RBE549 - Homework 9

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Problem 1

Here we are presented with the Logistic function ($L(\Sigma) = \frac{1}{1+e^{-\Sigma}}$) and the Hyperbolic tangent function ($tanh(\Sigma) = \frac{sinh(\Sigma)}{cosh(\Sigma)}$). We are asked to derive an expression for $tanh(\Sigma)$ in terms of $L(\Sigma)$.
To do this, first we recognize that $L(\Sigma)$ is symmetric about the origin; therefore $L(-\Sigma) = 1 - L(\Sigma)$. Thus we can show that:

$$1 - \frac{1}{1+e^{-\Sigma}} = \frac{1}{1+e^{\Sigma}} \tag{1}$$

We can then expand the definition of $tanh(\Sigma)$ by recognizing the definition of $sinh$ and $cosh$:

$$tanh(\Sigma) = \frac{sinh(\Sigma)}{cosh(\Sigma)} = \frac{e^{\Sigma} - e^{-\Sigma}}{e^{\Sigma} + e^{-\Sigma}} \tag{2}$$

We can then begin working this equation to get it in a familiar form relative to $L(\Sigma)$.

$$= \frac{e^{\Sigma} + e^{-\Sigma} - 2e^{-\Sigma}}{e^{\Sigma} + e^{-\Sigma}} = 1 + \frac{-2e^{\Sigma}}{1+e^{-2\Sigma}} = 1 - \frac{2}{e^{2\Sigma}+1} \tag{3}$$

We can then take our definiton of $L(x)$ ($L(x) = \frac{1}{1+e^{-x}}$) by letting $x = -2\Sigma$ and substituting:

$$= 1 - \frac{2}{e^{2\Sigma}+1} = 1 - 2L(-2\Sigma) = 1 - 2(1 - L(2\Sigma)) = 2L(2\Sigma) - 1 \tag{4}$$

Problem 2

In this problem we are posed with a question about Intersection over Union (IoU). IoU is defined as two regions A and B, $\frac{|A \cap B|}{|A \cup B|}$. We wish to show that $IoU(A, B) = 1 - \frac{|A \ xor \ B|}{|A \cup B|}$.
We will define the *xor* operator as the following:

$$\textit{xor(A,B)} = |AxorB| = |A \cap !B| \cup |!A \cap B| \tag{5}$$

...where we reach this as realizing that *xor* covering the set of "one or the other but not both".
To do this, we need to look at the different sets that $|A \cup B|$ represents. It breaks down to:

- A set wherein both A and B exists
- A set wherein A exists but not B
- A set wherein B exists but not A

...we can represent these sets by stating that $|A \cup B| = |A \cap B| \cup |A \cap !B| \cup |!A \cap B|$. With IoU defined as $\frac{|A \cap B|}{|A \cup B|}$, we can identify key set descriptions about our intersection. With $|A \cap B|$ as our set of intersection of A and B, and $|A \cup B| - |A \cap !B| \cup |!A \cap B|$ as the union of A and B subtracted the union of (the area that is not B but is A) and (the area that is not A and is B),
Expressing this in set notation, we can say that our $IoU(A, B)$ can be expanded into:

$$\frac{|A \cup B| - |A \cap !B| \cup |!A \cap B|}{|A \cup B|} \tag{6}$$

We can then begin to separate shared terms, specifically the $|A \cup B|$:

$$= \frac{|A \cup B|}{|A \cup B|} - \frac{|A \cap !B| \cup |!A \cap B|}{|A \cup B|} = 1 - \frac{|A \cap !B| \cup |!A \cap B|}{|A \cup B|} \tag{7}$$

Now utilizing the previously defined *xor* operator to replace $|A \cap !B| \cup |!A \cap B|$:

$$= 1 - \frac{|A \cap !B| \cup |!A \cap B|}{|A \cup B|} = 1 - \frac{|AxorB|}{|A \cup B|} \tag{8}$$

Problem 3

In this problem, we are tasked with looking at the classic LeNet-5 neurnal network, specifically looking at the third convolutional layer C_3. We are asked the number of connections in the layer, and the number of trainable parameters in the layers. We can caclulate both.
First, we need to calculate the number of trainable parameters. We have:

- C_3 has 16 kernels of size (5×5)
- The 16 kernels' receptive fields are divided into 6 of 3 features, 9 of 4 features, and 1 of 6 features.

Using these values, we can calulate a total number of parameters of:

$$5 * 5 * (6 * 3 + 9 * 4 + 1 * 6) + 16 = 1516 \tag{9}$$

With these $1,516$ trainable parameters, we can then calculate the number of connections, as we have an output feature map size of (10×10), so:

$$10 * 10 * 1,516 = 151,600 \tag{10}$$

...for $151,600$ total connections in C_3.