

GAMES w/o

Dice or Cards

Combinatorial

Game  
Theory

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# Ordering Games

$$G < O$$

$$G = O$$

$$G > O$$

$$G \parallel O$$

fuzzy

R wins  
2<sup>nd</sup> wins

Left wins

1<sup>st</sup> wins

$$G < H$$

H is better for L  
(even if part of a bigger  
game)

than G

$$0 < H - G$$

$$G = H$$

G and H have  
the same outcome  
— even if part of a  
bigger game

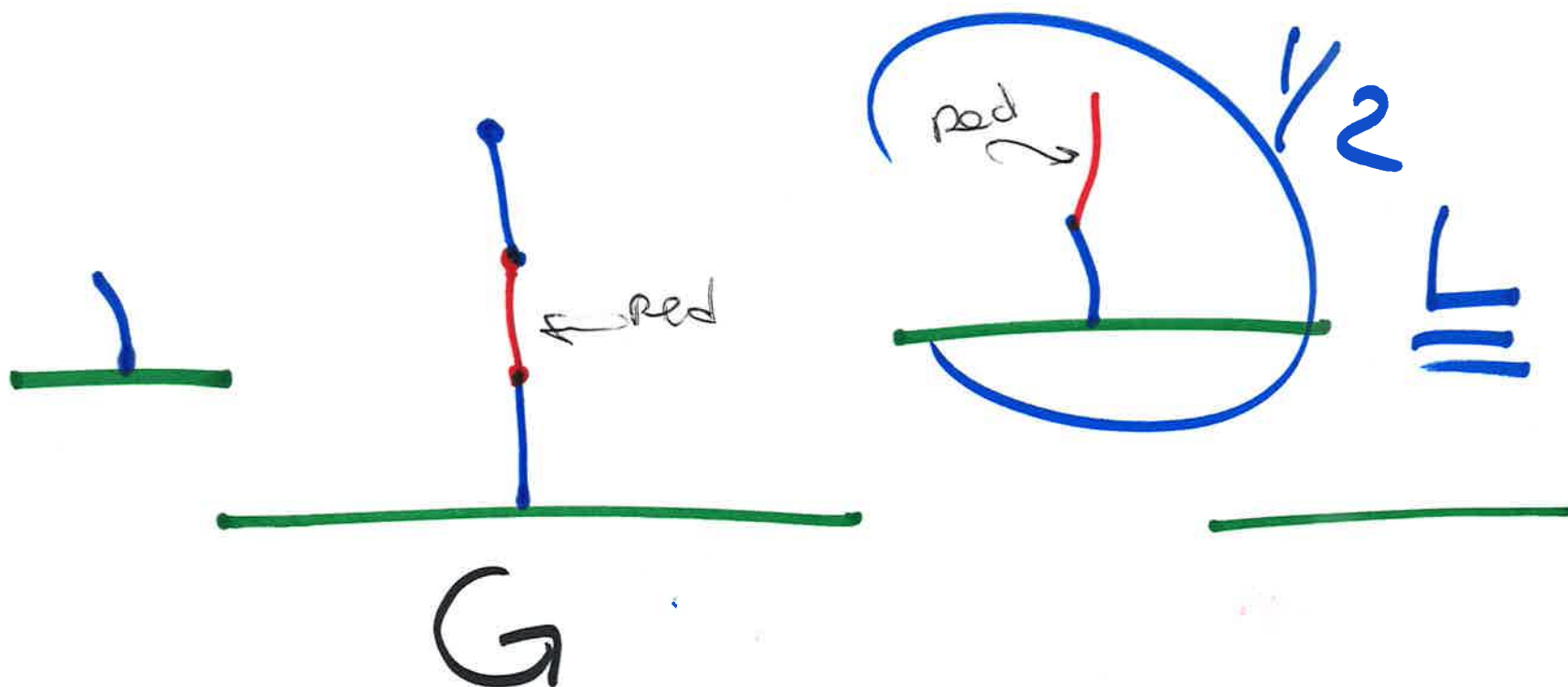
$$H - G = 0$$

$$G \leq H$$

$$G < H \text{ OR } G = H$$

H is at least as good for  
L as G.

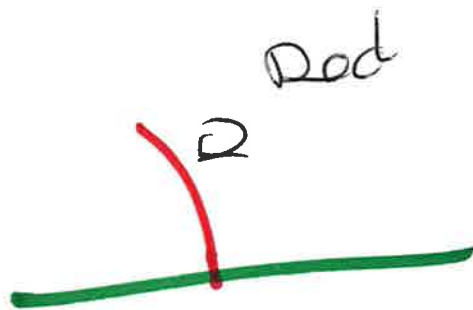
L wins  $H - G$   
going 2<sup>nd</sup>



{ Left moves | Right moves }

$$\{\cancel{0}, \frac{1}{2} | 1\} \quad \underline{\underline{0 < \frac{1}{2}}}$$

# PROBLEM

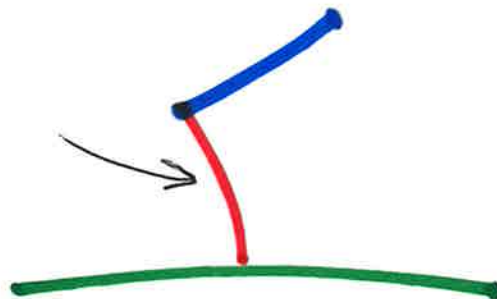


G

$\wedge$

$\wedge$

Red

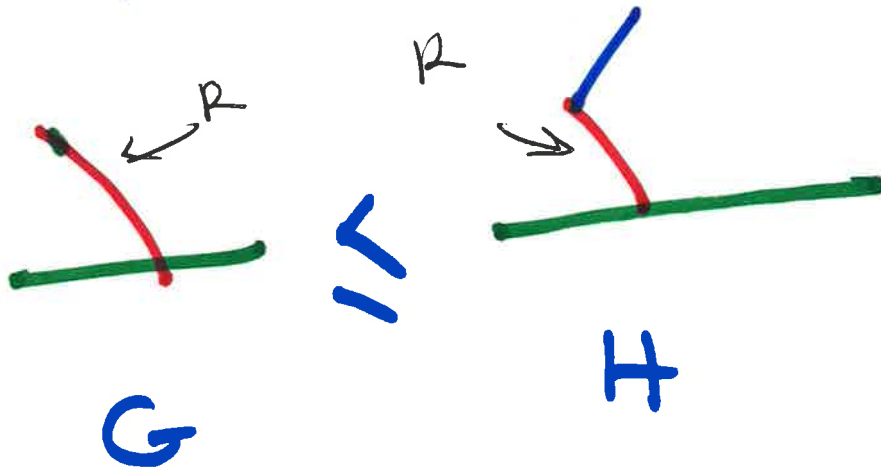


H

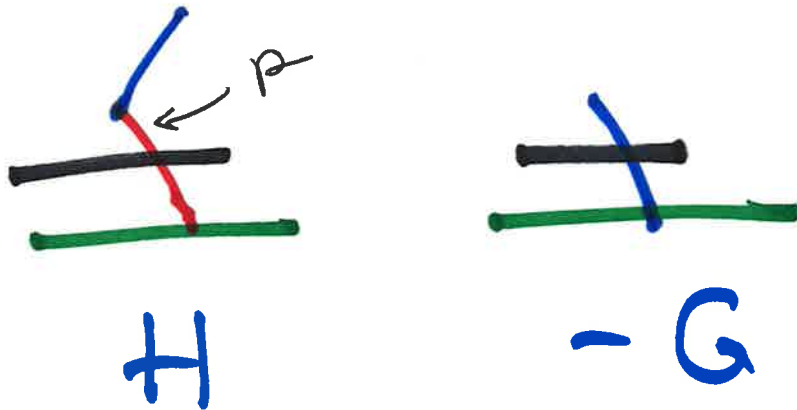
H - G

win L  
2<sup>nd</sup>  
|

# PROBLEM SOLUTION

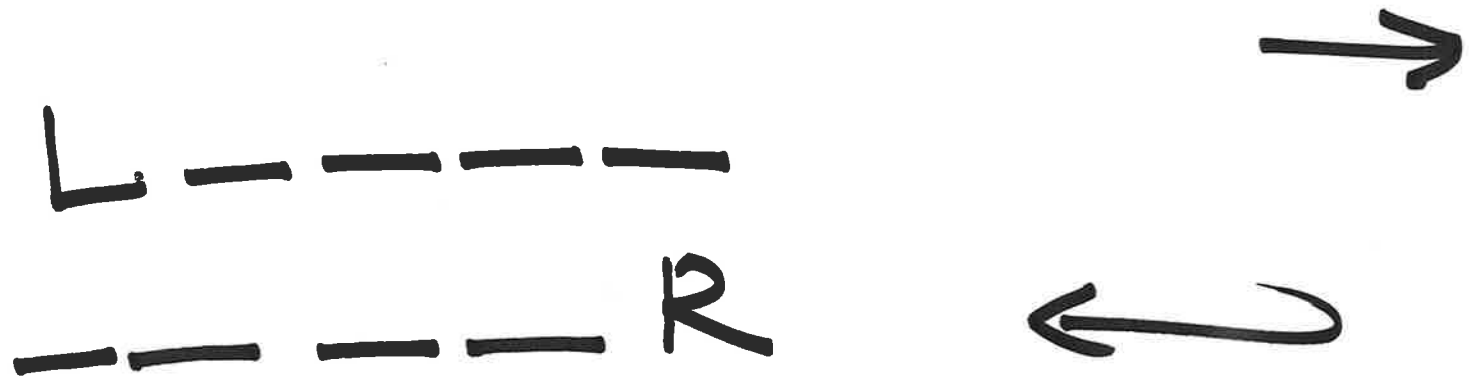


$$H - G \geq 0$$





# Ski Jumps / w.o. Jumps



= 0

1<sup>st</sup> Lose  
2<sup>nd</sup> win

# Ski Jumps w/ Jumps

L ———  
———R  
———



G

$$0 < G < 1$$

# Not Numbers

green



$$G = *$$

$$* + * = 0$$

$$* \parallel 0$$



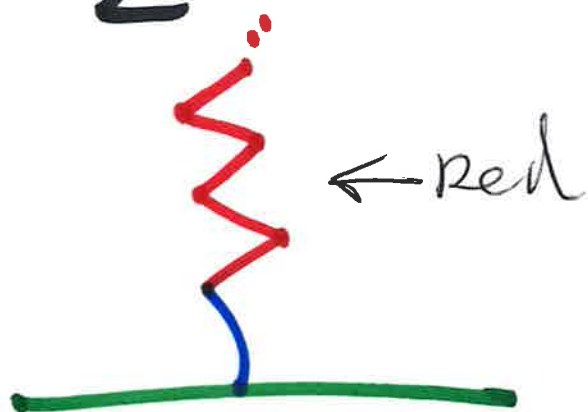
$\uparrow$

$\cup p$

$$\uparrow = \{0, 1, *\}$$

$$0 < \uparrow < \frac{1}{2^n}$$

$$\frac{1}{2^n} - \uparrow \geq 0$$



$$\frac{1}{2^n}$$

$$\{ * 10 \}$$

$$\rightarrow$$

Left wins 1<sup>st</sup> or 2<sup>nd</sup>

You try this ...

↑?

{0|\*}

L FIRST → O L wins

L 2<sup>nd</sup> R 1<sup>st</sup>

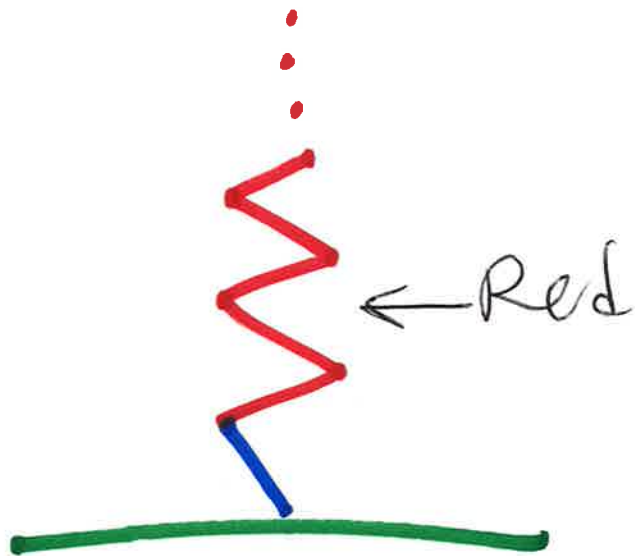
→ \*

{0|0}

→ O

✓  
{0|0}

Solution: 3.4



$\{ * 10 \}$

$L \ 1^{st}$

$\rightarrow \frac{1}{2^n} + * \rightarrow 0$

$R$

$\frac{1}{2^n} L$  wins

$R$  cuts  $\partial R$

small case  $\partial$  same

Fin



