We first show that the number of self-conjugate partitions of n is equal to the number of partitions of n into distinct odd summands. Given a Young diagram, denote (i, j) as the square at the i-th row and j-th column. The square at the NW corner is $(1, 1)$.

Let $n = \lambda_1 + \lambda_2 + \ldots + \lambda_k$, with $\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_k \geq 1$, be self-conjugate. Let Y be the set of squares forming the diagram. Then

$$Y = \{(i, j) : 1 \leq i \leq k, 1 \leq j \leq \lambda_i\}.$$

Note that $(i, j) \in Y \iff (j, i) \in Y$. For each positive integer m, define $Y_m = \{(i, j) \in Y : m = \min\{i, j\}\}$. Then $Y_m \neq \emptyset \iff (m, m) \in Y_m$, in which case $|Y_m|$ is odd. Also,

$$|Y_1| > |Y_2| > \ldots > |Y_r| > |Y_{r+1}| = |Y_{r+2}| = \ldots$$

where r is chosen so that $|Y_r| > 0$ and $Y_m = \emptyset$ for $m > r$. We see that $\cup_{i=1}^{n} Y_i = Y$ and $Y_i \cap Y_j = \emptyset$ if $i \neq j$. Since $n = |Y|$, we have that $n = |Y_1| + |Y_2| + \ldots + |Y_r|$ is a partition with distinct odd summands. This process is reversible. Given $n = x_1 + x_2 + \ldots + x_r$ with $x_1 > x_2 > \ldots > x_r \geq 1$ and each x_i odd, we construct a symmetric diagram, Y. For each $1 \leq i \leq r$, and $i \leq j < i + \frac{x_i+1}{2}$, we add (i, j) and (j, i) to Y. $\square$

The required generating function is

$$G(q) = (1 + q)(1 + q^3)(1 + q^5) \ldots = \prod_{i=1}^{\infty} (1 + q^{2i-1}) = \prod_{i=1}^{\infty} \frac{1 - q^{4i-2}}{1 - q^{2i-1}}.$$