Assignment #3

1. Use the Matrix Inversion Lemma (Woodbury's identity) to derive a recursion for the inverse of the estimated input autocorrelation matrix $[\widehat{\mathbf{R}}(k)]^{-1}$ based on the sample-average recursion
$$\widehat{\mathbf{R}}(k) = \frac{(k-1)\widehat{\mathbf{R}}(k-1) + \mathbf{r}_k\mathbf{r}_k^H}{k}$$
where $\mathbf{r}_k$, $k = 1, 2, \ldots$, are the array input vectors.

2. Show that $\alpha_1 = \frac{\alpha_1'}{\mathrm{SINR}^{\mathrm{opt}}(1-\alpha_1')+1}$ where $\mathrm{SINR}^{\mathrm{opt}}$ is the maximum attainable SINR by the antenna array and α_1, α_1' are as defined in the lectures.

3. Design a simulation to estimate $\mathrm{E}\{\alpha_1\}$, $\mathrm{E}\{\alpha_2\}$, and $\mathrm{E}\{\alpha_3\}$ (α_1, α_2, α_3 as defined in the lectures). Consider BPSK transmissions of one user of interest and three interferers. Assume $M = 10$ and arbitrary in $(-90^o, 90^o)$ but fixed angles of arrival.

    (a) For some fixed SNR values for the user signals, plot your estimated means of α_1, α_2, and α_3 as a function of N from $N = 10$ to 200.

    (b) Keep the SNRs of the interferences as in Part (a) and plot the estimated means of α_1, α_2, and α_3 as a function of the SNR of the user of interest for $N = 10, 50, 100, 200$.

    Comment on your findings.