

Quiz Week 6

April 2, 2013

1. Find the nim sum of 1, 5, and 11

Proof. First write out each number in binary and take the sum without carry:

$$\begin{array}{rcl} 1 & = & 0001 \\ 5 & = & 0101 \\ 11 & = & \underline{1011} \\ & & 1111 \end{array}$$

Converting back from binary we have $1111 = 1 \times 8 + 1 \times 4 + 1 \times 2 + 1 \times 1 = 15$ so the nimsum of 1, 5, and 11 is 15. \square

2. Find the mex of $\{0, 1, 2, 3, 8, 4, 11, 5\}$

Proof. By reordering the set we get $\{0, 1, 2, 3, 4, 5, 8, 11\}$ and we see that the first excluded number is 6 so $\text{mex}\{0, 1, 2, 3, 8, 4, 11, 5\} = 6$ \square

3. For the subtraction game $\{1, 2, 5\}$, find $G(7)$

Proof. We know $G(0) = 0$, $G(1) = \text{mex}\{G(0)\} = \text{mex}0 = 1$, $G(2) = \text{mex}\{G(1), G(0)\} = \text{mex}\{1, 0\} = 2$, and continuing this we get

0	1	2	3	4	5	6	7
0	1	2	0	1	2	0	1

so $G(7) = 1$. \square

4. Find $G(4)$ for the subtraction game $\{1, 2, 3\}$

Proof. We know $G(0) = 0$, $G(1) = \text{mex}\{G(0)\} = \text{mex}0 = 1$, and continuing this we get

0	1	2	3	4
0	1	2	3	0

so $G(4) = 0$. \square