COM SCI M238 (Quantum Computing)

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1 9.22 0th

- first quantum computer working in 2016
- rn = 127 qubits
- google projected to have 1mil qubits by 2029
- largest simulation of a quantum computer by a classical computer: nasa, 70 (perfect) qubits, half a year
  - only need 37 for practical uses
- 1% err rate
  - current err corrections reqs ~1000 qubits to support 1 perfect qubit
- moores law of quantum computing
  - err rate improves linearly per qubit
- quantum volume for the decade: 100 qubits * 1000 ops = 100k ops
  - need to push decoherence time
- good problem: small input, lots of calculation, small output, easily verifiable
- double slit experiment
  - numbers of photons that come thru when either slit is covered are not additive
  - complex α_1 and α_2 for probability for either, can be negative, amplitude leq 1
  - eg $\alpha_1 = 1/\sqrt{2}$ and $\alpha_2 = -1/\sqrt{2}$
  - probability = $|\alpha|^2 = 1/2$
  - probability when both are uncovered = $|\alpha_1 + \alpha_2|^2 = 0$
- borns rule: when measured, a state with amplitude α is observed with probability $|\alpha|^2$

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	classical	quantum
software	boolean algebra	linear algebra
hardware	classical mechanics	quantum mechanics

2.1 4 postulates that define the interface between us and the qubits

1. state space rule

- $|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$
- $|1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$
- $\left(\frac{1}{\sqrt{2}} |0\rangle + \frac{1}{\sqrt{2}} |1\rangle \right)$

2. composition rule

- tensor product

3. step rule

- unitary matrix
- $U\left|\psi\right\rangle = \left|\varphi\right\rangle$
- $\left|\psi\right\rangle$ and $\left|\varphi\right\rangle$ are unit vectors of size 2^n
- U is $2^n \times 2^n$ but can be programmed in polynomial amount of code

4. measurement rule

- bit $\xrightarrow{\text{load}}$ quantum $\xrightarrow{\text{compute}}$ quantum $\xrightarrow{\text{measure}}$ bit

2.2 from classical computing to probabilistic computing to quantum computing

- classical
  - step: $\begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$
    * rows: from 00, 01, 10, 11
    * cols: to 00, 01, 10, 11
- probabilistic: model of the world with uncertainties
  - state is a vector, taking a step = multiply by probability matrix
  - step: $\begin{bmatrix} 0 & 0 & 0 & 1/4 \\ 1 & 2/3 & 1/3 & 1/4 \\ 0 & 1/3 & 1/3 & 1/4 \\ 0 & 0 & 1/3 & 1/4 \end{bmatrix}$
  - tensor product: $\begin{bmatrix} p \\ 1-p \end{bmatrix} \otimes \begin{bmatrix} q \\ 1-p \end{bmatrix} \to \begin{bmatrix} pq \\ p(1-q) \\ (1-p)q \\ (1-p)(1-q) \end{bmatrix}$
  - "not gate": $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} p \\ q \end{bmatrix} = \begin{bmatrix} q \\ p \end{bmatrix}$
  - fair flip: $\begin{bmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{bmatrix} \begin{bmatrix} p \\ q \end{bmatrix} = \begin{bmatrix} \frac{1}{2}(p+q) \\ \frac{1}{2}(p+q) \end{bmatrix} = \begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix}$
  - $\begin{bmatrix} a \\ b \end{bmatrix} \otimes \begin{bmatrix} c \\ d \end{bmatrix} = \begin{bmatrix} ac \\ ad \\ bc \\ bd \end{bmatrix}$ can never equal $\begin{bmatrix} 1/2 \\ 0 \\ 0 \\ 1/2 \end{bmatrix}$
    * $\frac{1}{4} = \frac{1}{2} \cdot \frac{1}{2} = (ac)(bd) \neq (ad)(bc) = 0 \cdot 0 = 0$
- comparison between probabilitic and quantum | | probabilistic | quantum | | –: | :-: | :-: | | | real | complex | | state | vector of probabilities | vector of amplitudes | | | $\sum p_i = 1$ | $\sum |a|^2 = 1$ | | step | stochastic matrix | unitary matrix |
- quantum
  - fair flip: $H = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$
    * $H\left|0\right\rangle = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \frac{1}{\sqrt{2}}\left|0\right\rangle + \frac{1}{\sqrt{2}}\left|1\right\rangle$

    * $H\left|1\right\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}\begin{bmatrix}0\\ 1\end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\ 1\end{bmatrix} = \frac{1}{\sqrt{2}}\left|0\right\rangle + \frac{1}{\sqrt{2}}\left|1\right\rangle$
    * distinguishing $\begin{bmatrix}\frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}}\end{bmatrix}$ and $\begin{bmatrix}\frac{1}{\sqrt{2}}\\ \frac{-1}{\sqrt{2}}\end{bmatrix}$:
      · $H\begin{bmatrix}\frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}}\end{bmatrix} = \left|0\right\rangle$
      · $H\begin{bmatrix}\frac{1}{\sqrt{2}}\\ \frac{-1}{\sqrt{2}}\end{bmatrix} = \left|1\right\rangle$
  - $f\colon \{0,1\} \to \{0,1\}$

2.3 encoding a function to be invertible

- encoding of f: $U_f\colon \{0,1\}^2 \to \{0,1\}^2$
  - $U_f(x,b) = (x, b \oplus f(x))$ invertible
  - $(U_f \circ U_f)(x,b) = U_f(U_f(x,b)) = U_f(x, b \oplus f(x)) = (x, b \oplus f(x) \oplus f(x)) = (x,b)$