CS/RBE 549 Computer Vision Name: **Sol U. Tion**
Fall 2019 Wed 11 Dec 2019

Exam #2 Solutions

1. **Optical Flow (25 pts):** Suppose the image brightness is given by

$$I(x, y, t) = I_0 + \sqrt{(x - c_1 t)^2 + (y - c_2 t)^2}$$

a. (5 pts) What are I_x, I_y, and I_t?

$$I_x = \frac{(x - c_1 t)}{\sqrt{(x - c_1 t)^2 + (y - c_2 t)^2}}, \qquad I_y = \frac{(y - c_2 t)}{\sqrt{(x - c_1 t)^2 + (y - c_2 t)^2}},$$

$$I_t \frac{-c_1(x - c_1 t) - c_2(y - c_2 t)}{\sqrt{(x - c_1 t)^2 + (y - c_2 t)^2}}$$

b. (10 pts) Express the Optical Flow Constraint Equation $I_x u + I_y v + I_t = 0$ in the simplest terms possible for this image sequence.

$$\frac{(x - c_1 t)}{\sqrt{(x - c_1 t)^2 + (y - c_2 t)^2}} u + \frac{(y - c_2 t)}{\sqrt{(x - c_1 t)^2 + (y - c_2 t)^2}} v + \frac{-c_1(x - c_1 t) - c_2(y - c_2 t)}{\sqrt{(x - c_1 t)^2 + (y - c_2 t)^2}} = 0$$

Canceling the denominator and factoring gives

$$(u - c_1)(x - c_1 t) - (v - c_2)(y - c_2 t) = 0$$

c. (10 pts) The equation from b. must hold for all x, y, and t. Find a constant solution for u and v that makes this true, that is, such that u and v do not depend on x, y, and t.

$$u = c_1, \qquad v = c_2$$

2. **Hough Velocity Space (25 pts):** The Optical Flow Constraint Equation can be used to define a Hough-type velocity space that can combine observations of the optical flow from many points.

a. (5 pts) What are the coordinates in this space?
The Optical Flow Constraint Equation $I_x u + I_y v + I_t = 0$ suggests u and v.

b. (10 pts) What does the observation of a single point $I(x, y, t)$ and its derivatives in Image space map into in the Hough velocity space?
The line $I_x u + I_y v + I_t = 0$

c. (10 pts) P_1 and P_2 are points in the image. A small image patch in the neighborhood of P_1 has brightness $I(x, y, t) = 200 + 40x - 80y$. A small image patch in the neighborhood of P_2 has brightness $I(x, y, t) = 100 + 30x + 60y + 240t$. What image velocity is consistent with both of these image patches?
At P_1 we have $I_x = 40$, $I_y = -80$, $I_t = 0$, which maps to the line $u - 2v = 0$
At P_2 we have $I_x = 30$, $I_y = 60$, $I_t = 240$, which maps to the line $u + 2v + 8 = 0$
Solution is $u = -4$, $v = -2$

3. **Stereo via Singular Value Decomposition (30 pts):** Assume the usual stereo geometry, where the left and right cameras are offset by baseline $\vec{B}$ that is perpendicular to the common focal vector $\vec{F}$. Then the stereo imaging equations are

$$\vec{X}_L = \frac{|\vec{F}|^2}{\vec{F} \cdot \vec{X}^W}\left(\vec{X}^W + \frac{\vec{B}}{2}\right), \qquad \vec{X}_R = \frac{|\vec{F}|^2}{\vec{F} \cdot \vec{X}^W}\left(\vec{X}^W - \frac{\vec{B}}{2}\right)$$

In the presence of imaging errors or noise, these equations might not hold exactly. They can be approximated by

$$\vec{X}_L - \frac{|\vec{F}|^2}{\vec{F} \cdot \vec{X}^W}\left(\vec{X}^W + \frac{\vec{B}}{2}\right) \approx \vec{0}, \qquad \vec{X}_R - \frac{|\vec{F}|^2}{\vec{F} \cdot \vec{X}^W}\left(\vec{X}^W - \frac{\vec{B}}{2}\right) \approx \vec{0}$$

a. (15 pts) Show that these equations can be written as a 4x4 matrix operating on a column vector in homogeneous coordinates.

$$\begin{bmatrix} -f & 0 & x_L & -fb/2 \\ 0 & -f & y_L & 0 \\ -f & 0 & x_R & fb/2 \\ 0 & -f & y_R & 0 \end{bmatrix} \begin{bmatrix} x^W \\ y^W \\ z^W \\ 1 \end{bmatrix} \approx \vec{0}$$

Hint: Combine the approximate imaging equations into a single matrix equation, multiply to eliminate the denominators, and simplify, not necessarily in that order!

Rewrite the left projection equation as $(\vec{F} \cdot \vec{X}^W)\vec{X}_L - |\vec{F}|^2\left(\vec{X}^W + \frac{\vec{B}}{2}\right) \approx \vec{0}$**. Expand components to**

$$f z^W x_L - f^2 x^W - f^2 \frac{b}{2} \approx 0$$
$$f z^W y_L - f^2 y^W \approx 0$$
$$f z^W z_L - f^2 z^W \approx 0$$

Drop the bottom equation, which reduces to 0=0 because $z_L = f$. We can divide the top 2 equations by f to further simplify. Doing the same thing to the right projection equation gives

$$z^W x_L - f x^W - f\frac{b}{2} \approx 0$$
$$z^W y_L - f y^W \approx 0$$
$$z^W x_R - f x^W + f\frac{b}{2} \approx 0$$
$$z^W y_R - f y^W \approx 0$$

This leads immediately to

$$\begin{bmatrix} -f & 0 & x_L & -fb/2 \\ 0 & -f & y_L & 0 \\ -f & 0 & x_R & fb/2 \\ 0 & -f & y_R & 0 \end{bmatrix} \begin{bmatrix} x^W \\ y^W \\ z^W \\ 1 \end{bmatrix} \approx \vec{0}$$

b. (5 pts) The above equation can be written as $A\tilde{X}' \approx \vec{0}$. We can use SVD to find the singular vector $\tilde{X}'$ that minimizes $|A\vec{X}|^2$ subject to $|\vec{X}|^2 = 1$. Express world point $\vec{X}^W = [x, y, z]^T$ in terms of $\tilde{X}' = [x', y', z', w']^T$.

$$\vec{\boldsymbol{X}}^W = \frac{\mathbf{1}}{\boldsymbol{w}'}\widetilde{\boldsymbol{X}}' = \left[\frac{\boldsymbol{x}'}{\boldsymbol{w}'}, \frac{\boldsymbol{y}'}{\boldsymbol{w}'}, \frac{\boldsymbol{z}'}{\boldsymbol{w}'}, \mathbf{1}\right]^{\boldsymbol{T}}$$

c. (10 pts) When $y_L = y_R$, show that a. gives $z^W = \frac{fb}{d}$, where d is the disparity.

The 2nd and 4th equations become $\boldsymbol{z^W y - f y^W \approx 0}$**, so setting** $\boldsymbol{y^W = z^W y/f}$ **assures that** $\boldsymbol{z^W y - f y^W = 0}$ **and we can ignore these equations. Subtracting the remaining 3rd from 1st equation gives**

$$(\boldsymbol{x_L - x_R})\boldsymbol{z^W} - \boldsymbol{fb} \approx \mathbf{0}, \text{ i.e., } \boldsymbol{z^W} = \frac{\boldsymbol{fb}}{\boldsymbol{x_L - x_R}} = \frac{\boldsymbol{fb}}{\boldsymbol{d}}$$

4. **Infra-Red Vision (10 pts):** As is well-known, the comic book hero Superman has Infra-Red (heat) vision. Although the physics of heat vision are never quite explained, presumably, Superman's eyes are sensitive to photons in the infra-red spectrum, which are emitted uniformly in all directions from warm objects. Unlike X-rays, infra-red light can be focused, although objects tend to lack distinct features because nearby parts of a single object tend to have the same or similar temperature. *Briefly* discuss some of the challenges imaging the world using Infra-Red Vision.

   Many possible good answers. Here are a few:
   Shading – Most objects emit infra-red uniformly in all directions, so shading does not indicate surface orientation.
   Color – Although infra-red spans many wavelengths, they do not correspond to our usual sense of color.
   Objects – Similar objects may appear different if they are at different temperatures.
   Texture – Because infre-red produces a heat map, there may be little or no texture to objects.
   Etc.

5. **Object Detection (10 pts):** Based on your experience in Computer Vision, you are hired to design a Stop Sign detection system. List 5 types of knowledge / sources of information that you could use.
   a. Color = red
   b. Shape = octagon
   c. Surface shape = flat
   d. Location = along roads
   e. Texture = includes letters
   f. Corners and other features present
   Etc.

6. **Teamwork (1 pt):** On a scale of 1 to 5, with 1 being the lowest, 3 is neutral and 5 being the highest, rate how well your project team is working together.
   No correct answer.