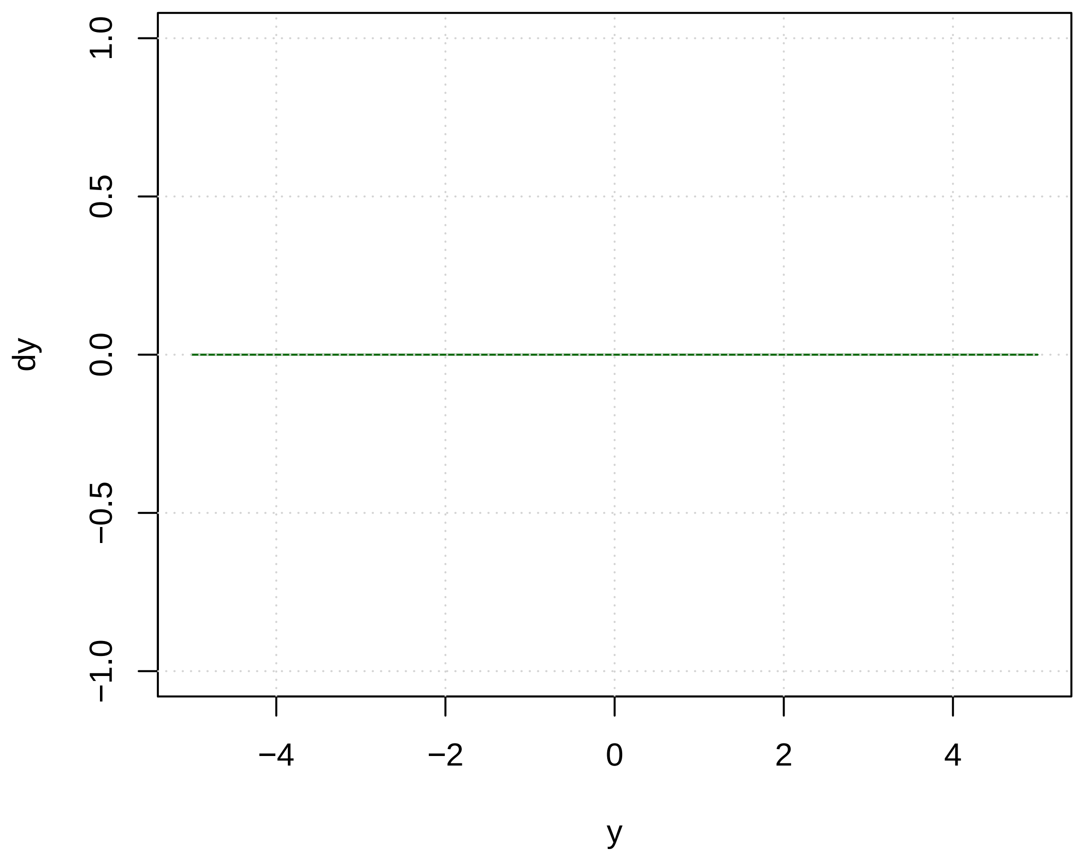
Basic Properties of fixed points, stability, and existence and uniqueness of solutions

A particle travels on the half-line $x > 0$ with a velocity given by $\dot{x} = -x^c$.

a. Find all values of c such that the origin is a fixed point:

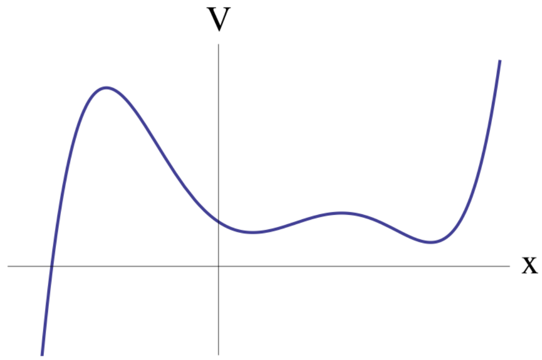


b. Now assume that c is chosen such that x = 0 is stable. Can the particle ever reach the origin in finite time? Specifically, how long does it take for the particle to travel from x = 1 to x = 0, as a function of c?

Sketching Phase portraits for Vector fields and Gradient Systems

Gradient Systems

Consider the dynamical system $\dot{x} = -\frac{dV}{dx}$ with V(x) plotted below.

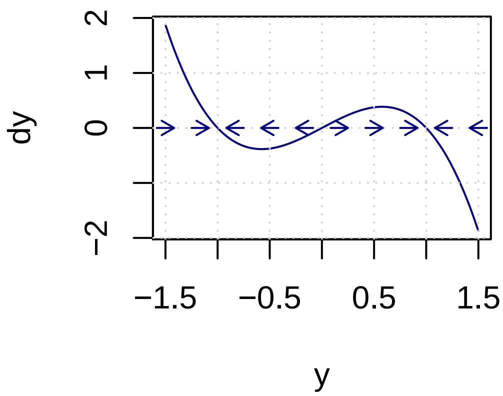


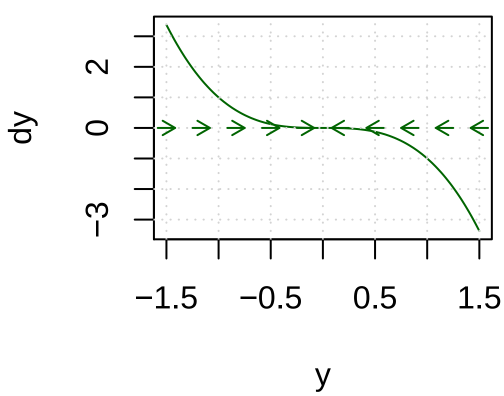
Consider the dynamical system $\dot{x} = ax - x^3$ where a is a real number that can be positve, negative, or zero.

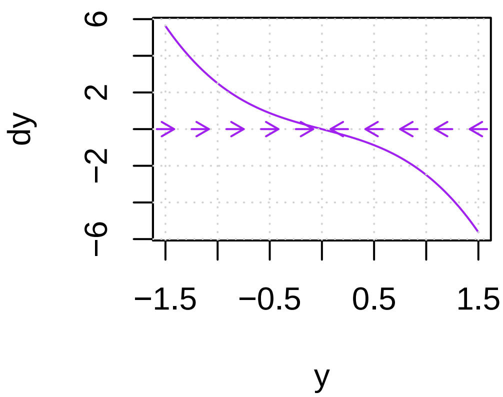
a. For all 3 cases, find the fixed point, classify stability, and sketch the graph of x(t) for different initial conditions:

`Case 1:`

$\dot{x} = ax - x^3$







`Case 1:`

$\dot{x} = ax - x^3$

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$\dot{x} = ax - x^3$

b. For all 3 cases, calculate and plot the potential function V(x)

Relationship between solution curves, vector fields, and phase portraits

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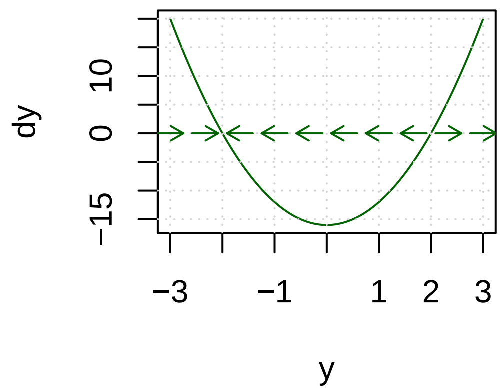
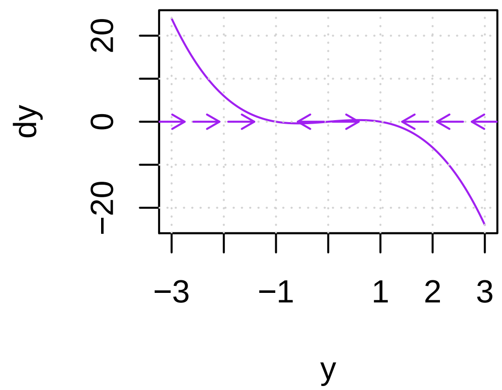
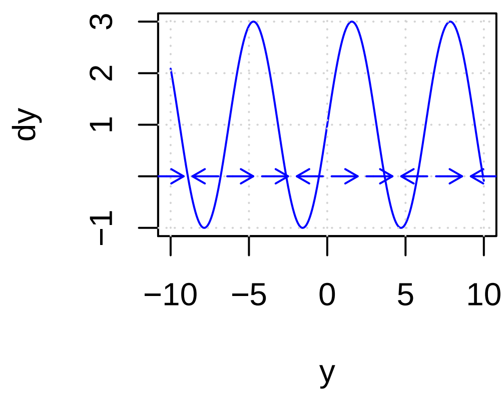
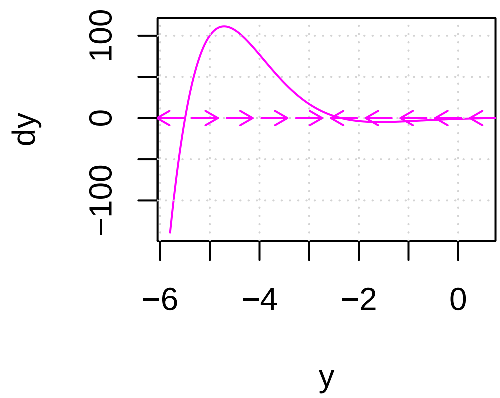
For the following equations, sketch the vector fields on the real line; if possible, find all fixed points, classify their stability, and sketch the graph of x(t) for different initial conditions:

a. $\dot{x} = 4x^2 - 16$
b. $\dot{x} = x - x^3$
c. $\dot{x} = 1 + 2\sin x$
d. $\dot{x} = e^{-x}(\sin x - \cos x)$

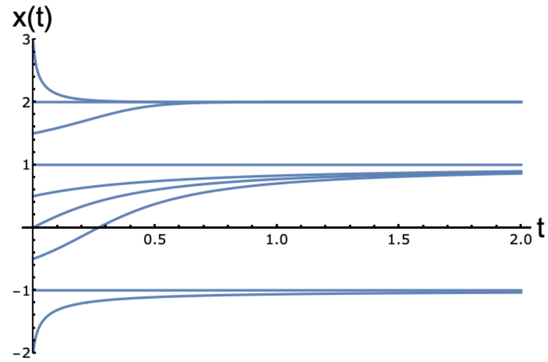








In the figure below several solution curves x(t) to the system $\dot{x} = f(x)$ are illustrated. Which of the following phase portraits are consistent with these solution curves.



Consider the following system where $r, a \in R$:

$$\dot{x} = rx + ax^2 - x^3$$

a. For each a, there is a bifurcation diagram in the parameter r. As a varies, these bifurcations can undergo qualitative changes. Sketch all of the qualitatively different bifurcation diagrams that can be obtained by varying a.

b. Summarize your results by plotting a "phase diagram" in the parameters r and a. That is, plot the regions in the (r,a) plane that correspond to qualitatively different class of vector fields. Bifurcations occur on the boundaries of these regions; identify the types of bifurcations that occur.