First note that the LHS can be rewritten as

$$\sum_{k=0}^{n} \binom{n}{k} \binom{n}{n-k}.$$

Suppose $2n$ people, n boys, and n girls apply to a certain depertment in a university. All the applicants are equally qualified, but there are enough positions for only n students. Clearly, there are $\binom{2n}{n}$ ways to select n students for admittance. The number of boys chosen, k, can be any integer between 0 and n inclusive. For each such k. We can choose k boys, and $n - k$ girls for admittance. The boys can be chosen is $\binom{n}{k}$ ways, and girls, $\binom{n}{n-k}$ ways. Hence, the LHS counts seperate cases based on the number of boys to admit, then sums over all these mutually exclusive and exhaustive cases. This completes the proof. $\square$