CS/RBE 549 Computer Vision
Fall 2022

Name:_____

HW #3

1. Use OpenCV to compute the Sobel edges of the grayscale image of HW0. Also compute the Marr-Hildreth edges (zero-crossings of $\nabla^2 g_\sigma * f$) for various values of σ. Try 1, 2, 4, 8, 16. Do you get closed contours?

2. Show that if $f(x)$
   a. is an odd function, that is, $f(x) = -f(-x)$, and
   b. it is pure imaginary, i.e. no real component,

   then its Fourier Transform is also odd and real. Hint: Decompose the Fourier Transform into 2 integrals: one from -∞ to 0 and another from 0 to ∞.

3. If an edge detector is as follows:

   Convolve image $f(\vec{x})$with $g_{\sigma_1} = \frac{1}{\sqrt{2\pi\sigma_1^2}} e^{-\frac{1|\vec{x}|^2}{2\,\sigma_1^2}}$ to form $h_1(\vec{x})$.

   Convolve $f(\vec{x})$with g_{σ_2} to form $h_2(\vec{x})$.

   Compute $h_3(\vec{x}) = \frac{h_2(\vec{x}) - h_1(\vec{x})}{\sigma_2 - \sigma_1}$.

   Find zero-crossings of $h_3(\vec{x})$.
   a. Describe how $h_3(\vec{x})$ can be computed by a single convolution with some kernel $g(\vec{x})$. What is the convolutional kernel $g(\vec{x})$?
   b. Sketch $g(\vec{x})$.
   c. What is the Fourier transform $G(\vec{u})$ of $g(\vec{x})$?
   d. As $\sigma_2 \to \sigma_1$, is this a good edge detector, that is, do zero-crossings of h_3 occur at edges? Why or why not? Hint: Consider $G(\vec{u})$ as $\sigma_2 \to \sigma_1$.

4. The following operators are proposed to detect diagonally oriented edges:

1	1	0
1	0	-1
0	-1	-1

NE

0	1	1
-1	0	1
-1	-1	0

NW

(a) How are these operators related to the Sobel H and V operators?
(b) Suggest two different ways in which to combine the NW and NE operators into a single measure of edge strength. What are the relative strengths and weaknesses of each?
(c) Express the NW operator as the convolution of [0 1;-1 0] and another different 2×2 operators

5. Your teammate proposes to convolve the Sobel vertical edge detector mask V with the 3x3 blurring mask B.

-1	0	1
-2	0	2
-1	0	1

V

2	3	2
3	4	3
2	3	2

B

a) What is the resulting combined convolutional mask for applying V first, then B?

b) Does convolving with B first then V produce the same result as convolving with V first then B? Why or why not?

c) Is the convolution $V * B$ separable into a convolution of an *x*-only mask (that is, a mask with dimensions M×1) followed by a *y*-only (with dimensions 1×N) mask? If not, explain why not. If so, what are the *x* and *y* masks?