

GAMES w/o
Chance

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- ① New Ways of
simplifying games
- ② Examples from Nim
- ③ Examples from $\uparrow, \downarrow, *$
General Reversible
- ④ TOADS AND FROGS

$$\{1, 2, 7 \mid 8, 10\}$$

$$\{\uparrow \mid 2\}$$

$$0 < \uparrow < \frac{1}{2^n}$$

$$\uparrow = \{0 \mid *\}$$

$$* = \{0 \mid 0\}$$

$$\{\cancel{1}, \cancel{2}, \underline{7} | \underline{8}, \cancel{10}\}$$

$$7 > 1$$

$$8 < 10$$

$$7 > 2$$

$$\{7 | 8\} = \underline{\underline{7\frac{1}{2}}}$$

$$1 = \{ \uparrow | 2 \} \quad 0 < \uparrow < \frac{1}{2^n}$$

$$\uparrow < 2$$

It's a number

$$\{ \uparrow | 2 \} - 1 = 0$$

$$*1 + *2 + *3 = 0$$

EXTRA COINS?

DON'T HOLD

$$\{ *0, *1, \underline{\underline{*3}}, \underline{\underline{*7}} \}$$

$$\underline{\underline{*2}} = \{ *0, *1 \}$$

$$\{ *1, *0, *2, *101 \}$$

$$= *3$$

Reversible Moves

$$G = \{a, b, \dots \mid e, f, \dots\}$$

Reverses through a if there is a right move a^R with $a^R \in G$ in which case replace a by the left moves of

a^R
Reverses through e if there is $e^L \in G$... replace e by right moves of e^L ...

Don't
Panic



EXAMPLE $\uparrow + *$

$$\uparrow = \{0 | *\} \quad * = \{0 | 0\}$$

$$\begin{array}{ccccc} \{0 | *\} & + & \{0 | 0\} \\ \uparrow & & + & & * \end{array}$$

$$= \{*, \uparrow | 0, \uparrow\}$$

$$\uparrow + * = \{*, \uparrow / 0, \cancel{*}\}$$

$0 < \uparrow$
 $0 < \uparrow < \frac{1}{2}n$
 0

$$\uparrow = \{0 | *\}$$

$$\uparrow^R = *$$

$$* = \{0 | 0\}$$

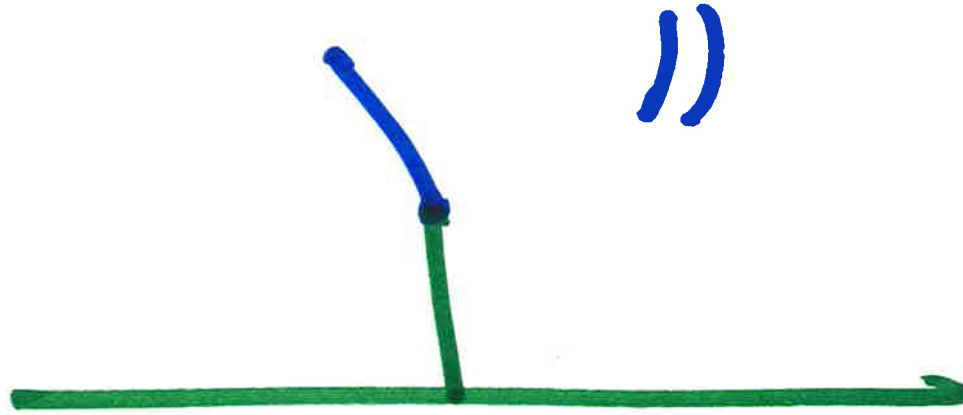
$$* < \uparrow + *$$

$$0 < \uparrow$$

$$\uparrow$$

$$\uparrow + * = \{0, * | \emptyset\}$$

$\{0, * | 0\}$



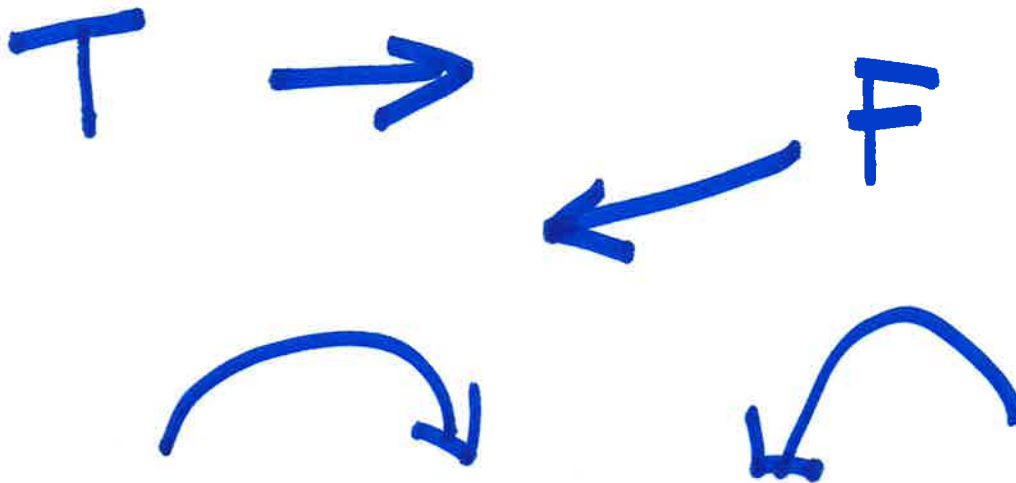
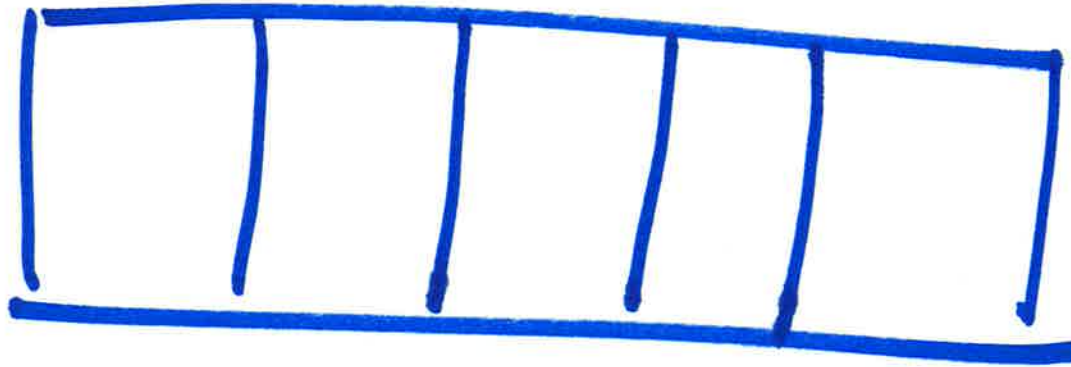
=


$$\uparrow + * = \uparrow *$$

$$\begin{aligned} & \uparrow * + \uparrow * \\ &= \uparrow + \uparrow + * + * \\ & \quad \quad \quad \underbrace{\hspace{1.5cm}}_{=0} \end{aligned}$$

$$\begin{aligned} &= \uparrow + \uparrow \\ &= \uparrow \parallel \end{aligned}$$

TOADS & FROGS



$G = \{\uparrow, |, \downarrow\}$ $\uparrow = \{0|*\}$
 $R \downarrow$ $\downarrow = \{*|0\}$
 Left move * 

* better for L than G

$G \geq *$ $G + * \leq 0$

$* = \{0|0\}$ $G = \{\uparrow|0\}$

① Dominated moves

② Reversible

ENOUGH??
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