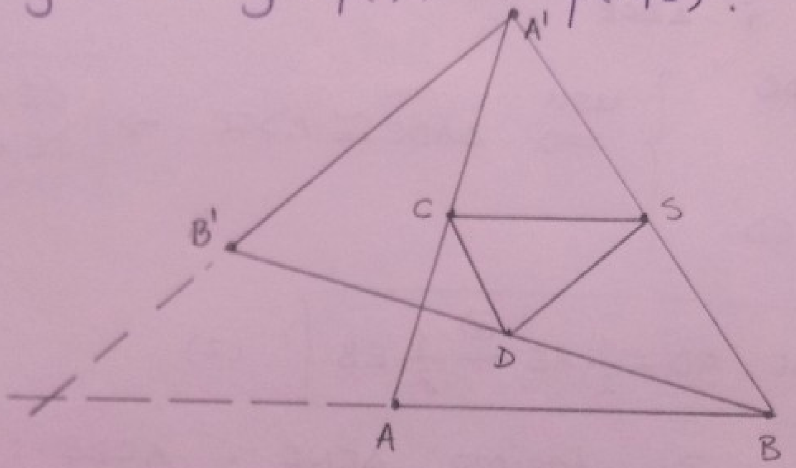
Definicija: Duž čije su krajnje tačke središte dvije stranice nekog trougla naziva se SREDNJA LINIJA tog trougla koja odgovara njegovoj trećoj stranici.

Teorema: Ako su P i Q središta stranica AB i AC redom, tada je
$$PQ = \frac{1}{2}BC \quad i \quad PQ \parallel BC.$$

$\boxed{2/5.}$ Ako su AB i $A'B'$ dvije podudarne duži i C i D središta duži AA' i BB' takve da je $CD = \frac{1}{2}AB$, odrediti mjeru ugla koji određuju $p(A,B)$ i $p(A',B')$.

Rješenje:



$CD = \frac{1}{2}AB$ (*)

$AB = A'B'$ (**)

Neka je S - središte duži $A'B$. (1)

C - središte duži AA' (2)

D - središte duži BB' (3).

* Iz (1) i (2) $\Rightarrow$ CS je srednja linija $\Delta A'AB$ $\Rightarrow$

$$CS = \frac{1}{2}AB \overset{(*)}{=} CD \qquad (4)$$

$$CS \parallel AB \qquad (5)$$

* Iz (1) i (3) $\Rightarrow$ DS je srednja linija $\Delta BB'A'$ $\Rightarrow$

$$DS = \frac{1}{2}A'B' \overset{(**)}{=} \frac{1}{2}AB \overset{(*)}{=} CD \qquad (6)$$

$$DS \parallel A'B' \qquad (7)$$

* Iz (4) i (6) $\Rightarrow$ ΔCSD je jednakostranični Δ. (8)

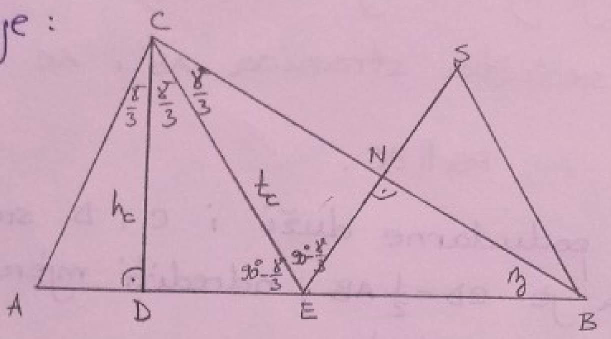
Iz (5) i (7) vrijedi

$\measuredangle AB, A'B' = \measuredangle CSD$ (ugl. s paralelnim kracima)

$= 60^\circ$ (zbog (8)).

216. Izračunati uglove trougla, ako visina i težišna linija iz tjemena C dijeli $\measuredangle ACB$ na tri jednaka dijela.

Rješenje:



CD - visina $\triangle ABC$

CE - težišna linija $\triangle ABC$

$AE = EB = \frac{1}{2} AB$ (*)

* Posmatrajmo $\triangle ADC$ i $\triangle DCE$

$$\left.\begin{aligned} \measuredangle ADC &= 90° = \measuredangle EDC \\ CD &= CD \\ \measuredangle ACD &= \frac{\gamma}{3} = \measuredangle ECD \end{aligned}\right\} \overset{usu}{\Longrightarrow} \triangle ADC \cong \triangle DCE \Rightarrow \boxed{\begin{aligned} CE &= AC \\ DE &= AD \end{aligned}} \quad (1)$$

$$DE = AD \Longrightarrow \boxed{DE = AD = \frac{1}{2} AE \overset{(*)}{=} \frac{1}{2} EB} \quad (2)$$

* Neka je $EN \perp BC$. Posmatrajmo $\triangle ENC$ i $\triangle CDE$:

$$\left.\begin{aligned} \measuredangle NEC &= 90° - \frac{\gamma}{3} = \measuredangle DEC \\ CE &= CE \\ \measuredangle NCE &= \frac{\gamma}{3} = \measuredangle DCE \end{aligned}\right\} \overset{usu}{\Longrightarrow} \triangle ENC \cong \triangle CDE \Rightarrow \boxed{\begin{aligned} CD &= CN \\ DE &= EN \end{aligned}} \quad (3)$$

$$DE = EN \Rightarrow \boxed{EN = DE \overset{(2)}{=} \frac{1}{2} EB} \quad (4)$$

Dakle, iz (4) $\Rightarrow \boxed{EB = 2EN}$ (5)

* Na $pp[E,N)$ odredimo tačku S tako da $E-N-S$ i

$$\boxed{ES = 2EN \overset{(5)}{=\!=} EB} \quad (6)$$

* Posmatrajmo $\triangle ENB$ i $\triangle SNB$:

$$\left.\begin{aligned} NB &= NB \\ \measuredangle ENB &= 90° = \measuredangle SNB \\ EN &= NS \end{aligned}\right\} \overset{sus}{\Longrightarrow} \triangle ENB \cong \triangle SNB \Rightarrow \boxed{\begin{aligned} EB &= BS \\ \measuredangle EBN &= \beta = \measuredangle NBS \end{aligned}} \quad (7)$$

Iz (6) i (7) je $ES = EB = BS \Rightarrow \triangle EBS$ je jednakostranični trougao

$\Rightarrow \boxed{\sphericalangle EBS = 60^\circ}$

Osim toga, vrijedi

$$\sphericalangle EBS = \sphericalangle EBN + \sphericalangle NBS \overset{(7)}{=\!=} 2\beta \Rightarrow \beta = \frac{1}{2} \sphericalangle EBS$$

$$\boxed{\beta = 30^\circ}$$

* Posmatrajmo $\triangle CDB$:

$$\sphericalangle CDB + \sphericalangle DBC + \sphericalangle BCD = 180^\circ$$

$$90^\circ + 30^\circ + \sphericalangle BCD = 180^\circ \Rightarrow \boxed{\sphericalangle BCD = 60^\circ}$$

Osim toga, $\sphericalangle BCD = \sphericalangle BCE + \sphericalangle ECD = \frac{\gamma}{3} + \frac{\gamma}{3} = \frac{2\gamma}{3}$ $\Rightarrow \gamma = \frac{3}{2} \cdot 60^\circ$

$$\boxed{\gamma = 90^\circ}$$

* Iz $\triangle ABC$ vrijedi :

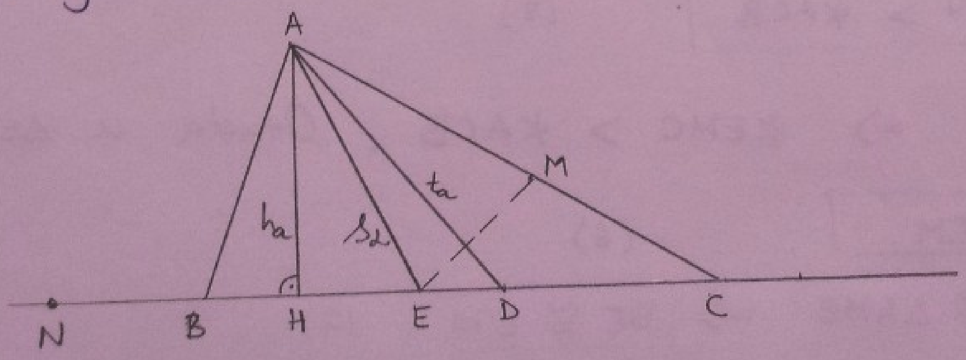
$$\alpha + \beta + \gamma = 180^\circ \Rightarrow \alpha = 180^\circ - 30^\circ - 90^\circ$$

$$\boxed{\alpha = 60^\circ}$$

2/7. U $\triangle ABC$ je $AB < AC$. Obilježimo sa E, D, H redom tačke u kojima simetrala ugla, težišna linija i visina iz tjemena A sijeku pravu $p(B,C)$. Dokazati da je

a) $\sphericalangle AEB < \sphericalangle AEC$ i b) $BE < CE$.

Rješenje:



* Budući da je $AB < AC \Rightarrow (\exists M)(M \in \overline{AC})$ $\overline{AB} \cong AM$ i vrijedi $A-M-C$
* Posmatrajmo $\triangle ABE$ i $\triangle AME$

$$\left.\begin{array}{l} AM \cong AB \\ \sphericalangle MAE \cong \sphericalangle BAE \quad (A_L - \text{simetrala } \sphericalangle BAC) \\ AE \cong AE \end{array}\right\} \overset{SUS}{\Longrightarrow} \triangle ABE \cong \triangle AME \Rightarrow \boxed{\sphericalangle AEB \cong \sphericalangle AEM} \quad (1)$$

* Kako je M unutrašnja tačka $\sphericalangle AEC$ i vrijedi $A-M-C \Rightarrow$ poluprava $pp[E, M)$ je u unutrašnjosti $\sphericalangle AEC$, pa je

$$\boxed{\sphericalangle AEM < \sphericalangle AEC.} \quad (2)$$

* Iz (1) i (2) $\Rightarrow$ $\sphericalangle AEB < \sphericalangle AEC$.

b) Neka je N tačka prave $p(B,C)$ tako da je $N-B-C$.
Zbog, $\triangle ABE \cong \triangle AME$ (dokazano u a)) slijedi da

$$\sphericalangle ABE \cong \sphericalangle AME . \quad (1)$$

* Uglovi
  $\sphericalangle ABE$ i $\sphericalangle ABN$ su naporedni uglovi (2)
  $\sphericalangle AME$ i $\sphericalangle EMC$ su naporedni uglovi . (3)

Budući da su naporedni uglovi podudarnih uglova, također, podudarni iz (1), (2), (3) slijedi

$$\boxed{\sphericalangle ABN \cong \sphericalangle EMC .} \quad (4)$$

* Kako je $\sphericalangle ABN$ spoljašnji ugao $\triangle ABC$ to je

$$\boxed{\sphericalangle ABN > \sphericalangle ACB} \quad (5)$$

Iz (4) i (5) $\Rightarrow$ $\sphericalangle EMC > \sphericalangle ACB$. Otuda u $\triangle EMC$ vrijedi $\boxed{CE > EM.}$ (6)

* Zbog $\triangle ABE \cong \triangle AME \Rightarrow \boxed{BE \cong EM.}$ (7)

Iz (6) i (7) $\Rightarrow$ $\boxed{CE > BE}$