RBE595 - Homework 1

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In this assignment we are provided with a proposed system of a drone tracked by a series of motion capture cameras. We are asked to formulate the system dynamics and Kalman Filter for this system, and then provided a set of measurements from the system for testing out implementation.

CSV files are provided that provide measurements - some in velocity, others direct position.

The system dynamics are given as:

$$\mathbf{x} = \begin{bmatrix} p \\ \dot{p} \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \\ \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} \tag{1}$$

...and inputs in the form of

$$\mathbf{u} = m\ddot{p} \tag{2}$$

...wherein m is our drone mass, at 27 grams. We are instructed to assume that all noise is additive Gaussian noise with 0 mean and a diagonal covariance matrix since the form of $\Sigma = \sigma^2 I$. The noise has $\sigma = 0.05$ m for low noise, $\sigma = 0.20$ m for high noise, and $\sigma = 0.05$ m/s for velocity noise.

Task 1

In this task we are asked to determine the matricies used in the process and measurement models, ignoring the noise matricies.

First we start with our state and known equations for our state and motion:

$$\mathbf{x} = \begin{bmatrix} p \\ \dot{p} \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \\ \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} \tag{3}$$

...and we know the state model for motion is:

$$\begin{aligned} x &= x + \dot{x}\Delta t + \frac{1}{2}\ddot{x}\Delta t^2 \\ y &= y + \dot{y}\Delta t + \frac{1}{2}\ddot{y}\Delta t^2 \\ z &= z + \dot{z}\Delta t + \frac{1}{2}\ddot{z}\Delta t^2 \\ \dot{x} &= \dot{x} + \ddot{x}\Delta t \\ \dot{y} &= \dot{y} + \ddot{y}\Delta t \\ \dot{z} &= \dot{z} + \ddot{z}\Delta t \end{aligned} \tag{4}$$

We wish to find the matricies in the following equation:

$$\dot{x} = \mathbf{F}\mathbf{x} + \mathbf{G}\mathbf{u} + E\eta \tag{5}$$

Since we are ignoring process noise, we can drop $E\eta$ from the equation. Solving for the $\mathbf{F}$ matrix:

$$\mathbf{F} = \begin{bmatrix} 1 & 0 & 0 & \Delta t & 0 & 0 \\ 0 & 1 & 0 & 0 & \Delta t & 0 \\ 0 & 0 & 1 & 0 & 0 & \Delta t \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{6}$$

...this is derived from an understanding of our state equations above.
For our $\mathbf{G}$ matrix, we first look at our $\mathbf{u}$ state equation:

$$\mathbf{u} = m\ddot{p} \tag{7}$$

...thus $\ddot{p}$ can be isolated:

$$\ddot{p} = \frac{\mathbf{u}}{m} \tag{8}$$

...so our G equation would be:

$$\mathbf{G} = \begin{bmatrix} \frac{\Delta t^2}{2} & 0 & 0 \\ 0 & \frac{\Delta t^2}{2} & 0 \\ 0 & 0 & \frac{\Delta t^2}{2} \\ \frac{\Delta t}{m} & 0 & 0 \\ 0 & \frac{\Delta t}{m} & 0 \\ 0 & 0 & \frac{\Delta t}{m} \end{bmatrix} \tag{9}$$

...thus leaving us with the following:

$$\ddot{x} = \begin{bmatrix} 1 & 0 & 0 & \Delta t & 0 & 0 \\ 0 & 1 & 0 & 0 & \Delta t & 0 \\ 0 & 0 & 1 & 0 & 0 & \Delta t \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} + \begin{bmatrix} \frac{\Delta t^2}{2} & 0 & 0 \\ 0 & \frac{\Delta t^2}{2} & 0 \\ 0 & 0 & \frac{\Delta t^2}{2} \\ \frac{\Delta t}{m} & 0 & 0 \\ 0 & \frac{\Delta t}{m} & 0 \\ 0 & 0 & \frac{\Delta t}{m} \end{bmatrix} \begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{bmatrix} \tag{10}$$

Now we need to solve for the covariance extrapolation equation, given by:

$$\mathbf{P}_{n+1,n} = \mathbf{F}\mathbf{P}_{n,n}\mathbf{F}^T + \mathbf{Q} \tag{11}$$

...wherein $\mathbf{Q}$ is the process noise covariance matrix, $\mathbf{P}_{n,n}$ is the covariance matrix at time n, and $\mathbf{P}_{n+1,n}$ is the covariance matrix at time $n+1$. $\mathbf{F}$ is the state transition matrix, which we have already solved for.

First, we must specify $\mathbf{P}$:

$$\mathbf{P} = \begin{bmatrix} p_x & p_{x\dot{x}} & p_{x\ddot{x}} & p_{xy} & p_{x\dot{y}} & p_{x\ddot{y}} & p_{xz} & p_{x\dot{z}} & p_{x\ddot{z}} \\ p_{\dot{x}x} & p_{\dot{x}\dot{x}} & p_{\dot{x}\ddot{x}} & p_{\dot{y}x} & p_{\dot{x}\dot{y}} & p_{\dot{x}\ddot{y}} & p_{\dot{x}z} & p_{\dot{x}\dot{z}} & p_{\dot{x}\ddot{z}} \\ p_{\ddot{x}x} & p_{\ddot{x}\dot{x}} & p_{\ddot{x}\ddot{x}} & p_{\ddot{x}y} & p_{\ddot{x}\dot{y}} & p_{\ddot{x}\ddot{y}} & p_{\ddot{x}z} & p_{\ddot{x}\dot{z}} & p_{\ddot{x}\ddot{z}} \\ p_{yx} & p_{y\dot{x}} & p_{y\ddot{x}} & p_y & p_{y\dot{y}} & p_{y\ddot{y}} & p_{yz} & p_{y\dot{z}} & p_{y\ddot{z}} \\ p_{\dot{y}x} & p_{\dot{y}\dot{x}} & p_{\dot{y}\ddot{x}} & p_{\dot{y}y} & p_{\dot{y}\dot{y}} & p_{\dot{y}\ddot{y}} & p_{\dot{y}z} & p_{\dot{y}\dot{z}} & p_{\dot{y}\ddot{z}} \\ p_{\ddot{y}x} & p_{\ddot{y}\dot{x}} & p_{\ddot{y}\ddot{x}} & p_{\ddot{y}y} & p_{\ddot{y}\dot{y}} & p_{\ddot{y}\ddot{y}} & p_{\ddot{y}z} & p_{\ddot{y}\dot{z}} & p_{\ddot{y}\ddot{z}} \\ p_{zx} & p_{z\dot{x}} & p_{z\ddot{x}} & p_z & p_{z\dot{y}} & p_{z\ddot{y}} & p_{zz} & p_{z\dot{z}} & p_{z\ddot{z}} \\ p_{\dot{z}x} & p_{\dot{z}\dot{x}} & p_{\dot{z}\ddot{x}} & p_{\dot{z}y} & p_{\dot{z}\dot{y}} & p_{\dot{z}\ddot{y}} & p_{\dot{z}z} & p_{\dot{z}\dot{z}} & p_{\dot{z}\ddot{z}} \end{bmatrix} \tag{12}$$

...wherein:

- p_x is the variance of the position in the x direction
- p_y is the variance of the position in the y direction

Since we can assume that state estimation errors in X, Y, and Z are not correlated, we can replace their sections with $\mathbf{0}$:

$$\mathbf{P} = \begin{bmatrix} \mathbf{p}_x & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{p}_y & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{p}_z \end{bmatrix} = \begin{bmatrix} p_x & p_{x\dot{x}} & p_{x\ddot{x}} & 0 & 0 & 0 & 0 & 0 & 0 \\ p_{\dot{x}x} & p_{\dot{x}\dot{x}} & p_{\dot{x}\ddot{x}} & 0 & 0 & 0 & 0 & 0 & 0 \\ p_{\ddot{x}x} & p_{\ddot{x}\dot{x}} & p_{\ddot{x}\ddot{x}} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & p_y & p_{y\dot{y}} & p_{y\ddot{y}} & 0 & 0 & 0 \\ 0 & 0 & 0 & p_{\dot{y}x} & p_{\dot{y}\dot{y}} & p_{\dot{y}\ddot{y}} & 0 & 0 & 0 \\ 0 & 0 & 0 & p_{\ddot{y}x} & p_{\ddot{y}\dot{y}} & p_{\ddot{y}\ddot{y}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & p_z & p_{z\dot{z}} & p_{z\ddot{z}} \\ 0 & 0 & 0 & 0 & 0 & 0 & p_{\dot{z}z} & p_{\dot{z}\dot{z}} & p_{\dot{z}\ddot{z}} \\ 0 & 0 & 0 & 0 & 0 & 0 & p_{\ddot{z}z} & p_{\ddot{z}\dot{z}} & p_{\ddot{z}\ddot{z}} \end{bmatrix} \tag{13}$$