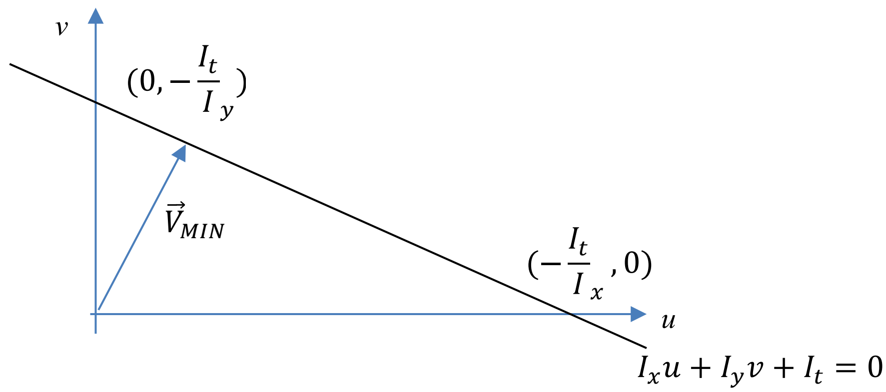
CS/RBE 549 Computer Vision Name:______________________________

Fall 2022

HW #10

1. **Smallest Optical Flow (30%):** What velocity $\vec{V}_{MIN}$ that satisfies the Optical Flow Constraint Equation $I_x u + I_y v + I_t = 0$ has the smallest magnitude $|\vec{V}|$? Hint: This can be solved geometrically as was outlined in class by considering the OFCE in u, v space.



2. **Optical Flow (30%):** Suppose the image brightness is given by

$$I(x, y, t) = I_0 + k\left(\tan^{-1}\left(\frac{x}{y}\right) - st\right)$$

   a. (10%) What are I_x, I_y, and I_t? Hint: You should find that these derivatives have a relatively simple form.

   b. (10%) Express the Optical Flow Constraint Equation $I_x u + I_y v + I_t = 0$ in the simplest terms possible for this image sequence.

   c. (10%) Show that the rotating flow field $u = sy, v = -sx$ is a solution to the OFCE

3. **Iterative Optical Flow (40%):** We learned in class an iterative method for computing optical flow, where at each iteration, the optical flow $u(x, y), v(x, y)$ is updated according to

$$\begin{bmatrix} u(x,y) \\ v(x,y) \end{bmatrix}^{\text{new}} = \begin{bmatrix} \lambda I_x^2 + 4 & \lambda I_x I_y \\ \lambda I_x I_y & \lambda I_y^2 + 4 \end{bmatrix}^{-1} \begin{bmatrix} \sum_{n \in \text{neighbors}(x,y)} u^{\text{old}}(n) - \lambda I_x I_t \\ \sum_{n \in \text{neighbors}(x,y)} v^{\text{old}}(n) - \lambda I_y I_t \end{bmatrix}$$

   a. (10%) Show that this is equivalent to

$$\begin{bmatrix} u(x,y) \\ v(x,y) \end{bmatrix}^{\text{new}} = \frac{1}{4\lambda I_x^2 + 4\lambda I_y^2 + 16} \begin{bmatrix} \lambda I_y^2 + 4 & -\lambda I_x I_y \\ -\lambda I_x I_y & \lambda I_x^2 + 4 \end{bmatrix} \begin{bmatrix} \sum_{n \in \text{neighbors}(x,y)} u^{\text{old}}(n) - \lambda I_x I_t \\ \sum_{n \in \text{neighbors}(x,y)} v^{\text{old}}(n) - \lambda I_y I_t \end{bmatrix}$$

b. (20%) Show that this is equivalent to update equations

$$u^{\text{new}}(x,y) = \bar{u}^{\text{old}} - \frac{I_x}{I_x^2 + I_y^2 + \frac{4}{\lambda}} \left(I_x \bar{u}^{\text{old}} + I_y \bar{v}^{\text{old}} + I_t \right)$$

$$v^{\text{new}}(x,y) = \bar{v}^{\text{old}} - \frac{I_y}{I_x^2 + I_y^2 + \frac{4}{\lambda}} \left(I_x \bar{u}^{\text{old}} + I_y \bar{v}^{\text{old}} + I_t \right)$$

where $\bar{u}^{\text{old}}, \bar{v}^{\text{old}}$ are the averages of the 4 neighbors of $u(x,y), v(x,y)$. Hint: You only need to show this for u^{new} because v^{new} follows an identical derivation.

c. (10%) In the case that $\lambda = 0$, what do the update equations reduce to?