Hw. 2 Problem 1

December 31, 2017

Find the image of the set $U = \{z \in \mathbb{C} : -\frac{\pi}{2} < \mathrm{Re} z < \frac{\pi}{2}\}$ under the function $f(z) = \sin z$.

1. What is the image of the line segment $L_1 = (-\frac{\pi}{2}, \frac{\pi}{2})$ under f?

   Notice that the real function, $\sin x$, is continuous and increasing on L_1, with $\sin(-\frac{\pi}{2}) = -1$ and $\sin(\frac{\pi}{2}) = 1$. Thus, $f(L_1) = (-1, 1)$.

2. What is the image of the imaginary axis $L_2 = \{iy : y \in \mathbb{R}\}$ under f?

3. What is the image of the vertical line $L_3 = \{-\frac{\pi}{2} + iy : y \in \mathbb{R}\}$ under f?

4. What is the image of the vertical line $L_4 = \{\frac{\pi}{2} + iy : y \in \mathbb{R}\}$ under f?

5. Given your above observations, what do you guess the image of the set U is under f?