Hw. 1 Problem 2

December 17, 2017

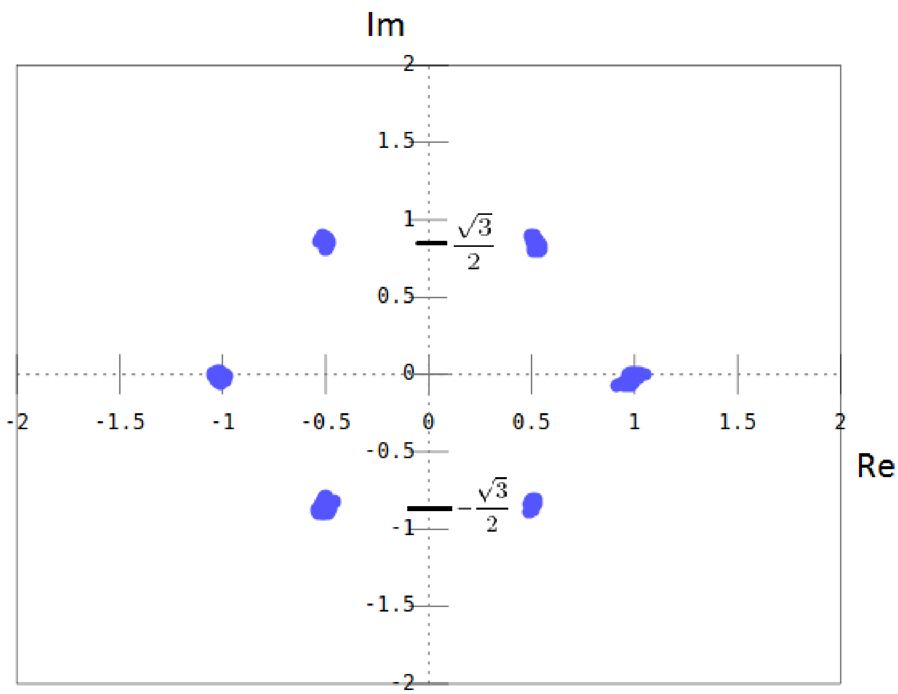
The 6th roots of unity areall the complex solutions, z, to $z^6 - 1 = 0$, or equivalently, $z^6 = 1 = e^{i0}$. Any such solution can be uniquely written as

$$e^{i(0+\frac{2\pi k}{6})} = e^{i\frac{\pi}{3}k}$$

When $k \in \mathbb{Z}, 0 \leq k < 6$. The 6th roots of unity are therefore,

$$\begin{aligned}
e^{i0} &= 1, \\
e^{i\frac{\pi}{3}} &= \cos\frac{\pi}{3} + i\sin\frac{\pi}{3} = \frac{1}{2} + i\frac{\sqrt{3}}{2}, \\
e^{i\frac{2\pi}{3}} &= \cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3} = -\frac{1}{2} + i\frac{\sqrt{3}}{2}, \\
e^{i\frac{3\pi}{3}} &= e^{i\pi} = \cos\pi + i\sin\pi = -1, \\
e^{i\frac{4\pi}{3}} &= \cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3} = -\frac{1}{2} - i\frac{\sqrt{3}}{2}, \\
e^{i\frac{5\pi}{3}} &= \cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3} = \frac{1}{2} - i\frac{\sqrt{3}}{2}.
\end{aligned}$$

Graph:



Im
2
1.5
1
$\frac{\sqrt{3}}{2}$
0.5
0
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
Re
-0.5
$-\frac{\sqrt{3}}{2}$
-1
-1.5
-2