Assignment #4

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1. Consider again the antenna-array and signal model of Problem 3 of Assignment #3.

   Fix the SNR of the user of interest at 12dB and the SNRs of the interferers at 10dB, 12dB, and 14dB. Run the normalized LMS, RLS, and Constant-Modulus (CM) algorithms (the latter is blind).

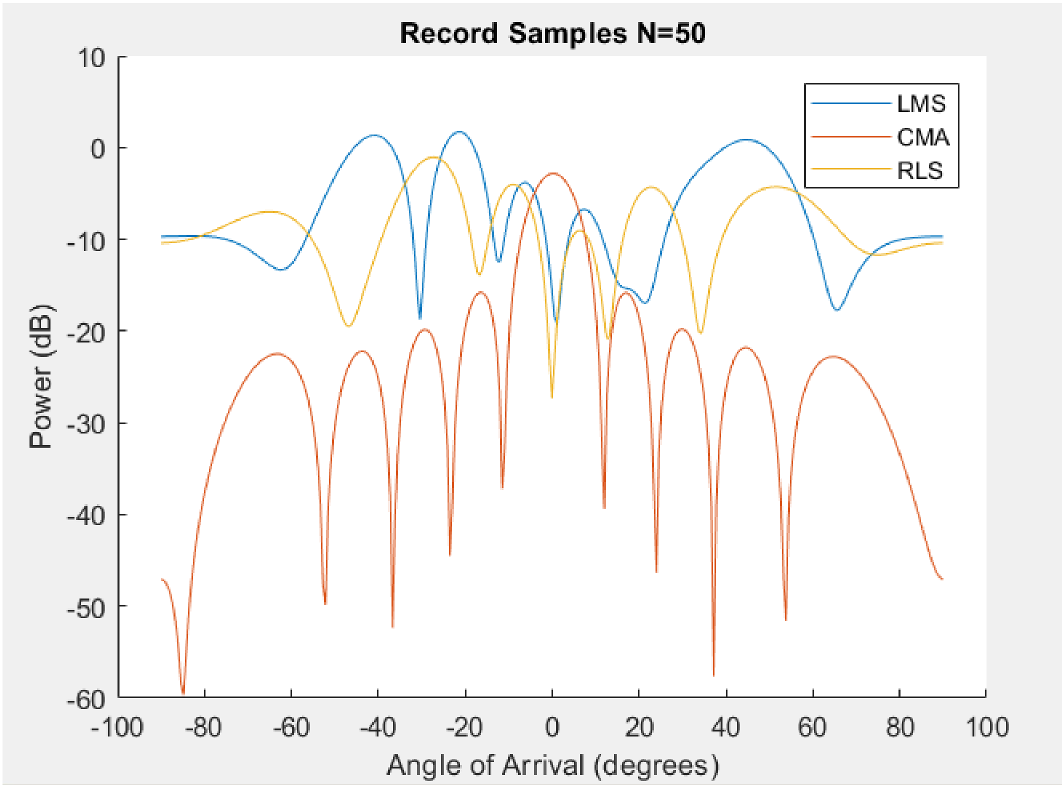
   Plot the beam pattern $10\log_{10}|y(\theta)|^2$ from -90^o to 90^o after 50, 250, and 1000 iterations (a total of 9 curves). Discuss your findings.

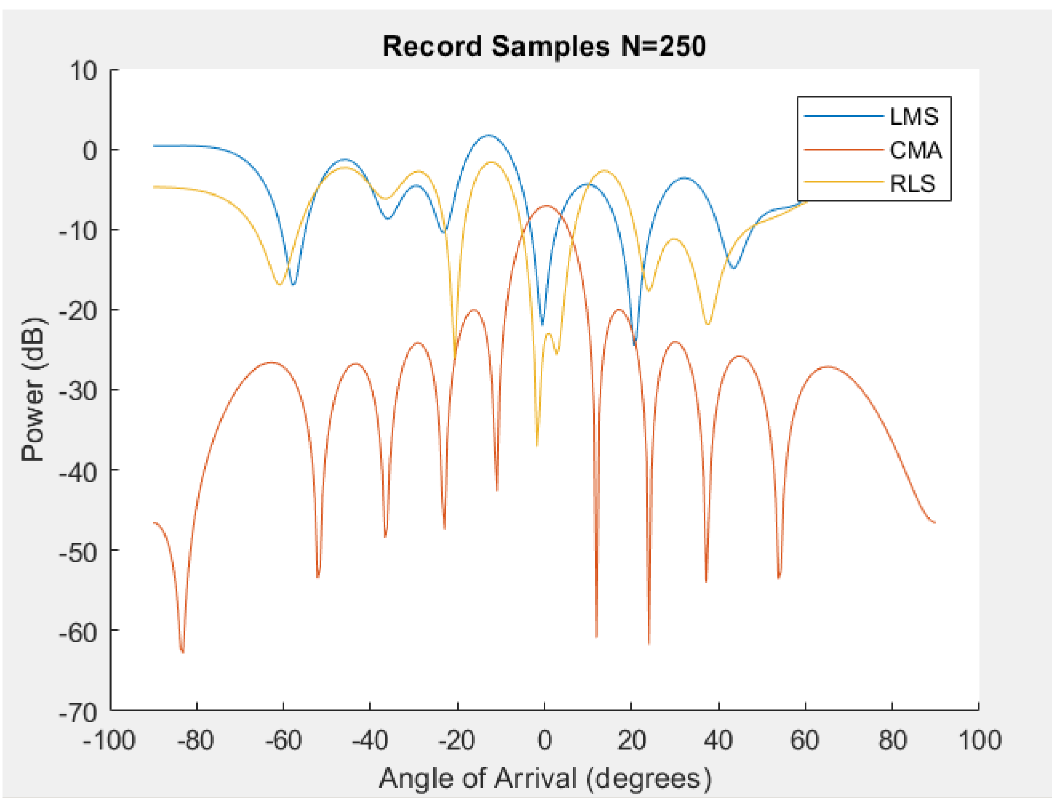
2. Design the minimum probability of error detector that operates on $b_n A T_b \mathbf{w}^H \mathbf{s}_\theta + v_n$.

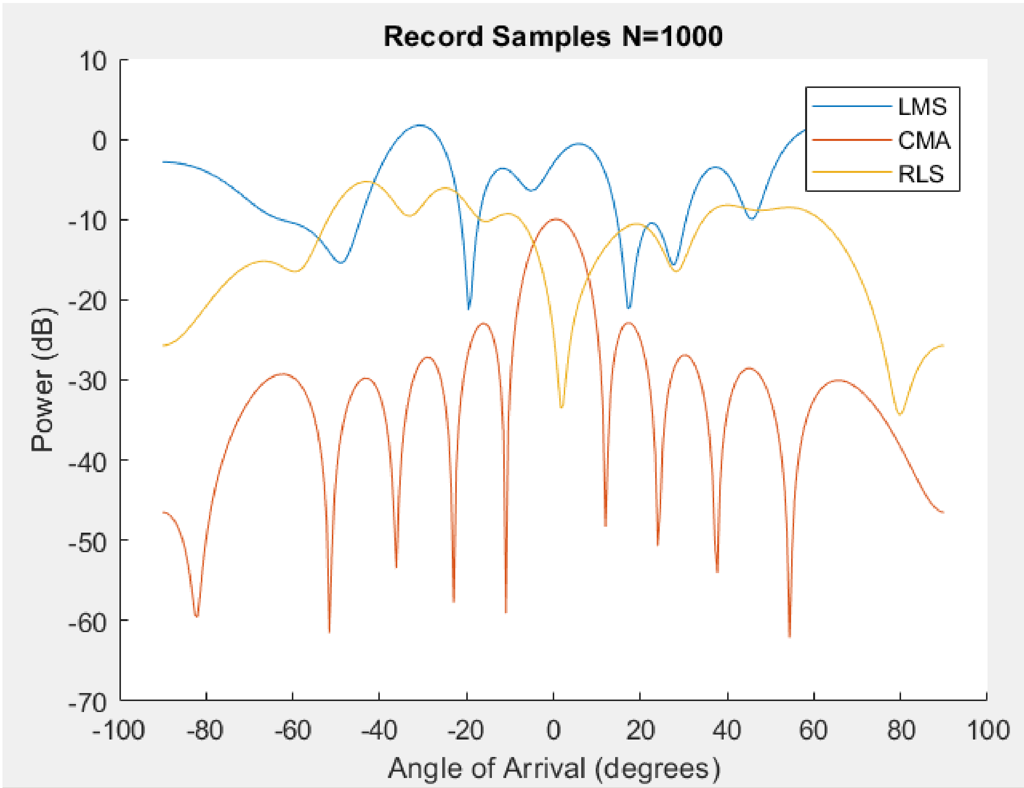
   Give the complete derivation. Calculate the BER in the form of a $Q(\cdot)$ function ($Q(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt$). Evaluate the BER for the MF beamformer $\mathbf{w} = \mathbf{s}_\theta$.

Part 1

Normalized LMS and RLS seam to provide similar responses until we use a large sample size. At that point the LMS and RLS seems to diverge in response. The CM algorithm seems to respond similarly in all record sizes.


Record Samples N=50
LMS
CMA
RLS
Power (dB)
Angle of Arrival (degrees)
10
0
-10
-20
-30
-40
-50
-60
-100
-80
-60
-40
-20
0
20
40
60
80
100


Record Samples N=250
LMS
CMA
RLS
Power (dB)
Angle of Arrival (degrees)
10
0
-10
-20
-30
-40
-50
-60
-70
-100
-80
-60
-40
-20
0
20
40
60
80
100




```
%%% Data Generation
clear all;
M=10; % Number of elements in antenna array
N=[50,250,1000]; % Data record sets sizes
K=3; % number of interfers signals
I=eye(M); % identity matrix
sigma=1; % variance
c=1; % constant
u=0.0000001; % small gain factor for LMS
beta=0.95;% forgetting factor, with 20 sample memory
eps0=1;% RLS initial value  constnat for R^-1(1)
E0=10^(12/10);%12dB
%interfers signals
E1=10^(10/10);%10dB
E2=10^(12/10);%12dB
E3=10^(14/10);%14dB
theta=-pi/2:0.01:pi/2;%range of theta for user signal analysis
%th0=20; %angle of arrival for user signal
%th1=-80;
%th2=80;
%th3=60;
th_i=[-31,62,19,-68]/180*pi;
```

```
fc=2*10^6;%carrier frequency
lambda_c=fc/(3*10^8);%carrier wavelength
d=lambda_c/2;%nyquist distance
%create array response vectors for each incoming interfer
S0=zeros(M,1);S1=zeros(M,1);S2=zeros(M,1);S3=zeros(M,1);
for m=1:M
S0(m)=exp((-1i*2*pi*(m-1)*d)*sin(th_i(1)));
S1(m)=exp((-1i*pi*(m-1)*d)*sin(th_i(2)));
S2(m)=exp((-1i*pi*(m-1)*d)*sin(th_i(3)));
S3(m)=exp((-1i*pi*(m-1)*d)*sin(th_i(4)));
end

wcma = zeros(M,N(3));
wcma(:,1)=S1/M;
wlms=zeros(M,N(3));
wrls=zeros(M,N(3));
R=(1/eps0)*I;

%%% Simulation
for i=2:N(3)
    % get three new random bits for each signal
    b=complex(sign(randn(K,1)));
    b0=complex(sign(randn(1)));

    %BPSK Signal
    n = sqrt(sigma)*complex(randn(M,1), randn(M,1))/sqrt(2);

r=b0*sqrt(E0)+b(1)*sqrt(E1)*S1+b(2)*sqrt(E2)*S2+b(3)*sqrt(E3)*S3+n;
    R=(1/beta)*(R-((R*r*ctranspose(r)*R)/(beta+ctranspose(r)*R*r)));

    wlms(:,i)=wlms(:,i-1)-(c/(norm(r)^2))*r*(ctranspose(r)*wlms(:,i-
1)-conj(b0));
    wcma(:,i)=wcma(:,i-1)-u*r*ctranspose(r)*wcma(:,i-1)*(abs(wcma(:,i-
1)'*r)^2-E0);
    wrls(:,i)=wrls(:,i-1)-R*r*(r'*wrls(:,i-1)-conj(b0));
end

%% Data Analysis
for k = 1:3
    for th=1:length(theta)
        x=zeros(M,1);
        for m=1:M
            x(m)=exp((-1i*pi*(m-1))*sin(theta(th)));
        end
       ylms(th)=wlms(:,N(k))'*x;
       ycma(th)=wcma(:,N(k))'*x;
       yrls(th)=wrls(:,N(k))'*x;
    end
    ylms_db = 10*log10((abs(ylms).^2));
    ycma_db = 10*log10((abs(ycma).^2));
    yrls_db = 10*log10((abs(yrls).^2));
    theta_deg = 180*theta/pi;
```

```
    figure
    hold on
    plot(theta_deg,ylms_db)
    plot(theta_deg,ycma_db)
    plot(theta_deg,yrls_db)
    title(sprintf('Record Samples N=%d',N(k)));
    legend('LMS','CMA','RLS');
    xlabel('Angle of Arrival (degrees)');
    ylabel('Power (dB)');
end
```

Part 2

Design of minimum Probability of error detector that operates on $b_n A T_b \vec{w}^H \vec{s}_\theta + v_n$

Where v_n is complex with 0 mean and $Var(v)$:

$$\mathcal{N}\left(0, \frac{N_0}{2} \|\vec{w}\|^2 T_b\right)$$

if Hypothesis, H_0, H_1 are:

$$H_0: b=-1 \quad x_n = -AT_b \vec{w}^H \vec{s}_\theta + v_n$$
$$H_1: b=1 \quad x_n = AT_b \vec{w}^H \vec{s}_\theta + v_n$$

Then probability of x given H_0 & H_1 is defined:

$$f(x|H_0) = \frac{1}{\sqrt{2\pi \frac{N_0}{2} \|\vec{w}\|^2 T_b}} \cdot e^{-\left(\frac{|x + AT_b \vec{w}^H \vec{s}_\theta|^2}{2\frac{N_0}{2} \|\vec{w}\|^2 T_b}\right)} \quad (1)$$

$$f(x|H_1) = \frac{1}{\sqrt{2\pi \frac{N_0}{2} \|\vec{w}\|^2 T_b}} \cdot e^{-\left(\frac{|x - AT_b \vec{w}^H \vec{s}_\theta|^2}{2\frac{N_0}{2} \|\vec{w}\|^2 T_b}\right)} \quad (2)$$

$$\frac{f(x|H_0)}{f(x|H_1)} \underset{H_1}{\overset{H_0}{\gtrless}} 1 \quad \left.\right\} \begin{array}{l}\text{if} > 1 \text{ then } H_0. \\ \text{Elseif} < 1 \text{ then } H_1\end{array}$$

$$\ln\left(\frac{|x - AT_b \vec{w}^H \vec{s}_\theta|^2 - |x + AT_b \vec{w}^H s_\theta|^2}{2\frac{N_0}{2} \|\vec{w}\|^2 T_b} \gtrless 1\right)$$

$$\Rightarrow |x - AT_b \vec{w}^H \vec{s}_\theta|^2 - |x + AT_b \vec{w}^H s_\theta|^2 \gtrless 0$$

is complex.

$$(x - AT_b \vec{w}^H \vec{s}_\theta)(x - AT_b \vec{w}^H \vec{s}_\theta)^* - (x + AT_b \vec{w}^H \vec{s}_\theta)(x + AT_b \vec{w}^H \vec{s}_\theta)^*$$

$$= \underline{-4} Re[x \underline{AT_b} \vec{w}^H \vec{s}_\theta] \underset{H_1}{\overset{H_0}{\gtrless}} 0 \rightarrow R_e[x \vec{w}^H \vec{s}_\theta] \underset{H_0}{\overset{H_1}{\gtrless}}$$

if $\vec{w}$ is real & positive then:

$$R_e[x] \underset{H_0}{\overset{H_1}{\gtrless}} 0$$

$v_n = \mathcal{N}(0, \frac{N_0}{2} \|\vec{w}\|^2 T_b)$

$\vec{w}^H \vec{s}_\theta = \in \mathbb{R}^+$

Statistical decision: $R_e[x_n] = R_e[b_n AT_b \vec{w}^H \vec{s}_\theta + v_n]$

$$= b_n AT_b \vec{w}^H \vec{s}_\theta + \underline{R_e[v_n]}$$

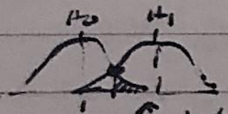
$Pr(H_0) = Pr(H_1) = 1/2$: probability of signal

apply Baye's probability to find probability of error

$$P_e = Pr(H_1 | H_0) \cdot Pr(H_0) + Pr(H_0 | H_1) \cdot Pr(H_1)$$

$$P_e = Pr(H_1 | H_0) = Pr(Re[x_n] > 0 | H_0)$$

$$= \int_0^{\infty} \frac{1}{\sqrt{2\pi \frac{N_0}{4} \|W\|^2 T_b}} \cdot e^{-\frac{(x + AT_b \vec{w}^H \vec{s}_\theta)^2}{2 \frac{N_0}{4} \|W^2\| T_b}} \, dt$$



Solve integral using change of variable

$$z^2 = \frac{(x + AT_b \vec{w}^H \vec{s}_\theta)^2}{\frac{N_0}{4} \|\vec{w}\|^2 T_b} \quad \therefore, \; z = \frac{x + AT_b \vec{w}^H \cdot s_\theta}{\sqrt{\frac{N_0}{4} \|\vec{w}\|^2 T_b}}$$

Rewrite integral:

$$\int_{\frac{AT_b \cdot \vec{w}^H \vec{s}_\theta}{\sqrt{\frac{N_0}{4} \|W\|^2 T_b}}}^{\infty} \frac{1}{\sqrt{2\pi}} e^{\frac{-z^2}{2}} \, dz$$

$\Rightarrow$ is defined as a $Q(\cdot)$ where: $Q(x) = \int_x^{\infty} \frac{1}{\sqrt{2\pi}} \cdot e^{\frac{-T^2}{T}} \, dt$

$$Q = \sqrt{\frac{4 \cdot A^2 T_b^2 (\vec{w}^H \vec{s}_\theta)^2}{N_0 \|\vec{w}\|^2 T_b}} \quad ; \; \vec{w} = \vec{s}_\theta$$

$$BER = Q\left(\sqrt{\frac{4A^2 T_b \cdot M}{N_0}}\right)$$