depend upon n_x and n_y respectively. Now, the radius is the result of the magnitude of the vector. This means we can do the following:

$$|f, g| = \sqrt{f^2 + g^2} = \sqrt{(2n_x)^2 + (2n_y)^2} = \sqrt{4(\sqrt{1 - n_y^2})^2 + (2n_y)^2} = \sqrt{4 - 4n_y^2 + 4n_y^2} = \sqrt{4} = 2 \tag{22}$$