CS/RBE 549 Computer Vision Name:_____
Fall 2019 Wed 11 Dec 2019

Exam #2

DO NOT OPEN THIS EXAMINATION UNTIL YOU ARE TOLD TO DO SO!

Write your name at the top of this page now.

This examination is open book and open notes.

No electronic computing / communications devices are permitted, except for access to electronic notes.

Write all your answers on the examination in the space provided. You may use the back of the examination for extra space. Partial credit will be given, but you must justify your work.

The examination will end exactly 90 minutes after it begins.
Good luck!

Problem 1: /25
Problem 2: /25
Problem 3: /30
Problem 4: /10
Problem 5: /10
Problem 6: /01

Total: /101

1. **Optical Flow (25 pts):** Suppose the image brightness is given by

$$I(x, y, t) = I_0 + \sqrt{(x - c_1 t)^2 + (y - c_2 t)^2}$$

   a. (5 pts) What are I_x, I_y, and I_t?

   b. (10 pts) Express the Optical Flow Constraint Equation $I_x u + I_y v + I_t = 0$ in the simplest terms possible for this image sequence.

   c. (10 pts) The equation from b. must hold for all *x*, *y*, and *t*. Find a constant solution for *u* and *v* that makes this true, that is, such that *u* and *v* do not depend on *x*, *y*, and *t*.

2. **Hough Velocity Space (25 pts):** The Optical Flow Constraint Equation can be used to define a Hough-type velocity space that can combine observations of the optical flow from many points.

   a. (5 pts) What are the coordinates in this space?

   b. (10 pts) What does the observation of a single point $I(x, y, t)$ and its derivatives in Image space map into in the Hough velocity space?

   c. (10 pts) P_1 and P_2 are points in the image. A small image patch in the neighborhood of P_1 has brightness $I(x, y, t) = 200 + 40x - 80y$. A small image patch in the neighborhood of P_2 has brightness $I(x, y, t) = 100 + 30x + 60y + 240t$. What image velocity is consistent with both of these image patches?

3. **Stereo via Singular Value Decomposition (30 pts):** Assume the usual stereo geometry, where the left and right cameras are offset by baseline $\vec{B}$ that is perpendicular to the common focal vector $\vec{F}$. Then the stereo imaging equations are

$$\vec{X}_L = \frac{\left|\vec{F}\right|^2}{\vec{F}\cdot\vec{X}^W}\left(\vec{X}^W + \frac{\vec{B}}{2}\right), \qquad \vec{X}_R = \frac{\left|\vec{F}\right|^2}{\vec{F}\cdot\vec{X}^W}\left(\vec{X}^W - \frac{\vec{B}}{2}\right)$$

In the presence of imaging errors or noise, these equations might not hold exactly. They can be approximated by

$$\vec{X}_L - \frac{\left|\vec{F}\right|^2}{\vec{F}\cdot\vec{X}^W}\left(\vec{X}^W + \frac{\vec{B}}{2}\right) \approx \vec{0}, \qquad \vec{X}_R - \frac{\left|\vec{F}\right|^2}{\vec{F}\cdot\vec{X}^W}\left(\vec{X}^W - \frac{\vec{B}}{2}\right) \approx \vec{0}$$

a. (15 pts) Show that these equations can be written as a 4x4 matrix operating on a column vector in homogeneous coordinates.

$$\begin{bmatrix} -f & 0 & x_L & -fb/2 \\ 0 & -f & y_L & 0 \\ -f & 0 & x_R & fb/2 \\ 0 & -f & y_R & 0 \end{bmatrix} \begin{bmatrix} x^W \\ y^W \\ z^W \\ 1 \end{bmatrix} \approx \vec{0}$$

Hint: Combine the approximate imaging equations into a single matrix equation, multiply to eliminate the denominators, and simplify, not necessarily in that order!

b. (5 pts) The above equation can be written as $A\tilde{X}' \approx \vec{0}$. We can use SVD to find the singular vector $\tilde{X}'$ that minimizes $\left|A\vec{X}\right|^2$ subject to $\left|\vec{X}\right|^2 = 1$. Express world point $\vec{X}^W = [x, y, z]^T$ in terms of $\tilde{X}' = [x', y', z', w']^T$.

c. (10 pts) When $y_L = y_R$, show that a. gives $z^W = \frac{fb}{d}$, where d is the disparity.

4. **Infra-Red Vision (10 pts):** As is well-known, the comic book hero Superman has Infra-Red (heat) vision. Although the physics of heat vision are never quite explained, presumably, Superman's eyes are sensitive to photons in the infra-red spectrum, which are emitted uniformly in all directions from warm objects. Unlike X-rays, infra-red light can be focused, although objects tend to lack distinct features because nearby parts of a single object tend to have the same or similar temperature. *Briefly* discuss some of the challenges imaging the world using Infra-Red Vision.

5. **Object Detection (10 pts):** Based on your experience in Computer Vision, you are hired to design a Stop Sign detection system. List 5 types of knowledge / sources of information that you could use.

   a.

   b.

   c.

   d.

   e.

6. **Teamwork (1 pt):** On a scale of 1 to 5, with 1 being the lowest, 3 is neutral and 5 being the highest, rate how well your project team is working together.