

MATH 18 SI Leader Demo

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1 Dimension

If V is a vector space, dimension of V ($\dim(V)$) is the number of vectors in any basis.

Example:

V is the span of $\begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$, $\begin{pmatrix} 3 \\ -1 \\ 1 \end{pmatrix}$, $\begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix}$. Find the dimension of V .

The matrix $A = \begin{pmatrix} 1 & 3 & 2 \\ 0 & -1 & -1 \\ 2 & 1 & -1 \end{pmatrix}$ has RREF: $\begin{pmatrix} 1 & 3 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix}$ There are two pivot

positions in this matrix. Since $V = \text{Col}(A)$, $\dim(V) = \dim(\text{Col}(A)) = 2$.

1.1 Practice Problem 1

Find the rank of the following matrix (hint: how do we find column space?)

$$A = \begin{pmatrix} 1 & 1 & 4 \\ 0 & 1 & 1 \\ 1 & 0 & 3 \end{pmatrix}$$

Solution:

Row Reduction steps:

$$\begin{pmatrix} 1 & 1 & 4 \\ 0 & 1 & 1 \\ 1 & 0 & 3 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 1 & 4 \\ 0 & 1 & 1 \\ 0 & -1 & -1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 1 & 4 \\ 0 & 0 & 0 \\ 0 & -1 & -1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 & 4 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

From this RREF, what can we conclude about the column space and rank?
(hint: pivots are important)

$$\begin{pmatrix} 1 & 0 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$