

# Design and Analysis of Virtual Synchronous Machines in Inductive and Resistive Weak Grids

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**Abstract**—The term Virtual Synchronous Machine (VSM) commonly refers to the emulation of synchronous machines operation by using power electronics converters. However, a wider application of this control technique is still pending on further refinement of its design aspects and digital implementation. In this paper, a systematic approach to design the VSM parameters is introduced. First, a simplified third-order small-signal model of a VSM based on the conventional model of synchronous generators is developed. Unlike the existing works published in literature, the proposed model takes into account the effects of inductive and resistive weak grids. The model is then used to design the controller parameters by appropriately placing the poles of the closed-loop system. VSM robustness is studied for weak grids dominated by either inductance or resistance, and the aspects of digital control implementation are addressed. All the control system features and improvements were tested on a 15 kW prototype of a battery-supported VSM connected to a weak grid.

**Index Terms**—AC-DC power converters, control systems, virtual synchronous machine, weak grids.

## I. INTRODUCTION

MODERN electricity networks are facing a challenge of a large scale deployment and integration of distributed energy resources based on different generation technologies. This fact has motivated changes in electricity network regulations that have become more restrictive with respect to the operation of conventional generation [1] and power converters [2]. The challenge is even more pronounced and present in weak grids because the network voltage and frequency are not constant as it is typically assumed in power converter controllers based on a Phase Locked Loop (PLL) [3]. A common solution for Voltage Source Converters (VSCs) connection to microgrids and weak grids are droop controllers, which have been studied in depth in [4].

A different approach to VSCs integration to power systems is to emulate the dynamics of synchronous machines and the term VSM is typically used to described its principles [5].

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Some authors included internal current and voltage controllers in VSMs since current limiting and other issues can be implicitly solved [6]. In all these solutions the effects of the internal controller parameters have an important impact on the performance of the VSM. Therefore, the process of the tuning of controller parameters may be challenging [7]. Direct emulation of synchronous machines is also possible, although additional features need to be added to the VSM. This includes current limiting [8], power quality [9], operation during voltage sags [10], and transition between control modes [11]. However, the aspects related to the digital control implementation such as discretization have received less attention in recent publications [12].

The design of the VSM control parameters is not a trivial task because of the non-linearity present in this type of control. This issue has been mostly addressed in the literature by using small-signal analysis techniques that have been proven to be adequate for studying the dynamics of VSMs [13]. Dong and Chen [14] presented a method to design the parameters of a VSM. Several factors such as additional damping were included in the model and an analytic solution for the parameters was obtained for second- and third-order models. Another comprehensive study to select the parameters of a VSM was presented by Wu *et al.* [15]. A frequency-domain approach was proposed to accurately set stability margins. Other design methods based on frequency-domain alternatives can also be found in the literature [16]. Shintai *et al.* [17] presented a model that was used to place the poles of the active power controller. Also, a method to decouple active and reactive power controls was explained. Other authors have also investigated the connection of VSMs to weak grids [18]. However, in these references the effects of the weak grid on the performance of the VSM have been studied by assuming only inductive weak grids. In contrast to the existing published works, a third-order model that takes into account the coupling effects of the weak grid (either resistive or inductive) is considered in this paper.

Shuai *et al.* [19] presented a stability analysis of a VSM by using the bifurcation theory and a small-signal analysis was carried out. The effect of the coupling impedance was studied in detail. Liu *et al.* [20] presented an enhanced VSM that reduced oscillations in VSM-based microgrids. The proposed method damped active and reactive power transients and improved the overall microgrid performance. The use of adaptive schemes for the selection of the VSM parameters is also reported in the literature [21]. D'Arco *et al.* [22], [23] presented a

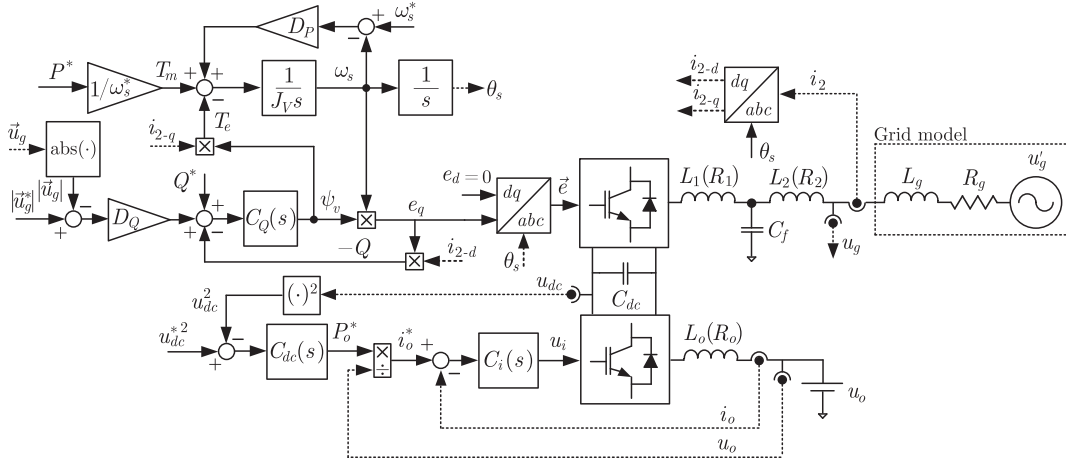


Fig. 1. Battery-supported VSM connected to a weak grid. The AC-DC VSC emulates a synchronous machine and provides voltage support and virtual inertia to the grid. The DC-DC VSC maintains the DC-link voltage constant.

parametric analysis of VSMs. The sensitivity of the closed-loop poles against variations in the system parameters was studied and the damping was optimised. State-space modelling was used, and the model provided an accurate description of the VSM dynamics. However, this type of model is too complex to be used at the design stage as the solutions for the VSM parameters cannot be obtained analytically. Dong and Chen [24] studied a method to reduce oscillations by adding a damping term to the VSM. Such approach yielded more degrees of freedom to the parameter selection, thus facilitating the design procedure.

In contrast to the previous studies, this paper features the conventional model commonly used to study synchronous machines for VSMs [25]. The contributions of this paper can be summarised as follows:

- 1) A simplified third-order small-signal model is developed. It will be demonstrated that the second order model commonly used to adjust the VSM parameters is a simplified version of this one. This model is based on the conventional synchronous generator model and takes into account the coupling between active- and reactive-power control loops. Compared to the works presented in the literature, the effects of the weak grid (both inductive and resistive) are taken into account during the design procedure.
- 2) An analytical solution for the steady-state values of the VSM is presented. This solution is an adaptation of the conventional method used for synchronous generators.
- 3) VSM robustness against grid impedance variations is explored for the case of resistive- and inductive-dominated weak grids. In particular, the impact of resistive weak grids is typically neglected in the literature.

All the proposed control improvements were tested on a 15 kVA prototype of a VSM with a 47.5 kWh Li-ion battery. The issues related to the discrete-time implementation, commonly ignored in the literature, are analysed here in detail.

## II. APPLICATION AND CONTROLLER OVERVIEW

### A. Application Description

Fig. 1 shows the electrical and control diagram of a battery-supported VSM connected to a weak grid that was proposed

in [12]. The grid-side converter provides voltage support and virtual inertia to the grid by emulating a synchronous generator. Meanwhile, the battery charger controls the DC-link voltage and manages the battery. The grid-side converter is connected to the PCC via a *LCL* filter, where  $L_1$  and  $R_1$  model the converter-side inductor,  $L_2$  and  $R_2$  model the grid-side inductor, and  $C_f$  is the filter capacitor. The effects of the weak grid are modelled by using  $L_g$  and  $R_g$ .

The grid-side current ( $i_2$ ) and the grid voltage ( $u_g$ ) are measured and used in the control system. A Synchronous Reference Frame (SRF) synchronized with the converter output voltage ( $e$ ) is used for the controller implementation. The DC-link voltage ( $u_{dc}$ ) is controlled by the DC-DC converter with a controller that sets the reference for the current controller ( $i_o^* = P_o^*/u_o$ ). The voltage generated by the battery converter is  $u_i$  and the voltage of the battery is  $u_o$ . The inductor of the DC-DC converter is modelled with  $L_o$  and  $R_o$ .

### B. VSM Overview in a SRF

This section introduces the VSM control strategy that will be used in this paper. This formulation is an adaptation of the one previously proposed by Zhong and Weiss [26], and it can be also found in [12], [27].

The VSM virtual shaft can be modelled as:

$$J_V \cdot d\omega_s/dt = T_m - T_e + D_P(\omega_s^* - \omega_s) + T_d, \quad (1)$$

where  $\omega_s$  is the synchronous frequency and  $\omega_s^*$  is its set-point value,  $J_V$  is the virtual inertia,  $T_m$  is the motor torque,  $T_e$  is the electromagnetic torque,  $T_d$  is an additional damping term, and  $D_P$  is the droop coefficient. The main reason for including  $D_P$  is to provide the necessary power-frequency droop in steady state. However, this term also has an effect on the system damping [22], [23]. Depending on the application, this term may not provide sufficient damping and additional damping terms (included in  $T_d$ ) should be added to the virtual shaft. This issue is studied in Section IV-C. The shaft rotation angle ( $\theta_s$ ) can be calculated as

$$d\theta_s/dt = \omega_s. \quad (2)$$

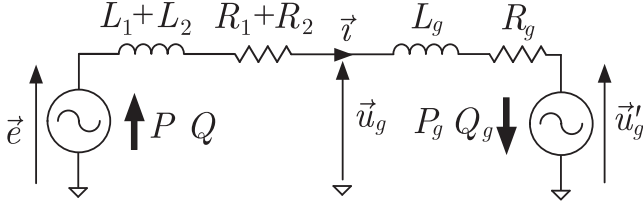


Fig. 2. Electrical diagram of a VSM connected to a weak grid. The filter capacitor of the LCL filter is neglected.

The SRF is synchronized with the  $q$ -axis of the VSM output voltage ( $\vec{e}$ ) to simplify the formulation, thus:

$$e_q = \psi_v \omega_s \text{ and } e_d = 0, \quad (3)$$

where  $\psi_v$  will be called “virtual flux”.

By using the instantaneous power theory, the active and reactive powers delivered by the VSC are:

$$P = \omega_s \psi_v i_q \text{ and } Q = \omega_s \psi_v i_d, \quad (4)$$

where  $i_d$  and  $i_q$  are calculated with the power-invariant Park's Transformation [28]. For the controller implementation, these variables will be calculated according to  $i_2$  and  $u_g$ .

The electromagnetic torque for the shaft equation is:

$$T_e = P / \omega_s = \psi_v i_q, \quad (5)$$

while the motor torque can be expressed in terms of the active power set-point ( $P^*$ ), yielding:

$$T_m = P^* / \omega_s \approx P^* / \omega_s^*, \quad (6)$$

since  $\omega_s \approx \omega_s^*$  in steady state.

An integral controller acting on the virtual flux has been chosen to control the reactive power, thus

$$\psi_v = K_Q \int (Q^* + Q_D^* - Q) dt, \quad (7)$$

where  $Q^*$  is the reactive-power set point and  $K_Q$  is the integral controller gain. Grid-voltage support is provided by means of a droop controller that adds the term  $Q_D^*$  to the reactive-power set-point, thus

$$Q_D^* = D_Q (|\vec{u}_g^*| - |\vec{u}_g|), \quad (8)$$

where  $D_Q$  is the grid-voltage droop coefficient,  $|\vec{u}_g|$  is the grid voltage space vector module, and  $|\vec{u}_g^*|$  is its set point.

As shown before, the VSM active power injection is controlled by manipulating the VSM frequency, while the reactive power is controlled by modifying the virtual flux. This control strategy is effective in the case of inductively dominated grids. However, in the case of resistively-dominated weak grids, a coupling effect appears [29]. This effect is represented in the small signal model developed in this section and studied in detail in Section VI.

The effect of the filter capacitor below the fundamental frequency can be neglected because it is typically designed so that its effect has impact only in higher frequency range [30]. The electrical model depicted in Fig. 2 represents the AC-system dynamics if the filter capacitor is neglected, where  $P$  and  $Q$  are the active and reactive powers generated by the VSM, while  $P_g$

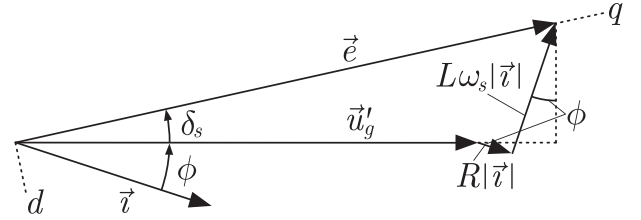


Fig. 3. Space-vector diagram of a VSM connected to the grid.

and  $Q_g$  are the active and reactive powers delivered to the grid. The filter equations are [31]:

$$L \cdot di_d/dt = -Ri_d - u'_{g-d} + \omega_s Li_q, \quad (9)$$

$$L \cdot di_q/dt = -Ri_q - u'_{g-q} - \omega_s Li_d + \omega_s \psi_v, \quad (10)$$

where  $L = L_1 + L_2 + L_g$  and  $R = R_1 + R_2 + R_g$ .

### III. VSM SMALL-SIGNAL MODEL

#### A. Small-Signal Virtual Shaft Dynamics

The small-signal model of the virtual shaft described in (1) can be written as:

$$J_V \cdot d\Delta\omega_s/dt = \Delta T_m - \Delta T_e - D_P \Delta\omega_s + \Delta T_d, \quad (11)$$

where “ $\Delta$ ” stands for “incremental operator”.

Assuming that  $di_d/dt = di_q/dt = 0$  in (9) and (10):

$$i_d = \frac{\psi_v \omega_s^2 L - R |\vec{u}'_g| \sin \delta_s - \omega_s L |\vec{u}'_g| \cos \delta_s}{R^2 + \omega_s^2 L^2}, \quad (12)$$

$$i_q = \frac{\psi_v \omega_s R + \omega_s L |\vec{u}'_g| \sin \delta_s - R |\vec{u}'_g| \cos \delta_s}{R^2 + \omega_s^2 L^2}, \quad (13)$$

where  $\vec{u}'_g = |\vec{u}'_g|(\sin \delta_s + j \cos \delta_s)$  and  $\delta_s = \theta_s - \theta'_g$ , as shown in Fig. 3 ( $\theta'_g$  is the global angle of  $\vec{u}'_g$ ). Under this premise the synchronous resonance of the connection filter is neglected [32]. Therefore, the order of the model is reduced and the system dynamics are easier to analyse. However, the proposed model is only accurate if the frequency of VSM poles is smaller than that of the synchronous resonance.

The electrical torque equation in (5) must be linearised to obtain  $\Delta T_e$ . Therefore:

$$\Delta T_e = \Delta(\psi_v i_q) = K_{T\delta} \Delta\delta_s + K_{T\psi} \Delta\psi_v \quad (14)$$

with

$$K_{T\delta} = \left. \frac{\partial T_e}{\partial \delta_s} \right|_{x_o} = \frac{|\vec{u}'_g| \omega_s^2 L \psi_v^o \cos \delta_s^o + |\vec{u}'_g| R \psi_v^o \sin \delta_s^o}{R^2 + (\omega_s^o)^2 L^2}, \quad (15)$$

$$K_{T\psi} = \left. \frac{\partial T_e}{\partial \psi_v} \right|_{x_o} = \frac{\omega_s^o L |\vec{u}'_g| \sin \delta_s^o - R |\vec{u}'_g| \cos \delta_s^o + 2\psi_v^o \omega_s^o R}{R^2 + (\omega_s^o)^2 L^2}, \quad (16)$$

where  $x_o$  is the operating point and the superscript “o” stands for steady-state value. The gain  $K_{T\delta}$  represents the effect of active-power command ( $\delta_s$ ) on the active power injection (torque), while  $K_{T\psi}$  represents the coupling effect between reactive-power command ( $\psi_v$ ) and the active power injection.

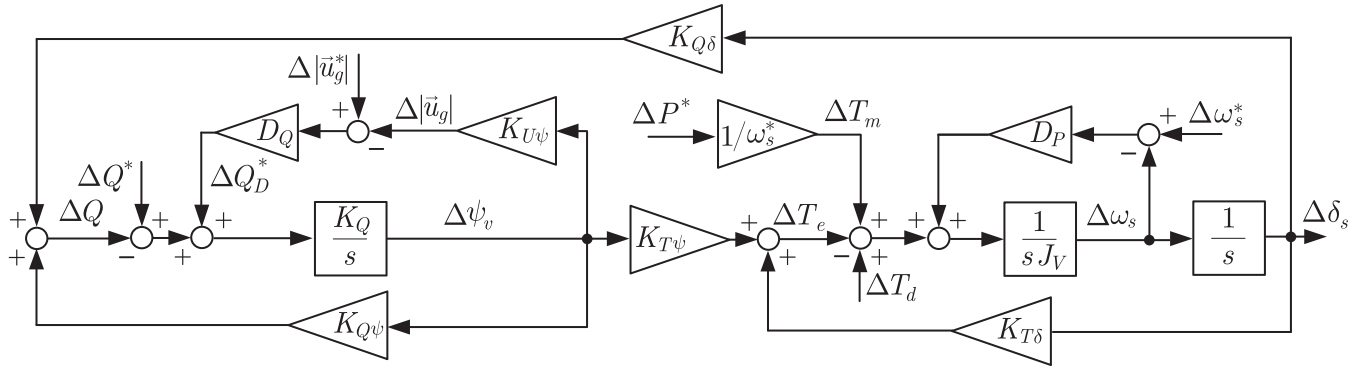


Fig. 4. Equivalent small-signal block diagram of a VSM in a weak grid. Gains  $K_{T\delta}$ ,  $K_{T\psi}$ ,  $K_{Q\delta}$ ,  $K_{Q\psi}$ , and  $K_{U\psi}$  are obtained by linearising the VSM model. Gains  $J_V$ ,  $K_Q$ ,  $D_P$ , and  $D_Q$  have to be designed according to the specific application.

### B. Small-Signal Reactive Power Dynamics

The dynamics of  $Q$  are also non-linear (see (4)). The small-signal model can be written as:

$$\Delta Q = \Delta(\omega_s \psi_v i_d) = K_{Q\delta} \Delta \delta_s + K_{Q\psi} \Delta \psi_v, \quad (17)$$

where

$$K_{Q\psi} = \left. \frac{\partial Q}{\partial \psi_v} \right|_{x_o} = \frac{2\psi_v^o L (\omega_s^o)^3 - |\vec{u}'_g| (\omega_s^o)^2 L \cos \delta_s^o - R |\vec{u}'_g| \omega_s^o \sin \delta_s^o}{R^2 + (\omega_s^o)^2 L^2}, \quad (18)$$

$$K_{Q\delta} = \left. \frac{\partial Q}{\partial \delta_s} \right|_{x_o} = \frac{\psi_v^o |\vec{u}'_g| (\omega_s^o)^2 L \sin \delta_s^o - R |\vec{u}'_g| \omega_s^o \psi_v^o \cos \delta_s^o}{R^2 + (\omega_s^o)^2 L^2}. \quad (19)$$

Gain  $K_{Q\psi}$  represents the effect of reactive-power controller command ( $\psi_v$ ) on the reactive power, while gain  $K_{Q\delta}$  represents the coupling effect between the command for active power controller ( $\delta_s$ ) and the reactive power.

### C. Small-Signal Grid Voltage Dynamics

If the grid is weak, the steady-state equation that links  $\vec{e}$  with  $\vec{u}_g$  can be written as

$$\vec{u}_g = \frac{L_g \omega_s j + R_g}{L \omega_s j + R} \vec{e} + \frac{(L_1 + L_2) \omega_s j + (R_1 + R_2)}{L \omega_s j + R} \vec{u}'_g. \quad (20)$$

Recalling that  $e_d = 0$ , the small-signal relation between  $|\vec{u}_g|$  and  $|\vec{e}| = e_q$  becomes:

$$\Delta |\vec{u}_g| = \sqrt{(L_g^2 \omega_s^{o2} + R_g^2) / (L^2 \omega_s^{o2} + R^2)} \cdot \Delta e_q. \quad (21)$$

In (21), input  $\Delta u'_g$  is not included since it is not needed for the design the VSM parameters. To quantify the VSM effect over the grid voltage, the small-signal equation that links  $\Delta e_q$  with  $\Delta \psi_v$  in (3) can be written as:

$$\Delta e_q = \Delta(\psi_c \omega_s) \approx \omega_s^o \Delta \psi_v + \psi_v^o \Delta \omega_s, \quad (22)$$

since  $\psi_v$  and  $\omega_s$  operate close to their nominal values. For simplicity, it is assumed that  $\omega_s^o \Delta \psi_v \gg \psi_v^o \Delta \omega_s$ . This means

that the effects of frequency variations over the grid voltage are neglected. By merging (21) and (22):

$$\Delta |\vec{u}_g| \approx K_{U\psi} \Delta \psi_v, \quad (23)$$

where

$$K_{U\psi} = \omega_s^o \sqrt{(L_g^2 \omega_s^{o2} + R_g^2) / ((L^2 \omega_s^{o2} + R^2))}. \quad (24)$$

This expression includes the effects of inductive and resistive components of the weak grid.

### D. Closed-Loop Transfer Functions

Fig. 4 shows the equivalent block diagram of the model developed in Section III-A, III-B, and III-C. The closed-loop transfer functions of this system can be obtained by applying block algebra to the system in Fig. 4 [33], yielding

$$F_T(s) = \frac{\Delta T_e(s)}{\Delta T_m(s)} = \frac{b_1^T s + b_0^T}{a_3 s^3 + a_2 s^2 + a_1 s + a_0}, \quad (25)$$

$$F_Q(s) = \frac{\Delta Q(s)}{\Delta Q^*(s)} = \frac{b_2^Q s^2 + b_1^Q s + b_0^Q}{a_3 s^3 + a_2 s^2 + a_1 s + a_0}, \quad (26)$$

where the coefficients are:

$$a_0 = K_Q K_{T\delta} (D_Q K_{U\psi} + K_{Q\psi}) - K_{Q\delta} K_{T\psi} K_Q,$$

$$a_1 = D_P K_Q (D_Q K_{U\psi} + K_{Q\psi}) + K_{T\delta},$$

$$a_2 = J_V K_Q (D_Q K_{U\psi} + K_{Q\psi}) + D_P,$$

$$a_3 = J_V,$$

$$b_0^T = K_Q K_{T\psi} K_{Q\delta} + K_{T\delta} K_Q (K_{Q\psi} + K_{U\psi} D_Q),$$

$$b_1^T = K_{T\delta},$$

$$b_0^Q = K_Q (D_P K_{Q\psi} + K_{T\delta} K_{Q\psi} - K_{T\delta} K_{T\psi}),$$

$$b_1^Q = K_Q K_{Q\psi} D_P,$$

$$b_2^Q = K_Q K_{Q\psi} J_V.$$



### E. Operating Point

The operating point of the VSM can be obtained by using conventional methods commonly applied to synchronous machines [25], although some modifications are required. First of all, by assuming the ideal grid ( $\vec{u}_g$ ) has constant frequency ( $\omega_g$ ) and voltage levels:

$$\omega_s^o = \omega_g \text{ and } |\vec{u}_g^o| = |\vec{u}_g|. \quad (27)$$

To simplify the calculations, the active ( $P_g^o$ ) and reactive ( $Q_g^o$ ) powers delivered to  $\vec{u}_g^o$  are taken as inputs. Therefore, if the power-invariant Park's Transformation is used [34]:

$$|\vec{i}^o| = \sqrt{(P_g^o)^2 + (Q_g^o)^2} / |\vec{u}_g^o|, \quad (28)$$

while the power-factor angle can be calculated as:

$$\phi^o = \cos^{-1} \left( P_g^o / \sqrt{(P_g^o)^2 + (Q_g^o)^2} \right). \quad (29)$$

From Fig. 3, the ideal grid voltage angle ( $\delta_s^o$ ) is [25]:

$$\delta_s^o = \tan^{-1} \left( \frac{L\omega_s^o |\vec{i}^o| \cos \phi^o - R |\vec{i}^o| \sin \phi^o}{|u_g^o| + R |\vec{i}^o| \cos \phi^o + L\omega_s^o |\vec{i}^o| \sin \phi^o} \right). \quad (30)$$

The VSM output power ( $P$ ) is required to calculate the virtual flux in (5). Therefore:

$$P^o = P_g^o + |\vec{i}^o|^2 R, \quad (31)$$

and the operating point for the virtual flux becomes:

$$\psi_v^o = P^o / (\omega_s^o i_q^o) = P^o / (\omega_s^o |\vec{i}^o| \cos(\delta_s^o + \phi^o)). \quad (32)$$

## IV. CLOSED-LOOP POLE ASSIGNMENT ALTERNATIVES

### A. Alternative 1: Third-Order Model Pole Assignment

With this alternative the poles of  $F_T(s)$  are placed by selecting the VSM parameters. It is assumed that  $D_Q$  is fixed by the application, while  $D_P$ ,  $J_V$ , and  $K_Q$  are the control parameters. Some remarks regarding the method proposed to select the VSM parameters:

- 1) Droop coefficient ( $D_P$ ) is often set by the application requirements. In this case, an additional damping term is needed in the VSM controller [22], [24]. The selection of this damping term is addressed in Section IV-C.
- 2) The variable  $J_V$  is chosen to solve the third-order equation required to assign the closed-loop poles because it does not have the constraints of  $D_P$ .

Transfer function  $F_T(s)$  can be parametrised with a pair of complex poles ( $\zeta$  is the damping factor and  $\omega_n$  is the natural frequency) and a single pole ( $\omega_c$  is the cut-off frequency):

$$F_T(s) = \frac{N_T(s)}{s^3 + (\omega_c + 2\zeta\omega_n)s^2 + (\omega_n^2 + 2\zeta\omega_n\omega_c)s + \omega_c\omega_n^2}, \quad (33)$$

where  $N_T(s)$  is the numerator polynomial. If coupling between active and reactive power is neglected, the complex poles ( $\omega_n$  and  $\zeta$ ) are mainly related to the dynamics of the virtual shaft, while the single pole ( $\omega_c$ ) is mainly related with the dynamics of the reactive-power controller. These issues will be clarified

in the following section where a simplified method to select the position of the closed-loop poles is studied.

The factors in the denominator of (25) can be identified with those of (33) and, after some algebraic calculations, the following equation can be obtained:

$$J_V^3 + c_2 J_V^2 + c_1 J_V + c_0 = 0, \quad (34)$$

where

$$c_0 = -K_{T\delta} / (AB)^2, \quad c_1 = (\omega_n^2 + 2\zeta\omega_n\omega_c) / (AB)^2,$$

$$c_2 = -(2\zeta\omega_n + \omega_c) / (AB),$$

$$A = \omega_c\omega_n^2 / (K_{T\delta} - K_{Q\delta}K_{T\psi}), \quad B = D_Q K_{U\psi} + K_{Q\delta}.$$

The expression in (34) is a third-order equation that can be solved by using the Cardano's method [35], yielding:

$$J_V = S_A + S_B - c_2/3, \quad (35)$$

where

$$S_A = (R_A + (Q_A^3 + R_A^2)^{1/2})^{1/3},$$

$$S_B = (R_B - (Q_B^3 + R_B^2)^{1/2})^{1/3},$$

$$Q_A = (3c_1 - c_2)/9, \quad R_A = (9c_1c_2 - 27c_0 - 2c_2)/54.$$

With  $J_V$  already found,  $D_P$  and  $K_Q$  can be calculated from the rest of the polynomial factors, yielding

$$D_P = (J_V(\omega_n^2 + 2\zeta\omega_n\omega_c) - K_{T\delta}) / (J_V AB), \quad (36)$$

$$K_Q = \omega_c\omega_n^2 / (K_{T\delta}B - K_{Q\delta}K_{T\psi}). \quad (37)$$

The third order equation (34) may have complex solutions for  $J_V$  and this should be avoided. The condition to guarantee that  $J_V$  is real can be written as follows [35]:

$$c_2^2 - 3c_1 > 0. \quad (38)$$

Equation (38) can be simplified, yielding:

$$(2\zeta\omega_n + \omega_c)^2 - 3(\omega_n^2 + 2\zeta\omega_n\omega_c) > 0. \quad (39)$$

Condition (39) only depends on the parameters of the closed-loop poles. Therefore, to guarantee that the solution for  $J_V$  is real it should be checked that the combination of  $\omega_n$ ,  $\zeta$ , and  $\omega_c$  used in the design meets (39). In addition to that, one should note that the proposed model has been developed by neglecting the synchronous resonance of the connection filter [16]. Therefore, the closed-loop poles should be significantly smaller than that particular resonant frequency.

### B. Alternative 2: Simplified Pole Assignment

With this alternative, a simplified virtual-shaft model is used to select  $J_V$  and  $D_P$ , first. With these parameters already set,  $F_T(s)$  is used to design  $K_Q$ . A simplified virtual-shaft transfer function ( $\hat{F}_T(s)$ ) can be obtained assuming that  $K_{T\psi} = K_{Q\delta} = 0$  in Fig. 4, thus

$$\hat{F}_T(s) = K_{T\delta} / (J_V s^2 + D_P s + K_{T\delta}). \quad (40)$$

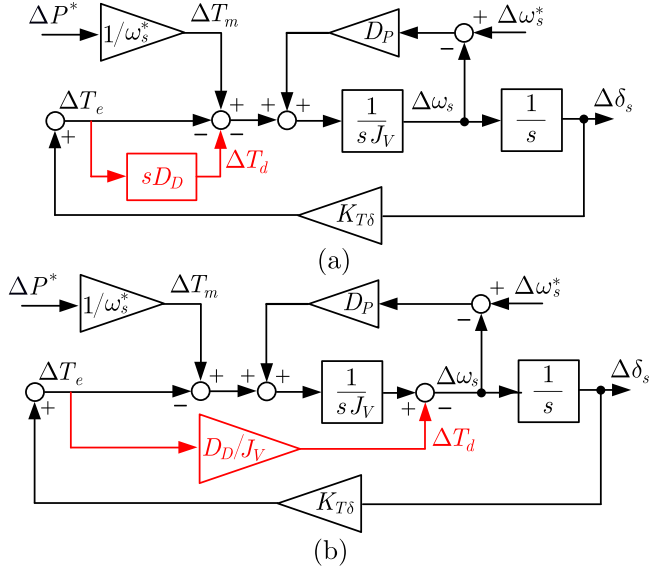


Fig. 5. Additional damping term that needs to be included if the droop coefficient is imposed by specific application requirements. (a) Conceptual idea and (b) practical implementation.

Defining the closed-loop poles with  $\zeta$  and  $\omega_n$  and identifying terms with a canonical second-order model [36]:

$$D_P = 2K_{T\delta}\zeta/\omega_n \text{ and } J_V = K_{T\delta}/\omega_n^2. \quad (41)$$

It can be deduced from these results that the complex poles of the third-order model are mainly related to the active-power dynamics.

With  $D_P$  and  $J_V$  already set, the remaining pole of  $F_T(s)$  ( $\omega_c$ ) can be placed with  $K_Q$  by using (37). Therefore, it can be seen that the simple pole of the third-order model is mainly linked to the reactive-power dynamics.

This simplified model is similar to the one that is commonly used in the literature [14], [15]. Therefore, this verifies that the proposed third-order model is a generalised case of that one.

### C. Additional Damping Term

If the value of  $D_P$  is imposed by the application, a possible solution is to provide additional damping by adding the following damping torque to the virtual shaft, as suggested in [24]:

$$T_d = D_D \cdot dT_e/dt. \quad (42)$$

The block diagram of the VSM including the additional term is depicted in Fig. 5(a). It can be seen that this term has an effect similar to that of a PD controller. However, in order to simplify its implementation and to avoid the derivative terms, the solution as in Fig. 5(b) is proposed.

By manipulating the block diagram, the following transfer function can be obtained:

$$\hat{F}_T = K_{T\delta}/(J_V s^2 + (D_P + D_D)s + K_{T\delta}). \quad (43)$$

From (43), the value of  $D_D$  and  $J_V$  can be calculated.

$$D_D = 2K_{T\delta}\zeta/\omega_n - D_P \text{ and } J_V = K_{T\delta}/\omega_n^2. \quad (44)$$

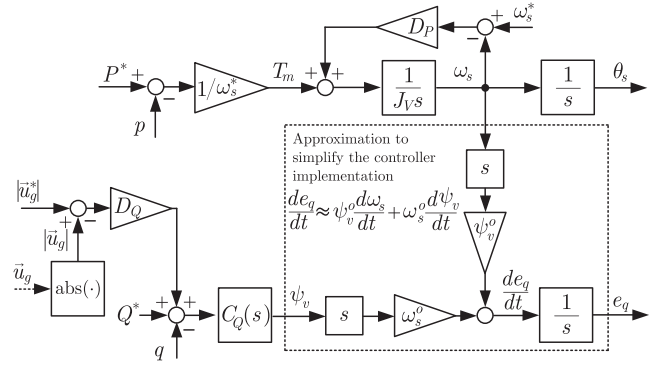


Fig. 6. Modified version of the VSM. The reactive-power controller integral is moved to the q-axis voltage command ( $e_q$ ). A continuous-time (s) representation is used to simplify the explanation.

Now, the damping coefficient of the closed-loop poles can be tuned by using  $D_D$  even if the droop coefficient ( $D_P$ ) is already set. This modification can be also included in the formulation of the third-order model. In that case, the coefficients presented in Section III-D should be recalculated taking into account this new term.

## V. CONTROLLER IMPLEMENTATION

In this section, discrete-time implementation aspects of VSMs are described. This section represents an improved version of a part of conference paper [12].

### A. Alternative Formulation for the Implementation

The electrical torque produced by the VSM can be written in terms of the electric power equation, giving:

$$T_e = p(t)/\omega_s \approx p(t)/\omega_s^*, \quad (45)$$

$$p(t) = u_{g-d} \cdot i_{2-d} + u_{g-q} \cdot i_{2-q} \approx \omega_s \psi_v i_q, \quad (46)$$

if the power-invariant Park's Transformation is used [34] and  $i_2$  is taken as the control variable. The use of (46) to compute  $p(t)$  instead of the internal variables makes the controller more practical since it will work in closed loop. Similarly, for the reactive power [28]:

$$q(t) = u_{g-q} \cdot i_{2-d} - u_{g-d} \cdot i_{2-q} \approx \omega_s \psi_v i_d, \quad (47)$$

if the power stored in the passive elements of the LCL filter is neglected [31]. Fig. 6 (left) shows the VSM controller when (46) and (47) substitute the original equations. It is worth pointing out that this controller resembles a Power Synchronisation Control (PSC) [32], except for the coupling term between  $\omega_s$  and  $de_q/dt$ .

### B. VSM Discretization

The VSM controller will be implemented in its incremental form [12], [37]. Therefore, all integrals are moved to the command signals ( $e_q$  and  $\omega_s$ ), as shown in Fig. 6 (right). This operation is typically done in discrete time, but here it is done in continuous time to facilitate the explanation, thus:

$$de_q/dt = d(\psi_v \omega_s)/dt = \omega_s \cdot d\psi_v/dt + \psi_v \cdot d\omega_s/dt. \quad (48)$$



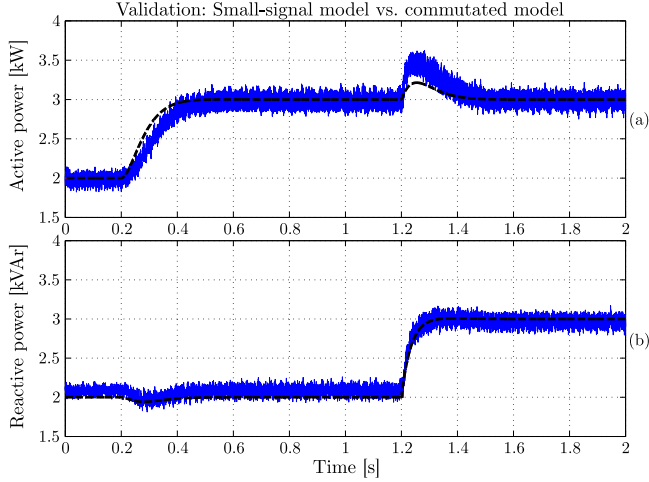


Fig. 10. Validation of the small-signal model (including inductive grid impedance). (a) Active and (b) reactive power injected by the VSM. (blue) Commutated model developed in SimPowerSystems and (black) linearised model.

- 1) The first is to reduce the transient speed of the VSM in order to avoid interactions. From the practical point of view, this means that  $\omega_n$  and  $\omega_c$  should be considerably smaller than  $\omega_s^*$ .
- 2) The second is to add a compensator to damp the resonance. A common solution is the application of the resistive virtual impedance [38], the use of a decoupling system [39], or the introduction of a current loop with a high-pass filter [32]

Reduction of the synchronous resonance effect in VSMs is highly recommended for practical applications in order to avoid its unwanted interactions with other elements connected to the grid.

Fig. 10 shows the results obtained by using the simulation model developed in Simulink and SimPowerSystems and the linearised model. Step changes were applied to the active and reactive power commands. The controller used for the simulation was the same one that was used to obtain the experimental results. The simulation included all the effects related to switching devices and discrete-time implementation, such as PWM, dead-time, and calculus delay. It can be seen the proposed model captured the essential dynamics of the VSM. However, there were minor differences in the coupling, especially in the reactive-power step change. This was expected because of the use of more realistic component models in the simulation.

### C. Stability Limits for Weak Grids

The closed-loop system can become unstable under weak-grid conditions since the small-signal-model gains (e.g.  $K_{T\delta}$ ) are calculated according to the operating point and the system parameters [25]. At least, two scenarios can take place when the active- or the reactive-power loops change their sign. In Fig. 4, it can be seen that this happens when  $K_{Q\psi}$  and  $K_{T\delta}$  become negative, respectively. Other sources of instability may also appear, but their effects are difficult to assess by analysing the sign of the gains of the small-signal model. Therefore:

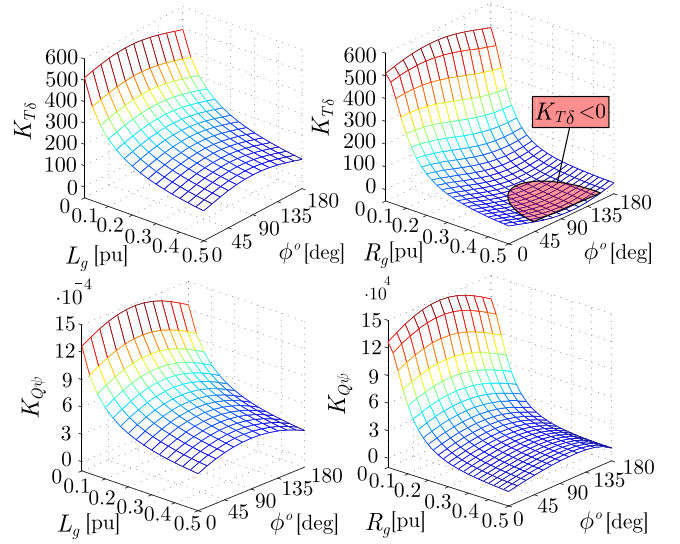


Fig. 11. Value of (top)  $K_{T\delta}$  and (bottom)  $K_{Q\psi}$ , changing (left)  $L_g$  and (right)  $R_g$ . The power-factor angle is modified from 0 to 180 deg.

- 1) The simplified virtual shaft in (6) will become unstable if  $K_{T\delta} < 0$ . From (15),  $K_{T\delta} > 0$  is guaranteed if:

$$-\omega_s^o \cdot L/R < \tan \delta_s^o < \omega_s^o \cdot L/R, \quad (52)$$

where  $\delta_s^o$  is calculated according to the operating point. The result in (52) is consistent with the poles depicted in Fig. 8(c) and (d). If  $L$  increases, the closed-loop poles move away from instability. However, when  $R$  increases, the damping factor decreases.

- 2) From Fig. 4, the sign of the reactive-power controller will change if  $K_{Q\psi} < 0$ , and this will happen once the numerator of (18) becomes zero. In this case:

$$L\omega_s^o > \cos(\delta_s^o + \phi^o)(L\omega_s^o \cos \delta_s^o + R \sin \delta_s^o)/2, \quad (53)$$

where  $\psi_v^o$  has been substituted by its operating point calculated with (32). Also, it has been assumed that  $P_g^o \approx P^o$  to simplify the expression. Note that (53) will be always fulfilled if  $L\omega_s^o > R/2$ , and this is consistent with the results already obtained in this section.

Fig. 11(a) to (d) show the value of  $K_{T\delta}$  and  $K_{Q\psi}$  when the VSM generates its nominal apparent power, for different values of  $\phi^o$ ,  $R_g$ , and  $L_g$ . For inductive grids,  $K_{T\delta} > 0$  and  $K_{Q\psi} > 0$ , always. However, for resistive grids  $K_{T\delta}$  becomes negative, eventually, while  $K_{Q\psi}$  approaches zero. This is realistic since the active power-angle and reactive power-voltage equations that are used in the VSM are only valid for inductive grids.

## VII. EXPERIMENTAL RESULTS

### A. Experimental Platform

The proposed control system was tested at IMDEA Energy facilities [40]. The main parameters of hardware setup are shown in Table I. Resonances were neglected since parasitic resistances provided sufficient damping. The grid was emulated by another VSC connected via a  $LC$  filter ( $L'_1 = L_1$  and  $C'_f = C_f$ ) that was operated in open-loop. Weak-grid scenarios were emulated



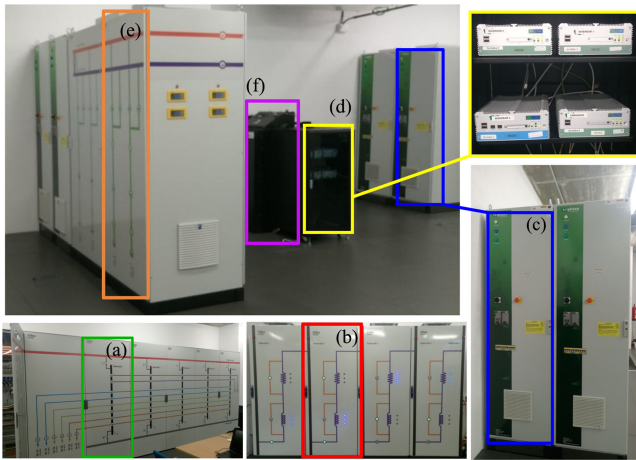


Fig. 14. Experimental results (from the prototype). Step-change of the grid frequency, weak-grid. (left) Inductive ( $L_g = 0.2$  pu,  $R_g = 0.04$  pu) and (right) resistive ( $L_g = 0.07$  pu,  $R_g = 0.3$  pu) weak grid.

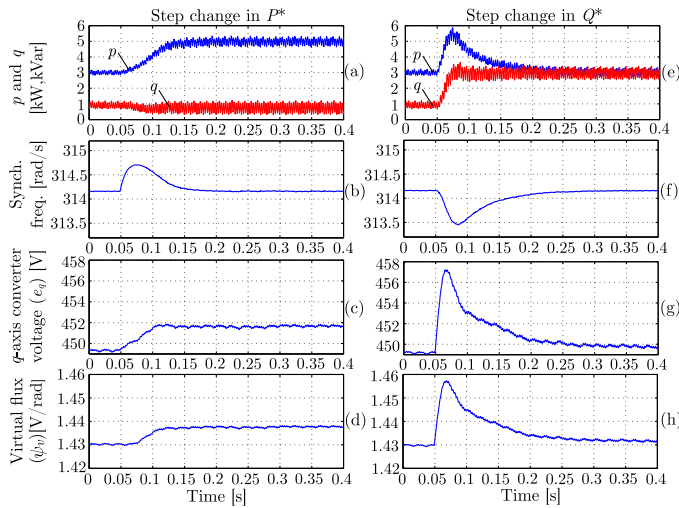


Fig. 15. Experimental results. Output current generated by the VSM when a step change was applied to  $P^*$ .

the reactive power transient there was a coupling between active and reactive power. This was expected because  $e_q$  is affected by both the frequency and voltage commands (see Fig. 1). To avoid this effect the coupling term in the VSM voltage command can be eliminated, yet this was not the case under studied in this paper.

2) *Operation in a Weak-Grid Scenario:* Fig. 14 (left) shows results obtained when the VSM was connected to an inductive weak grid ( $L_g = 0.2 \text{ pu}$  and  $R_g = 0.04 \text{ pu}$ ). The maximum value of inductance available in the laboratory was used. The grid frequency suddenly changed from 50 Hz to 49.8 Hz. Therefore, the converter delivered 5.5 kW due to the active-power droop coefficient. Fig. 14 (right) shows the same test, but in this case the grid had a resistive characteristic ( $R_g = 0.3 \text{ pu}$  and  $L_g = 0.07 \text{ pu}$ ). The maximum value of resistance available in the laboratory was used. The VSM remained connected to the grid, but the transient was slow and less damped. The coupling between the active and reactive powers increased due to the resistive weak-grid effect.

3) *Output Current Waveform*: Fig. 15 shows the output current for a step change of 4 kW in  $P^*$ . The current contains low-frequency harmonics (THD is 4.6%). To solve this issue, a

dedicated controller can be added [27], but it was not included in this case in order to validate the proposed model without modifying the VSM dynamics.

# VIII. CONCLUSION

This paper has addressed the application of grid-connected VSMs that provide voltage support and virtual inertia functionalities to a weak grid. A simplified small-signal model was developed and a method for the VSM parameters design proposed. This model successfully captures the low-frequency dynamics of the VSM apart from the synchronous resonance effect.

It has been shown that VSMs are robust against varying operating point. In addition to that, VSM robustness against variations of the grid impedance ( $L_g$  and  $R_g$ ) was explored analytically and validated experimentally. This has been studied in two different ways: firstly the trajectory of the closed-loop was evaluated and secondly the signs of the relevant gains in the small-signal model were analysed against any changes of grid impedance. In both cases, it was shown that, unlike inductively-dominated grids, resistively-dominated weak grids may destabilise VSMs. All the control algorithms were tested on a 15 kW prototype where the controller was discretized and implemented in its incremental form.

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