

Chapter 9 - Hypothesis Testing for Single Populations

Contents

1. Hypothesis test for Population Mean	1
1.1. Requirements for applying method	1
1.2. Test Statistic	1
1.3. p -value Approach	2
1.4. Critical Value Approach	3

1. Hypothesis test for Population Mean

Null Hypothesis:

$H_0 : \mu = \mu_0$, where μ_0 is the hypothesized value (such as $H_0 : \mu = 10$).

Alternative hypothesis:

$H_a : \mu > \mu_0$ (one-sided), such as $H_a : \mu > 10$.

$H_a : \mu < \mu_0$ (one-sided), such as $H_a : \mu < 10$.

$H_a : \mu \neq \mu_0$ (two-sided), such as $H_a : \mu \neq 10$.

1.1. Requirements for applying method

- The sample is a simple random sample.
- The population is approximately normal distributed or $n \geq 30$.

1.2. Test Statistic

Definition

The test statistic is a calculated value using sample data used in making a decision about the null hypothesis.

Found by converting sample statistic (e.g. \bar{x} or \hat{p}) to find a standardized score (e.g. z or t) with the assumption that the null hypothesis is true.

Note:

For a hypothesis test about a population mean, the test statistic is calculated as:

- with a known σ

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

- with an unknown σ

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

1.2.1. Understanding the Test Statistic

- Tells us how far away sample statistic (\bar{x}) is

1.3. p -value Approach

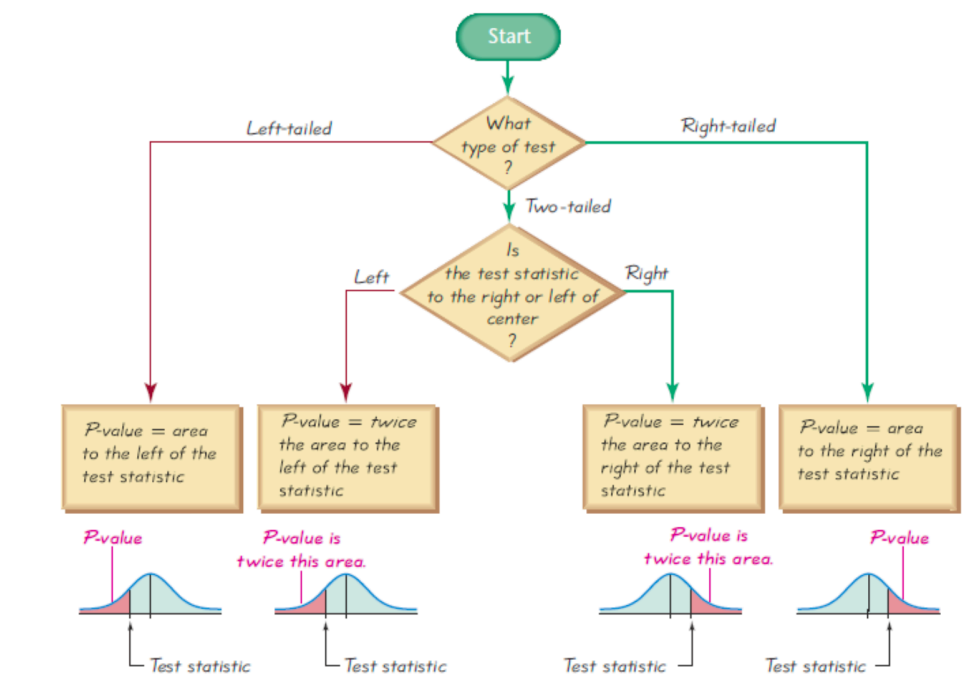
Definition

The p -value (or probability value) is the probability of getting the observed, or more extreme value of the test statistic from the sample data, assuming that the null hypothesis is true.

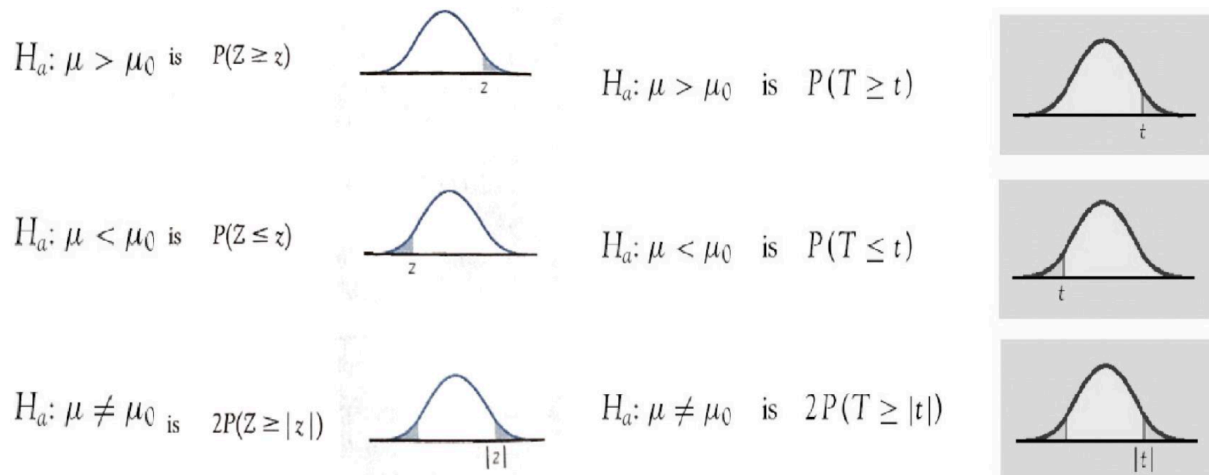
Notes:

- Small p -values are evidence against H_0 because they say that the observed result would be unlikely to occur if H_0 is true.
- Large p values fail to give evidence against H_0 .
- The definition of “extreme” depends on how the hypothesis is being tested.

1.3.1. Finding p -value



1.3.2. Finding p -value for Hypothesis Test about a μ



1.4. Critical Value Approach

Definition - Rejection Region

The rejection region is the set of all values of the test statistic that cause us to reject the null hypothesis.

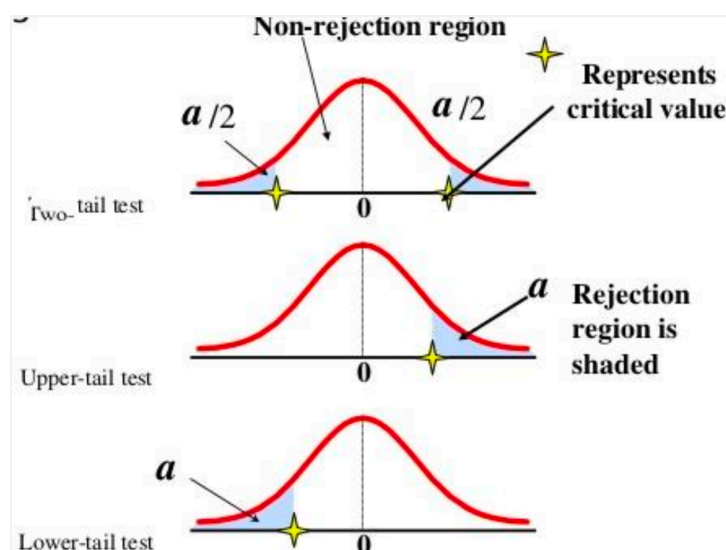
Definition - Critical Value

A critical value is any value that separates the rejection region (where we reject the null hypothesis) and non-rejection region. The critical values depend on the nature of the alternative hypothesis, the sampling distribution that applies, and the significance level of α .

Note:

The critical value approach requires first to determine a critical value for the test statistic in order to reach a statistical conclusion.

1.4.1. Rejection Region



1.4.2. Example:

Approximate the p -value and critical value for the following hypothesis and their corresponding test statistics.

a. $H_0 : \mu = 0$ $H_a : \mu > 0$ for sample of $n = 25$ and $t = 1.75$ with $\alpha = 0.05$

- One tailed (right)

 p -value Approach

using t-table:

1.75 is between (1.711, 2.064)

$\therefore p$ -value is between (0.025, 0.05)

since p -value $< \alpha = 0.05$

\therefore reject H_0