Zadatak 1. *Dokazati da* $(A \setminus B) \cup (A \cap B) \cup (B \setminus A) = A \cup B$

Primjer 1. *Neka su* A, B, C *i* D *proizvoljni skupovi. U kom odnosu su skupovi* $(A \cap B) \cup (C \cap D)$ *i* $(A \cup C) \cap (B \cup D)$*?*

Rješenje: Za proizvoljno x vrijedi:

$$\begin{aligned} x \in (A \cap B) \cup (C \cap D) &\iff x \in A \cap B \vee x \in C \cap D \\ &\iff (x \in A \wedge x \in B) \vee (x \in C \wedge x \in D) \\ &\Longrightarrow (x \in A \vee x \in C) \wedge (x \in B \vee x \in D) \\ &\iff x \in (A \cup C) \cap (B \cup D)\ . \end{aligned}$$

Dakle, vrijedi $(A \cap B) \cup (C \cap D) \subseteq (A \cup C) \cap (B \cup D)$.
Da li vrijedi jednakost u ovoj vezi? Odgovor je NE! Primjer za to su skupovi $A = \{0\}$, $B = \{1\}$, $C = \{1\}$ i $D = \{0\}$. Sada imamo

$$A \cap B = \varnothing\ ,\ C \cap D = \varnothing\ ,\ A \cup C = \{0, 1\}\ ,\ B \cup D = \{0, 1\}\ .$$

Dakle,

$$(A \cap B) \cup (C \cap D) = \varnothing\ ,\ (A \cup C) \cap (B \cup D) = \{0, 1\}\ ,$$

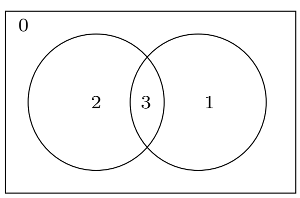
to jest

$$(A \cap B) \cup (C \cap D) = \varnothing \subset \{0, 1\} = (A \cup C) \cap (B \cup D)\ .$$

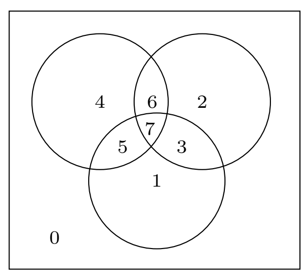
♣

A	B	C	$A \setminus C$	$B \setminus C$	Lijevo	$A \cap B$	Desno	Venn	
Ne	Ne	Ne	Ne	Ne	Ne	Ne	Ne	0	
Da	Ne	Na	Da	Ne	Ne	Ne	Ne	4	
Ne	Da	Ne	Ne	Da	Ne	Ne	Ne	2	
Ne	Ne	Da	Ne	Ne	Ne	Ne	Ne	1	
Da	Da	Ne	Da	Da	Da	Da	Da	6	✓
Da	Ne	Da	Ne	Ne	Ne	Ne	Ne	5	
Ne	Da	Da	Ne	Ne	Ne	Ne	Ne	3	
Da	Da	Da	Ne	Ne	Ne	Da	Ne	7	

$$\begin{aligned} x \in A \cup B &\Longrightarrow x \in A \vee x \in B && \big(\text{aksiom unije}\big) \\ &\Longrightarrow (x \in A \wedge x \in B) \vee x \in B && \big(\text{pretpostavka } A = A \cap B\big) \\ &\Longrightarrow x \in B && \big(\text{tautologija } ((p \wedge q) \vee q) \Rightarrow q\big) \end{aligned}$$



(a)



(b)