EC ENGR 113 (Digital Signal Processing)

October 3, 2022

1 9.26 1m

- syllabus

1.1 ece102: signals & systems

- signals were modeled as functions of time
  - $x(t) : t \in \mathbb{R}$ (continuous time)
  - $x : \mathbb{R} \to \mathbb{R}, \mathbb{C}$
  - sampling $t = nT_s$
    * T_s = sampling interval
    * n = integer
    * $t - x$ plot $\xrightarrow{\text{sampling}}$ $n - x$ plot
    * $x(t)$ bandlimited ($|X(f)| \le f_{\max}$) then if sampling rate is $f_s > 2f_{\max}$ (nyquist rate) then it is possible to recover the original signal
    * sampling at lower rate then there will be aliasing
- sytems
  - $x(t) \to \boxed{S} \to y(t)$
  - $x[n] \to \boxed{S} \to y[n]$

1.2 three ways tor write out signals

- mathematical expression
  - $x[n] = 3\cos(2n), n \in \mathbb{Z}$
- tabular list of significant samples
  - $x[n] = \{-1, 8, 0, 5, -2\}, n \in \mathbb{Z}$
    * underspecified signal: loss of indices
    * typically put an arrow underneath the 0 index
    * convention: any sample not listed has 0 amplitude
- plotting

1.3 signal operations

- arithmetic
  - $g[n] = x[n] + A$
  - $g[n] = Bx[n]$
  - signal addition: $g[n] = x_1[n] + x_2[n]$
  - signal multiplication: $g[n] = x_1[n] \cdot x_2[n]$
- time shifting
  - ece102
    * $x(t) \to x(t - \tau)$ delays signal by shifting it right
    * $x(t) \to x(t + \tau)$ advances signal by shifting left
  - ece113

    - $g[n] = x[n-k], k > 0$
- time scaling
  - ece102
    - $x(at), a > 1$: contraction
    - $x(at), 0 < a < 1$: dilation
  - ece113
    - $g[n] = x[2n]$ (downsampling)
    - $g[n] = x\left[\frac{1}{2}n\right]$ (upsampling)
    - $g[n] = x[-n]$ (time reversal)

1.4 basic signals

- $\delta(t)$ from ece102 $\longrightarrow$ hard concept to grasp
- $\delta[t]$ from ece113 $\longrightarrow$:)
  - $\delta[n] = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$
  - $\delta[n]$ does not have complexities of $\delta(t)$
- sampling property of δ
  - $x[n] * \delta[n] = x[0] \cdot \delta[0] = x[0]$
  - $x[n] * \delta[n-k] = x[k] \cdot \delta[0] = x[k]$

2 9.28 1w

2.1 sampling properties hold in discrete domain as well

- $\sum_{n=-\infty}^{\infty} x[n] \cdot \delta[n-k] = \sum_n x[k]\delta[n-k] = x[k] \sum_n \delta[n-k] = x[k]$

2.2 systems

- unit step system $u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$
  - $u[n] - u[n-1] = \delta[n]$
  - $\delta[n]$ is the first derivative of $u[n]$
  - ece102: $\delta(t) = \frac{du(t)}{dt}$
  - $u[n] = \sum_{k=0}^{\infty} \delta[n-k]$
- unit ramp signal $r[t] = \begin{cases} n & n \geq 0 \\ 0 & n < 0 \end{cases}$
  - 102: $r(t) = tu(t)$
  - $r[n] = n \cdot u[n] = \sum_{k=1}^{\infty} u[n-k]$

2.3 how wan se express $x[n]$ using basic signals?

- generic signal representation using the "canonical basis" of $\{\delta[n-k] \forall k\}$

- $x[n] = \{1, 2, (0), 1, 1\}$
- $x[n] = \delta[n+2] + 2\delta[n+1] + \delta[n-1] - \delta[n-2]$

2.4 sinusoidal signals

- $x(t) = A\cos(\omega_0 t + B)$, $\omega_0 = 2\pi f_0$
- $x[n] = A\cos(\Omega_0 n + \theta)$, $\Omega_0 = 2\pi F_0 \in [-\pi, \pi)$ where F_0 is the normalized frequency
- $x[n]A\cos(2\pi f_0 n T_0 + \theta) = A\cos\left(2\pi \frac{f_0}{f_s} n + \theta\right)$
- $F_n := \frac{f_0}{f_s} < 1$

2.5 complex exponential

- $x[n] = Ae^{j(2\pi F_0 n + \theta)}$

2.6 additional signal properties

1. duration of signal
2. real or complex
3. periodicity: a signal is periodic if $\exists N \in \mathbb{Z} : x[n+N] = x[n] \; \forall n$

- $x[n]$ is also periodic with $2N, 3N$, etc.
- $x[n+kN] = x[n] \; \forall k, n$
- fundamental period: smallestpositive N such that $x[n+N] = x[n]$
- e.g. $x[n] = \cos(2\pi F_0 n) \longrightarrow \exists k \in \mathbb{Z} : 2\pi F_0 N = 2\pi k \longrightarrow F_0 N \in \mathbb{Z}, N = \frac{k}{F_0}$
- is $x[n] = \cos(0.2n)$ periodic?
  - no
  - $2\pi F_0 = 0.2 \longrightarrow \frac{k}{F_0} = 10k\pi \notin \mathbb{Z}$