# RBE549 - Homework 7

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## Problem 1

In this problem we are shown a number of shapes and then wish to find the $r(\theta)$ for the given shapes. The image below shows this:

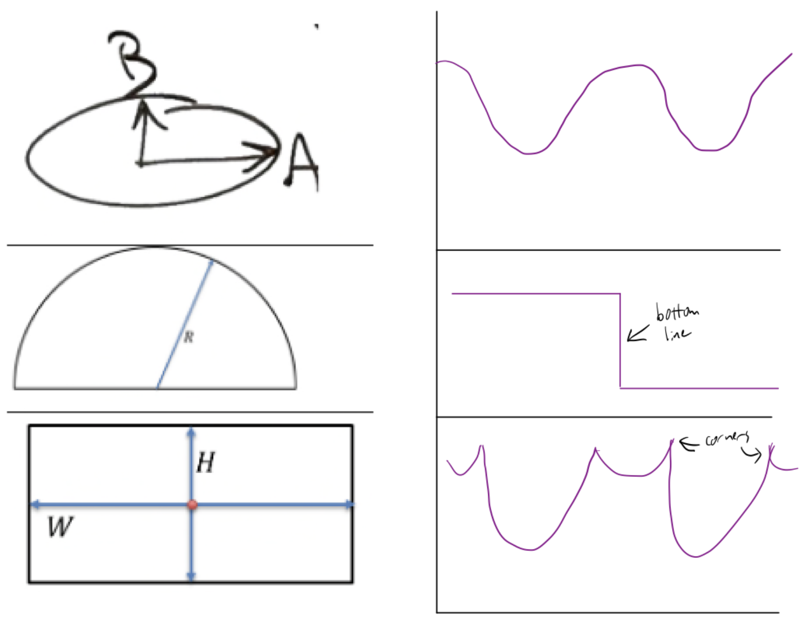


Figure 1: $r(\theta)$

## Problem 2

In this problem we are aske a hypothetical problem. Supposing that we have a 2-class classification problem with centroid points $\mu A$ and $\mu B$, and that each classes are equally likely, we need to show that the Nearest Mean classifier decision boundary is midway along the line segment connecting $\mu A$ to $\mu B$.

First, we can look at the problem in 1, 2, and 3 dimensions. For a $1D$ classifier, the resulting points would fall on a line and the resulting bounary would be between the centroids. If we looked at a $2D$ classifier, we would see the resulting decision boundary would be a line. Finally, for a $3D$ classifier, the decision boundary would be a plane. In each we depict the boundaries at a set distance $d$ between centroids.

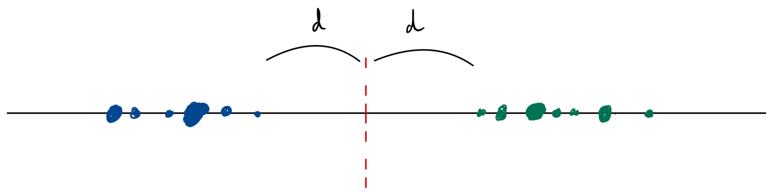


Figure 2: 1$D$ Classifier

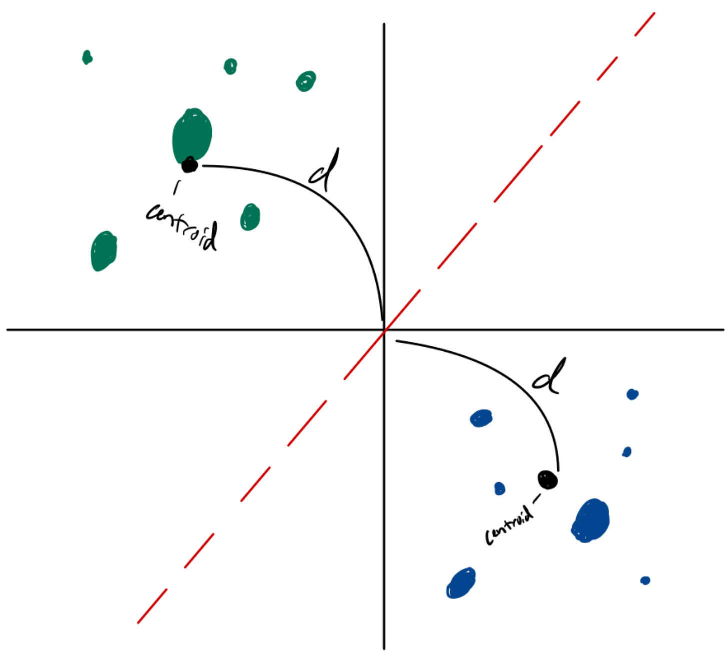


Figure 3: 1$D$ Classifier

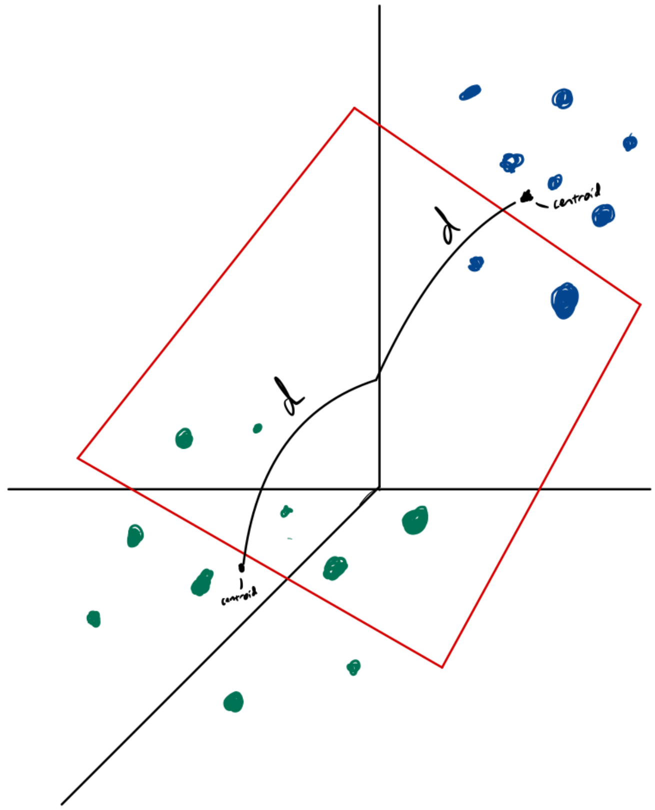


Figure 4: $3D$ Classifier

Since the problem states that the two classes $A$ and $B$ are equally likely, it means the the resulting centroids would be equidistant from a properly calculated Nearest Mean decision boundary. If either class had a higher probability than the other, the decision boundary as calculated via Nearest Mean would be closer to the centroid of that class.

# Problem 3

In this problem we are asked to represent an object by its boundary $(x(s), y(s)), 0 \leq s \leq S$ where $S$ is the length of the object's boundary and $s$ is distance along that bonudary from some arbitrary starting point. Combine $x$ and $y$ into a single complex function $z(a) = x(s) + jy(s)$. The Discrete Fourier Transform (DFT) of $z$ is:

$$Z(k) = \sum_{s=0}^{S-1} e^{-2*\pi j \frac{ks}{S}} z(s), 0 \leq k \leq S-1 \tag{1}$$

We can use the coefficients $Z(k)$ to represent the object boundary. The limit onf $s$ is $S-1$ because for a closed contour $Z(S) = z(0)$. The Inverse Discrete Fourier Transform is:

$$z(s) = \frac{1}{S} \sum_{k=0}^{S-1} e^{+2\pi j \frac{ks}{S}} Z(k), 0 \leq s \leq S-1 \tag{2}$$

## A

Here we suppose that the object is translated by $(\Delta x, \Delta y)$, such that $z'(s) = z(s) + \Delta x + j\Delta y$. How is $z'$'s DFT $Z'(k)$ related to $Z(k)$?

Stating again, that we start with:

$$Z(k) = \sum_{s=0}^{S-1} e^{-2*\pi j \frac{ks}{S}} z(s), 0 \leq k \leq S-1 \tag{3}$$

Which we can define $z'(s)$ as:

$$z'(s) = z(s) + \Delta x + j\Delta y \tag{4}$$

If we plug this into our original equation, we get...

$$Z'(k) = \sum_{s=0}^{S-1} e^{-2\pi j + \frac{ks}{S}} z(s) + \sum_{s=0}^{S-1} e^{-2\pi j + \frac{ks}{S}} (\Delta x + j\Delta y) \tag{5}$$

Here we see that we have a segment that is equivalent to oru defined $Z(k)$, so we can simplify by expressing:

$$Z(k) + \sum_{s=0}^{S-1} e^{-2\pi j + \frac{ks}{S}} (\Delta x + j\Delta y) \tag{6}$$

With $\Delta x$ and $j\Delta y$ isolated, we can use a table of known Fourier Transforms to identify the resulting conversion. Based on our problem's definition $\Delta X$ and $\Delta y$ are both constants, so we can state that:

$$Z'(k) = Z(k) + \sigma(k) \tag{7}$$

## B

What object has $z(s) = R\cos(\frac{2\pi s}{S} + jR\sin(\frac{4\pi s}{S}))$? Below we see a graph drawing the shape, and included code to generate it. Random values were chosen for the $S$ and $r$ for the sake of plotting it.

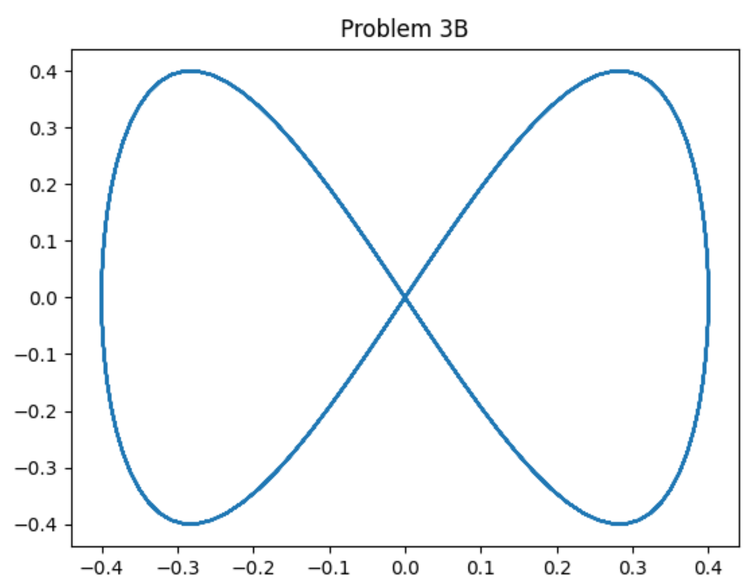


Figure 5: Our resulting shape

```python
import numpy as np
from numpy import cos, sin, pi
import matplotlib.pyplot as plt

figure = plt.figure()
plt.title("Problem 3B")

S = 10
r = 4

theta = [theta for theta in np.arange(0, S, 0.01)]
X = [
        r * cos(2*pi*theta)/S
        for theta in theta
    ]
Y = [
        r * sin(4*pi*theta)/S
        for theta in theta
    ]

# Plot the results
plt.plot(X, Y)
plt.savefig("./imgs/prob3_b.png")
```

## C