RBE549 - Homework 7

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Due date: November 1, 2022

Problem 1

In this problem we are shown a number of shapes and then wish to find the $r(\theta)$ for the given shapes. The image below shows this:

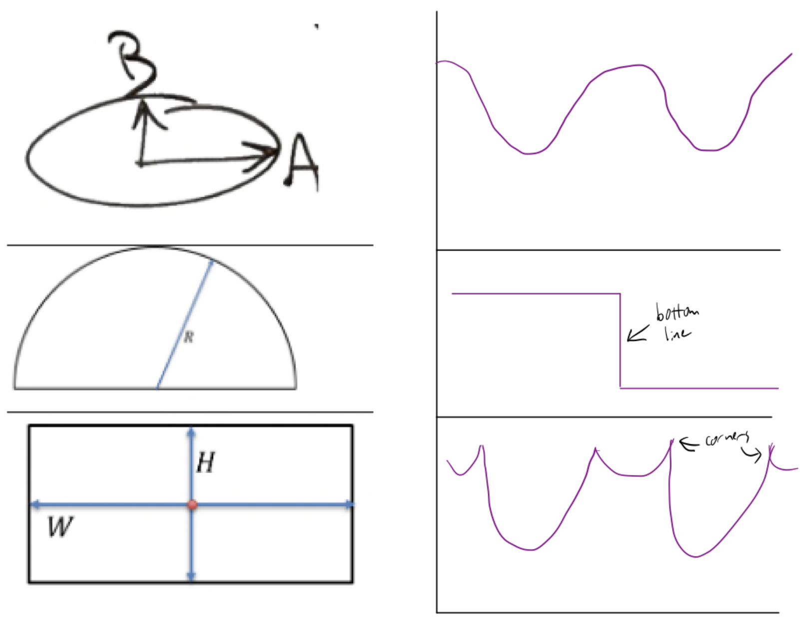

Figure 1: $r(\theta)$

Problem 2

In this problem we are aske a hypothetical problem. Supposing that we have a 2-class classification problem with centroid points μA and μB, and that each classes are equally likely, we need to show that the Nearest Mean classifier decision boundary is midwasy along the line segment connecting μA to μB.

First, we can look at the problem in 1, 2, and 3 dimensions. For a $1D$ classifier, the resulting points would fall on a line and the resulting bounary would be between the centroids. If we looked at a $2D$ classifier, we would see the resulting decision boundary would be a line. Finally, for a $3D$ classifier, the decision boundary would be a plane. In each we depict the boundaries at a set distance d between centroids.

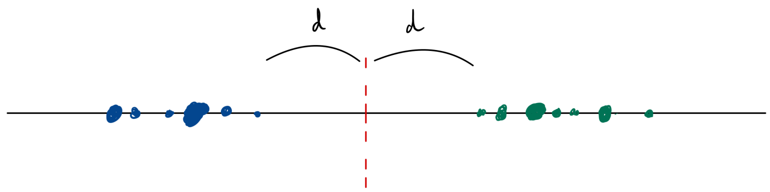

Figure 2: $1D$ Classifier

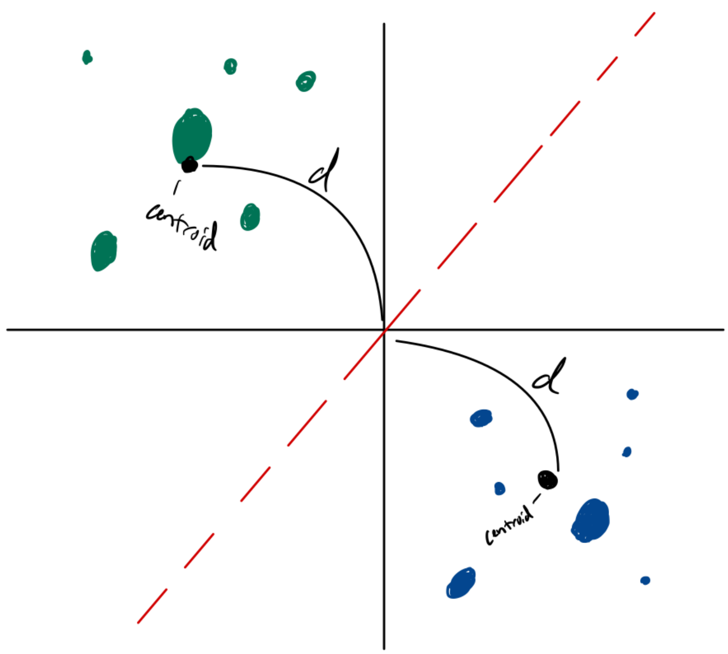

Figure 3: $1D$ Classifier

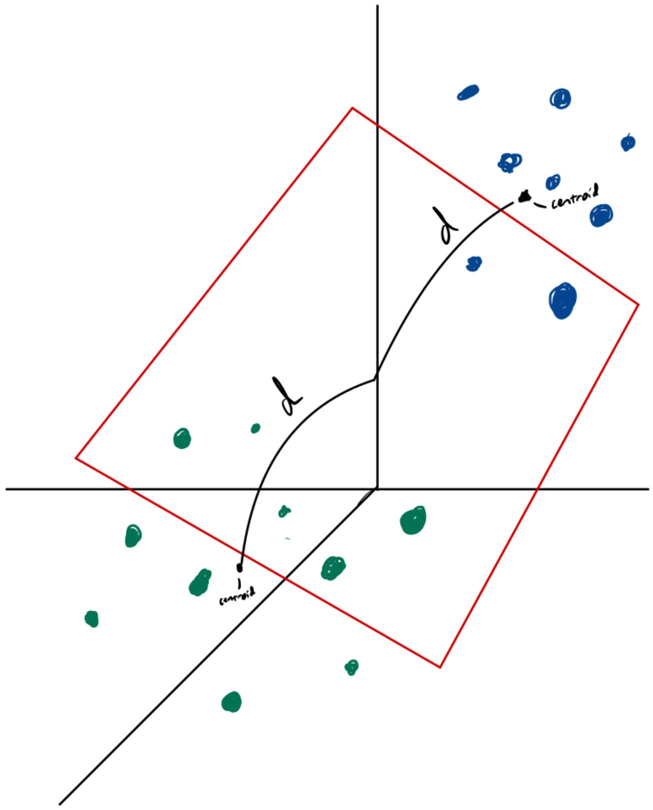

Figure 4: $3D$ Classifier

Problem 3

In this problem we are asked to represent an object by its boundary $(x(s), y(s)), 0 \leq s \leq S$ where S is the length of the object's boundary and s is distance along that bonudary from some arbitrary starting point. Combine x and y into a single complex function $z(a) = x(s) + jy(s)$. The Discrete Fourier Transform (DFT) of z is:

$$Z(k) = \sum_{s=0}^{S-1} e^{-2*\pi j \frac{ks}{S}} z(s), 0 \leq k \leq S-1 \tag{1}$$

We can use the coefficients $Z(k)$ to represent the object boundary. The limit onf s is $S-1$ because for a closed contour $Z(S) = z(0)$. The Inverse Discrete Fourier Transform is:

$$z(s) = \frac{1}{S} \sum_{k=0}^{S-1} e^{+2\pi j \frac{ks}{S}} Z(k), 0 \leq s \leq S-1 \tag{2}$$

A

B

What object has $z(s) = R\cos(\frac{2\pi s}{S} + jR\sin(\frac{4\pi s}{S}))$? Below we see a graph drawing the shape, and included code to generate it:

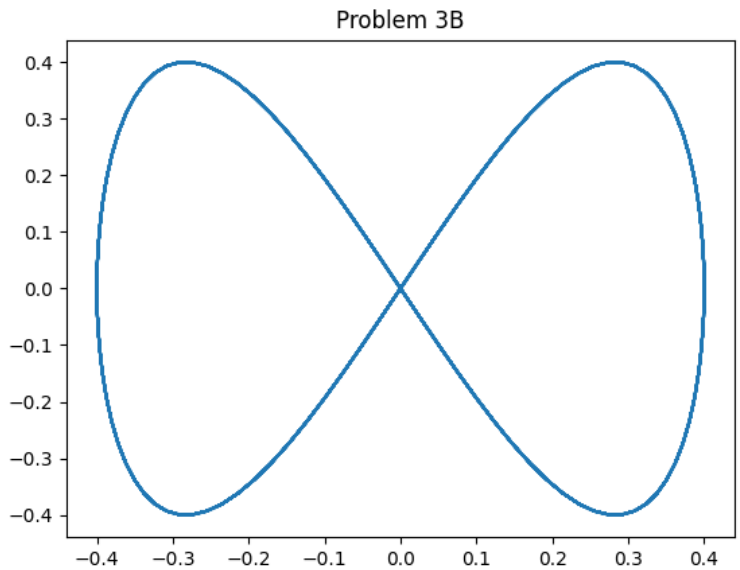

Figure 5: Our resulting shape

```
import numpy as np
from numpy import cos, sin, pi
import matplotlib.pyplot as plt

figure = plt.figure()
plt.title("Problem 3B")

S = 10
r = 4

theta = [theta for theta in np.arange(0, S, 0.01)]
X = [
        r * cos(2*pi*theta)/S
        for theta in theta
    ]
Y = [
        r * sin(4*pi*theta)/S
        for theta in theta
    ]

# Plot the results
plt.plot(X, Y)
plt.savefig("./imgs/prob3_b.png")
```

C