

RBE549 - Homework 7

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Due date: November 1, 2022

Problem 1

In this problem we are shown a number of shapes and then wish to find the $r(\theta)$ for the given shapes. The image below shows this:

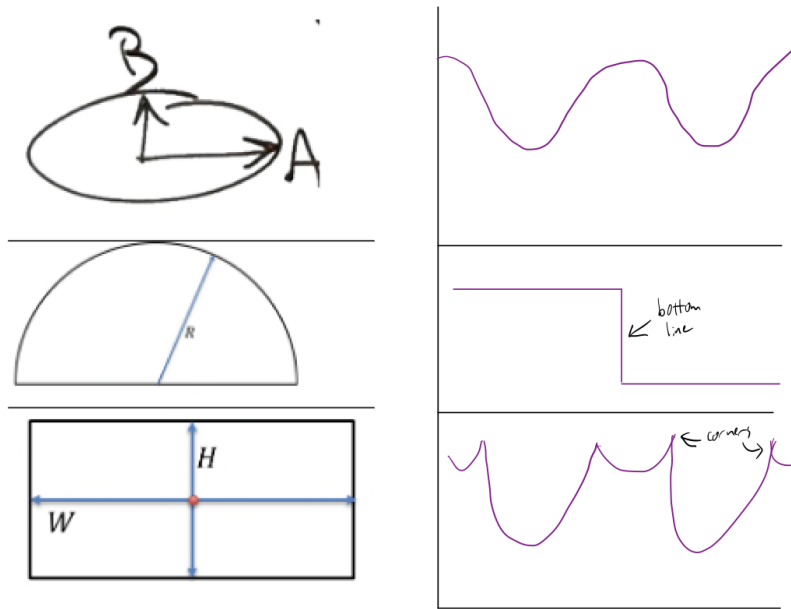


Figure 1: $r(\theta)$

Problem 2

In this problem we are asked a hypothetical problem. Supposing that we have a 2-class classification problem with centroid points μA and μB , and that each class is equally likely, we need to show that the Nearest Mean classifier decision boundary is midway along the line segment connecting μA to μB .

First, we can look at the problem in 1, 2, and 3 dimensions. For a 1D classifier, the resulting points would fall on a line and the resulting boundary would be between the centroids. If we looked at a 2D classifier, we would see the resulting decision boundary would be a line. Finally, for a 3D classifier, the decision boundary would be a plane. In each we depict the boundaries at a set distance d between centroids.

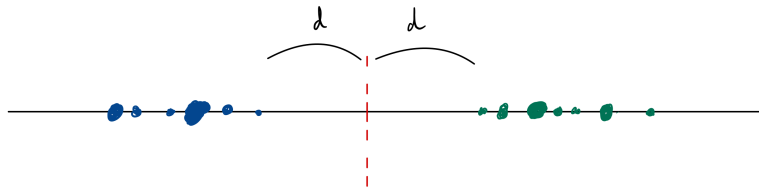


Figure 2: 1D Classifier

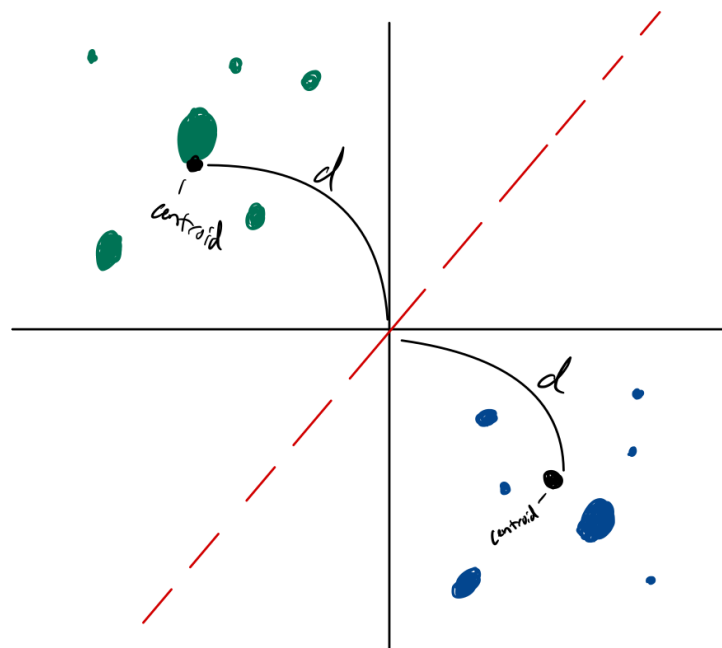


Figure 3: 1D Classifier

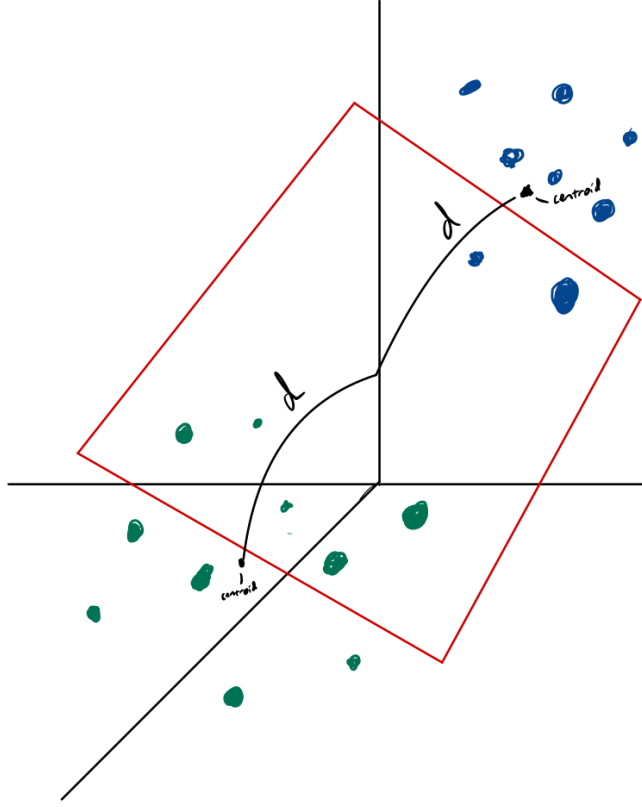


Figure 4: 3D Classifier

Problem 3

In this problem we are asked to represent an object by its boundary $(x(s), y(s))$, $0 \leq s \leq S$ where S is the length of the object's boundary and s is distance along that boundary from some arbitrary starting point. Combine x and y into a single complex function $z(s) = x(s) + jy(s)$. The Discrete Fourier Transform (DFT) of z is:

$$Z(k) = \sum_{s=0}^{S-1} e^{-2\pi j \frac{ks}{S}} z(s), 0 \leq k \leq S-1 \quad (1)$$

We can use the coefficients $Z(k)$ to represent the object boundary. The limit on s is $S-1$ because for a closed contour $z(S) = z(0)$. The Inverse Discrete Fourier Transform is:

$$z(s) = \frac{1}{S} \sum_{k=0}^{S-1} e^{+2\pi j \frac{ks}{S}} Z(k), 0 \leq s \leq S-1 \quad (2)$$

A

Here we suppose that the object is translated by $(\Delta x, \Delta y)$, such that $z'(s) = z(s) + \Delta x + j\Delta y$. How is z' 's DFT $Z'(k)$ related to $Z(k)$?

B

What object has $z(s) = R \cos(\frac{2\pi s}{S} + jR \sin(\frac{4\pi s}{S}))$? Below we see a graph drawing the shape, and included code to generate it:

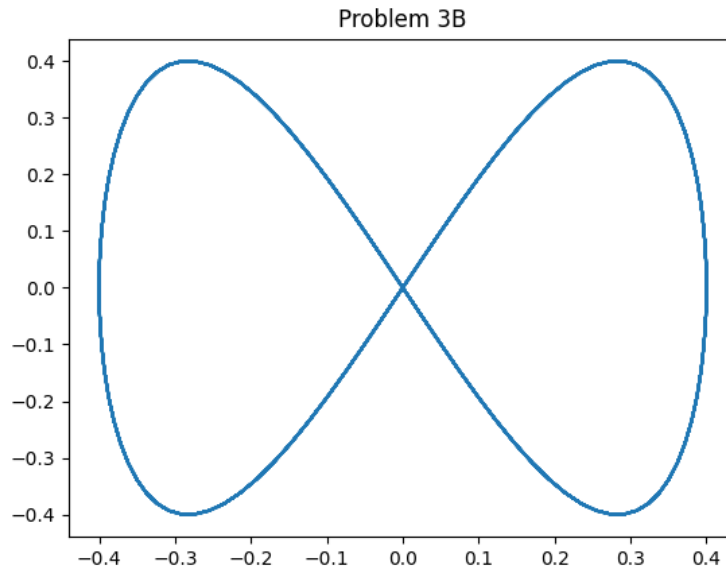


Figure 5: Our resulting shape

```
import numpy as np
from numpy import cos, sin, pi
import matplotlib.pyplot as plt

figure = plt.figure()
plt.title("Problem 3B")

S = 10
r = 4

theta = [theta for theta in np.arange(0, S, 0.01)]
X = [
    r * cos(2*pi*theta)/S
    for theta in theta
]
Y = [
    r * sin(4*pi*theta)/S
    for theta in theta
]

# Plot the results
plt.plot(X, Y)
plt.savefig("./imgs/prob3_b.png")
```

C