CS 181 (Introduction to Formal Languages and Automata Theory)

March 30, 2022

1 Deterministic finite automata (DFAs)

1.1 Basic notions

Definition 1.1.1

An **alphabet** is any finite set of symbols.

Example 1.1.2. Binary alphabet: $\{\mathtt{0}, \mathtt{1}\}$

Example 1.1.3. English alphabet: $\{\mathtt{a}, \mathtt{b}, \dots, \mathtt{c}\}$

Definition 1.1.4

A **string** is any finite sequence of symbols from a given alphabet.

Example 1.1.5. `001010110101`

Example 1.1.6. `abracadabra`

Example 1.1.7. ε (empty string)

Definition 1.1.8

A **language** is a set of strings over a given alphabet.

Example 1.1.9. $\varnothing$ (empty language)

Example 1.1.10. $\{\varepsilon\}$

Example 1.1.11. $\{\mathtt{acclaim}, \mathtt{aim}, \mathtt{brim}, \dots\}$

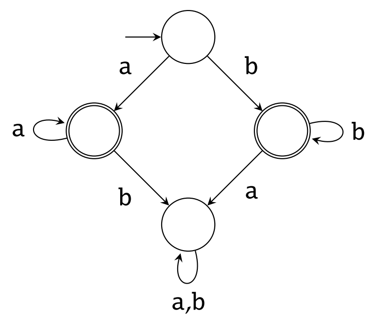
Example 1.1.12. $\{\mathtt{0}, \mathtt{1}, \mathtt{00}, \mathtt{11}, \dots\}$

Definition 1.1.13

A **computational device** is a mechanism that inputs a string and either accepts or rejects it.

1.2 Deterministic finite automata

- Choose an alphabet: $\{\mathtt{a}, \mathtt{b}\}$.
- Draw states.
- Choose start state and accept states.
- Draw transitions (out of every state on every symbol).

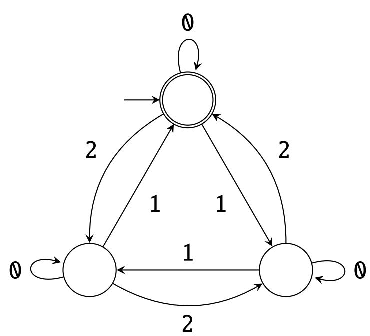


Input	Output
ε	reject
ab	reject
aaa	accept
bb	accept

In words, this machine accepts nonempty strings of all a's or all b's.

> **Definition 1.2.1**
>
> The **language** of a DFA is the set of all strings it accepts.

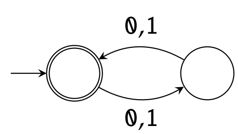
Example 1.2.2.



Input	Output
00...0	accept
12	accept
111	accept
20	reject
1	reject

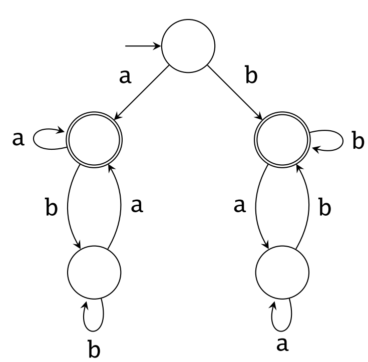
Alphabet: $\{0, 1, 2\}$, language: $\{w : 3 \mid \sum w_i\}$

Example 1.2.3.



Alphabet: $\{0, 1\}$, language: $\{w : 2 \mid |w|\}$

Example 1.2.4.

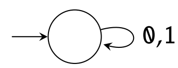


Alphabet: $\{a, b\}$, language: $\left\{w : w \neq \varepsilon \wedge w_1 = w_{|w|}\right\}$

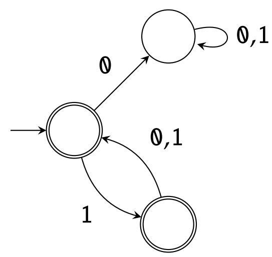
1.3 Designing DFAs

We will be using the binary alphabet $\{\texttt{0}, \texttt{1}\}$.

Example 1.3.1. Language: $\varnothing$



Example 1.3.2. Language: $\{w : \text{every odd position is a } \texttt{1}\}$



Example 1.3.3. Language: $\{w : w \text{ ends in } \texttt{0}\}$