

Quiz Week 5

March 29, 2013

1. *Simplify* $\{2, 2\frac{1}{2}|3, 6\}$.

Proof. We know that Left want the game to be more positive, so $2\frac{1}{2}$ dominates 2. Similarly, we know that Right wants the game to be more negative, so 3 dominates 6. Thus, the position simplifies to $\{2\frac{1}{2}|3\}$ and since the Left options are less than the Right options, we know that this simplifies to the simplest number between the two which is $2\frac{3}{4}$. □

2. *Simplify* $\{0, \uparrow|1\}$

Proof. We know that $\uparrow > 0$ so the best left option is \uparrow . We also know that $\uparrow < \frac{1}{2^n}$ so, in particular, $\uparrow < \frac{1}{2}$. Therefore, $\{0, \uparrow|1\} = \{\uparrow|1\}$ and $\frac{1}{2}$ is in between the two and is the simplest number between them so $\{0, \uparrow|1\} = \frac{1}{2}$. □

3. *Simplify* $\{\uparrow|2\}$

Proof. We know that $\uparrow < \frac{1}{2^n}$ so in particular $\uparrow < 1$. Therefore, $\uparrow < 1 < 2$ and 1 is the simplest number between them so $\{\uparrow|2\} = 1$. □

4. *Simplify* $\{\uparrow|0\}$

Proof. Recall that $\uparrow = \{0|\star\}$. If Left moves to \uparrow , then Right moves to \star , but now $\star < \{\uparrow|0\}$ (Check this! We have shown that $\{\uparrow|0\} + \star$ is positive). If Left goes first, Left moves to \uparrow giving $\uparrow + \star$ which is a second player win, namely Left. Therefore, by reversing through \uparrow to the Left option of \uparrow (which is 0), we get $\{\uparrow|0\} = \{0|0\} = \star$. □

5. *Simplify the toads and frogs position* $FT\square F$

Proof. Since the leftmost frog can not move (frogs move to the left), we can just ignore this spot and the game simplifies to $T\square F$. If Left (toad) moves first, it has 1 move to $\square TF$, in which case Rights (frogs) only move is to $FT\square$. Since Right has no more moves but Left does, this means Left wins going first. By

the same argument, Right wins going first. Therefore, this is a first player win game and thus has value \star .

□