Ruprecht-Karls University of Heidelberg
Institut für Theoretische Astrophysik
Prof. Dr. Ralf S. Klessen
Tutor: Marcelo Barazza
Exercise 4

Teammates:
H.Lüdemann, F.Walberg
D.Wolf

1 Tree code for gravity calculation

In this exercise we should compare the performance of a tree algorithm with the exact calculation of the gravitational forces. All the particles in the system are randomly generated and have the mass $m = 1$. For the gravitational constant we also adopt $G = 1$.
The gravitational potential used for the force calculation is Plummer-softened with the softening length $\epsilon = 0.001$. This leads to

$$\Phi(\vec{r}) = -\frac{m}{(r^2 + \epsilon^2)^{\frac{1}{2}}}. \tag{1}$$

The force on this particle resulting from another particle of the mass M is the given by

$$\vec{F} = M \cdot \vec{\nabla}\Phi(\vec{r}) = \frac{mM\vec{r}}{(\vec{r}^2 + \epsilon^2)^{\frac{3}{2}}}. \tag{2}$$

To see how correct the forces of the tree algorithm are we compare them to the exact forces by calculating the relative error for every particle

$$\eta = \frac{|\vec{a}_{tree} - \vec{a}_{exact}|}{|\vec{a}_{exact}|} \tag{3}$$

and then determining the mean of all these values. The results of the simulations with different parameters are listed in table 1. Because of very long execution times the exact calculation was not run for 40000 particles.

Table 1: Results of the simulation for different parameter settings.

Particles	Time exact/s	Angle	Time tree/s	Node interactions	Mean error
5000	1.037e+03	0.2	1.744e+02	4010	3.441e-04
		0.4	4.353e+01	981	2.442e-03
		0.8	9.078	203	1.461e-02
10000	3.584e+03	0.2	4.39e+02	5163	3.832e-04
		0.4	9.728+01	1160	2.726e-03
		0.8	1.933e+01	228	1.361e-02
20000	1.292e+04	0.2	1.022e+03	6230	3.578e-04
		0.4	2.194e+02	1315	2.363e-03
		0.8	4.179e+01	214	1.125e-02
40000	–	0.2	2.714e+03	7513	–
		0.4	5.631e+02	1497	–
		0.8	1.045e+02	273	–

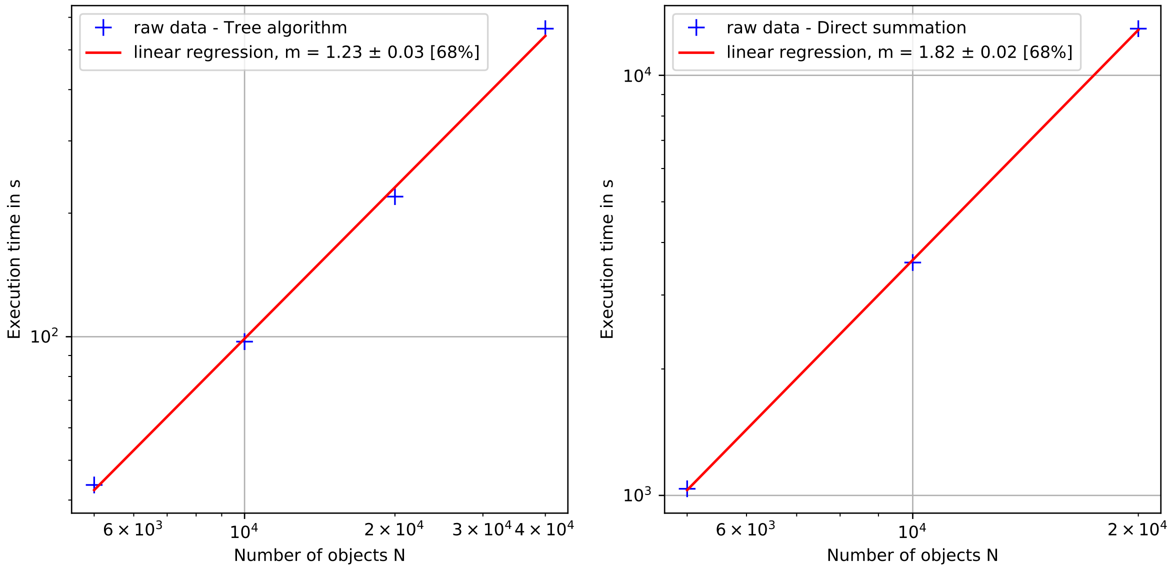

Figure 1: Execution time with linear regression for the two methods.

For a better illustration the execution times of both methods are shown in the figures 1. Using logarithmic axis we can fit a regression line to the data and get information about the complexity of the calculations. For exact calculation we get a slope of $m_{exact} = 1.82$ which is close to the expected value of 2 for an algorithm of complexity $O(n^2)$. The tree algorithm gives a slope of $m_{tree} = 1.23$ which is as expected for a complexity of $O(nlog(n))$. It is to be noted that these values are not very exact because only very few data points were used.
With these values we can estimate the amount of time needed for the simulation of 10^{10} particles. The exact method would take about 10^{19} s and the tree method about 10^{12} s.

As an additional illustration we have also plotted the error distribution in figure 2 and 3. As expected the errors decreased when the angle is decreased.

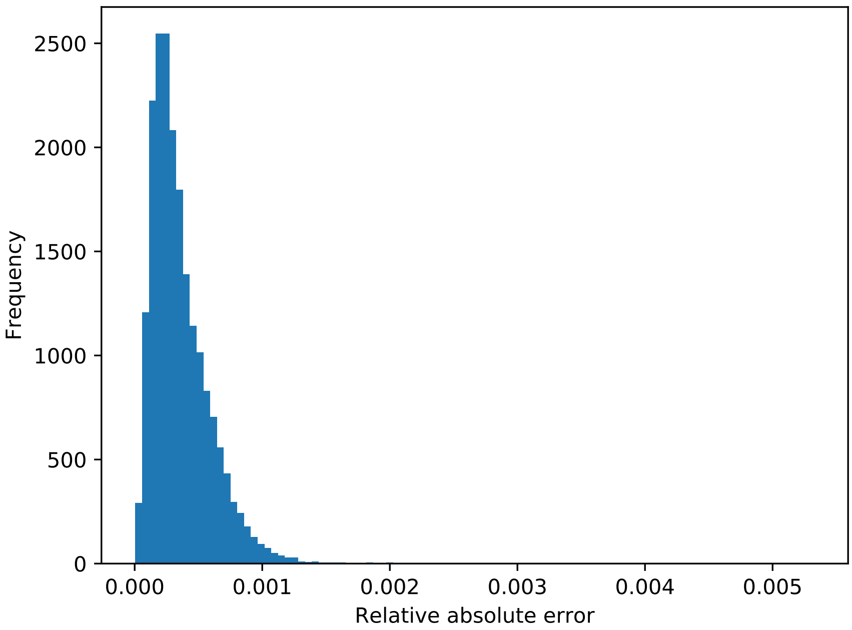

Figure 2: Distribution of the absolute relative errors for an angle of 0.2.

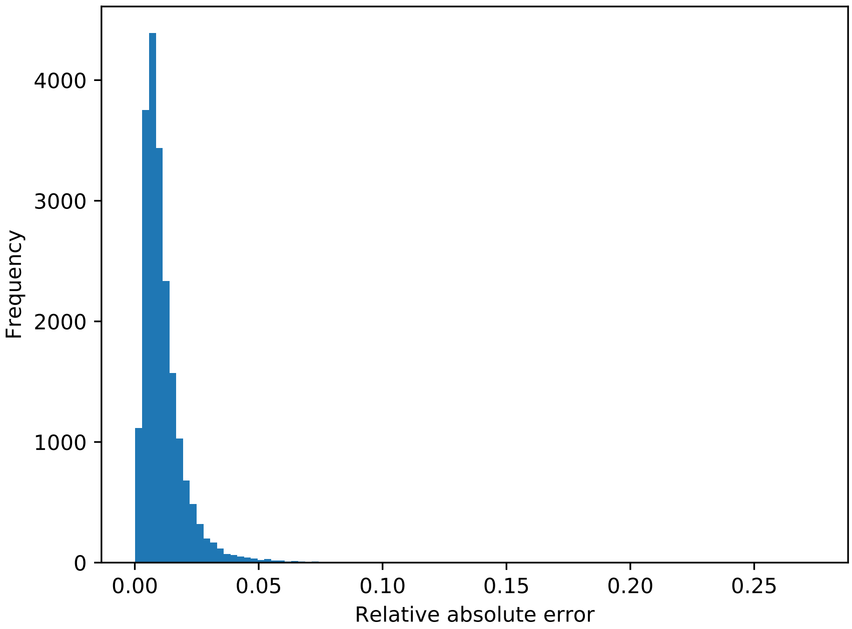

Figure 3: Distribution of the absolute relative errors for an angle of 0.8.