A Review of Game Theory Applications in Transportation Analysis

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***Abstract*—Game theory has been widely used in various fields since its birth. Particularly, it plays an important role in transportation analysis and provides powerful tools to solve many difficult problems. This paper provides an overview of the numerous applications of game theory in transportation analysis and classifies these applications into two types. We aim to provide a clear framework of these applications and also a feasible solving process of problems. At last, we point out some drawbacks of game theory based applications and discuss how to adjust game theory method to deal with complex situations in future practice.**

Index Terms—Transportation Analysis, Game Theory

I. INTRODUCTION

GREAT progress has already and is still taking place in transportation analysis all over the world since the establishment of game theory. Transportation researchers have been using different models from game theory to solve different problems. These numerous achievements come from various angles and it is hard to find the internal relations between them, so it is of great need to have a comprehensive overview and thus make a clear classification among these applications. In this paper, we divide the applications into two types, and in each type, the applications can be further classified.

After the classification, we find the characteristics of the process in which researchers use game theory to analyze transportation. Then, we propose a general research process to which researchers can refer to, when they are using game theory in their future work. The process could be described as four steps: Problem analysis, model formulation, game solution and practical verification.

At last, we should notice that there are also some points needed to be improved when applying game theory to transportation analysis. For this reason, further studies should be made.

To give a detailed explanation, the rest of this paper is organized as follows: Section II gives some necessary backgrounds of game theory; Section III describes how the applications are classified and presents a general research process, which would be used for reference, according to the numerous previous researches. Section IV discusses on the drawbacks of game theory's application in transportation analysis and proposes a few suggestions about further study on this subject. Finally, Section V concludes the paper.

II. BACKGROUNDS OF GAME THEORY

A. Origin and Development of Game Theory

In the early 20^{th} century, game theory was established as a formal discipline. In 1928, the great mathematician John von Neumann proposed a non-cooperative zero-sum game of two people. Then in 1944, the book *Theory of Games and Economic Behavior* written by John von Neumann and an economist Oskar Morgenstern symbolized the establishment of game theory.

In the 1950s, cooperative game theory enjoyed its peak and non-cooperative game theory began to develop. During this time, a legendary figure John F. Nash deserved special mention, for his two essays in 1950 [1] and 1951 [2] firstly used rigorous mathematical language and simple words to accurately define Nash Equilibrium, which was a significant milestone in game theory history.

In 1972, the famous professional journal *International* Journal *of Game Theory* was founded, which signed the maturity of game theory.

From then on, game theory was firstly widely used in economics. As time went on, it was gradually applied in other subjects, such as politics, military, biology, transportation, computer science, etc.

B. Basic Elements and Common Models of Game Theory

There are four basic elements in a game:

1) Players: The participants in the game who can decide their own strategies. An important assumption is that the players are rational, which means that they don't leave things to chance and don't take advantage of others' mistakes.

2) Strategies set: A collection of all the strategies available for all players. According to whether a player's strategies are finite, we can divide games into two groups: finite games and infinite games.

3) Payoff functions: After each player in the game chooses a strategy from their own strategies set, a relevant result (a group of data) is provided to show each player's gain or loss. A good payoff is the fundamental goal of a player, and is the main basis of a player's judgment and behavior.

4) Orders: When different players are about to decide, there is a need to decide the orders. Sometimes players make their decisions at the same time to make sure that the game is fair; sometimes players make decisions one after another; most times in reality players may choose their strategies more than once. Different orders result in quite different situations.

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Next we list six common models of game theory that are frequently used in transportation analysis. So far, researchers usually focus on non-cooperative game theory, the common models are as follows:

1) Ordinary non-cooperative game: The establishment of ordinary non-cooperative game model is no longer a difficult thing. The basic idea is to analyze the problem, abstract the three elements of modeling, establish model, analyze the properties of equilibrium solution and solve the problem.

2) Generalized Nash equilibrium game: Each player's decision affects not only other players' payoffs, but also their feasible strategies set.

3) Cournot game: It is the earliest version of the application of Nash equilibrium and a classic case in game theory, which is also well-known as a special case of prisoner's dilemma model.

4) Stackelberg game: It is the simplest model of a leader-follower game, in which the follower can know well about the leader's behavior and previous game information and the leader can predict the follower's action before making a decision.

5) Bounded rationality game: While game theory has acquired great success, some people have questioned the assumption that the players are completely rational. Bounded rational game is much closer to reality because it is based on the assumption that each player desires the best result but can only get limited payoffs.

6) Repeated game: Made up of a few repeated basic games, each stage of a repeated game is a complete game. Although repeated game is just a repetition of basic games in the form, but the result could be quite different.

III. Classification of Game Theory Applications in Transportation Analysis and a General Research Process

A. Classification of Game Theory Application in Transportation Analysis

From the study on relevant researches, we classify the applications into two types: Macro-policy analysis and Micro-behavior simulation. The former focuses on overall situations, in which a large number of players take part in the game in a complex and large place. And the latter concentrates on part situations, in which only few players are in the game in a very limited place.

1) Macro-policy analysis

In this part, based on the fact that players in transportation can be divided into two types as authorities (governmental, municipal, regional organizations or companies that operate public transport services or toll roads) and travelers (users of transportation system), we make a further classification as follows: games between travelers and authorities, games between authorities and games between travelers.

a) Games between travelers and authorities

This type of game may be the most common case in overall applications, where the goals of the two types of players are not rigorously contradictory.

Holguin-Veras and Cetin discuss the optimal tolls for multi-class traffic [3], in which they use a Stackelberg game as the model. The toll agency is the leader and the equilibrating traffic as the followers.

Takama and Preston develop a new agent-based model for simulating to forecast the consequences of a user charging scheme for the Upper Derwent Valley in the Peak District National Park [4].

Albert also takes advantage of game theory to analyze the relationship between the operator of a toll road and a traveler. [5]

The applications above are all about road tolls, they all try to find a proper toll-policy, by the application of which can the road efficiency be well improved. Besides, there are a large number of other game theory applications in transportation analysis.

Bell employs models in game theory to analyze vehicle routing problems [6], transportation network reliability [7] and the equilibrium of risk-averse users in traffic [8] and notices that the Nash equilibrium of the game can be regarded as the measurement of roads' reliability.

Thomas and White present a Markov decision model for the dynamic shortest path with anticipation and realize that the expected cost is highly related to the probability of how fast the congested situation becomes uncongested [9].

Schwanen and Ettema focus on parents' behavior when collecting children with cumulative prospect theory. From their work, they make a conclusion that unreliability has special effects on parents' trips when they collect their children at the end of their workday [10].

Hollander, Prashker and Mahalel concentrate on optimal parking policy with a Stackelberg game and suggest that by considering parking problem as a game, the government can avoid weakening transit ridership [11].

Massow and Canbolat examine the relationship of taxicab drivers' response and consumers' waiting time with a complex game model and recommend that high demand points should be put more emphasize on [12].

In China, there are also many researchers using game theory to analyze situations between travelers and authorities.

X. Zhang, H. Huang and H. M. Zhang make a research on optimal tolls and parking fees in a linear city by Nash equilibrium and propose a time-varying toll-policy for both morning and evening commutes [13].

D. Wang, Q. Yan and S. Luo apply a special game model to find out the urban traffic demand and underlying problems and point out that government should combine traffic participants' demands and management efficiency, and also give various punishments to the disobedient [14].

L. Li and B. Wu construct corresponding game models to compare the solutions to traffic congestion and find the Nash equilibrium. They also indicate that roads will be excessively used if they are free of charge, due to the principles of individual rationality, which provides us theoretical base on traffic congestion pricing [15].

Z. Li discusses dynamic traffic assignment and traffic signal control with a differential game model and justifies its efficiency in a simple traffic network [16].

J. Li and B. Fan introduce a well-defined general game model to study drivers' response to ITS guidance and conclude that guidance system is not only necessary but also significant [17].

B. Su, H. Chang, Y. Chen and D. He draws on a simplified game model to study Beijing urban public traffic networks from the viewpoint of complex networks. They believe that the game model between network manipulators sometimes can describe traffic situations better than those between nodes, which can be seen in their research [18].

b) Games between authorities

This type of game is about authorities who play against each other without involving the travelers.

Medda applies a game framework to discuss the allocation of risk between public and private sectors in transport, and finds out that we are faced with potential moral problems when guarantees value higher than financial loss [19].

Hideshima employs a leader-follower model to study the privatization of tram and bus lines and focuses on different strategies of transportation enterprise. But it cannot help one decide whether to enter or not. To do this, improved models are needed [20].

In China, C. Hsu, Y. Lee and C. Liao study the competition between high-speed and conventional rail system with game theory and calculate the Nash equilibrium, by which they solve the price problem, study the sensitivity of the equilibrium state to the parameters and finally point out that there is a demand for better numeric solutions to handle with complicated models [21].

F. Chen and X. Luo exploit systematic optimization theory to simulate the competition among different types of transport modes and find ways to harmonize this competition [22].

c) Games between travelers

This type of game is known as the one in which players are all travelers. Because we are now discussing overall application, so the examples below all include a number of travelers who can affect the whole situation by their own decisions.

Arslan, Marden and Shamma consider autonomous vehicles as self-interested decision makers and seek an optimization of a global utility function using a multiplayer game model. They introduce two new algorithms to fulfill the requirement of informational and computation algorithm [23].

Yamauchi, Tanimoto, Hagishima and Sagara describe the situation that a vehicle interrupts from the first lane to the second before the lane-closing section, which causes heavier traffic burden. They use a prisoner's dilemma game structure as the stochastic optimal velocity model [24].

Godbole, Sengupta and Hagenmeyer design safe and efficient lane-change strategies for automatic vehicles with a distributed hybrid control model, which help people to get a better understanding of the communication for inter-vehicle coordination and how single vehicles affect the whole transportation network [25].

2) Micro-behavior simulation

From previous research results, we find that applications in this type can be further divided into two groups: games between travelers and authorities (often strategies for traffic signal control at junctions) and games between travelers (often take place in a very limited place).

a) Games between travelers and authorities

This type of game often describes strategies for signal control or some other conflicts which require authorities and travelers to make decisions in a limited place.

Y. Li, R. Wu and W. Li propose a new effect method combining game theory and social rules to solve the problem between two adjacent traffic signal agents, and uses computer simulation to prove its efficiency [26].

J. Zhang, F. Gao and X. Zhao exploit cooperative game theory to find a perfect traffic signal timing-plan for two adjacent signals. When traffic flow is high, this plan is much more obviously effective than that under fixed time control. But this plan is only effective between two adjacent signals [27].

H. Xiang, H. Shu and G. Hao take advantage of a dynamic game model to find the best way for expressway emergency rescue, and give an example to elaborate the application of the model, in which they divide the whole rescue process into several stages and analyze the traffic state of each stage [28].

b) Games between travelers

This type of game is often used in situations where two or a few travelers need to make decisions to avoid collision and be efficient at the same time.

Bicchi and Pallottino make a research about how to plan optimal path among given waypoints for airplanes with a cooperative game model, and several suboptimal solutions are proposed. But there are also many subjects needed to be researched, such as situations where more than two planes are involved [29].

Tomlin, Pappas and Sastry also study about airplane route problems but utilize a non-cooperative game model. In their current research, they also use different flight models to extend their methodology to dynamic rather than kinematic models [30].

Arora, Raina and Mittal apply a general game theory to find out how to avoid collision between two AGVs (Automated Guided Vehicles) at a junction, in which they find out that based on the fact that two controllers are optimal for various initial conditions, collision free can be acquired computationally [31].

Y. Ji, J. Hu, L. Li and other three scholars take advantage of computer simulating to study the conflict between pedestrians and right-turning vehicles in signalized intersections with a majority-game based conflict model and the result shows that this approach can well match up with the situations in reality. At last, they also indicate that it is hard to evaluate all possible situations in such intersections by video data alone [32].

Ghods, Amir Hosein1and Rahimi-Kian, Ashkan find the optimal coordination of ramp metering and variable speed limits in a large-scale freeway traffic network, the game theoretic model is employed to solve this problem. It's proved that the parallel computation nature of the game algorithm results in finding the optimal plans over the rolling horizon in a short time [33].

From the above study, we can make a clear classification of applications of game theory in transportation analysis, as is shown in Table I.

TABLE I. GAME THEORY APPLICATIONS IN TRANSPORTATION ANALYSIS

Type	Players	Common Models	Transportation Analysis Examples
Macro-policy Analysis	Travelers and Authorities	Ordinary Non-cooperative Game Generalized Nash Equilibrium Game Stackelberg Game	Road/Parking Tolls Policy Vehicle Routing Problems Transportation Network Reliability Urban Traffic Demand Drivers' Response to Guidance Transport Modes Competition Risk Allocation Local Competition's Effects to Overall Situation
	Authorities	Ordinary Non-cooperative Game Cournot Game	
	Travelers	Ordinary Non-cooperative Game Generalized Nash Equilibrium Game Stackelberg game	
Micro-behavior Simulation	Travelers and Authorities	Ordinary Non-cooperative Game Generalized Nash Equilibrium Game Stackelberg Game	Adjacent Traffic Signals Strategy Emergency Rescue Conflicts between Two Airplanes Collision Avoidance between Vehicles Conflict between Pedestrians and Vehicles
	Travelers	Ordinary Non-cooperative Game Bounded Rationality Game Repeated Game	

B. A General Research Process

By studying the numerous examples by researchers at home and abroad, we propose a general process of transportation analysis using game theory. The process includes four steps: Problem analysis, model formulation, game solution and practical verification. The process is shown in Fig. 1.

In the first step, researchers should collect background information about the problem they are faced with, and be sure about the goal of the research; then in the second step, they should find or create a proper game model for the problem and define the elements of the game; in the third step, researchers may solve the game by mathematics, computer simulation or other methods available; then in the last step, it is necessary for them to test the solution in reality or by computer simulation to demonstrate that the game model they apply is effective.

For example, an analysis on ramp metering could apply general game theory research process, as shown in Fig. 2. The traffic flow characteristics nearing on-ramp and off-ramp are considered to optimizing motorway traffic control strategy; players, strategies set, payoff functions and orders are all included on conflicts between main-stream flow and on-ramp or out-ramp traffic flow, so this phenomenon could be modeled by game theory; game solutions applied by ramp metering can be obtained through mathematical calculation or computer simulation; finally, simulating results would be verified by practice because traffic behavior nearing ramp could be achieved form city express way easily, as shown in Fig. 2.

IV. DRAWBACKS OF GAME THEORY'S APPLICATION IN TRANSPORTATION ANALYSIS AND SUGGESTIONS ABOUT FURTHER STUDIES

Although game theory has already solved many problems in transportation analysis, which can be seen from the overview above, there are still drawbacks when using some game models to solve practical problems. The most worthy of concern is that in most models that we apply, we assume that all the players are completely rational. Only upon this condition can the solution be carried out smoothly. But this assumption is not practical because players in reality are all bounded rational, for they can be affected by many other factors and cannot get all of the useful information.

Another problem is that for a huge system including large numbers of players, it is quite difficult to formulate a proper model and solve it because of its big scale.

What's more, after formulating a game model, the calculation needed to be done is usually quite complex.

Based on the above-mentioned drawbacks, we can focus our further studies on the points below:

1) Try to apply bounded-rationality game models to describe situations in transportation analysis, because this can represent practical problems more appropriately.

2) When dealing with huge systems, try to classify the players (especially travelers) into different types to simplify the modeling.

3) How to expand the existing algorithms, or find a new algorithm for solving game models still needs more efforts and should be the next central issue.

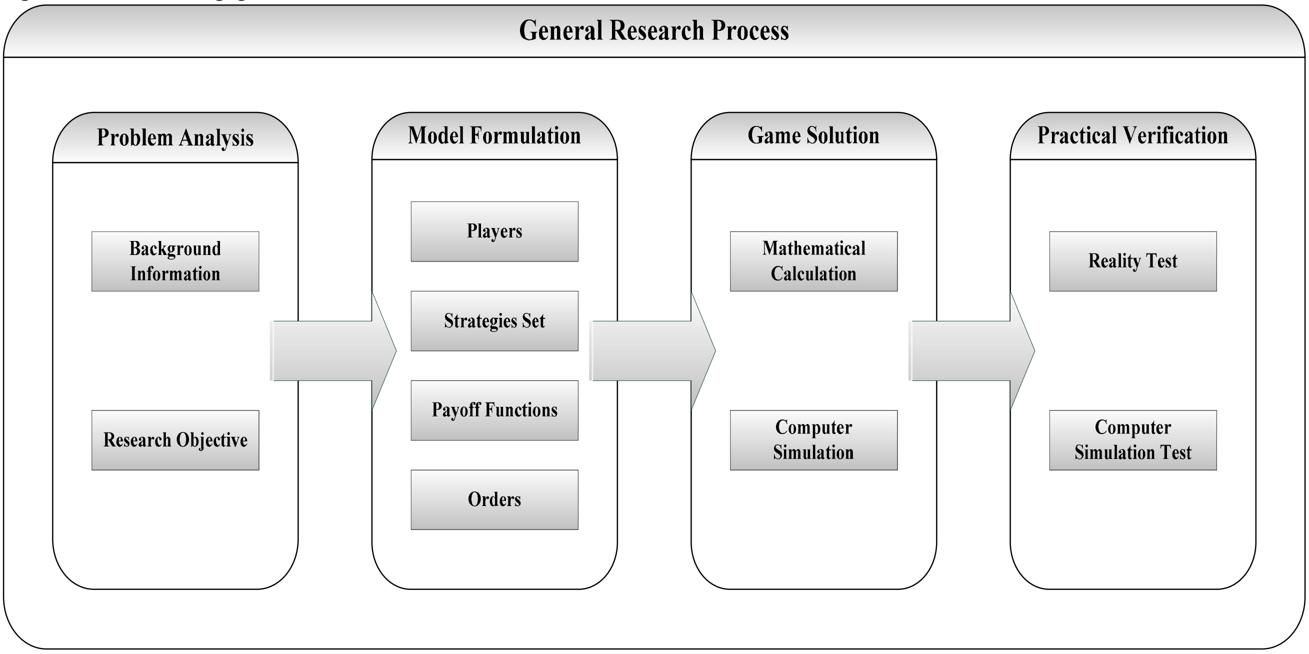

Figure 1. A general research process

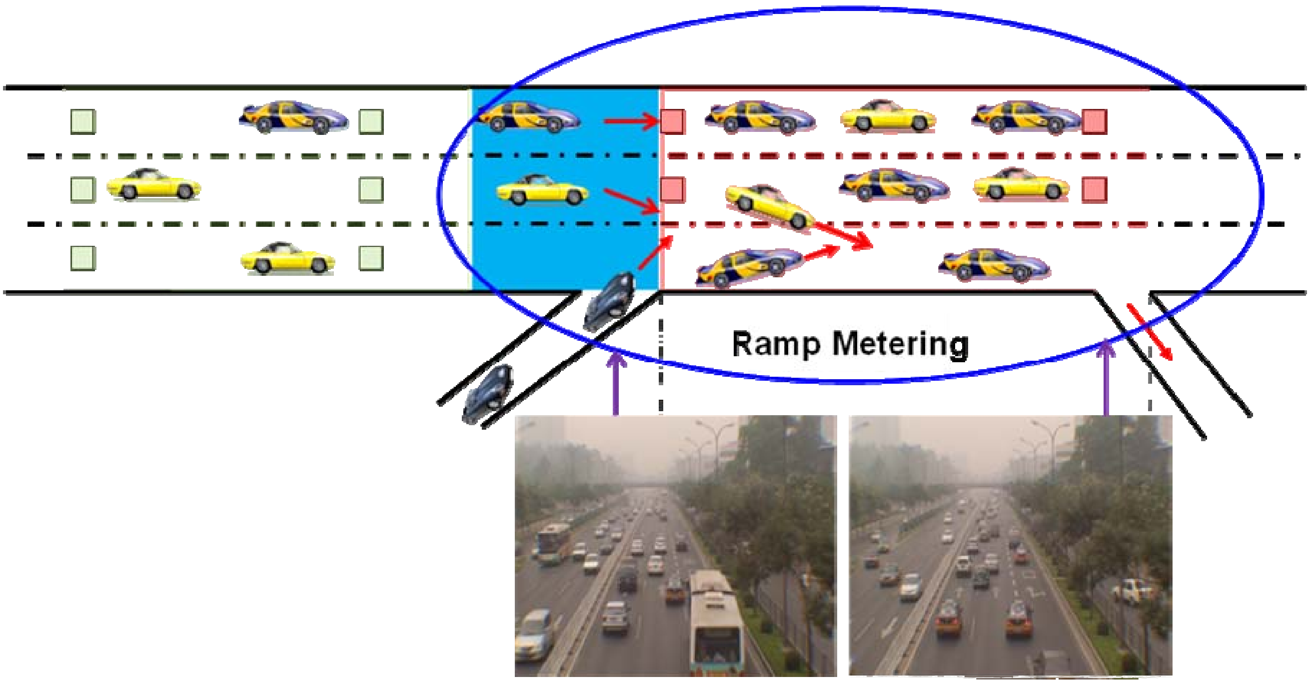

Figure 2. Using general game theory research process for ramp metering analysis

V. CONCLUSIONS

As game theory has been utilized in transportation analysis more and more widely, it is of great significance to make an overview of these applications, thus set up a general research process and find the direction for further studies.

This paper firstly provides some necessary background knowledge on game theory, then makes a review of previous work and classifies these applications into two types. In the third section, we propose a research process that can be generally used when analyzing transportation problems with game theory. At last, we point out some drawbacks of currently existing applications of game theory aiming at discussing transportation issues, and suggest three points that are worth paying attention to in further studies.

REFERENCES

[1] Nash, J. F. "Equilibrium points in n-person games." Proceedings National Academy of Sciences, Vol.36, pp. 48-49, 1950.

[2] Nash, J. F. "Non-cooperative games." Annals of Mathematics, Vol.54, pp. 286-295, 1951.

[3] Holguin-Veras, J., Cetin, M. "Optimal tolls for multi-class traffic: analytical formulations and policy implications." Transportation Research Part A: Policy and Practice, Vol.43, pp. 445-467, 2009.

[4] Takama, T., Preston, J. "Forecasting the effects of road user charge by stochastic agent-based modelling." Transportation Research Part A: Policy and Practice, Vol.42, No.4, pp. 738-749, 2008.

[5] Albert, G. "Analysis of the influence of tolls in a congested transport system." Research report, Technion – Israel Institute of Technology, 2001.

[6] Bell, M.G.H. "Games, heuristics, and risk averseness in vehicle routing problems." J. Urban Plan. Dev., Vol.130, No.1, pp. 37-41, 2004.

[7] Bell, M.G.H. "A game theory approach to measuring the performance reliability of transport network." Transportation Research Part B, Vol.34, pp. 533-545, 2000.

[8] Bell, M.G.H., Cassir, C. "Risk-averse user equilibrium traffic assignment: an application of game theory." Transportation Research Part B, Vol.36, pp. 671-681, 2002.

[9] Thomas B. W., White C. C. "The dynamic shortest path problem with anticipation." European Journal of Operational Research, Vol.176, pp. 836-854, 2007.

[10] Schwanen, T., Ettema, D. "Coping with unreliable transportation when collecting children: Examining parents' behavior with cumulative prospect theory." Transportation Research Part A: Policy and Practice, Vol.43, pp. 511-525, 2009.

[11] Hollander, Y., Prashker, J. N., Mahalel, D. "Determining the desired amount of parking using game theory." Journal of Urban Planning and Development-ASCE, Vol.132, No.1, pp. 53-61, 2006.

[12] Massow, M., Canbolat, M. S. "Fareplay: An examination of taxicab drivers' response to dispatch policy." Expert Systems with Applications, Vol.37, No.3, pp. 2451-2458, 2010.

[13] Zhang, X., Huang, H., Zhang, H. M. "Integrated daily commuting patterns and optimal road tolls and parking fees in a linear city." Transportation Research Part B, Vol.42, pp. 38-56, 2008.

[14] Wang, D., Yan, Q., Luo, S. "Game-theory based analysis and research of the urban traffic demand management model and underlying problems." Proceedings of the 8th International Conference of Chinese Logistics and Transportation Professionals - Logistics: The Emerging Frontiers of Transportation and Development in China, Chengdu, China, pp. 160-167, 2008.

[15] Li, L., Wu, B. "Analysis of traffic congestion's inherence based on game theory." Proceedings of the 8th International Conference of Chinese Logistics and Transportation Professionals - Logistics: The Emerging Frontiers of Transportation and Development in China, Chengdu, China, pp. 4226-4231, 2008.

[16] Li, Z. "A differential game modeling approach to dynamic traffic assignment and traffic signal control." 2003 IEEE International Conference on Systems, Washington, USA, pp. 849-855, 2003.

[17] Li, J., Fan, B. "Driver behavior analysis on response to ITS guidance." Intelligent Transportation Systems, 2003. Proceedings, Vol.2, pp. 1411-1416, 2003.

[18] Su, B., Chang, H., Chen, Y., et. al.. "A game theory model of urban public traffic networks." Physica A, Vol.379, pp. 291-297, 2007.

[19] Medda, F. "A game theory approach for the allocation of risks in transport public private partnerships." International Journal of Project Management, Vol.25, No.3, pp. 213-218, 2007.

[20] Hideshima, E. "An analysis for privatization process of tram and bus lines as a leader-follower game in product differentiation." 2002 IEEE International Conference on Systems, Man and Cybernetics. Yasmin Hammamet, Tunisia, 4 pp., 2002.

[21] Hsu, C., Lee, Y., Liao, C. "Competition between high-speed and conventional rail systems: A game theoretical approach." Expert Systems with Applications, Vol.37, No.4, pp. 3162-3170, 2010.

[22] Chen, F., Luo, X. "Analysis on the game among transportation modes in comprehensive passenger transportation corridor." International Conference on Transportation Engineering, Chengdu, China, pp. 358-363, 2007.

[23] Arslan, G., Marden, J. R., Shamma, J. S. "Autonomous vehicle-target assignment: a game-theoretical formulation." Journal of Dynamic Systems, Measurement and Control, Vol.129, No.5, pp. 584-596, 2007.

[24] Yamauchi, A., Tanimoto, J., Hagishima, A., et. al.. "Dilemma game structure observed in traffic flow at a 2-to-1 lane junction." Physical Review E, Vol.79, pp. 036104(6 pages), 2009.

[25] Godbole, D. N., Sengupta, R., Hagenmeyer, V. "Distributed hybrid control for automated vehicle lane changes." 37th IEEE Conference on Decision and Control, Tampa, USA, pp. 2639-2644, 1998.

[26] Li, Y., Wu, R., Li, W. "The coordination between traffic signal control agents based on Q-learning." Proceedings of the 5th World Congress on Intelligent Control and Automation, Hangzhou, China, pp. 2690-2693, 2004.

[27] Zhang, J., Gao, F., Zhao, X. "Traffic signal coordination control for two intersections applying cooperative game theory." Proceedings of the 2nd International Conference on Transportation Engineering, Chengdu, China, pp. 4366-4371, 2009.

[28] Xiang, H., Shu, H., Hao, G. "Expressway emergency rescue model based on dynamic game theory." Proceedings of the 2nd International Conference on Transportation Engineering, Chengdu, China, pp. 1649-1654, 2009.

[29] Bicchi, A., Pallottino, L. "On optimal cooperative conflict resolution for air traffic management systems." IEEE Transactions on Intelligent Transportation Systems, VOL.1, NO.4, pp. 221-232, 2000.

[30] Tomlin, C., Pappas, G. J., Sastry, S. "Noncooperative Conflict Resolution." Proceedings of the 36th Conference on Decision & Control, San Diego, USA, pp. 1816-1821, 1997.

[31] Arora, S., Raina, A. K., Mittal, A. K. "Collision avoidance among AGVs at junctions." Proceedings of the IEEE Intelligent Vehicles Symposium, Dearborn, USA, pp. 585-589, 2000.

[32] Ji, Y., Hu, J., Li, L., et. al.. "Majority-game based conflict modeling for pedestrians and right-turning vehicles in signalized intersection." Proceedings of IEEE Intelligent Vehicle Symposium, Eindhoven, Holland, pp. 1191-1196, 2008.

[33] Amir Hosein, G., Ashkan, R-K. "A game theory approach to optimal coordinated ramp metering and variable speed limits." Proceedings of Chinese Control and Decision Conference, CCDC, China, pp. 91-96, 2008.