COM SCI M238 (Quantum Computing)

September 29, 2022

1 9.22 0th

- first quantum computer working in 2016
- rn = 127 qubits
- google projected to have 1mil qubits by 2029
- largest simulation of a quantum computer by a classical computer: nasa, 70 (perfect) qubits, half a year
  - only need 37 for practical uses
- 1% err rate
  - current err corrections reqs ~1000 qubits to support 1 perfect qubit
- moores law of quantum computing
  - err rate improves linearly per qubit
- quantum volume for the decade: 100 qubits * 1000 ops = 100k ops
  - need to push decoherence time
- good problem: small input, lots of calculation, small output, easily verifiable
- double slit experiment
  - numbers of photons that come thru when either slit is covered are not additive
  - complex α_1 and α_2 for probability for either, can be negative, amplitude leq 1
  - eg $\alpha_1 = 1/\sqrt{2}$ and $\alpha_2 = -1/\sqrt{2}$
  - probability = $|\alpha|^2 = 1/2$
  - probability when both are uncovered = $|\alpha_1 + \alpha_2|^2 = 0$
- borns rule: when measured, a state with amplitude α is observed with probability $|\alpha|^2$

2 9.27 1t

	classical	quantum
software	boolean algebra	linear algebra
hardware	classical mechanics	quantum mechanics

2.1 4 postulates that define the interface between us and the qubits

1. state space rule

- $|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$
- $|1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$
- $\left(\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle\right)$

2. composition rule

- tensor product

3. step rule

- unitary matrix
- $U|\psi\rangle = |\varphi\rangle$
- $|\psi\rangle$ and $|\varphi\rangle$ are unit vectors of size 2^n
- U is $2^n \times 2^n$ but can be programmed in polynomial amount of code

4. measurement rule

- bit $\xrightarrow{\text{load}}$ quantum $\xrightarrow{\text{compute}}$ quantum $\xrightarrow{\text{measure}}$ bit

2.2 from classical computing to probabilistic computing to quantum computing

- classical
  - step: $\begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$
    * rows: from 00, 01, 10, 11
    * cols: to 00, 01, 10, 11
- probabilistic: model of the world with uncertainties
  - state is a vector, taking a step = multiply by probability matrix
  - step: $\begin{bmatrix} 0 & 0 & 0 & 1/4 \\ 1 & 2/3 & 1/3 & 1/4 \\ 0 & 1/3 & 1/3 & 1/4 \\ 0 & 0 & 1/3 & 1/4 \end{bmatrix}$
  - tensor product: $\begin{bmatrix} p \\ 1-p \end{bmatrix} \otimes \begin{bmatrix} q \\ 1-p \end{bmatrix} \to \begin{bmatrix} pq \\ p(1-q) \\ (1-p)q \\ (1-p)(1-q) \end{bmatrix}$
  - "not gate": $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} p \\ q \end{bmatrix} = \begin{bmatrix} q \\ p \end{bmatrix}$
  - fair flip: $\begin{bmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{bmatrix} \begin{bmatrix} p \\ q \end{bmatrix} = \begin{bmatrix} \frac{1}{2}(p+q) \\ \frac{1}{2}(p+q) \end{bmatrix} = \begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix}$
  - $\begin{bmatrix} a \\ b \end{bmatrix} \otimes \begin{bmatrix} c \\ d \end{bmatrix} = \begin{bmatrix} ac \\ ad \\ bc \\ bd \end{bmatrix}$ can never equal $\begin{bmatrix} 1/2 \\ 0 \\ 0 \\ 1/2 \end{bmatrix}$
    * $\frac{1}{4} = \frac{1}{2} \cdot \frac{1}{2} = (ac)(bd) \neq (ad)(bc) = 0 \cdot 0 = 0$
- comparison between probabilistic and quantum

	probabilistic	quantum
value	real	complex
state	vector of probabilities	vector of amplitudes
	$\sum p_i = 1$	$\sum \lvert a\rvert^2 = 1$
step	stochastic matrix	unitary matrix

- quantum
  - fair flip: $H = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$
    * $H\left|0\right\rangle = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \frac{1}{\sqrt{2}}\left|0\right\rangle + \frac{1}{\sqrt{2}}\left|1\right\rangle$
    * $H\left|1\right\rangle = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \frac{1}{\sqrt{2}}\left|0\right\rangle + \frac{1}{\sqrt{2}}\left|1\right\rangle$
    * distinguishing $\begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}$ and $\begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} \end{bmatrix}$:
      · $H \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix} = \left|0\right\rangle$
      · $H \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} \end{bmatrix} = \left|1\right\rangle$
  - $f\colon \{0,1\} \to \{0,1\}$

2.3 encoding a function to be invertible

- encoding of f: $U_f\colon \{0,1\}^2 \to \{0,1\}^2$
  - $U_f(x,b) = (x, b \oplus f(x))$ invertible
  - $(U_f \circ U_f)(x,b) = U_f(U_f(x,b)) = U_f(x, b \oplus f(x)) = (x, b \oplus f(x) \oplus f(x)) = (x,b)$

3 9.29 1th

3.1 complex numbers

- $a + ib$
- $\overline{a+ib} = a - ib$
- $e^{i\theta} = \cos\theta + i\sin\theta$
- $\overline{e^{i\theta}} = \cos\theta - i\sin\theta = e^{-i\theta}$

3.2 hilbert space

- complex vector space w inner product
- $\left\langle \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix}, \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix} \right\rangle = \overline{\alpha_1}\beta_1 + \overline{\alpha_2}\beta_2 = \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix}^* \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix} = \begin{bmatrix} \overline{\alpha_1} & \overline{\alpha_2} \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix}$
- write $\left|\psi\right\rangle = \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix}$ and $\left\langle\varphi\right| = \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix}$
- bra-ket notation: $\left\langle\psi\right| = \begin{bmatrix} \overline{\alpha_1} & \overline{\alpha_2} \end{bmatrix}$
- inner product: $\left\langle\psi\middle|\varphi\right\rangle$
- $\left|+\right\rangle = \frac{1}{\sqrt{2}}(\left|0\right\rangle + \left|1\right\rangle)$
- $\left|-\right\rangle = \frac{1}{\sqrt{2}}(\left|0\right\rangle - \left|1\right\rangle)$

- $H\ket{0} = \ket{+}$
- $H\ket{1} = \ket{-}$
- $\braket{0|1} = 1^* \cdot 0 + 0^* \cdot 1 = 0$
- $\braket{+|-} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \end{bmatrix} \cdot \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \frac{1}{2}(1 \cdot 1 + 1 \cdot (-1)) = 0$
- outer product: $\ket{\psi}\bra{\varphi} = \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} \begin{bmatrix} \overline{\beta_1} & \overline{\beta_2} \end{bmatrix} = \begin{bmatrix} \alpha_1\overline{\beta_1} & \alpha_1\overline{\beta_2} \\ \alpha_2\overline{\beta_1} & \alpha_2\overline{\beta_2} \end{bmatrix}$
- a matrix U is unitary iff $UU^* = I$ (equivalent to $U^*U = I$)

3.3 partial measurement

- start state: $\ket{\psi} = \alpha_{00}\ket{00} + \alpha_{01}\ket{01} + \alpha_{10}\ket{10} + \alpha_{11}\ket{11}$
- measure qubit $1 \longrightarrow 0$
- new sate: $\frac{\alpha_{00}\ket{00} + \alpha_{01}\ket{01}}{\sqrt{|\alpha_0 0|^2 + |\alpha_0 1|^2}} = \ket{0} \otimes \frac{\alpha_{00}\ket{0} + \alpha_{01}\ket{1}}{\sqrt{|\alpha_0 0|^2 + |\alpha_0 1|^2}}$

3.4 generalization of tensor product

- outer product: $\ket{\psi}\bra{\varphi} = \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} \otimes \begin{bmatrix} \overline{\beta_1} & \overline{\beta_2} \end{bmatrix} = \begin{bmatrix} \alpha_1\overline{\beta_1} & \alpha_1\overline{\beta_2} \\ \alpha_2\overline{\beta_1} & \alpha_2\overline{\beta_2} \end{bmatrix}$
- $\begin{bmatrix} \alpha_{00} & \alpha_{01} \\ \alpha_{10} & \alpha_{01} \end{bmatrix} \otimes B = \begin{bmatrix} \alpha_{00}B & \alpha_{01}B \\ \alpha_{10}B & \alpha_{01}B \end{bmatrix}$
- $\ket{00} = \ket{0} \otimes \ket{0} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \otimes \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$
- $\underbrace{\ket{101}}_{\text{5 in decimal}} = \ket{1} \otimes \ket{0} \otimes \ket{1} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$
- inner product $\rightsquigarrow$ matrix product
- outer product $\rightsquigarrow$ matrix product + tensor product
- $\otimes$ associative: $A \otimes (B + C) = A \otimes B + A \otimes C$
- $(A \otimes B)(C \otimes D) = (AC) \otimes (BD)$
- "floating scalar rule": $(\alpha A) \otimes B = A \otimes (\alpha B) = \alpha(A \otimes B)$
- $\ket{\psi} \cdot \bra{\varphi} \cdot \ket{\gamma} = \ket{\psi} \cdot \braket{\varphi|\gamma} = \braket{\varphi|\gamma} \cdot \ket{\psi}$