

Consensus Network Based Distributed Energy Management for PV-Based Charging Station

Jing Zhang

*Beijing Electric Vehicle Charging/
Battery Swap Engineering and
Technology Research Center
China Electric Power Research Institute
Beijing, China
Email: zhangjingl@sgcc.com.cn*

Ruiming Yuan, Zhenyu Jiang

*State Grid Jibei Electric
Power Company Limited
Electric Power Research Institute
Beijing, China
Email: ydollars@sina.com
jzy1343803@163.com*

He Yin

*Univ. of Michigan-Shanghai
Jiao Tong Univ. Joint Institute
Shanghai Jiao Tong Univ.
Shanghai, China
Email: yyy@sjtu.edu.cn*

Tianjin Chen

*XJ Power Corporation
Xuchang, Henan
Email: chentianjindy@126.com*

Taoyong Li

*Beijing Electric Vehicle Charging/Battery Swap
Engineering and Technology Research Center
China Electric Power Research Institute
Beijing, China
Email: litaoyong@epri.sgcc.com.cn*

He Zhao

*Beijing Electric Power Company
State Grid Corporation of China
Beijing, China
Email: zhaoha@bj.sgcc.com.cn*

Abstract—A distributed energy management for a photovoltaic (PV) based electric vehicles (EVs) charging station is proposed and discussed in this paper. The PV-based charging station and EV owners are modeled as independent players with different preferences where the preferences of the players are described through linear and logarithmic functions. Then, a non-cooperative power distribution game is set up and a generalized stackelberg equilibrium is found at any control instant. Through utilizing the consensus network based learning algorithm, the charging power of EVs can be updated in a distributed way. The single stage, dynamic case study and scalability analysis in the simulation show that the proposed energy management can be well applied in the PV-based charging station and have an excellent performance.

Index Terms—Energy management, Game theory, PV-based charging station, consensus network.

I. INTRODUCTION

Thanks to the concerns of carbon emissions and energy securities, electric vehicles (EVs) are believed to be a promising solution [1]. Due to the limited capacities of energy storage systems, all kind of EVs, i.e., plug-in EVs and pure EVs, need to be charged when their energy levels of storage batteries are low. Thus, it is inevitable to develop public charging stations in order to providing charging services to EV owners. However, the charging stations can be a threat for the grid because of the large power requirement for charging EVs simultaneously [2]. One possible solution to overcome this drawback is to implement an islanded charging station using photovoltaic (PV) energy, namely, a PV-based charging station. As a renewable and clean energy, PVs could be a local energy source and thus the energy gain from the main grid could be reduced or even vanished. Meanwhile, due to the unpredictable solar irradiation, energy storage systems are always utilized

to provide a continuous charging service. On the other hand, involving additional energy systems, i.e., PV and energy storage systems, will increase the system complexity and thus require a more comprehensive energy management strategy.

Energy management strategy is critical issue for achieving a high performance of PV-based charging stations. Different from charging EVs at home, charging EVs in a charging station requires fast charging technology in order to charge EVs in a short time. More importantly, EVs have different battery statuses, types and even charging requirements. With these uncertainties, developing an energy management strategy to satisfy both the charging station and EV owners becomes a critical problem. The energy management problem in a distributed charging station with renewable energy sources has been discussed by several literatures [3]–[6]. Most of them focuses on centralized control. Centralized controls in PV-based charging station can be categorized into rule-based decision making strategies and prediction-based decision making strategies. A rule-based decision making strategy implemented in a PV-based battery switch station is introduced in [3]. The objective is to provide available battery swapping service all the time. A pre-defined heuristic rule-based strategy is proposed to improve the self-consumption of PV energy and reduce the impact on the grid [4]. An online-learning algorithm based control is applied to maximize the self-consumption of PV system and decide the power supplied from the power grid through scheduling strategy [5]. A two stage prediction based strategy is proposed in [6]. The first stage is to predict the PV power as well as electricity consumption, and the second one is to schedule the EV charging.

Decentralized charging strategies, comparing with centralized ones, could reduce the required communications and com-

putational capacity of the control center and fragility to single point of failure [7], [8]. A decentralized energy management system is developed for regulating the energy flow among the photovoltaic system, the battery and the grid [9]. Their objectives are to achieve the efficient charging of electric vehicles. However, the charging power of EVs can not be determined by EV owners. The energy management problem in a community charging EVs is discussed in [10]. The EV charging power is determined in a cooperative distributed way. To the understanding of the authors, EV owners should be selfish and in non-cooperative manner when they are charging their EVs. Game theory is a famous mathematical tool to solve the decision making problem among selfish players [11]. In this paper, the energy management problem will be solved through being modeled into a non-cooperative game.

In this research, a non-cooperative stackelberg game is set up to solve the charging power distribution problem among selfish and individual players. The PV-based charging station and EV owners are modeled as players where the PV-based charging station is modeled as leader and EV owners are modeled as followers. Players in this game optimize their objectives through tuning their own control variables under a common power limitation constrain while after communication and negotiation through consensus algorithm, they finally achieve a generalized Stackelberg equilibrium. The contribution of this paper focuses on charging power distribution with following features,

- There is no centralized controller in the system, i.e., this strategy is fully distributed;
- Applying decentralized charging strategy to avoid single point of failure;
- No EV owner has to share its local information, i.e., SOC of batteries;
- The requirement of selfish EV owners and charging station are fully satisfied through achieving the equilibrium;

II. SYSTEM CONFIGURATION AND MODELING

The PV-based charging station is consisted of two parts, namely, the charging station system and the EV charging system. The physical and cyber systems are introduced in detail in this section.

A. Charging Station System

As shown in Fig. 1, the key components of a charging station system are a PV system, a battery storage system, a grid-connect system, and an EV charging system. All these systems are connected together through a 400V DC-bus. Meanwhile, from information and management point of view, there is a central energy management center. Notice that this control center gathers all the data except the data from EVs due to the individual privacy of the EV owners.

The PV system consists of PV panels and DC/DC converters. These PV panels are in series and parallel connection while the DC/DC converters are implemented through maximum power point tracking (MPPT) strategy. A detailed PV system model is introduced in [3] where the output power

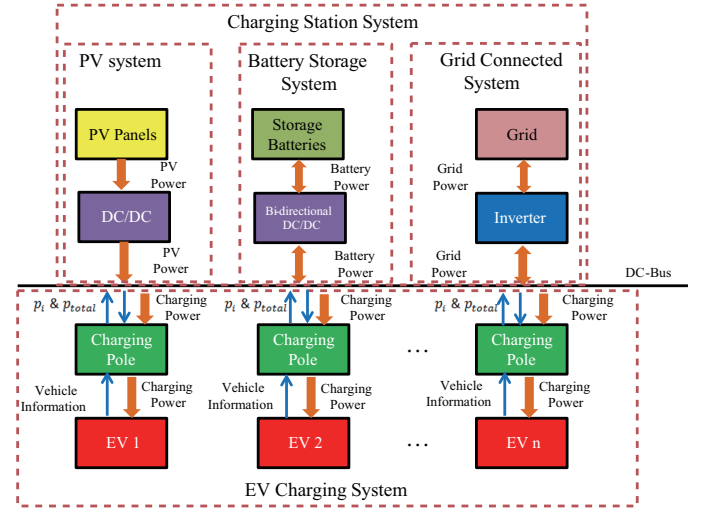


Fig. 1. System configuration of a charging station with PVs.

of a PV system depends on the irradiation and temperature. As mentioned in [3], two example days, i.e., a typical winter day and a summer day, are chosen here. The radiation and temperature profiles are converted into power profile, shown in Fig. 2. Since MPPT technology is applied on PV system, there is no degree of freedom in this system. Basically, the power generated from PV system highly depends on the weather condition which can be treated as an unpredictable input, i.e., an uncertainty.

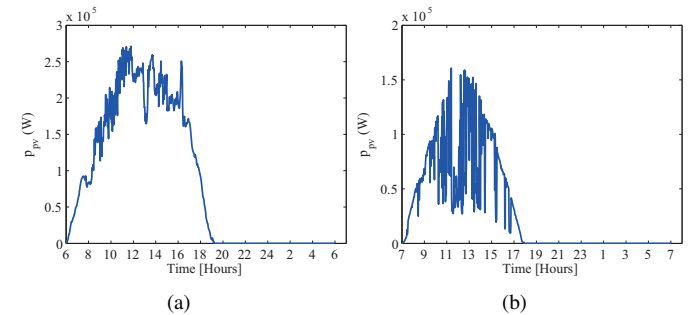


Fig. 2. (a) A typical summer day. (b) A typical winter day.

The battery storage system here consists of a battery pack and a bi-directional DC/DC converter. This battery pack here is utilized to filter the dynamic power generated by the PV system and support the charging service when irradiation is not strong enough. Thus, the control loop for this storage battery is to maintain the voltage level of the DC-bus. Similar to the PV system, there is no degree of freedom in battery storage system. The grid connected system consists of a bi-directional inverter (i.e., bi-directional DC/AC.), a transformer, and the main grid. The main objective of the grid connect system is to be a backup energy supplier for the PV-based charging station. With proper sizing and charging power control, the grid connect system may do not need to work at all. Note that the household load of the charging station is not considered

in this paper. It could be included in the future work.

Since the entire charging station (i.e., the PV system, the battery storage system, and the grid connect system) is treated as one single player, the objective of the charging station is to fully use the PV power as well as to maintain the capability of power supplying. Therefore, the objective of the charging station is designed to maintain the SOC of the battery storage system. Due to the fact that there is no actual degree of freedom in the charging station system, the total power available to the EVs, p_{total} , is virtually designed to be the control variable of the charging station. This variable will be selected as a virtual limitation for all the EVs and thus the utility function for the charging station can be written as,

$$u_s = -|\sum p_i + p_{pv}|, \quad (1)$$

$$p_{min,s} \leq \sum p_i \leq p_{total} \leq p_{max,s}, \quad (2)$$

where $p_{min,s}$ and $p_{max,s}$ are the lower and upper bounds for the p_{total} .

B. Electrical Vehicle Charging System

The EV charging system consists of charging poles and EVs where each charging pole could only charge one EV at any time instant. When an EV comes to the charging station, it will park at a randomly selected charging pole. Through communication between the charging pole and the battery management system in the EV, the EV owner can be treated as an independent player. It could determine its own charging power to maximize its objective function using local vehicle information while all the players should follow a common constrain.

The objective functions for EV owners are utilized to describe the satisfaction level of the EV owners from the charging service in the charging station. Researchers focus on many related factors to reflect this satisfaction level, such as improving the battery cycle life [12], avoiding the load variance [3], reducing the cost of charging [13], and increasing the total SOC of the vehicle [10]. In this paper, similar to [14], the objective function is designed to increase its charging power and considering its SOC, as follow,

$$u_i = \frac{P_i^*}{SOC_i} \ln(p_i + 1), \quad (3)$$

$$\sum_{i=1}^n p_i \leq p_{total}. \quad (4)$$

$$0 \leq p_i \leq P_i^*. \quad (5)$$

III. DISTRIBUTED ENERGY MANAGEMENT

The proposed distributed energy management algorithm is based on both the generalized non-cooperative game and the consensus network.

A. Generalized Non-cooperative Game

The energy management problem in this research is to determine the charging power among the EV owners in the charging station at a particular time instant. This problem

can be divided into two sub-problems, namely a two stage problem. In the first stage, the charging station will determine a virtual total charging limitation, p_{total} . This will become a common constrain to all the EV owners as shown in (4). In the second stage, the EV owners in the charging station will determine their charging power in a distributed fashion.

1) *First stage:* In the first stage, p_{total} can be determined based on SOC_b . Since the objective of the charging station is to maintain the SOC level of the storage battery, the p_{total} should be tuned accordingly. On the other hand, the charging station should provide services to EV owners. Thus, a simple linearised rule based energy management algorithm (p_{total} has an proportional relationship with SOC_b) is applied as follows,

$$p_{total} = p_{min,s}(1 + SOC_b), \quad (6)$$

where The upper and lower bound of p_{total} are $p_{min,s} = 0.25C_{max}V_{bus}N$ and $p_{max,s} = 0.5C_{max}V_{bus}N$, respectively. N is the number of the charging poles. C_{max} is the maximum C rate of all the EV battery.

2) *Second stage:* Given the p_{total} , the EV owners should determine their charging power in a distributed fashion. Since the charging station and EV owners are treated as selfish agents here, the charging station intends to minimize p_{grid} , while the EV owners try to be charged at a higher power. Since the charging station has advantage of accessing all the charging powers of EV owners, it can be treated as a leader while the EV owners are followers. Thus the energy management problem can be treated as a non-cooperative stackelberg game. Given the objective function discussed in previous section, each EV owner needs to determine its charging power to optimize its objection function. The solution provided here is based on Karush-Kuhn-Tucker (KKT) conditions of optimality. For each EV owner, its objective function can be written as Lagrangian L_i ,

$$L_i(p_i, \lambda_i) = u_i + \lambda_i G(p_i, \bar{\mathbf{p}}_{-i}), \quad (7)$$

where

$$G(p_i, \bar{\mathbf{p}}_{-i}) = \sum_{i=1}^n p_i - p_{total}. \quad (8)$$

λ_i is the Lagrange multiplier, and $\bar{\mathbf{p}}_{-i}$ is the vector formed by all the followers' decision variables, i.e., the charging powers here, except the one of the i th follower.

Since (7) is concave, the KKT conditions are the necessary and sufficient conditions for the existence the generalized Nash equilibrium (GNE). The KKT conditions of the i th follower's optimization problem are

$$\frac{\partial L_i}{\partial P_i} = -\frac{a_i}{p_i + 1} + \lambda_i = 0, \quad (9)$$

$$G(p_i, \bar{\mathbf{p}}_{-i}) \leq 0, \quad (10)$$

and it is known that the KKT conditions are satisfied with [13], [15]

$$\lambda_1 := \lambda_2 := \dots := \lambda_n := \bar{\lambda}. \quad (11)$$

Notice that if (11) holds, the GNE is the most socially stable one. When $\bar{\lambda} = 0$, i.e., $p_{total} > \sum_{i=1}^n p_i$, it is straightforward that

$$p_i = P_i^* \text{ for } \bar{\lambda} = 0. \quad (12)$$

Otherwise, combining (9) and (11) gives the solution for non-zero $\bar{\lambda}$, i.e., a balanced decision on competing the total available power p_{total} among the followers,

$$p_i = \frac{a_i(p_{total} + 1) - \sum_{i=1}^n a_i}{\sum_{i=1}^n a_i} \text{ for } \bar{\lambda} \neq 0. \quad (13)$$

$$a_i = \frac{P_i^*}{SOC_i} \quad (14)$$

Thus the existence of the GNE is proved. From (3), it can be derived that

$$\mathbf{F} = -\frac{du_i}{dp_i} = \begin{bmatrix} \frac{a_1}{p_1+1} \\ \vdots \\ \frac{a_n}{p_n+1} \end{bmatrix}. \quad (15)$$

The Jacobian of \mathbf{F} is

$$\mathbf{JF} = \begin{bmatrix} \frac{a_1}{(p_1+1)^2} & 0 & \cdots & 0 \\ 0 & \frac{a_2}{(p_2+1)^2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \frac{a_n}{(p_n+1)^2} \end{bmatrix}. \quad (16)$$

Since \mathbf{JF} is positive definite, \mathbf{F} is strictly monotone, and thus the above solution of p_i 's is unique.

B. Consensus Network Based Learning Algorithm

Based on the previous discussion, each EV owner only needs λ_i to determine its charging power, as shown in (9). If a centralized strategy is applied here, λ_i can be determined through a central controller. While the question here is how to determine the λ_i 's in a decentralized fashion. The solution provided here is applying consensus network technology. Since the required global information to assign the local optimal solution would be $\bar{\lambda}$, λ would be introduced as consensus variable for the i th EV to access the global information using a local sharing of information with neighbours based on consensus algorithm. The block diagram of the proposed consensus algorithm is shown in Fig. 3. The first step is initialization where λ_i 's are determined as,

$$\lambda_i = \frac{a_i}{P_i^* + 1}. \quad (17)$$

The second step is the consensus phase. In this phase, each EV updates λ_i following the rule,

$$\lambda_i(k+1) = \lambda_i(k) + \sum_{j=1}^n w_{ij}(\lambda_j(k) - \lambda_i(k)), \quad (18)$$

where w_{ij} 's are connectivity strengths. The w_{ij} 's are always chosen within $[0, 1/n]$ in order to make sure that the consensus values converge to the average of the initial values of all the nodes. (Note that the nodes should form a connected group, i.e., there is a bidirectional path between any two nodes.) In the next step, the virtual vehicle will tune its $\lambda_i(k+1)$ according

to the difference between the $\sum p_i$ and p_{total} . Then it goes back to step two until the difference between the $\sum p_i$ and p_{total} is small enough ($\lambda_i(k+1)$ for the virtual vehicle will stop changing.). After the $\lambda_i(k+1)$'s converge, each EV will update its charging power according to the $\lambda_i(k+1)$ and its own charging power boundaries,

$$p_i(k+1) = \frac{a_i}{\lambda_i(k+1)} - 1, \quad (19)$$

$$0 \leq p_i(k+1) \leq p_i^*. \quad (20)$$

Note that KKT multipliers for box constraints are not considered in this paper because if the solution in (19) is out of the range, the solution will be located on the boundary. This two stage energy management will begin once any EV join the system or leave the system. If no EV joins the system, this algorithm will still work once every ten minutes. Notice that this control instant here is the decision making control instant which is different from the control instant of the DC/DC converter, i.e., 1ms.

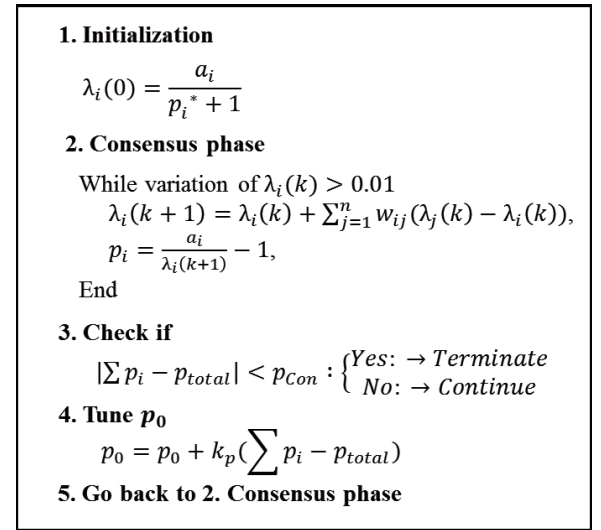


Fig. 3. The consensus algorithm block diagram.

IV. SIMULATION RESULTS AND ANALYSIS

A. Simulation Environment

An urban PV-based charging station is taken as an example to study the energy management strategy. As shown in Table I, most of the model parameters are listed in the table. In order to model the ageing of the EV battery, the capacities of EVs are treated as random values range from 65kWh to 85kWh. The initial SOC of EV batteries depend on the willingness to charge the EV for each EV owner. Thus the SOC of the EV battery follows a normal distribution range from 0.2 to 0.5 (Notice that the SOC working range of an EV is from 0.2 to 0.95.). The number of the coming EVs is assumed to be 100 per day. Since a fast charging technology is applied in the PV-based charging station, the charging rate can be tuning from 0.25 C to 0.5 C. The EVs are assumed to come to the PV-based charging station following Poisson distribution.

TABLE I
SPECIFICATION OF THE PV-BASED CHARGING STATION

Parameters	Value
Rated power of PV Cells	1200 kW
Rated capacity of an EV battery	64 kWh
Number of charging poles	20
Rated capacity of the grid-connecting system	1 MW
Capacity of the storage battery	3000 kWh

B. Single Stage Power Distribution

In the first case study, it shows how the game theory based control and consensus algorithm works during a single stage. The initial time instant when first six EVs are in the charging station is picked out as example, i.e., time 30 (min). Fig. 4 shows the convergence of the λ_i s given a constant p_{total} . When the λ_i s finally stabilize, each charging pole will determine the charging power for EVs according to (19) and (20). As shown in Table II, the power distribution for this six EVs case is given. Moreover, because only six EVs are competing for the available charging power, all EVs are charged at their highest charging powers. According to the power distribution in a single stage, it verifies that the EV with lowest SOC and highest capacitor will be charged at a highest power, i.e., EV₅. The consensus network based learning algorithm is also proved to be efficient.

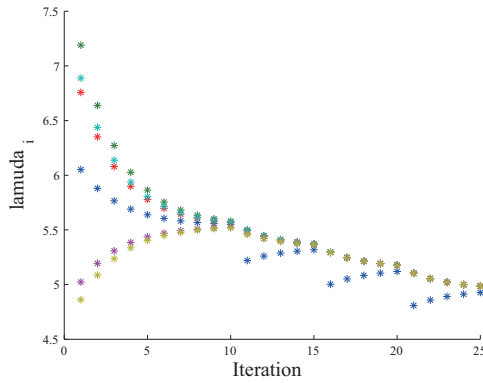


Fig. 4. (a) λ_i map at time 30.

TABLE II
THE POWER DISTRIBUTION AT TIME 30.

time (min)	p_1 (W)	p_2 (W)	p_3 (W)	p_4 (W)	p_5 (W)	p_6 (W)
30	3.85e04	3.52e04	3.84e04	4.00e04	3.90e04	3.59e04
p_b (W)	SOC_1	SOC_2	SOC_3	SOC_4	SOC_5	SOC_6
1.76e05	0.52	0.37	0.52	0.29	0.30	0.32
p_{total} (W)	C_1 (kWh)	C_2 (kWh)	C_3 (kWh)	C_4 (kWh)	C_5 (kWh)	C_6 (kWh)
3.67e05	7.69e04	7.04e04	7.67e04	8.01e04	7.81e04	7.19e04

C. Dynamic Power Distribution

Based on the single stage simulation discussed above, the entire dynamic charging procedure in one day, i.e., 24 hours, is presented here. Notice that due to there is nearly no radiation after 7p.m. (in summer days) in the PV radiation profile, there is no PV power for the rest time.

Since there are 100 EVs in this simulation, the entire charging profile is complex and the dynamic characteristics can not be clearly described. Thus, the first five EVs have been picked up to show the power distribution determined by game theory based energy management. As shown in Fig. 5 (a), it can be concluded that when EVs join or leave the PV-based charging station, the charging powers will change according to their preferences and decentralized rules at each control instant. As shown in Fig. 5 (b), SOC of the first five EVs are shown. It can be concluded that increasing rate of SOC has a direct relationship with the charging power while p_i is determined by both the current SOC and the p_i^* . As shown in Fig. 5 (c), the number of EVs in the PV-based charging station can be less than or equal to 20. It means that this dynamic simulation includes both the fully and non-fully used charging station cases. The SOC of the storage battery pack is shown in Fig. 5 (d). It proves that the utility function of the leader is well satisfied and the sizing of the PV-based charging station is well designed. The total power available to the EVs, i.e., p_{total} is shown in Fig. 5 (e). It can be concluded that with different SOC_b , the p_{total} is changing dynamically according to the pre-defined rule, i.e., the p_{total} is proportional to SOC_b .

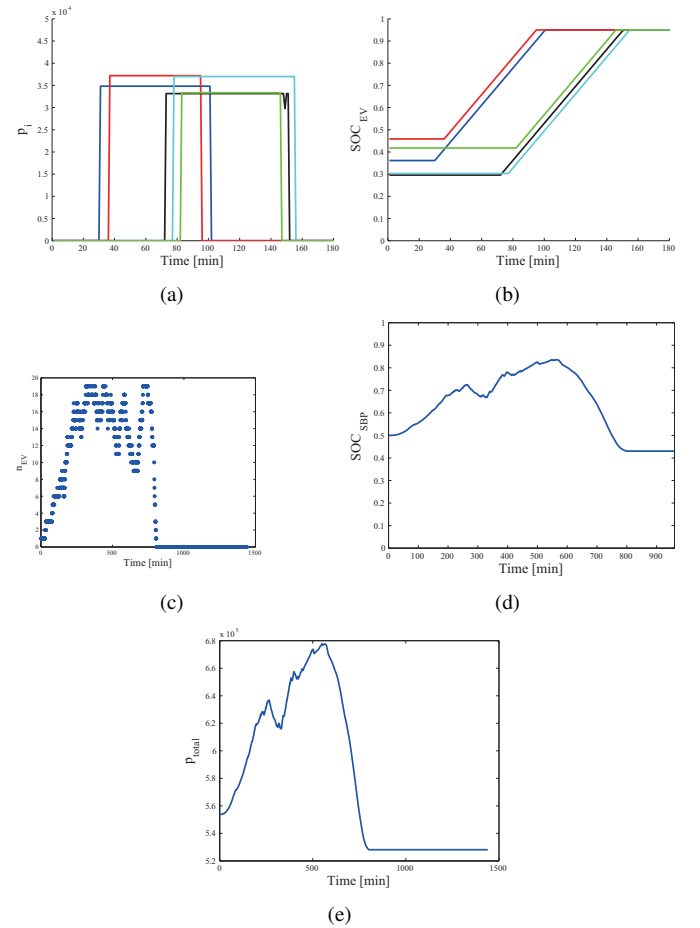


Fig. 5. (a) Picked charging power of five EVs. (b) Picked SOC of five EVs. (c) Number of EVs. (d) SOC of storage battery pack. (e) p_{total} .

D. Scalability Analysis

Note that the proposed strategy is a distributed one. The convergence rates for increasing number of EVs is thus an important concern. Since EVs here update their charging powers through consensus network based learning algorithm, a scalability analysis of the proposed strategy is discussed here. The maximum number of the EVs is designed to be 20 in this paper. However, in the scalability analysis, it varies from 3 to 100 and the iterations for the consensus network based learning algorithm are counted. Note that the stop threshold values for the while loop, i.e., q for $\text{norm}(\lambda_i) < q$, is changed from 0.1 to 0.001. As shown in Fig. 6, no specific pattern for the iterations versus numbers of EVs (i.e., exponential increase) can be caught under different threshold values. As the number of the required computations on each energy system is directly related to the number of the iterations, this result suggests that the proposed strategy is scalable with increasing the size of the system [16].

Note that different choices of q affects the convergence rate and iterations for convergence of the consensus network based learning algorithm. As shown in Fig. 6, a smaller q causes a more iterations for the learning algorithm but a higher accuracy of the learning algorithm. Actually, it is a trade-off relationship between the iterations for convergence and the accuracy of the learning algorithm. This value can be treated as a user defined one.

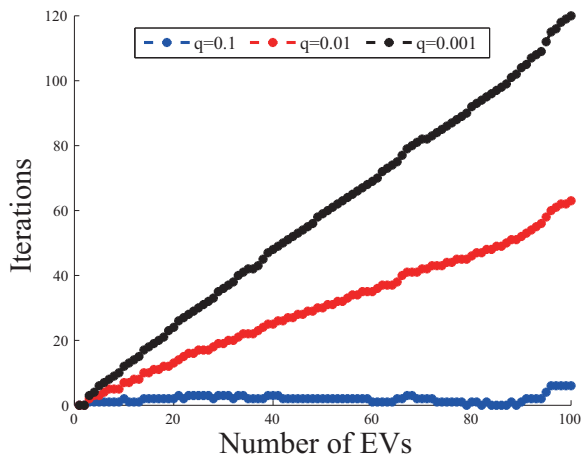


Fig. 6. Iterations under different number of EVs.

V. CONCLUSIONS

This paper proposes a game theory based distributed energy management of a PV-based charging station. The power distribution problem is modeled as a non-cooperative Stackelberg game. It is solved through reaching a generalized stackelberg equilibrium among PV-based charging station and EV owners at each control instant. The objective of the PV-based charging station and EV owners are designed to maintain the SOC of the storage battery tank and to charge with higher power, respectively. In the simulation, single stage case study shows the effectiveness of the power distribution and the consensus

network based learning algorithm. The dynamic case study shows the excellent entire performance of the proposed energy management strategy. The scalability analysis verifies the flexibility of the energy management strategy. The future work of this research could include shiftable load of the charging station.

ACKNOWLEDGMENT

This work was supported by National Key R&D Program of China-Comprehensive Demonstration Project of Smart Grid Supporting Low-carbon Winter Olympics (2016YFB0900500).

REFERENCES

- [1] D. B. Richardson, "Electric vehicles and the electric grid: A review of modeling approaches, impacts, and renewable energy integration," *Renewable and Sustainable Energy Reviews*, vol. 19, pp. 247–254, 2013.
- [2] K. Clement-Nyns, E. Haesen, and J. Driesen, "The impact of vehicle-to-grid on the distribution grid," *Electric Power Systems Research*, vol. 81, no. 1, pp. 185–192, 2011.
- [3] N. Liu, Q. Chen, X. Lu, J. Liu, and J. Zhang, "A charging strategy for pv-based battery switch stations considering service availability and self-consumption of pv energy," *IEEE Transactions on Industrial Electronics*, vol. 62, no. 8, pp. 4878–4889, 2015.
- [4] N. Liu, Q. Chen, J. Liu, X. Lu, P. Li, J. Lei, and J. Zhang, "A heuristic operation strategy for commercial building microgrids containing evs and pv system," *IEEE Transactions on Industrial Electronics*, vol. 62, no. 4, pp. 2560–2570, 2015.
- [5] N. Liu, F. Zou, L. Wang, C. Wang, Z. Chen, and Q. Chen, "Online energy management of pv-assisted charging station under time-of-use pricing," *Electric Power Systems Research*, vol. 137, pp. 76–85, 2016.
- [6] A. Mohamed, V. Salehi, T. Ma, and O. Mohammed, "Real-time energy management algorithm for plug-in hybrid electric vehicle charging parks involving sustainable energy," *IEEE Transactions on Sustainable Energy*, vol. 5, no. 2, pp. 577–586, 2014.
- [7] L. Gan, U. Topcu, and S. H. Low, "Optimal decentralized protocol for electric vehicle charging," *IEEE Transactions on Power Systems*, vol. 28, no. 2, pp. 940–951, 2013.
- [8] Z. Ma, D. S. Callaway, and I. A. Hiskens, "Decentralized charging control of large populations of plug-in electric vehicles," *IEEE Transactions on Control Systems Technology*, vol. 21, no. 1, pp. 67–78, 2013.
- [9] J. P. Torreglosa, P. García-Triviño, L. M. Fernández-Ramírez, and F. J. J. rado, "Decentralized energy management strategy based on predictive controllers for a medium voltage direct current photovoltaic electric vehicle charging station," *Energy Conversion and Management*, vol. 108, pp. 1–13, 2016.
- [10] N. Rahbari-Asr and M.-Y. Chow, "Cooperative distributed demand management for community charging of phev/pevs based on kkt conditions and consensus networks," *IEEE Transactions on Industrial Informatics*, vol. 10, no. 3, pp. 1907–1916, 2014.
- [11] R. B. Myerson, *Game theory*. Harvard university press, 2013.
- [12] H. Yin, C. Zhao, M. Li, C. Ma, and M. Y. Chow, "A game theory approach to energy management of an engine generator/battery/ultracapacitor hybrid energy system," *IEEE Transactions on Industrial Electronics*, vol. 63, no. 7, pp. 4266–4277, 2016.
- [13] W. Tushar, W. Saad, H. Poor, and D. Smith, "Economics of electric vehicle charging: A game theoretic approach," *IEEE Trans. Smart Grid*, vol. 3, no. 4, pp. 1767–1778, Dec 2012.
- [14] L. Ma, N. Liu, J. Zhang, W. Tushar, and C. Yuen, "Energy management for joint operation of chp and pv prosumers inside a grid-connected microgrid: A game theoretic approach," *IEEE Trans. Ind. Informat.*, to be published.
- [15] A. A. Kulkarni and U. V. Shanbhag, "On the variational equilibrium as a refinement of the generalized Nash equilibrium," *Automatica*, vol. 48, no. 1, pp. 45–55, 2012.
- [16] N. Rahbari-Asr, M.-Y. Chow, J. Chen, and R. Deng, "Distributed real-time pricing control for large scale unidirectional v2g with multiple energy suppliers," *IEEE Trans. Ind. Informat.*, to be published.