NE 318 Project Assignment, Spring 2018
Due on July 25, 2018 (end of day)

1 Heat Transfer Model Solution

In HW3 you determined the solution to the two-dimensional transient diffusion equation,

$$\frac{\partial \tilde{T}}{\partial \tilde{t}} = \frac{\partial^2 \tilde{T}}{\partial \tilde{x}_1^2} + \frac{\partial^2 \tilde{T}}{\partial \tilde{x}_2^2}$$

where $t_s = \frac{W^2}{\alpha}$, $\alpha = \frac{k}{\rho c_p}$ is the *thermal diffusivity* of the material, and $\varphi = \frac{W}{L}$. In the current context this model is derived from the *conservation of energy equation* for a stationary amorphous/isotropic incompressible material.

In this project, you will instead study the same model which has the general solution,

$$\tilde{T}(\tilde{t}, \tilde{x}_1, \tilde{x}_2) = \exp -\lambda^2 t \left(A \cos \kappa x_1 + B \sin \kappa x_1\right) \left(C \cos \sqrt{\lambda^2 - \kappa^2} x_2 + D \sin \sqrt{\lambda^2 - \kappa^2} x_2\right)$$

except for the use of slightly more complicated boundary and initial conditions. Given boundary conditions of no (conductive) flux at the $x_1 = 0, W$ boundaries and a fixed temperature of T_0 at the $x_2 = 0, W$ boundaries, *determine the particular solution to the model.*

2 Initial Condition Formulation

The initial condition of the temperature within the domain is,

$$T(t = 0, x_1, x_2) = \frac{T_1 - T_0}{W^4} \left(W^{-2} x_1^4 - 2W^{-1} x_1^3 + x_1^2 + W^2\right) \left(x_2^2 - W x_2\right) + T_0$$

where $T_1 > T_0$. This initial condition must be represented in a compatible form with respect to the particular solution of the model. You will find that this is a relatively tedious process if done "by hand" and the use of a symbolic math library. *Regardless of the solution method used, it must be described, documented, and self-contained.*

3 Visualization and Analysis

Visualize the (dimensionless) temperature and heat flux evolution of the domain starting from the initial condition to steady-state. Compare visualizations using: (i) surfaces versus contours for temperature and (ii) arrows versus streamlines for heat flux. Briefly discuss the benefits and drawbacks of each type of visualization.

When determining a compatible form of the initial condition with respect to the particular solution of the model, show *quantitatively* that any approximations made are within a reasonable error tolerance. Typically the 2-norm is used as an error metric.