

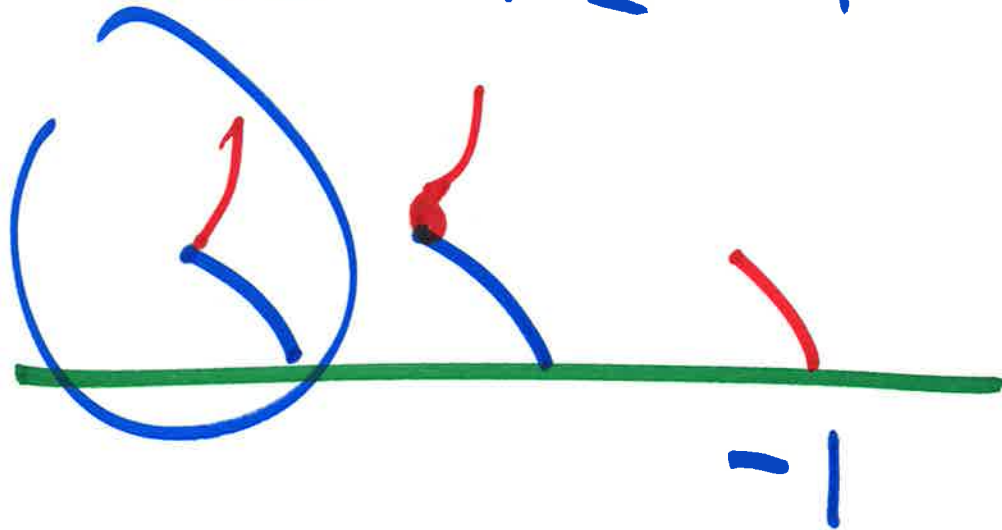
Games without

Chance

Tom MORLEY

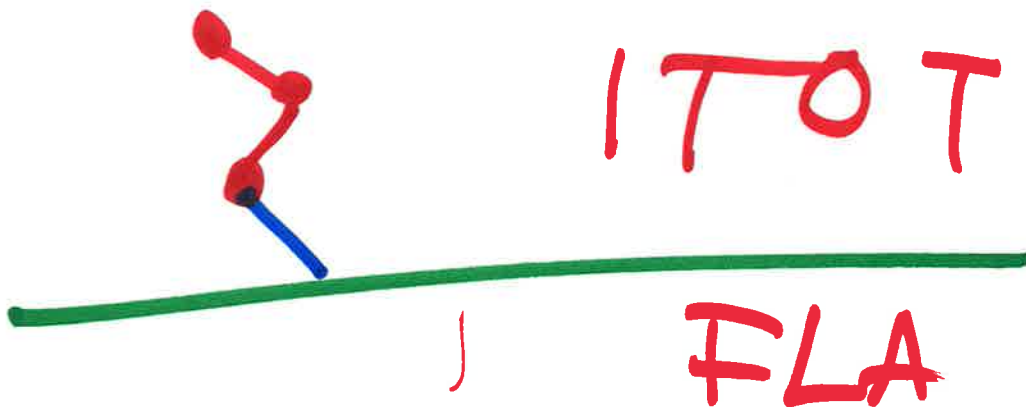
1. Some Numbers are Games
2. Some Games are Numbers
3. Simplicity
4. More Numbers
5. Quiz/HW

$$\frac{1}{2} + \frac{1}{2} - 1$$



Blue
Red

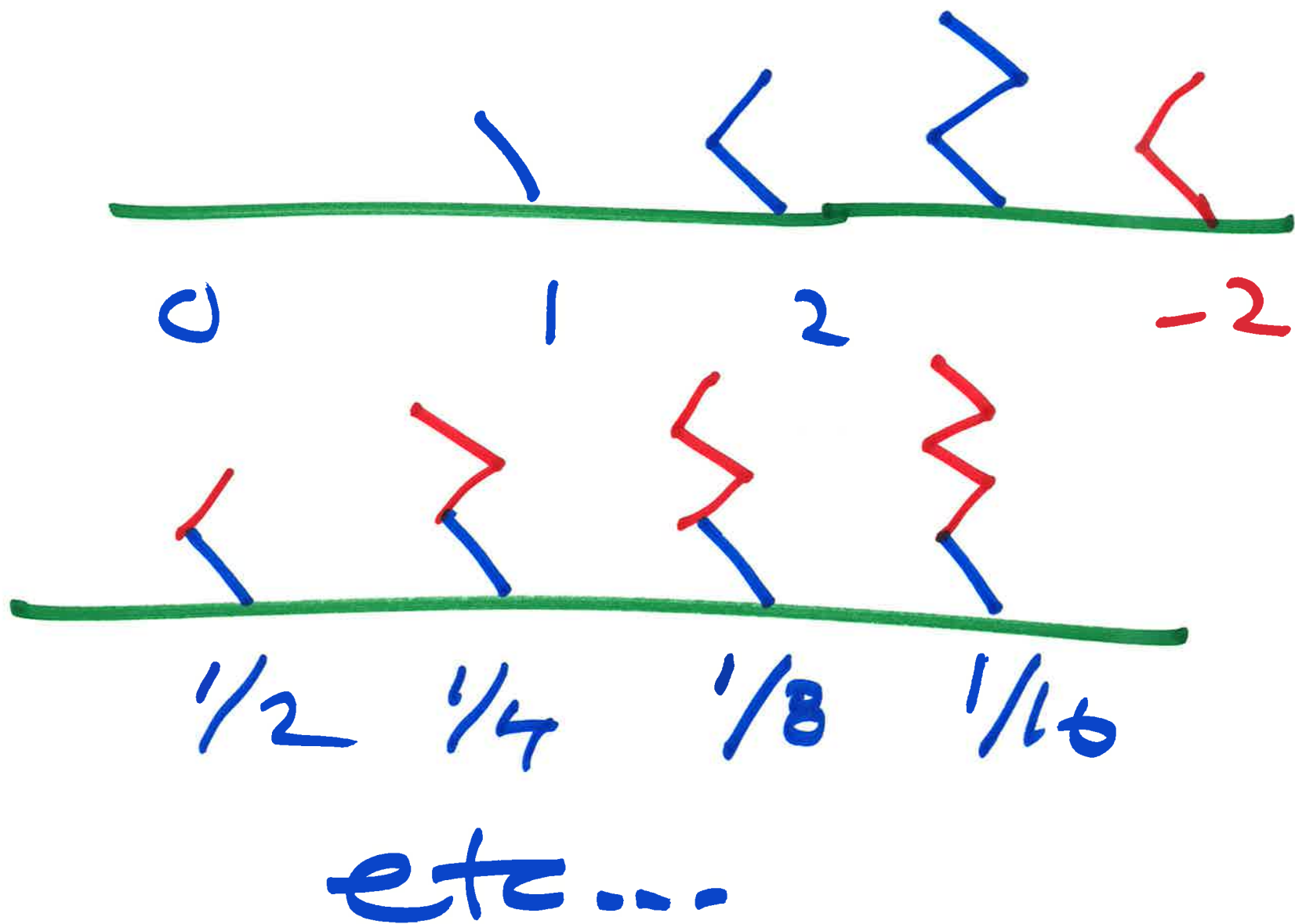
= 0



ITOT

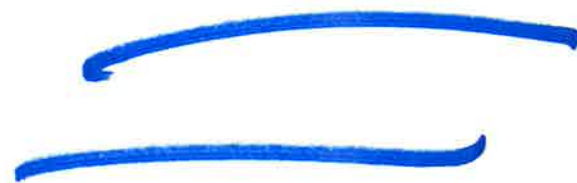
FLA

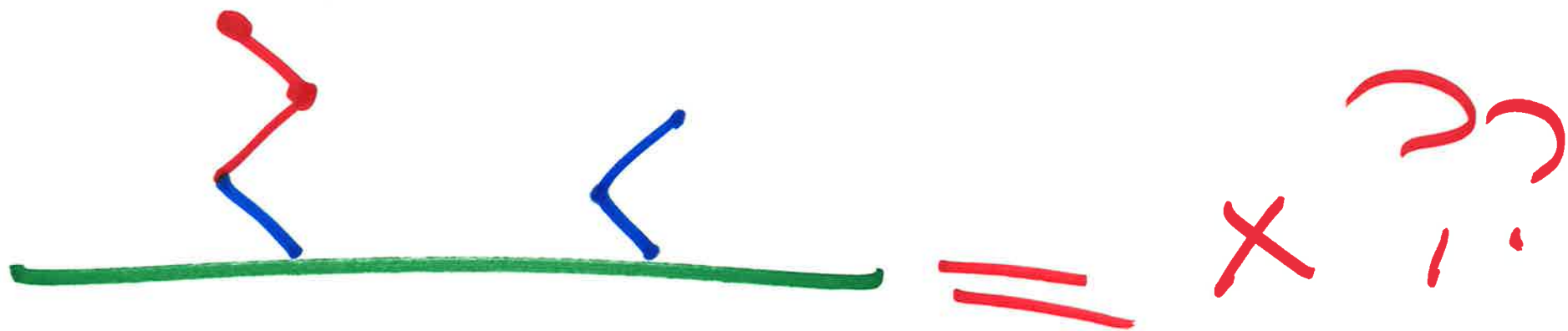
1/4



$$\frac{p}{2^n}$$

dyadic
rationals





$$\frac{1}{4} + 2 = 2\frac{1}{4}$$

$$\frac{9}{4}$$

$$0 = \{1\}$$

$$\underline{\underline{G=0}}$$

1st Loss

$$\{\underline{-3}, 1\} = 0$$

$$\begin{array}{l} L. \\ R \end{array} \begin{array}{c} \longrightarrow \\ \longrightarrow \end{array} \begin{array}{c} -3 \\ 1 \end{array}$$

RIGHT
Left.

{ left + options / right }

$$0 = \{ 1 \}$$

$$1 = \{ 0 | \}$$

$$2 = \{ 1 | \}$$

$$\frac{1}{2} = \{ 0 | 1 \}$$

$$\frac{1}{4} = \{ 0 | \frac{1}{2} \}$$

etc...

$$-2 = 51 - 13$$

$$51 | 43 = 2$$

$$\underline{\underline{51}} | 43 - 2 = 0$$

$$L \quad 1 - 2 = -1 \quad \text{RIGHT}$$

$$R \quad 4 - 2 \quad \text{or} \quad 51 | 43 - 1 \quad \text{Left wins}$$

$$\{-4, 1-1\} = x?$$

Ans: $= -2$

$$\{-4, 1-1\} + 2 = 0$$

$$\{-3 \mid 1\} = x$$

$$-3 < x < 1$$

which one?

Simplest one

$$\{1 \mid 13/8\} = \underline{\underline{1'1/4}}$$

$$1 \leq 1'1/4 \leq 13/8$$

Choice simplest

0

1

2

3

4

⋮

-1

-2

⋮

⋮

⋮

Simplest number

$\frac{1}{2}$ $-\frac{1}{2}$...

$3\frac{1}{2}$

$\frac{1}{4}$

⋮

$\frac{1}{8}$

$\frac{1}{16}$

⋮

⋮

⋮

TRY THIS AT HOME

$$\left\{ \frac{3}{\infty} / \frac{3}{5} \right\} = ?$$

$$\frac{3}{\infty} < x < \frac{3}{5}$$

$$\frac{1}{2}$$

$$\{1 \mid 3^{7/8}\} = x$$

$$\underline{\underline{2}}$$

Other Numbers

$P/2^n$ dyadic
rationals

$$\sqrt{2} = \left\{ \begin{array}{l} \# < \sqrt{2} \\ 1/4, \dots \end{array} \mid \begin{array}{l} \# > \sqrt{2} \\ 1/2, \dots \end{array} \right\}$$

$1/3$

$$G = \{0, 1, 2, 3, 4, \dots\}$$

$= \omega$ infinite

$$G = \{0, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \dots\}$$

ω infinitesimal

Surreal
Numbers

