Shapley Coalitions for Prosumers in NRG-X-Change Network

Paying Prosumers Using NRG-X-Change

What is NRG-X-Change? "In this paper we propose NRG-X-Change — anovel mechanism for trading of locally produced re-newable energy that does not rely on an energy mar-ket or matching of orders ahead of time. In our modellocally produced energy is continuously fed into thegrid and payment is received based on actual usage,rather than predicted, as consumption is measured bythe DSO (Distribution Service Operator) and billed in near real-time." (Mihail Mihaylov)

Modeling a Prosumer Market

We will leverage an NRGX-Change based market of prosumers on an micro-grid. Any excess energy that is not consumed by the micro-grid is not considered at this time. The goal of each prosumer would be to offset the demand of the micro-grid. In most cases the total demand would need to be supplemented by the larger Grid at some retail electricity price. The network will use the payout and consumption functions for NRGX-Change as each prosumer generates every month.

The prosumers (P) are indicated on the diagram as generators of electricity. In this scenario solar PV is shown as a generation source. The Distribution Service Operator (DSO) is in charge of consuming excess Net Energy that is generated by the prosumer and it is not consumed by the prosumer. A consumer (C) is the term for a residence that has to consumer more energy that it can produce. It will need to pay for electricity and the DSO would charge it at some price. The NRG-X-Change function will charge the consumer and pay the prosumer for electricity consumed or generated.

Prosumer Payment Function

The NRG-X-Change as described by the authors performs a dynamic payment to prosumers that are capable of meeting the demand of the micro-grid. The micro-grid is made of prosumers and consumers. As the load demand spikes the pricing for net generation also spikes to meet the demand. When there is too much generation on the grid the pricing drops encouraging prosumers to generate less and consumer to consume more. The payout function g(.) , utilizes a normalization component in the denominator to account for over or under generation distributing the payout along the curve. The payment is at its highest when generation meets the total demand and at its lowest as generation starts to saturate the market because of low demand.

$$g(x, t_p, t_c) = \frac{x^n * q_{t_p=t_c}}{e^{\frac{(t_p-t_c)^2}{a}}}$$

In [1]:

```
import math
# NRGXChange Payment g(.) Function
def g(price,X,tp,tc,a,n):
    q = price
    try:
        #print(f"n{n},tp{tp},tc{tc},a{a},q{q}")
        pay = (pow(X,n)*q)/math.exp(pow((tp-tc),2)/a)
    except OverflowError:
        pay = float('inf')
    return pay
```

Consumer Charge/Cost Function

Where x, is the net energy of the prosumer. q, is the maximum price allowed. t_p ,is the total produced energy of all prosumers. t_c is the total consumption of all the prosumers. a , is a scaling constant to adjust the pay out. Similarly lets consider the cost of energy for consumers to purchase based on pricing set by the h(.) function. In tandem these incentives are non-linear because of the distribution curve. The shape of that curve can be adjusted to the size of the network and the volatility of the network.

$$h(y, t_p, t_c) = \frac{y * r_{t_c >> t_p} * t_c}{t_c + t_p}$$

Where y is the withdrawn energy, and $r_{t_c >> t_p}$ is the maximum cost of energy delivered by the utility when the energy supply by prosumers is low. Again, t_p is the total production and t_c is the total consumption of the prosumers in the network. The minimum payment by the utility in the historical payment prices would indicate the minimum amount willing to charge customers for energy in order to cover the cost of delivering the energy. We will use the minimum price in our list for r.

In [2]:

```
# NRGXChange Charge h(.) Function
def h(price,c,tp,tc):
    y = c
    r = (0.01*price)
    try:
        cost = (y*r*tc)/(tc+tp)
    except OverflowError:
        cost = float('inf')
    return cost
```

Creating Coalitions Using Shapley Value

Review of Game Theory and Shapley Value

The game is in terms of a **characteristic function**, which specfies for every group of players the total payoff that the members of S can by signing an greement among themselves; this payoff is available for distribution among the members of the group. A coalitional game with transferable payoff is a pair $< N, v >$ where $N = \{1, \ldots, n\}$ is the set of players and for every subset S of I (called a coalition) $v(S) \in \mathbb{R}$ is the total payoff that is available for division among members of S (called the worth of S). We assume that the larger the coalition the larger the payoff (this property is called superadditivity).

An agreement amongst players is a list $(x_1, x_1, \ldots, x_n)$ where x_1, is the proposed payoff to individual i. Shapley value is interpreted in terms of **expected marginal contribution**. It is calculated by considering all the possible orders of arrival of the players into a room and giving each player his marginal contribution.

In [3]:

```
# Shapley Value Python Logic
# Authored by Susobhan Ghosh
# https://github.com/susobhang70
# Committed on 02/01/2020
from itertools import combinations
import bisect
#Create Combinatorial from List
def power_set(List):
    PS = [list(j) for i in range(len(List)) for j in combinations(List, i+1)]
    return PS
#Calculate Shapley from Characteristic Value list
def get_shapley(n,v):
    tempList = list([i for i in range(n)])
    N = power_set(tempList)
    shapley_values = []
    for i in range(n):
        shapley = 0
        for j in N:
            if i not in j:
                cmod = len(j)
                Cui = j[:]
                bisect.insort_left(Cui,i)
                l = N.index(j)
                k = N.index(Cui)
                temp = float(float(v[k]) - float(v[l])) *\
                           float(math.factorial(cmod) * math.factorial(n - cmod - 1)) / float(math.factorial(n))
                shapley += temp
        cmod = 0
        Cui = [i]
        k = N.index(Cui)
        temp = float(v[k]) * float(math.factorial(cmod) * math.factorial(n - cmod - 1)) / float(math.factorial(n))
        shapley += temp
        shapley_values.append(shapley)
    return shapley_values
```

In [4]:

```
#Test of Shapley Function Calculation
# Results in Shapley distribution of : Prosumer-1: $0.65, Prosumer-2: $0.39
n=2
p1_characteristic_value = 0.77
p2_characteristic_value = 0.51
p1_n_p2_characteristic_value = 1.04
v=[p1_characteristic_value,p2_characteristic_value,p1_n_p2_characteristic_value]
vals = get_shapley(n,v)
print(f"Prosumer-1: ${vals[0]}, Prosumer-2: ${vals[1]}")
```

```
Prosumer-1: $0.65, Prosumer-2: $0.39
```

Pecan Street Data

Pecan Street is a research and development orgnaization that gathers data from active homes, solar homes and electric vehicle owners. According to Spandana Vadam , the pecan street data can be used to build out prosumers and calculating shapley value for coalitional contributions.

The data ranges between 2015-09-23 and 2015-12-22 and is segmented by hour. The data is of 6 single family homes located in Austin, texas. Each home was installed with a PV system for the year of 2015. Each home has a 'Gen',power generated from PV systems, and a 'Use',whole-home electrical usage, value for every hour of the day during Fall, Spring and Winter.

Define Mathematical Model

The monthly average energy production is the individual energy production summed up across the entire Fall season and then devided by 3 for each month. The Gen_{ai} in kWh is the average generation of the i^th prosumer.

The generalized characteristic function, $v(i) = \frac{X*q}{e^{\frac{[Gen_a - Use_a]^2}{a}}}$. When we apply it specifically to a single prosumer we must use the single prosumers net generation for X and then consider all prosumer generation and all prosumer usage as a sum of the averages. Note, we will also choose the $q = \$10/kWh$ for the pricing and the scale factor , $a = 10^6$.

$$v(i) = \frac{(Gen_{ai} - Use_{ai}) * 10}{e^{\frac{[\sum_{i=1}^{6}(Gen_{ai}) - \sum_{i=1}^{6}(Use_{ai})]^2}{10^6}}}$$

Computing Shapley value with data collected from Pecan Street

i(House)	FALL			Spring			Winter		
Prosumer	Monthly average Energy Production for Fall (kWh) ***Gen_ai***	Monthly average Energy Consumption for Fall(kWh) ***Use_ai***	Energy offered by each prosumer (kWh) (X)	Monthly average Energy Production for Spring (kWh) ***Gen_ai***	Monthly average Energy Consumption for Spring (kWh) ***Use_ai***	Energy offered by each prosumer (kWh) (X)	Monthly average Energy Production for Winter (kWh) ***Gen_ai***	Monthly average Energy Consumption for Winter (kWh) ***Use_ai***	Energy offered by each prosumer (kWh) (X)
1	1473	1523	-50.00	1514	1431	83.00	1239	1138	101.00
2	1215	1056	159.00	1937	1279	658.00	1045	903	142.00
3	1006	367	639.00	1337	378	959.00	871	288	583.00
4	643	970	-327.00	903	1087	-184.00	575	810	-235.00
5	1737	676	1061.00	1996	749	1247.00	1447	406	1041.00
6	1518	1098	420.00	1560	1346	214.00	1273	677	596.00
Total	7592.00	5690.00	1902.00	9247.00	6270.00	2977.00	6450.00	4222.00	2228.00

In [5]:

```
from pandas import DataFrame
# Convert Pecan Data into dataset by trial to analyze
pecan_data = [
    {'id':1,'time':'Fall','generation':1473,'consumption':1523},
    {'id':2,'time':'Fall','generation':1215,'consumption':1056},
    {'id':3,'time':'Fall','generation':1005,'consumption':367},
    {'id':4,'time':'Fall','generation':643,'consumption':970},
    {'id':5,'time':'Fall','generation':1737,'consumption':676},
    {'id':6,'time':'Fall','generation':1518,'consumption':1098},
```

```python
    {'id':1,'time':'Spring','generation':1514,'consumption':1431},
    {'id':2,'time':'Spring','generation':1937,'consumption':1279},
    {'id':3,'time':'Spring','generation':1337,'consumption':378},
    {'id':4,'time':'Spring','generation':903,'consumption':1087},
    {'id':5,'time':'Spring','generation':1996,'consumption':749},
    {'id':6,'time':'Spring','generation':1560,'consumption':1346},
    {'id':1,'time':'Winter','generation':1239,'consumption':1138},
    {'id':2,'time':'Winter','generation':1045,'consumption':903},
    {'id':3,'time':'Winter','generation':871,'consumption':288},
    {'id':4,'time':'Winter','generation':575,'consumption':810},
    {'id':5,'time':'Winter','generation':1447,'consumption':406},
    {'id':6,'time':'Winter','generation':1273,'consumption':677}
]
pecan_df = DataFrame(pecan_data)
pecan_df['net_energy'] = pecan_df['generation'] - pecan_df['consumption']
```

In [6]:

```python
######################################################################
# Calculate the shapley value given the net energy and the average
# generation and consumption values
######################################################################
def get_nrg_payments( df,price,n,a=0,drop_negative_contribution=False):
    for t in df:
        # drop the prosumer from consideration if the energy offered
        # by the prosumer is a negative value
        if drop_negative_contribution:
            index_names = t[ t['net_energy'] < 0 ].index
            t.drop(index_names, inplace = True)

        tp = t['generation'].sum()
        tc = t['consumption'].sum()
        net_energy = t['net_energy']

        ## shapley without coalition, only depends on characteristic
        # equation Y=X^n
        t['shapley_wo_coalition'] = net_energy.apply(lambda x:
        g(price=price,X=x,tc=tc,tp=tp,n=n,a=a))

        ## Implement shapley function
        # Use the id of each prosumer in order to build a factorial powerset
        # the power set is used by summing the energy contributed as a coalition
        # the aggregate energy is then sent to NRG payment function
        # the response is the shapley contribution payout for that coalition
        List = t['id']
        PS = [list(j) for i in range(len(List)) for j in combinations(List, i+1)]
        shapley_contributions = []
        for coalition_combination in PS:
            # energy contribution of the coalitional combination
```

```
            contribution = t.loc[t['id'].isin(coalition_combination)]['net_energy'].sum()
            # get nrg payment for the contribution of the coalitional combination
            nrg_payment = g(price=price,X=contribution,tc=tc,tp=tp,n=n,a=a)
            shapley_contributions.append(nrg_payment)
        t['shapley_w_coalition'] = get_shapley(len(t['id']),shapley_contributions)


    return df

####################################################################
# print the tables for each season to show the shapley value w/wo
# coalitional contribution
####################################################################
def print_tables(df,n=1,title=''):
    tb = 1
    for t in df:
        t = t.reindex(columns=['id','time','generation','consumption',
        'net_energy','shapley_w_coalition','shapley_wo_coalition'])
        display(HTML(f"<h1>{title}</h1></br> Table {tb}: {t.iloc[0]['time']} Y=X^{n} </br>{t.to_html(index=False)}"))
        tb=tb+1
```

In [7]:

```
#Test of Example 1
# Results in Shapley distribution of : Prosumer-1: $0.65, Prosumer-2: $0.39
n=2
p1_characteristic_value = 0.77
p2_characteristic_value = 0.51
p1_n_p2_characteristive_value = 1.04
v=[p1_characteristic_value,p2_characteristic_value,p1_n_p2_characteristive_value]
vals = get_shapley(n,v)
print(f"Prosumer-1: ${vals[0]}, Prosumer-2: ${vals[1]}")
```

```
Prosumer-1: $0.65, Prosumer-2: $0.39
```

The characteristic function results in a shapley value for each of the prosumers for each of the time periods.

In [8]:

```
from IPython.display import display, HTML
pecan_df_by_t = [DataFrame(y) for x, y in pecan_df.groupby('time', as_index=False)]
x1_pecan_df_by_t = get_nrg_payments(pecan_df_by_t,price=10,a=1000000,n=1)
print_tables(x1_pecan_df_by_t)
```

Table 1: Fall Y=X^1

id	time	generation	consumption	net_energy	shapley_w_coalition	shapley_wo_coalition
1	Fall	1473	1523	-50	-13.474609	-13.474609
2	Fall	1215	1056	159	42.849257	42.849257
3	Fall	1005	367	638	171.936011	171.936011
4	Fall	643	970	-327	-88.123943	-88.123943
5	Fall	1737	676	1061	285.931203	285.931203
6	Fall	1518	1098	420	113.186715	113.186715

Table 2: Spring Y=X^1

id	time	generation	consumption	net_energy	shapley_w_coalition	shapley_wo_coalition
1	Spring	1514	1431	83	0.117525	0.117525
2	Spring	1937	1279	658	0.931705	0.931705
3	Spring	1337	378	959	1.357911	1.357911
4	Spring	903	1087	-184	-0.260538	-0.260538
5	Spring	1996	749	1247	1.765709	1.765709
6	Spring	1560	1346	214	0.303017	0.303017

Table 3: Winter Y=X^1

id	time	generation	consumption	net_energy	shapley_w_coalition	shapley_wo_coalition
1	Winter	1239	1138	101	7.054894	7.054894
2	Winter	1045	903	142	9.918762	9.918762
3	Winter	871	288	583	40.722806	40.722806
4	Winter	575	810	-235	-16.414853	-16.414853
5	Winter	1447	406	1041	72.714307	72.714307
6	Winter	1273	677	596	41.630862	41.630862

In [9]:

```
import numpy as np
```

```python
import matplotlib.pyplot as plt

########################################################################
# print the tables in a plot
########################################################################
def plot_tables(df,n=1):
    for t in df:
        plt.style.use('seaborn-whitegrid')
        fig = plt.figure()
        ax = plt.axes()
        t = t.sort_values(by='net_energy')
        plt.title(f"Shapley value for energy offered be each prosumer\nwith X^{n} function {t.iloc[0]['time'].upper()}")
        plt.xlabel("Energy offered by each prosumer (kWh)")
        plt.ylabel("Shapley Value obtained by each prosumer ($)\n With and Without Coalition");
        plt.plot(t['net_energy'], t['shapley_w_coalition'], ':xr', label='With Coalition', linewidth=1, alpha=0.3)
        plt.plot(t['net_energy'], t['shapley_wo_coalition'], ':ob', label='Without Coalition',linewidth=1, alpha=0.4)
        plt.legend();
        ax.plot();

plot_tables(x1_pecan_df_by_t)
```

Modify the Linearity of the Characteristic Function (X^n)

Negtaive Shapley value means tha the prosumer owes to a coalition. But Shapley values are not used to define how much money the prosumer owes. Therefore the prosumers with negative Shapley values will be ignored. Only those prosumers with positive Shapley values will be considered in this section, by excluding prosumers whose average production values are less than consumption values. In the end the results would show if the psoumer recieves higher profits by forming coalitions.

In [10]:

```python
from IPython.display import display, HTML
pecan_df_by_t = [DataFrame(y) for x, y in pecan_df.groupby('time', as_index=False)]
x1p5_pecan_df_by_t = get_nrg_payments(pecan_df_by_t,price=10,a=1000000,n=1.5,drop_negative_contribution=True)
print_tables(x1p5_pecan_df_by_t,n=1.5)
```

Table 1: Fall Y=X^1.5

id	time	generation	consumption	net_energy	shapley_w_coalition	shapley_wo_coalition
2	Fall	1215	1056	159	403.535400	111.793945
3	Fall	1005	367	638	1689.509355	898.573511
5	Fall	1737	676	1061	2874.425956	1927.061920

id	time	generation	consumption	net_energy	shapley_w_coalition	shapley_wo_coalition
6	Fall	1518	1098	420	1095.049136	479.950589

Table 2: Spring Y=X^1.5

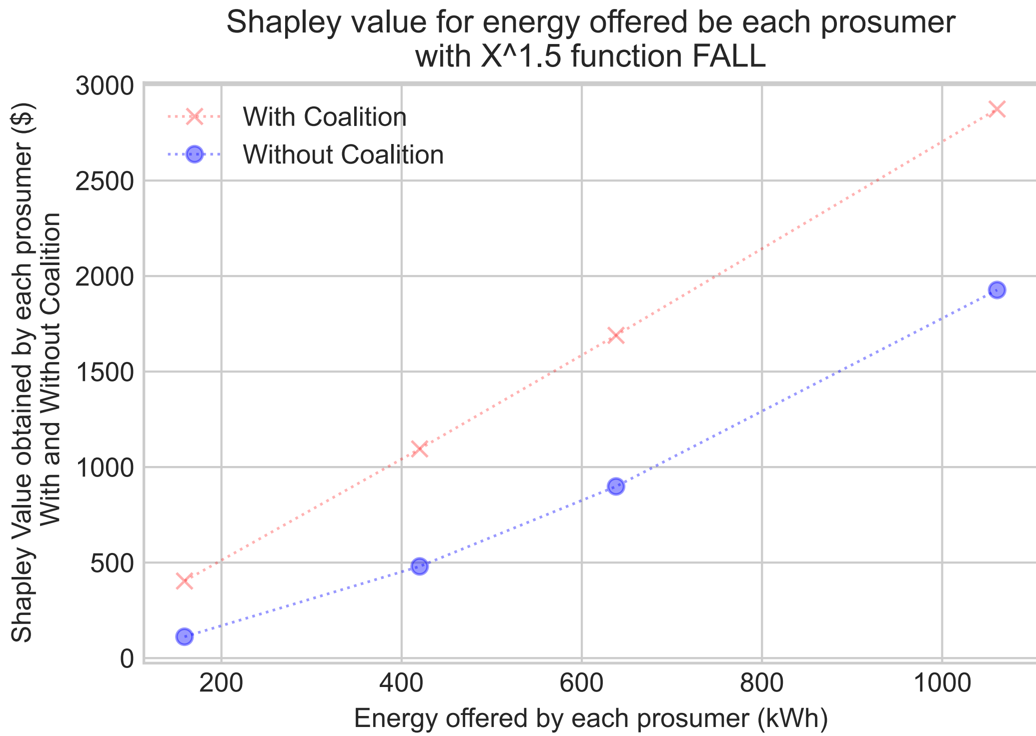
id	time	generation	consumption	net_energy	shapley_w_coalition	shapley_wo_coalition
1	Spring	1514	1431	83	2.017967	0.346084
2	Spring	1937	1279	658	16.744626	7.725075
3	Spring	1337	378	959	24.749423	13.592263
5	Spring	1996	749	1247	32.543414	20.154105
6	Spring	1560	1346	214	5.283899	1.432796

Table 3: Winter Y=X^1.5

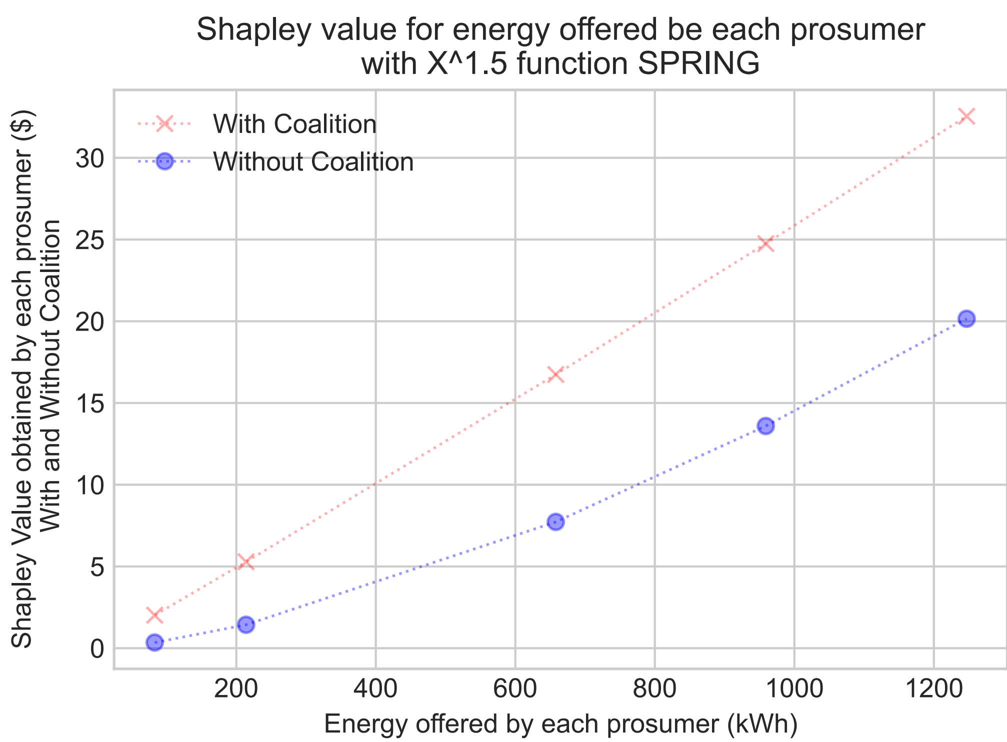
id	time	generation	consumption	net_energy	shapley_w_coalition	shapley_wo_coalition
1	Winter	1239	1138	101	110.611444	23.544609
2	Winter	1045	903	142	156.426309	39.250205
3	Winter	871	288	583	666.826403	326.521851
5	Winter	1447	406	1041	1219.240313	779.086201
6	Winter	1273	677	596	682.241348	337.503909

In [11]:

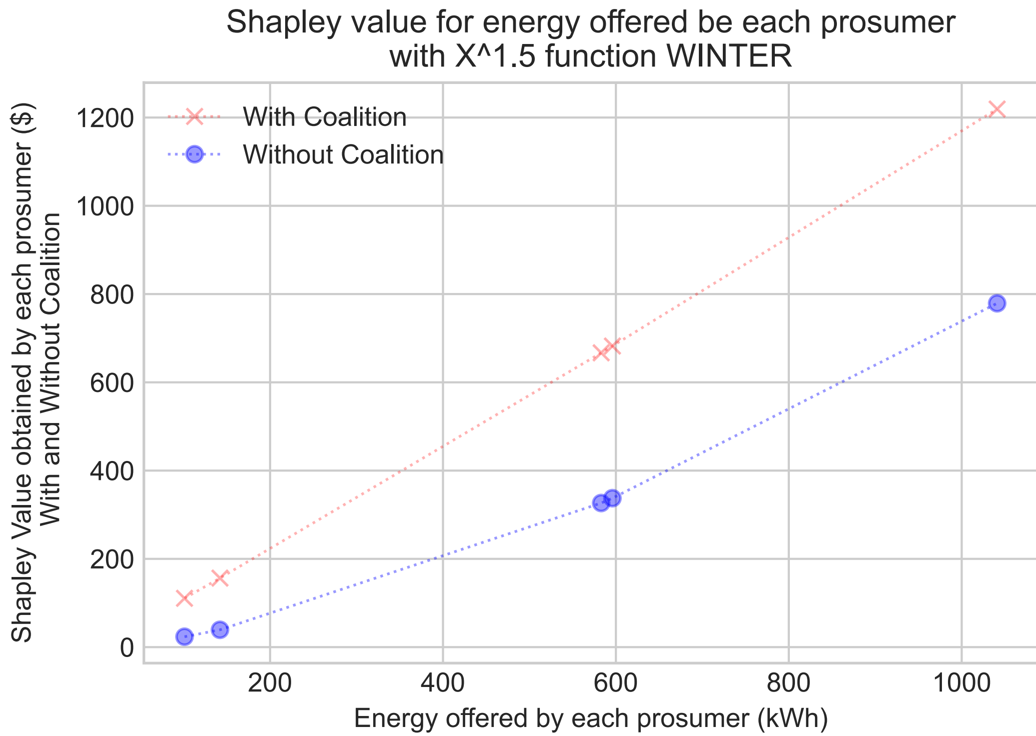
```
plot_tables(x1p5_pecan_df_by_t,n=1.5)
```



Shapley value for energy offered be each prosumer
with X^1.5 function FALL
With Coalition
Without Coalition
Shapley Value obtained by each prosumer ($)
With and Without Coalition
Energy offered by each prosumer (kWh)
0
500
1000
1500
2000
2500
3000
200
400
600
800
1000



Shapley value for energy offered be each prosumer
with X^1.5 function SPRING
With Coalition
Without Coalition
Shapley Value obtained by each prosumer ($)
With and Without Coalition
Energy offered by each prosumer (kWh)
0
5
10
15
20
25
30
200
400
600
800
1000
1200



Prosumer Synthesized Data from EIA.gov

The data gathered is from EIA.gov. The data was then used to synthesize typical prosumer consumption and generation over the course of 12 months. Volatitlity in the usage and generation was added as a normal distribution with a given variance to simulate real world conditions. The data is pulled from a local file and then sorted by timestamp. The fields are grouped by id and by time. Grouping by time allows for settlement calculations to occur at each time interval.

In [12]:

```
import pandas as pd
import numpy as np
#####################################################################
# Data Gathering
#####################################################################
# Data gathering and synthesization has been done in a seperate module.
# import the external data gathering set. Pull data from a dataset
# of randomly insantiated prosumer, synthesized from EIA.gov data trends
#
import p0_data_gather as p0

#####################################################################
# get dataset by calling the local file and queirying for
```

```
# parameters desired.
####################################################################
def get_eia_dataset():
    # Set path of dataset file containing all parameterized values
    data_set_path = 'data/prosumer_N10_all_20210305_1129.csv'
    # Set initial conditions for prosumers dataset
    dem_mean = 1100
    gen_mean = 1300
    # Set number of prosumers, an array of N[] values for multiple experiments
    number_of_prosumers_for_each_trial = [3,5]
    # Trials [] holds session data for each N itteration of prosumers
    eia_dataset_trials = []
    for N in number_of_prosumers_for_each_trial:
        # Call the data as a query with the instantiated values
        eia_dataset = p0.get_data(path=data_set_path,query=f'id > 0 & id<={N} & demand_std == {dem_mean*0.20} & generati
        eia_dataset['net_energy'] = eia_dataset['generation'] - eia_dataset['consumption']
        # Data Wrangling : Split data by time 't'
        ####################################################################
        # Wrangle data into monthly timesteps then add to an object for each trial
        eia_dataset_by_t = [pd.DataFrame(y) for x, y in eia_dataset.groupby('time', as_index=False)]
        eia_trial = {"N":N, "df":eia_dataset,"df_by_t":eia_dataset_by_t}
        eia_dataset_trials.append(eia_trial)
    return eia_dataset_trials

eia_dataset_trials = get_eia_dataset()
```

Running Trials

The trial for each dataset is a colelction of N prosumers. The trials are ran with the parameters for the characteristive function response X^n varying for the value of n. The other parameter that can be adjusted is the scalar a. The running trials leverages the EIA.gov dataset for a maximum price to pay out during the trial. This value can also be adjusted if needed in the future revisions.

In [16]:

```
import matplotlib as mpl
from mpl_toolkits.mplot3d import Axes3D

def run_trials(a,n=1):
    for eia_dataset_trial in eia_dataset_trials:
        N = eia_dataset_trial['N']
        df = eia_dataset_trial['df']
        df_by_t = eia_dataset_trial['df_by_t']
        price = df['price'].max()
        eia_dataset_trial['df_by_t'] = get_nrg_payments(df_by_t,price=price,a=a,n=n,drop_negative_contribution=True)
    return eia_dataset_trials
```

In [19]:

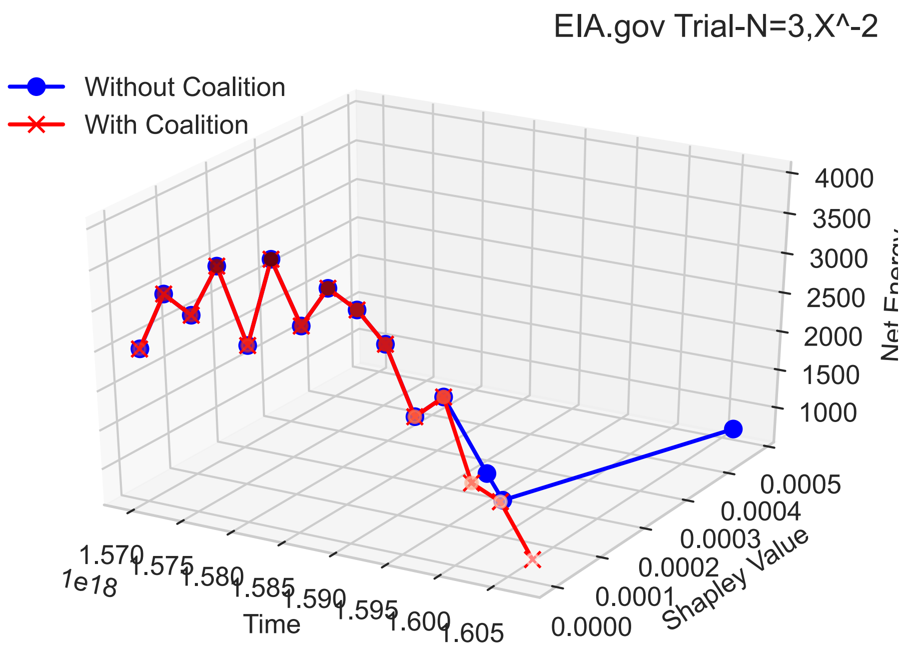
```
def plot_trials(trials,n):
    for eia_dataset_trial in eia_dataset_trials:
        X = [x for x in  eia_dataset_trial['df']['time'].unique().tolist()]
        Y_wo = [t['shapley_wo_coalition'].sum() for t in eia_dataset_trial['df_by_t']]
        Y_w = [t['shapley_w_coalition'].sum() for t in eia_dataset_trial['df_by_t']]
        Z = [t['net_energy'].sum() for t in  eia_dataset_trial['df_by_t']]
        ax = plt.axes(projection='3d')
        ax.set_title(f"EIA.gov Trial-N={eia_dataset_trial['N']},X^{n}",loc="right")
        ax.set_xlabel("Time")
        ax.set_ylabel("Shapley Value")
        ax.set_zlabel("Net Energy")
        ax.scatter3D(X, Y_wo, Z, c=Z,cmap='Blues');
        ax.plot3D(X, Y_wo, Z,'blue',label='Without Coalition',marker='o');
        ax.scatter3D(X, Y_w, Z, c=Z,cmap='Reds');
        ax.plot3D(X, Y_w, Z,'red',label='With Coalition',marker='x');
        ax.legend(loc='upper left')
        plt.show()
        #print_tables(df_by_t,n=1.5)
```

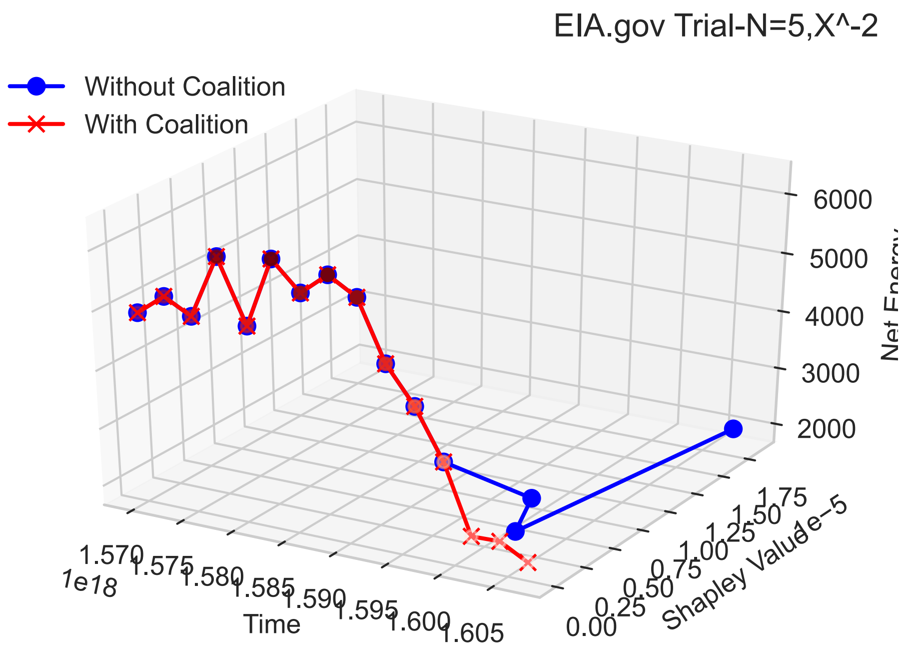
Consider Convex , Linear & Concave characteristic functions

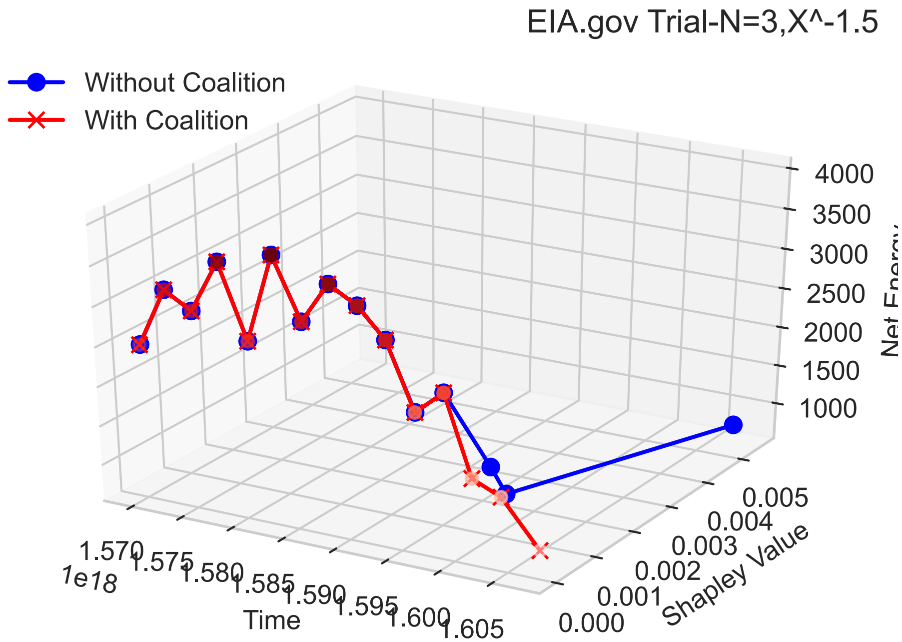
We consider a comunity of solar prosumers $P = 1, 2, 3..N$, who agree to form a coalition and produce energy. The number of possible coalitions are 2^n and the number of ways to build the grand coalition is $N!$. The coalitional contribution is compared at different N prosumers and at different characteristic function pay out to observer the impact as the network scales

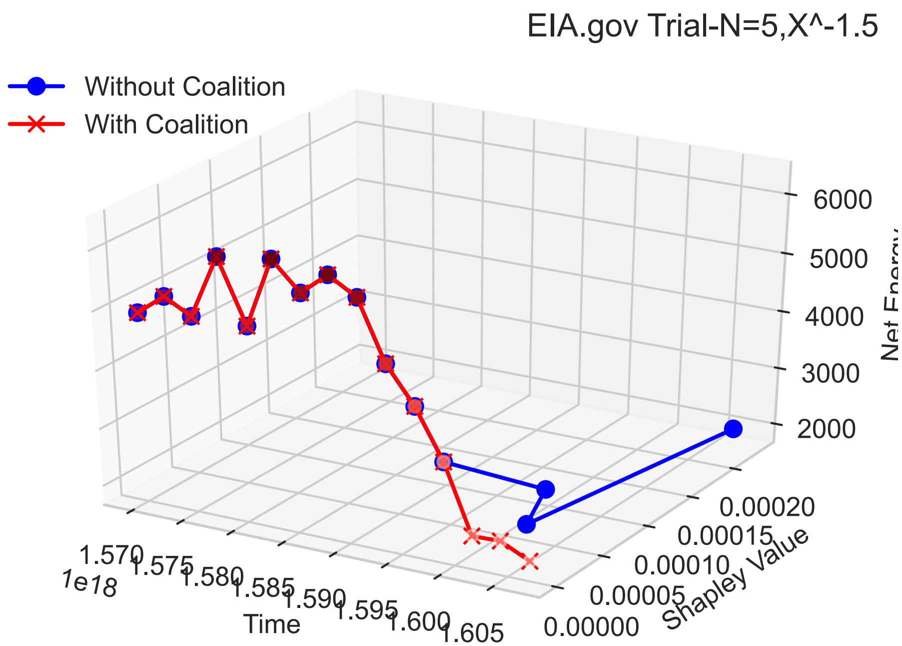
In [20]:

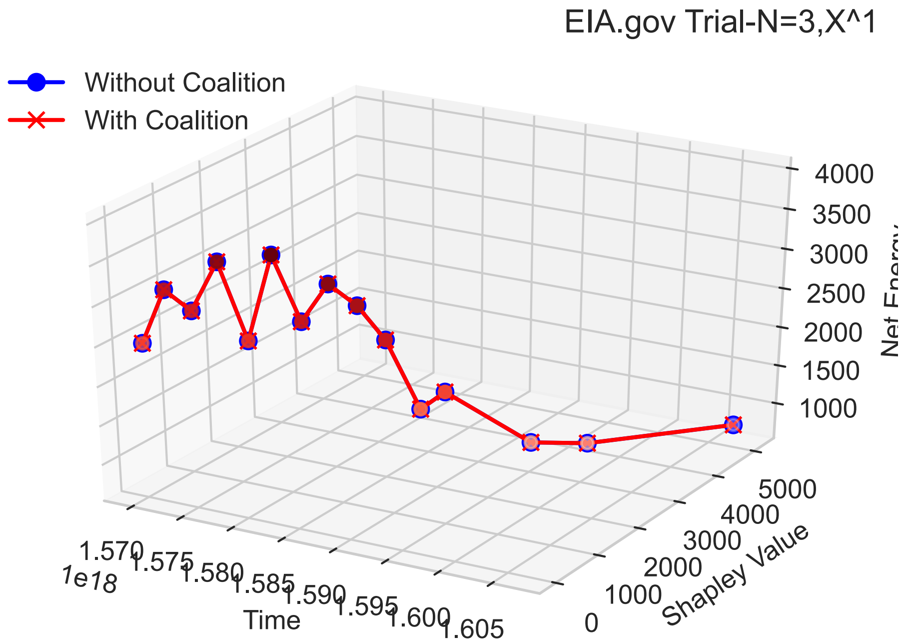
```
#####################################################################
# Consider Convex , Linear & Concave characteristic functions
#####################################################################
# Adjust the value of 'X^n' used in NRG payment function
X_n= [-2,-1.5,1,1.5,2]
for n in X_n :
    trials = run_trials(n=n,a=1000000)
    plot_trials(trials=trials,n=n)
```

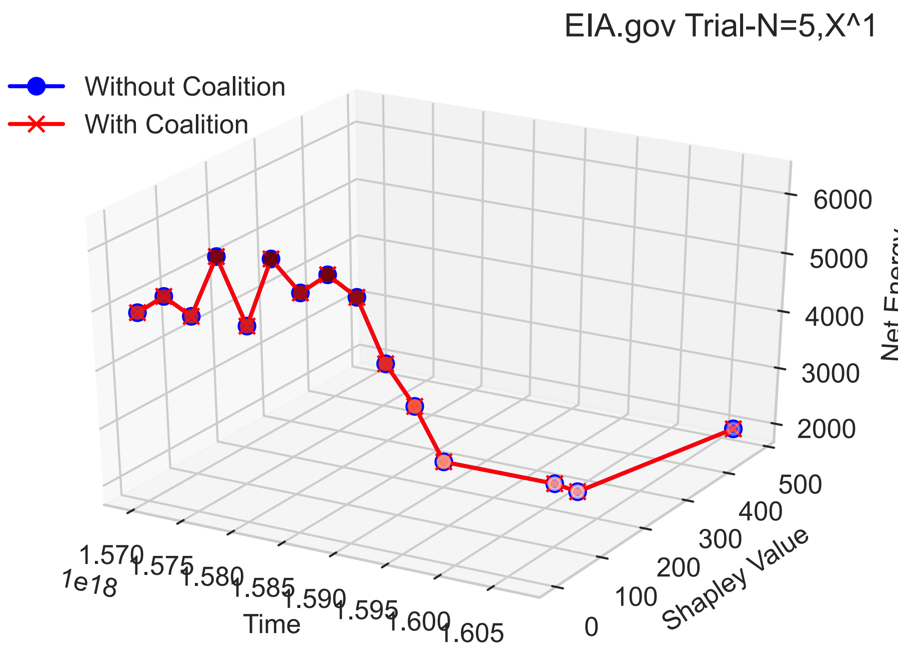

EIA.gov Trial-N=3,X^-2
Without Coalition
With Coalition
Time
1e18
1.570
1.575
1.580
1.585
1.590
1.595
1.600
1.605
Shapley Value
0.0000
0.0001
0.0002
0.0003
0.0004
0.0005
1000
1500
2000
2500
3000
3500
4000

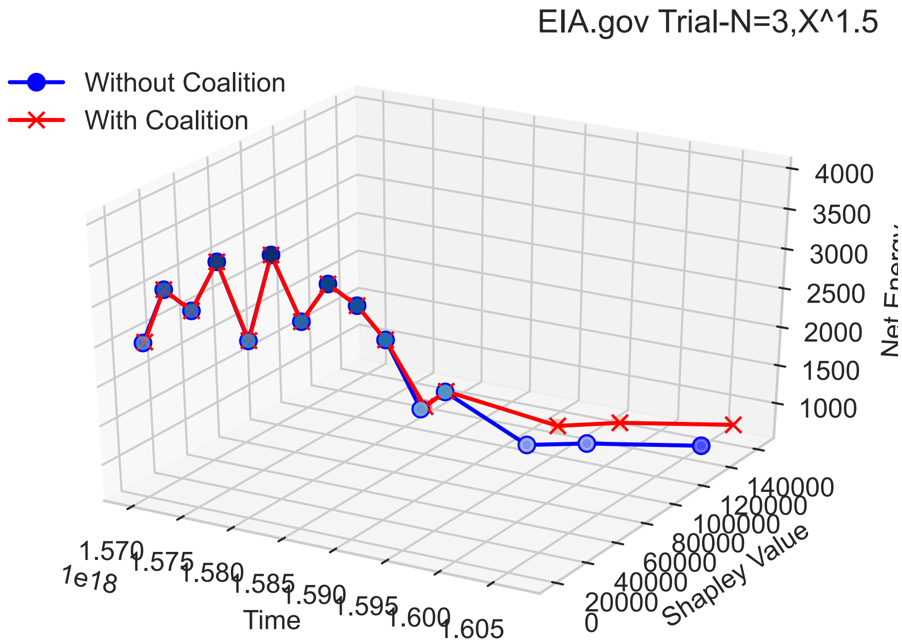

EIA.gov Trial-N=5,X^-2
Without Coalition
With Coalition
Time
1e18
1.570
1.575
1.580
1.585
1.590
1.595
1.600
1.605
Shapley Value
1e-5
0.00
0.25
0.50
1.75
2000
3000
4000
5000
6000

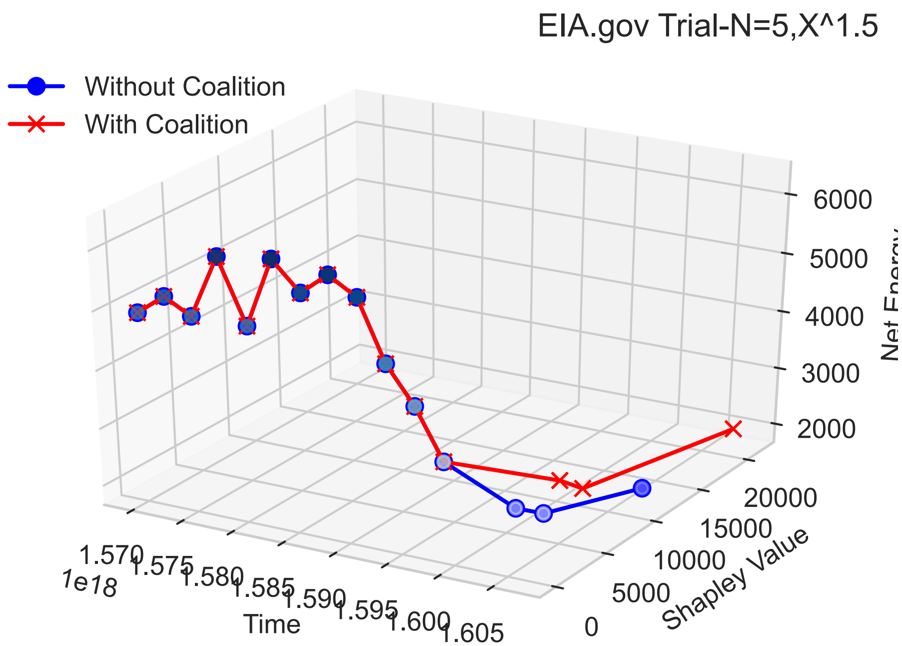

EIA.gov Trial-N=3,X^-1.5
Without Coalition
With Coalition
4000
3500
3000
2500
2000
1500
1000
0.005
0.004
0.003
0.002
0.001
0.000
Shapley Value
1.570
1.575
1.580
1.585
1.590
1.595
1.600
1.605
1e18
Time

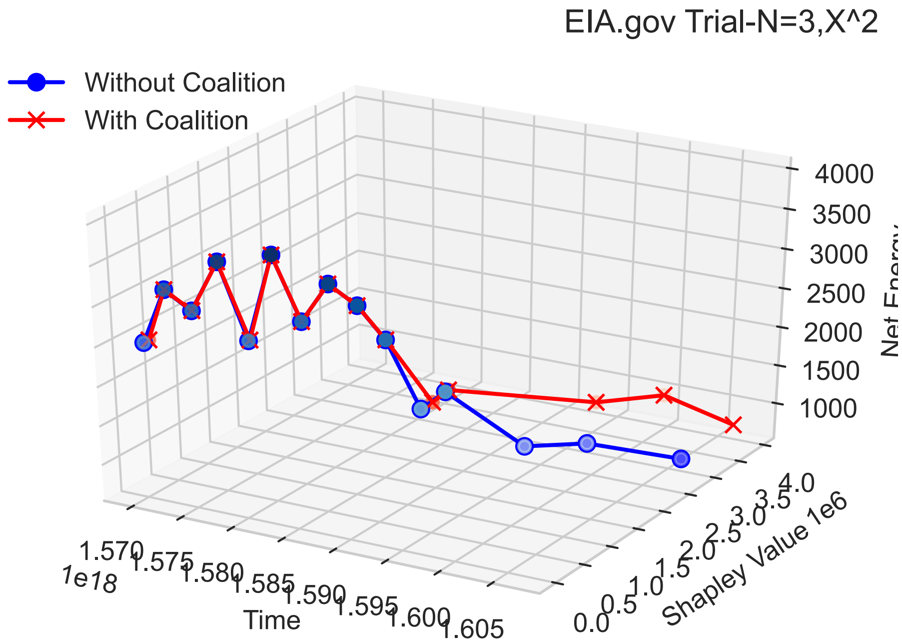

EIA.gov Trial-N=5,X^-1.5
Without Coalition
With Coalition
6000
5000
4000
3000
2000
0.00020
0.00015
0.00010
0.00005
0.00000
Shapley Value
1.570
1.575
1.580
1.585
1.590
1.595
1.600
1.605
1e18
Time

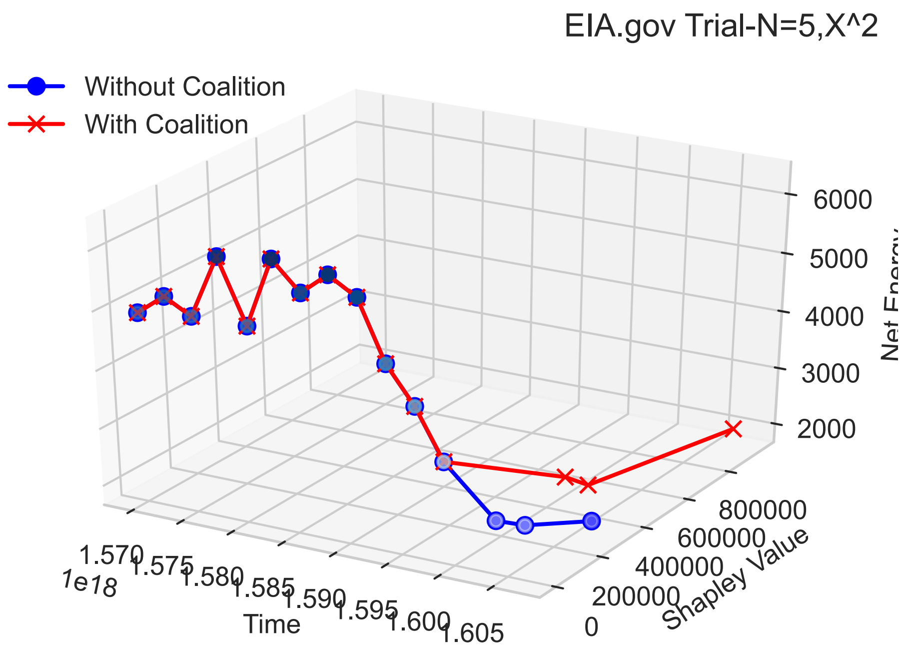

EIA.gov Trial-N=3,X^1
Without Coalition
With Coalition
4000
3500
3000
2500
2000
1500
1000
Net Energy
5000
4000
3000
2000
1000
0
Shapley Value
1.570
1.575
1.580
1.585
1.590
1.595
1.600
1.605
1e18
Time


EIA.gov Trial-N=5,X^1
Without Coalition
With Coalition
6000
5000
4000
3000
2000
Net Energy
500
400
300
200
100
0
Shapley Value
1.570
1.575
1.580
1.585
1.590
1.595
1.600
1.605
1e18
Time


EIA.gov Trial-N=3,X^1.5
Without Coalition
With Coalition
4000
3500
3000
2500
2000
1500
1000
140000
120000
100000
80000
60000
40000
20000
0
Shapley Value
1.570
1.575
1.580
1.585
1.590
1.595
1.600
1.605
1e18
Time


EIA.gov Trial-N=5,X^1.5
Without Coalition
With Coalition
6000
5000
4000
3000
2000
20000
15000
10000
5000
0
Shapley Value
1.570
1.575
1.580
1.585
1.590
1.595
1.600
1.605
1e18
Time


EIA.gov Trial-N=3,X^2
Without Coalition
With Coalition
Time
1e18
Shapley Value 1e6
Net Energy


EIA.gov Trial-N=5,X^2
Without Coalition
With Coalition
Time
1e18
Shapley Value
Net Energy