Chapter 9 - Hypothesis Testing for Single Populations

Contents

1. Hypothesis test for Population Mean

Null Hypothesis:

$H_0 : \mu = \mu_0$, where μ_0 is the hypothesized value (such as $H_0 : \mu = 10$).

Alternative hypothesis:

$H_a : \mu > \mu_0$ (one-sided), such as $H_a : \mu > 10$.
$H_a : \mu < \mu_0$ (one-sided), such as $H_a : \mu < 10$.
$H_a : \mu \neq \mu_0$ (two-sided), such as $H_a : \mu \neq 10$.

1.1. Requirements for applying method

- The sample is a simple random sample.
- The population is approximately normal distributed or $n \geq 30$.

1.2. Test Statistic

Definition
The test statistic is a calculated value using sample data used in making a decision about the null hypothesis.
Found by converting sample statistic (e.g. $\bar{x}$ or $\hat{p}$) to find a standardized score (e.g. z or t) with the assumption that the null hypothesis is true.

Note:
For a hypothesis test about a population mean, the test statistic is calculated as:

- with a known σ

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$$

- with an unknown σ

$$t = \frac{\bar{x} - \mu}{s / \sqrt{n}}$$

1.2.1. Understanding the Test Statistic

- Tells us how far away sample statistic ($\bar{x}$) is

1.3. p-value Approach

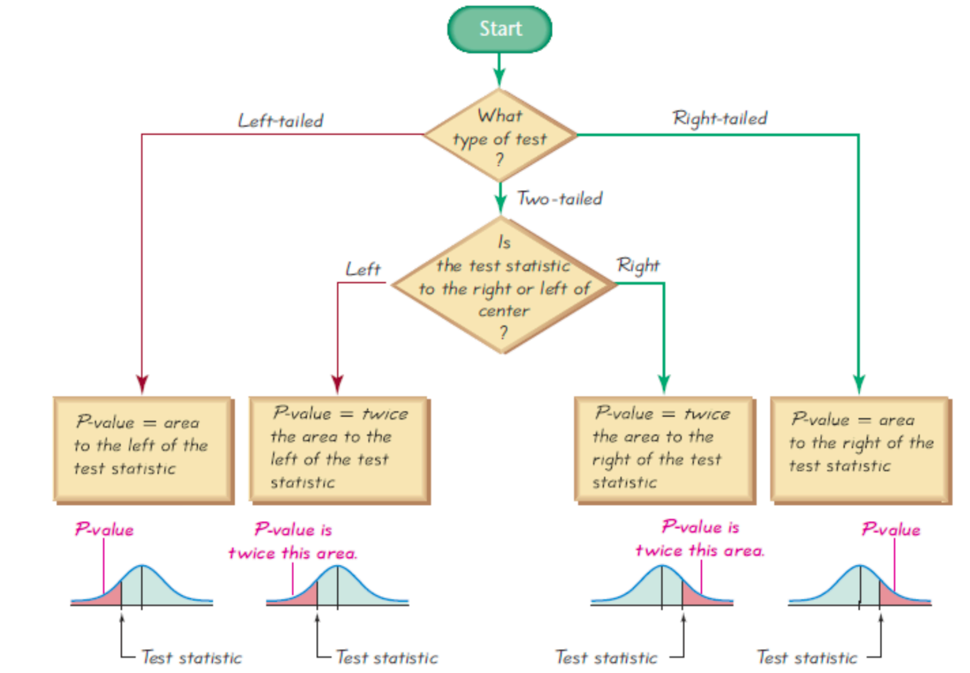
Definition

The p-value (or probability value) is the probability of getting the observed, or more extreme value of the test statisic from the sample data, assuming that the null hypothesis is true.

Notes:

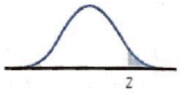
- Small p-values are evidence against H_0 because they say that the observed result would be unlikely to occur if H_0 is true.
- Large p values fail to give evidence against H_0.
- The definition of "extreme" depends on how the hypothesis is being tested.

1.3.1. Finding p-value

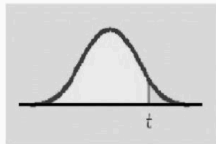


1.3.2. Finding p-value for Hypothesis Test about a μ

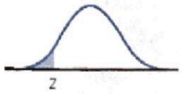
H_a: $\mu > \mu_0$ is $P(Z \geq z)$



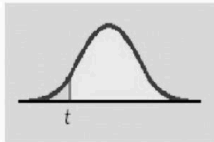
H_a: $\mu > \mu_0$ is $P(T \geq t)$



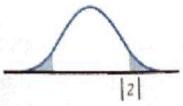
H_a: $\mu < \mu_0$ is $P(Z \leq z)$



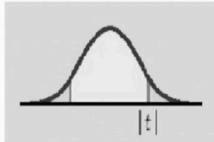
H_a: $\mu < \mu_0$ is $P(T \leq t)$



H_a: $\mu \neq \mu_0$ is $2P(Z \geq |z|)$



H_a: $\mu \neq \mu_0$ is $2P(T \geq |t|)$



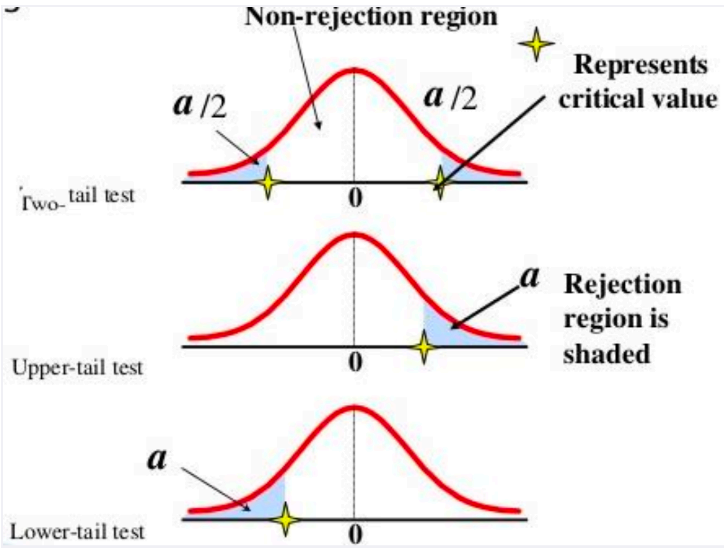
1.4. Critical Value Approach

Definition - Rejection Region
The rejection region is the set of all values of the test statistic that cause us to reject the null hypothesis.

Definition - Critical Value
A critical value is any value that separates the rejection region (where we reject the null hypothesis) and non-rejection region. The critical values depend on the nature of the alternative hypothesis, the sampling distribution that applies, and the significance level of α.

Note:
The critical value approach requires first to determine a critical value for the test statistic in order to reach a statistical conclusion.

1.4.1. Rejection Region



1.4.2. Example:

Approximate the p-value and critical value for the following hypothesis and their corresponding test statisics.
a. $H_0 : \mu = 0 \qquad H_a : \mu > 0$ for sample of $n = 25$ and $t = 1.75$ with $\alpha = 0.05$

- One tailed (right)

using the p-value approach
using t-table:
1.75 is between (1.711, 2.064)
$\therefore$ p-value is between (0.025, 0.05)
since p-value $< \alpha = 0.05$
$\therefore$ reject H_0

1.4.3. Second Example:

Approximate the p-value and critical value for the following hypothesis and their corresponding test statisics.
b. $h_0 : \mu = 20; \qquad H_a : \mu < 20$ for sample of $n = 18$, $t = -2.14$, and $\alpha = 0.05$

- One tailed (left)

using the p-value approach
using t-table:
2.14 is between (2.11, 2.567)
$\therefore$ p-value is between (0.01, 0.025)
$\therefore$ p-value is $< \alpha = 0.05$
$\therefore$ reject H_0

using the critical value approach
given $\alpha = 0.5,\ \mathrm{df} = 17$, using t-table
1.74, but that's for the right side.
since distribution is symmetric, we can flip the value.
critical value $= -1.74$
comparing the critical value and the t-statistic:
$-2.14 < -1.74$; the t-statistic is in the rejection region
$\therefore$ reject H_0

One more approach on next page.

with z-statistic instead of t-statistic (σ known case)

- One tailed (left)
- using the p-value approach

z = -2.14
using z-table:
-2.14 area = 0.0162
p-value = 0.0162
p-value $< \alpha$
$\therefore$ reject H_0

- using critical value approach

find value on z table closest to 0.5 (α)
≈ -1.645
$-2.14 < -1.645$
$\therefore$ reject H_0

1.4.4. Third Example

Approximate the p-value and critical value for the following hypothesis and their corresponding test statisics.
b. $h_0 : \mu = 120; \qquad H_a : \mu \neq 120$ for sample of $n = 81$, $t = 1.18$, and $\alpha = 0.05$

- Two-tailed test

using the p-value approach
using t-table:
1.18 is between (0.846, 1.292)
$\therefore$ p-value/2 is between (0.1, 0.2)
$\therefore$ p-value is between (0.2, 0.4)
$\therefore$ p-value $> \alpha = 0.05$
$\therefore$ Failed to reject H_0

using the critical value approach
using t-table:
since two tailed test, split α in half
df = 80
critical value = 1.99
t = 1.18 is in the non-rejection region (t < critical value)
$\therefore$ Failed to reject H_0