CS/RBE 549 Computer Vision
Fall 2022

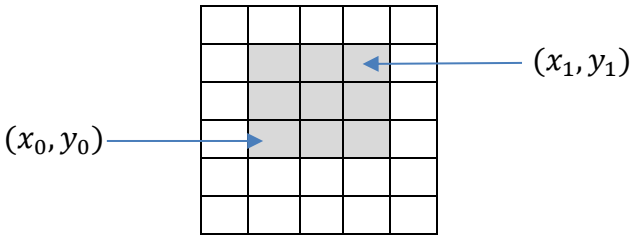
Name:_____

HW #5

1. **Integral Images (20%)** SURF gets much of its speed from using integral images to quickly approximate derivatives of $g_\sigma * I$, where I is the image. The integral image is defined as

$$I_\Sigma[i,j] = \sum_{i'=0}^{i' \le i} \sum_{j'=0}^{j' \le j} I[i',j']$$

   a. Show how to compute the sum over a rectangular region of the image using only 3 operations (+ and -) on the integral image. Let the rectangular region have lower left corner (x_0, y_0) and upper right corner (x_1, y_1).

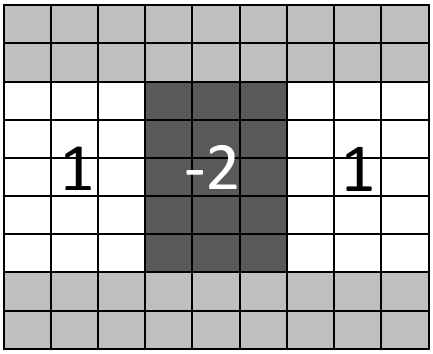


   The desired sum is

$$\sum_{i'=x_0}^{i' \le x_1} \sum_{j'=y_0}^{j' \le y_1} I[i',j']$$

   You can assume that any pixels outside the range $(0 \ldots M-1, 0 \ldots N-1)$ are 0.

   b. How can you compute the 9x9 D_{xx} approximation for SURF using I_Σ?



2. **Quadtrees (30%):** Ima Robot uses a quadtree to represent a 4×4 binary image. Each quadtree block starts at the upper left and proceeds clockwise.
   a. What image is represented by [[0110]10[0100]]?

b. The image in part a is shifted left by 1 pixel, with the right column (at i=3) replaced by 0s. What is the new image quadtree?

c. Describe a recursive algorithm for rotating a quadtree clockwise 90°.

3. **Distance (20%):** Show that if we rotate and translate a point cloud, the distance between any 2 points is the same in the original and transformed set of points. Let $\vec{x}_i^1 = R\vec{x}_i^0 + \vec{T}$ and show that $\|\vec{x}_1^0 - \vec{x}_0^0\| = \|\vec{x}_1^1 - \vec{x}_0^1\|$. Hint: Use the fact that for a rotation matrix *R*, $R^T R = I$.

4. **Segmentation (30%):** A 5x5 image has class labels as follows:

1	0	0	0	1
1	1	1	1	1
1	0	1	0	1
1	0	0	0	1
1	2	2	2	1

Assume that the outside world is region #-1 with class -1.

a. List all the regions and their classes. List all edges, indicating which regions it separates. E.g.,

Region	Class
-1	-1
0	1
. . .	. . .

Edge	Regions
0	-1, 0
1	-1,1
. . .	. . .

There is no single correct way to list the regions and edges. However, there is an exactly correct number of regions and edges.

b. How many vertices are there?

c. Is each region 4-connected or 8-connected?