

We know that for any Dyck path from  $(0, 0)$  to  $(2n, 0)$  satisfy the following:

- For each  $0 \leq i \leq 2n$ , the path passes through  $(i, k)$ , for some  $k \in \mathbb{Z}_{\geq 0}$ , and
- if  $i < 2n$ , then the path can next visit  $(i + 1, k + 1)$ , or, if  $k > 0$ ,  $(i + 1, k - 1)$ . We will call these cases “going up” and “going down”, respectively.

For  $0 < j \leq 2n$ , let the  $j$ -th symbol of our expression be ‘(’, if the path is going up in the  $j$ -th iteration, and ‘)’ otherwise. It is easy to see that this forms the required bijection.