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Schilichting (2015) consider self-similar solution to adiabatic hydrodynamic equations for an isothermal and adiabatic atmosphere. The hydrodynamic equations are

$$\frac{1}{\rho}\frac{D\rho}{Dt} + \frac{\partial u}{\partial z} = 0$$
$$\frac{Du}{Dt} + \frac{1}{\rho}\frac{\partial p}{\partial z} = 0$$
$$\frac{1}{p}\frac{Dp}{Dt} - \frac{\gamma}{\rho}\frac{D\rho}{Dt} = 0.$$

$$h = \text{the atmospheric scale height} = \frac{\kappa T}{Mg}$$
$$\kappa = \text{Boltzmann constant} = 1.38 \times 10^{23} J \cdot K^{1}$$
$$T = \text{mean atmospheric temperature [K]}$$
$$M = \text{molecular mass of a molecule [kg]}$$
$$g = \text{gravity}\left[\frac{m}{s^2}\right]$$
$$z = \text{height in the atmosphere from the ground}$$
$$\gamma = \text{adiabatic index}$$
$$p = \text{pressure}$$
$$\rho = \text{density}$$
$$\boldsymbol{u} = \text{particle velocity}$$
$$\dot{Z}(t) = \text{shock velocity}$$

Due to nature of the self-similar solution, the temporal and spatial parts are separated for the two atmospheric cases. In the isothermal case, by introducing

$$\rho(z,t) = \rho_0 \exp\frac{-Z(t)}{h} G(\xi)$$
$$p(z,t) = \rho_0 \exp\frac{-Z(t)}{h} \dot{Z}^2 P(\xi)$$
$$\boldsymbol{u}(z,t) = \dot{Z} U(\xi)$$

the solution for

$$\gamma = \frac{4}{3}, \quad \alpha = 5.669, \quad \xi_c = -0.356$$

to the right. Note that alpha is numerically determined.

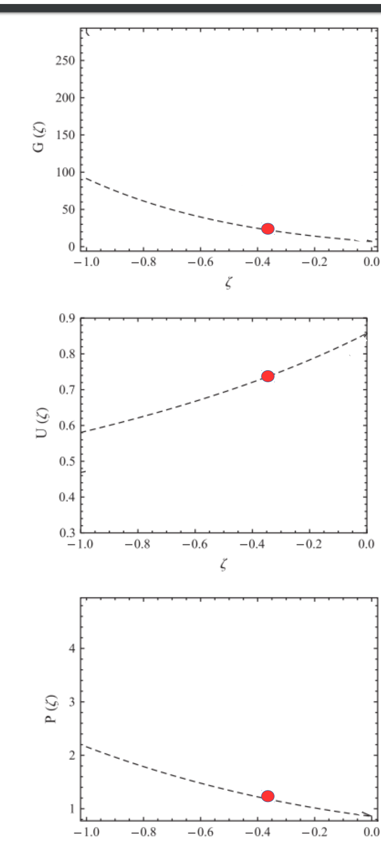

Fig. 2. Solutions for $G(\zeta)$, $U(\zeta)$ and $P(\zeta)$ for an adiabatic index $\gamma = 4/3$

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A similar solution is derived for the adiabatic atmosphere case.

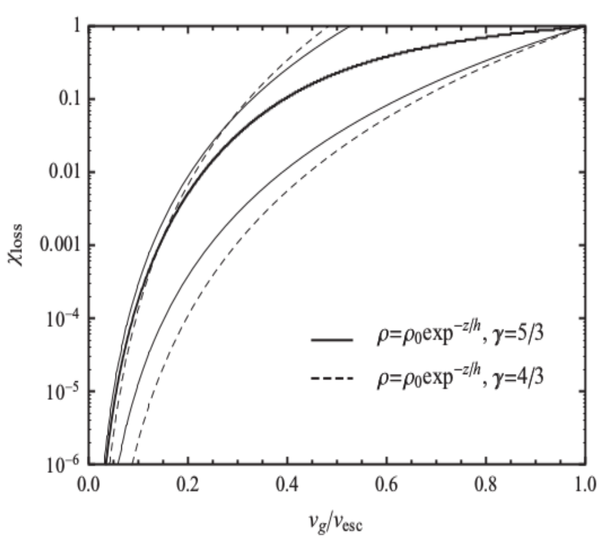
To quantify the atmospheric loss, the lowest height measured from the ground at which a parcel of fluid experiences the escape velocity is considered. The atmosphere is locally lost beyond this position. The atmospheric loss is influenced by the product of the acceleration factor

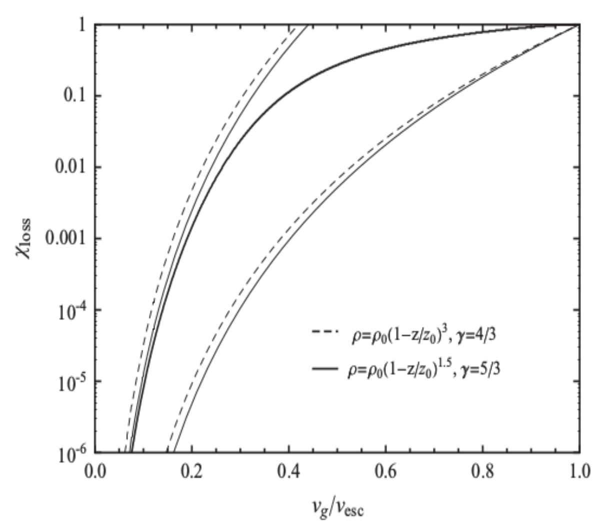
$$\beta = \frac{u_\infty}{u_0} = \frac{u_\infty}{u_{t_0}}\frac{u_{t_0}}{u_0}$$

$$\frac{u_{t_0}}{u_0} = \frac{U(\zeta \to -\infty)\dot{Z}(\zeta \to -\infty)}{U(\zeta = 0)\dot{Z}(\zeta = 0)},$$

$$\frac{u_\infty}{u_{t_0}} = \frac{U(\zeta \to +\infty)\dot{Z}(\zeta \to +\infty)}{U(\zeta \to -\infty)\dot{Z}(\zeta \to -\infty)}.$$

The local atmospheric loss is then





where the solid curve is the numerical solution from the

To relate the global ground motion to the impactor mass
scaling with radius of target and thereby assume momer
through the target. Therefore, ground velocity and shock velocity are shown to the right.

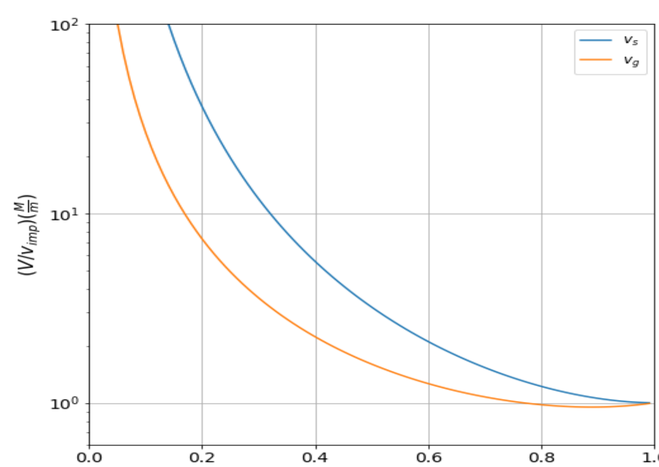


Fig. 7. Shocked fluid velocity v_s and the ground velocity v_g as a function of distance travelled by the shock, l, from the impact point to the other side of the planet, $l = 2R$.

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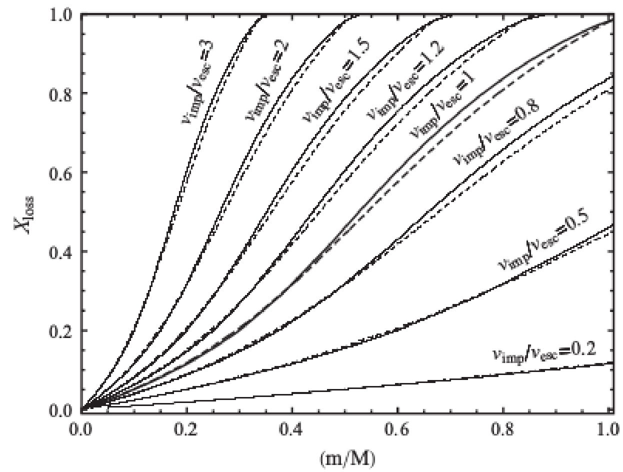
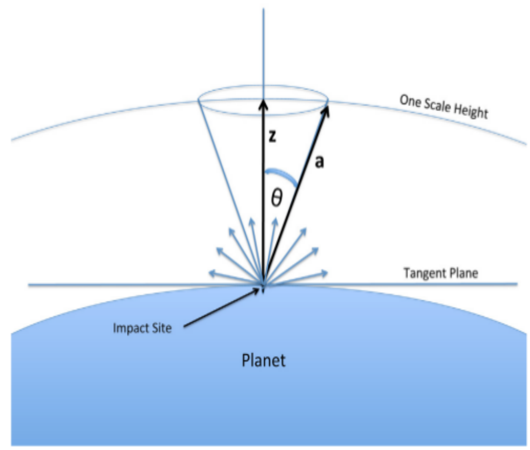
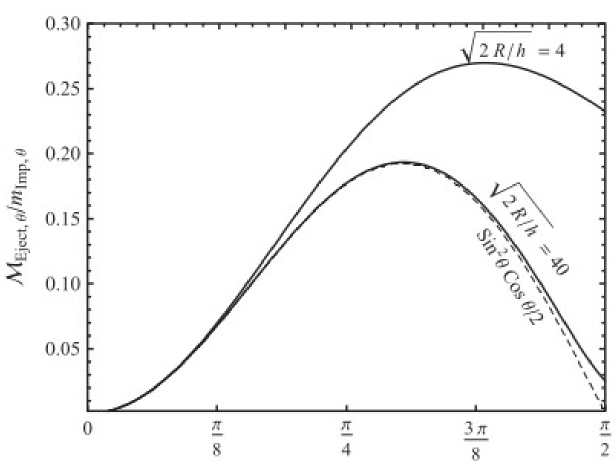
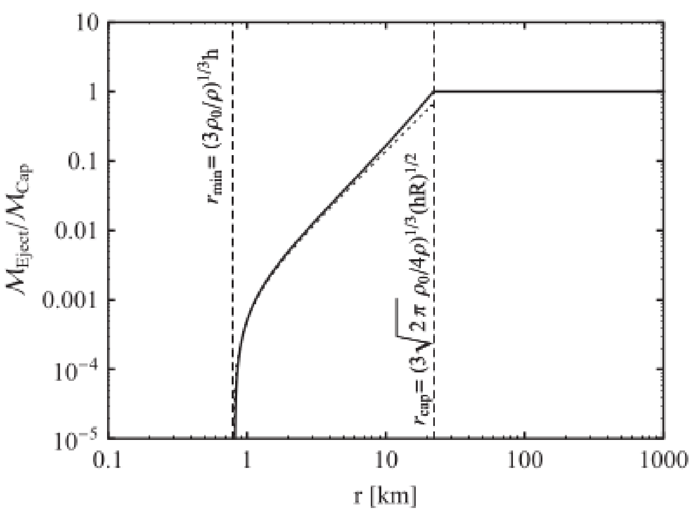
The minimum of the ground velocity is reached at approximately l/2R = 8/9. This then determines the ratio at which the entire atmosphere is lost. Considering different ratios, the plot to the right shows the influence on atmospheric loss. Another possibility of reaching these ratios is considering oblique impacts given that the shock wave that propagated through the target is characterized by the vertical component of the impact velocity.



To characterize the mass of atmosphere lost, Schilichting quantifies the mass of atmosphere contained based on angle and atmospheric height. Next, assuming that the projectile's impact velocity is close to the escape velocity, the mass of the impactor required to eject all of the atmospheric mass in the section of the cap subtended by theta. The ratio as a function of theta is shown below:







Furthermore, using the Earth's atmosphere as an example, the minimum impactor radius to eject the all of the atmosphere above the tangent plane is determined assuming projectile density of 2 g/cm^3. Next, the minimum impactor radius to eject the atmosphere at theta=0 degrees is determined. The plot below shows ratio of atmosphere mass ejected as a ratio of max mass ejected as a function of impactor radius.

However, in only considering vertical impacts, Shavalov (2009) shows that the atmospheric loss is less efficient. In contrast, for oblique impacts, the atmosphere expands more isotropically and therefore more atmosphere is lost. Compared to Shavalov (2009), for impact velocities of 15 km/s, 20 km/s, and 30 km/s, the ratio of mass of atmosphere ejected to mass of impactor is overestimated by factors of 10, 3, and 1 respectively.

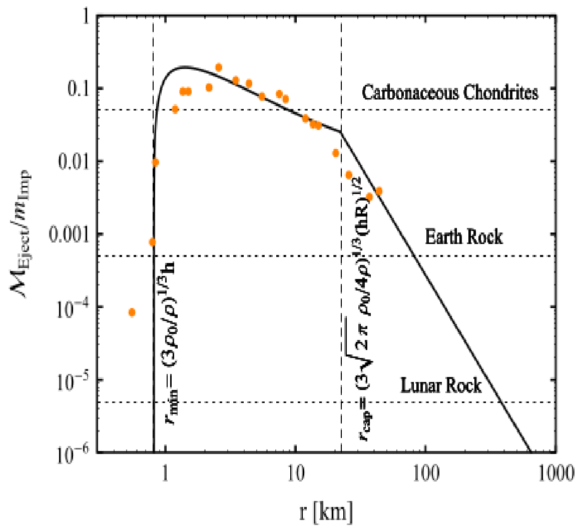


Fig. 16. Ratio of atmospheric mass ejected to impactor mass, $\mathcal{M}_{Eject}/m_{Imp}$. Numerical values are scaled to the current Earth. Small impactors with $r_* = \sqrt{3}r_{min}$ are the

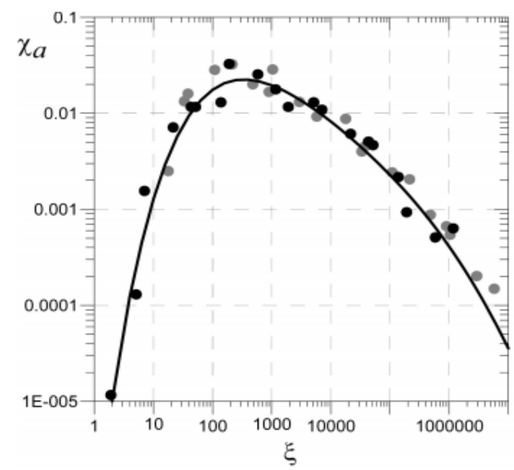


Fig. 3. Dimensionless atmospheric escape mass χ_a versus dimensionless erosional power ξ. Black circles show data for escape velocity of 11.2 km/s, gray circles show data for escape velocity of 5 km/s. Both data sets can be approximated by the curve (5) shown by black line.

Shuvalov (2009)