Fall 2018: EEL-6935 Smart Grid – Homework 01

Hector Lopez 9/14/18

1. Regularized least squares [20 pts]

Consider the regularized least-squares objective mentioned in class.

By setting the gradient equal to zero and solving for θ , find a closed form expression of the optimal θ* that minimizes this function.

$$J(\theta) = \|\Phi\theta - y\|_2^2 + \lambda\|\theta\|_2^2$$

$$= (\Phi^T\theta - y)^T(\Phi\theta - y) + \lambda\theta^T\cdot\theta$$

$$= \theta^T\Phi^T\Phi\theta - 2\cdot y^T\cdot\Phi\cdot\theta + y^T\cdot y + \lambda\theta^T\cdot\theta$$

$$\nabla_\theta J(\theta) = \nabla_\theta(\theta^T\Phi^T\Phi\theta) - \nabla_\theta(2y^T\Phi\theta) + \nabla_\theta(y^Ty) + \nabla_\theta(\lambda\theta^T\theta)$$

$$2\Phi^T\Phi\theta \quad - \quad 2\cancel{y^T\Phi}\ \Phi^T y \qquad + \lambda\cdot 2\theta$$

$$= 2\Phi^T\Phi\theta + 2\lambda\theta - 2\cancel{y^T\Phi}\ \Phi^T y$$

Set $\nabla_\theta J(\theta) = 0$ & solve for θ^*

$$2\Phi^T\Phi\theta + 2\lambda\theta - 2\overset{\Phi^T y}{\cancel{y^T\Phi}} = 0$$

$$\frac{2\Phi^T\Phi\theta + 2\lambda\theta}{2} = \frac{2\cancel{y^T\Phi}\ \Phi^T y}{2}$$

$$\Phi^T\Phi\theta + \lambda\theta = \Phi^T\cdot y$$

$$\theta(\lambda + \Phi^T\Phi) = \Phi^T y$$

$$\theta = \Phi^T y\,(\lambda + \Phi^T\Phi)^{-1}$$

$$\boxed{\theta^* = \Phi^T y\,(\lambda + \Phi^T\Phi)^{-1}}$$

2. Introduction to YALMIP [40 pts]

Throughout the class we will use the optimization library YALMIP. In this question, you will set up and install MATLAB, YALMIP and other needed libraries, and experiment with some basic optimization formulations.

(e) For the following problem, you'll need this data, which is also in the file q2.m provided with the problem set data. Use both YALMIP and the MATLAB linprog command to solve the following problem, with optimization variable z :

Include code for both version of this solution, along with the optimal z. Verify that they do in fact get the same answer.

```
Calculating Optimal solution with linprog :

Optimal solution found.


z_linprog =

    0.0227
   -1.4421

Check if Az<=b
Az= 0.579800 , b=0.579800 : Meets condition
Az= 0.760400 , b=0.760400 : Meets condition
Az= -2.356385 , b=0.529800 : Meets condition


Calculating Optimal solution with yalmip(sdpvar) :

Optimal solution found.


z_yalmip =

    0.0227
   -1.4421

Check if Az<=b
Az= 0.579800 , b=0.579800 : Meets condition
Az= 0.760400 , b=0.760400 : Meets condition
Az= -2.356385 , b=0.529800 : Meets condition
```

(f) Solve the following linear programming problem, using both linprog and YALMIP. Here you'll have to convert the program to standard form before using linprog; you could also use this standard form for the YALMIP solver but try to write a solution in YALMIP that more directly corresponds to the problem.

Linear Programming (2.F)

$$\min_z \; c^T z$$

Subject to $Az \leq b$

$$\min_{z \in R^3} \; z_1 + 3z_3 \quad \text{(objective function)}$$

(Convert to usual form.)

s.t. $z_1 \geq z_2$; $(z_1 - z_2 \geq 0) \times -1$

$2z_1 \leq z_2 + 5z_3$; $2z_1 - z_2 - 5z_3 \leq 0$

$z_2 \geq 2 + z_3$; $(z_2 - z_3 - 2 \geq 0) \times -1$

$z_3 \leq z_1$; $-z_1 + z_3 \leq 0$

$-z_1 + z_2 \leq 0$; $-z_1 + z_2 + 0 \cdot z_3 \leq 0$

$2z_1 - z_2 - 5z_3 \leq 0$; $2z_1 - z_2 - 5 \cdot z_3 \leq 0$

$-z_2 + z_3 + 2 \leq 0$; $0 \cdot z_1 - z_2 + z_3 \leq -2$

$-z_1 + z_3 \leq 0$; $-z_1 + 0 \cdot z_2 + z_3 \leq 0$

$$A = \begin{bmatrix} -1 & 1 & 0 \\ 2 & -1 & -5 \\ 0 & -1 & 1 \\ -1 & 0 & 1 \end{bmatrix}$$

z^* obj func.

$z_1 + 0 \cdot z_2 + 3z_3$

$z^* = [1 \; 0 \; 3]$

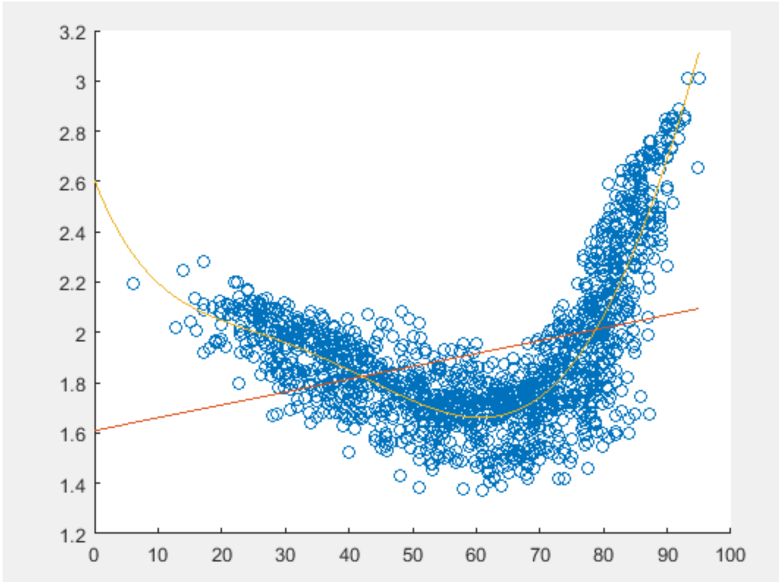
$b = [0 \; 0 \; -2 \; 0]$

Matlab: linprog(z^*, A, b) =

$z_2, z_1 = 2.5 \quad z_3 = 0.5$

3. Experiments with linear and non-linear regression [40 pts]

In this program, you'll recreate some of the figures and results from the class slides. All the data for this problem can be found in the data q3_max_temp.txt and q3_max_demand.txt. Include the code and resulting figures in the answers to all these questions (these are the standard items to include for all programming assignments). Use linear and non-linear regression, with the squared loss function, to predict peak demand from high temperature. Use 1) linear regression, and 2) nonlinear regression with polynomial features (of max degree d = 5). Plot the resulting fits over the input data (hint: to do this, create a set of evenly spaced input points over the entire range of the input, and compute the prediction at each of these points).



Appendix:

Problem 2.e:

```
%-----------------------------------
%
% HWK1 - Problem 2.e
% Compare the linprog an yalmip optimizer
% confirm that the solution meets criteria
%-----------------------------------

k = 2;
m = 3;

A = [-0.6885 -0.4129;
     1.3395 -0.5062;
     -0.9092 1.6197];

b = [0.5798; 0.7604; 0.5298];

c = [-0.2583; 0.7007];

fprintf('\nCalculating Optimal solution with linprog : \n');
z_linprog = linprog(c,A,b)


fprintf('Check if Az<=b \n');
Az= A*z_linprog;
```

```
for i = 1:size(Az,1)
    if(Az(i) <= b(i))
        fprintf('Az= %f , b=%f : Meets condition\n', Az(i),b(i))
    else
        fprintf('Az= %f , b=%f : Does not meet condition!! \n', Az(i),b(i))
    end
end

fprintf('\n\nCalculating Optimal solution with yalmip(sdpvar) : \n');

zz= sdpvar(k,1);
optimize(A*zz <= b , c'*zz);
z_yalmip = value(zz)

fprintf('Check if Az<=b \n');
Azz= A*z_yalmip;
for i = 1:size(Azz,1)
    if(Azz(i) <= b(i))
        fprintf('Az= %f , b=%f : Meets condition\n', Azz(i),b(i))
    else
        fprintf('Az= %f , b=%f : Does not meet condition!! \n', Azz(i),b(i))
    end
end
```

Problem 2.f:

```
%-------------------------------------
%
% HWK1 - Problem 2.f
% use the linprog to solve the
% minimization subject to the criteria
%-------------------------------------
% Solve the following linear programming problem:
% min (z1 + 3*z3)
%  subject to :
%   z1 >= z2 ,
%   2z1<= z2+5*z3,
%   z2 >= 2+ z3
%   z3 <= z1


A = [-1,1,0;
    2,-1,-5;
    0,-1,1;
    -1,0,1];
```

```
b = [0,0,-2,0];
f = [1,0,3];

z = linprog(f,A,b)

z = sedumi(A,b,f)
```

Problem 3 :

```
%-----------------------------------
%  HWK1 - Problem 3
%  Plot a linear and
%  nonlinear regression curve for the test data
%-----------------------------------
X=load('q3_max_temp.txt');
y=load('q3_max_demand.txt');
m=size(X,1);
n=size(X,2);
plot_x = 0:1:max(X);

Phi = [X,ones(m,1)];
theta = Phi\y; %norm
double(theta);
%lsq - using l-norm
lsq_y = theta(1)*plot_x + theta(2);

p = polyfit(X,y,5);
nonlin_y = polyval(p,plot_x);

%plot data
hold on;
plot(X,y,'o');
plot(plot_x,lsq_y);
plot(plot_x,nonlin_y);
hold off;
```