

Quiz Week 3

February 21, 2013

1. Is $G > 0$, $G < 0$, $G = 0$, $G||0$? G is 3 single green stalks.

Proof. Say Left goes first. Then Left cuts down one of the green stalks, Right cuts down one, and Left cuts down the last one, so Left wins. Similarly, Right wins when Right goes first. Therefore this is a first person win game so $G||0$. \square

2. Is $G > 0$, $G < 0$, $G = 0$, $G||0$? G is a green stalk with a blue stalk on top of it.

Proof. Notice that the first player to go can cut the green stalk and win. Thus, this is a first player win game so $G||0$. \square

3. Is $G > 0$, $G < 0$, $G = 0$, $G||0$? G is two green stalks, each with a red stalk on top of it.

Proof. If Left goes first, Left must cut a green stalk, in which case Right cuts the other green stalk and Right wins.

If Right goes first, Right cuts one of the red stalks, Left cuts a green stalk, and Right cuts the last green stalk and wins. Therefore, Right always wins this game so $G < 0$. \square

4. Is $G > 0$, $G < 0$, $G = 0$, $G||0$? $G = \uparrow + \star$.

Proof. Recall that $\uparrow = \{0|\star\}$ and $\star = \{0|0\}$. If Right moves to \star in the game \uparrow , The resulting game is $\star + \star = 0$ so Left has to move in 0 so Right wins. If Left goes first, Left can move to 0 in the game \star so we have $\uparrow + 0$ which is a Left win. Therefore, the first player to move wins so $G||0$. \square