

COM SCI M238 (Quantum Computing)

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1 9.22 0th

- first quantum computer working in 2016
- rn = 127 qubits
- google projected to have 1mil qubits by 2029
- largest simulation of a quantum computer by a classical computer: nasa, 70 (perfect) qubits, half a year
 - only need 37 for practical uses
- 1% err rate
 - current err corrections reqs ~1000 qubits to support 1 perfect qubit
- moores law of quantum computing
 - err rate improves linearly per qubit
- quantum volume for the decade: 100 qubits * 1000 ops = 100k ops
 - need to push decoherence time
- good problem: small input, lots of calculation, small output, easily verifiable
- double slit experiment
 - numbers of photons that come thru when either slit is covered are not additive
 - complex α_1 and α_2 for probability for either, can be negative, amplitude leq 1
 - eg $\alpha_1 = 1/\sqrt{2}$ and $\alpha_2 = -1/\sqrt{2}$
 - probability = $|\alpha|^2 = 1/2$
 - probability when both are uncovered = $|\alpha_1 + \alpha_2|^2 = 0$
- borns rule: when measured, a state with amplitude α is observed with probability $|\alpha|^2$

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	classical	quantum
software	boolean algebra	linear algebra
hardware	classical mechanics	quantum mechanics

2.1 4 postulates that define the interface between us and the qubits

1. state space rule

- $|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$
- $|1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$
- $\left(\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle\right)$

2. composition rule

- tensor product

3. step rule

- unitary matrix
- $U|\psi\rangle = |\varphi\rangle$
- $|\psi\rangle$ and $|\varphi\rangle$ are unit vectors of size 2^n
- U is $2^n \times 2^n$ but can be programmed in polynomial amount of code

4. measurement rule

- bit $\xrightarrow{\text{load}}$ quantum $\xrightarrow{\text{compute}}$ quantum $\xrightarrow{\text{measure}}$ bit

2.2 from classical computing to probabilistic computing to quantum computing

- classical

$$\text{– step: } \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

* rows: from 00, 01, 10, 11

* cols: to 00, 01, 10, 11

- probabilistic: model of the world with uncertainties

- state is a vector, taking a step = multiply by probability matrix

$$\text{– step: } \begin{bmatrix} 0 & 0 & 0 & 1/4 \\ 1 & 2/3 & 1/3 & 1/4 \\ 0 & 1/3 & 1/3 & 1/4 \\ 0 & 0 & 1/3 & 1/4 \end{bmatrix}$$

$$\text{– tensor product: } \begin{bmatrix} p \\ 1-p \end{bmatrix} \otimes \begin{bmatrix} q \\ 1-p \end{bmatrix} \rightarrow \begin{bmatrix} pq \\ p(1-q) \\ (1-p)q \\ (1-p)(1-q) \end{bmatrix}$$

$$\text{– “not gate”: } \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} p \\ q \end{bmatrix} = \begin{bmatrix} q \\ p \end{bmatrix}$$

$$\text{– fair flip: } \begin{bmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{bmatrix} \begin{bmatrix} p \\ q \end{bmatrix} = \begin{bmatrix} \frac{1}{2}(p+q) \\ \frac{1}{2}(p+q) \end{bmatrix} = \begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix}$$

$$\text{– } \begin{bmatrix} a \\ b \end{bmatrix} \otimes \begin{bmatrix} c \\ d \end{bmatrix} = \begin{bmatrix} ac \\ ad \\ bc \\ bd \end{bmatrix} \text{ can never equal } \begin{bmatrix} 1/2 \\ 0 \\ 0 \\ 1/2 \end{bmatrix}$$

$$* \frac{1}{4} = \frac{1}{2} \cdot \frac{1}{2} = (ac)(bd) \neq (ad)(bc) = 0 \cdot 0 = 0$$

- comparison between probabilistic and quantum | | probabilistic | quantum | | –: | ::: | ::: | | | real | complex | | state | vector of probabilities | vector of amplitudes | | $\sum p_i = 1$ | $\sum |a|^2 = 1$ | | step | stochastic matrix | unitary matrix |
- quantum

$$\text{– fair flip: } H = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$* H|0\rangle = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$

$$\begin{aligned}
& * H |1\rangle = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \frac{1}{\sqrt{2}} |0\rangle + \frac{1}{\sqrt{2}} |1\rangle \\
& * \text{distinguishing } \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix} \text{ and } \begin{bmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{bmatrix}: \\
& \quad \cdot H \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix} = |0\rangle \\
& \quad \cdot H \begin{bmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{bmatrix} = |1\rangle \\
& - f: \{0,1\} \rightarrow \{0,1\}
\end{aligned}$$

2.3 encoding a function to be invertible

- encoding of $f: \{0,1\}^2 \rightarrow \{0,1\}^2$
 - $U_f(x, b) = (x, b \oplus f(x))$ invertible
 - $(U_f \circ U_f)(x, b) = U_f(U_f(x, b)) = U_f(x, b \oplus f(x)) = (x, b \oplus f(x) \oplus f(x)) = (x, b)$