# Chapter 1

# Miscellaneous

## The guiding center coordinates

are conserved, namely they commute with the Hamiltonian, and we have

$$[X, Y] = [x + l^2\pi_y, y - l^2\pi_x] = -il^2$$

Notice that they travel on equipotential lines:

$$\begin{aligned} i\dot{X}_1 &= [X_1, V] = -il^2\partial_2 V \\ i\dot{X}_2 &= [X_2, V] = +il^2\partial_1 V \end{aligned}$$

## Spin-waves and magnons

The Schwinger-boson representation

$$\begin{aligned} S_+ &= a^\dagger b \\ S_- &= b^\dagger a \\ 2S_z &= a^\dagger a - b^\dagger b \end{aligned}$$

gives us the Holstein-Primakoff transformation if we use the constraint

$$a^\dagger a + b^\dagger b = 2S$$

The spin-wave is obtained by, e.g., expanding around $S_z = -S$, namely setting

$$b = b^\dagger = \sqrt{2S - a^\dagger a} \approx \sqrt{2S}$$

## The Bogoliubov transformation

In order to diagonalize a Hamiltonian of the form

$$\Psi_{i\dagger} A_{ij} \Psi_j$$

while keeping the commutation relations, we want

$$U^\dagger A U = \Lambda$$

where $\Lambda$ is a diagonal matrix, as well as

$$C_{ij} = \left[\Psi_i, \Psi_{j\dagger}\right] = \left[U_{ik}\Phi_k, U_{jl*}\Phi_{l\dagger}\right] = U_{ik}U_{jl*}C_{kl} \Rightarrow C = UCU^\dagger$$

where $C$ is, by assumption, a diagonal matrix. Therefore we have

$$CAU = UCU^\dagger AU = UC\Lambda$$

which means that $U$ diagonalizes the matrix $CA$, with eigenvalues $C\Lambda$.

## The three-plus-one decomposition

In Minkowski spacetime,

$$V^a = -V^b Z_b Z^a + W^a$$

such that $W^a$ is a spatial vector

$$\begin{aligned} W^a Z_a &= V^a Z_a + V^b Z_b Z^a Z_a \\ &= V^a Z_a - V^b Z_b \\ &= 0 \end{aligned}$$

Notice also that $Z^b \nabla_b Z^a$ is a spatial vector, since $Z^b \nabla_b Z^a Z_a = 0$.