

A) void f1(int n){

int i=2;

while(i < n){

// O(1)

i = i * i;

}

}

i increases by i in each iteration:

i → ii → iii → ... → n

2 → 2(2) → 2(2)(2) → ... → n

1 2 3 ... x

$$2^x = n$$

$$\log_2(2^x) = \log_2(n)$$

$$x = \log_2(n) \rightarrow \boxed{\Theta(\log_2(n))}$$

B) void f2(int n){

for(int i=1; i <= n; i++){ // runs n times

if((i % (int)sqrt(n)) == 0){

for(int k=0; k < pow(i,3); k++){

// O(1)

}

}

}

ex) n=9

for(i=1 → i <= n){ // runs 9 times

if(i % √9 == 0) ✓ for i=3, 6, 9

{

↑

(triggered 3 times)

ex) n=25

for(i=1 → i <= n){ // runs 25 times

if(i % √25 == 0) ✓ for i=5, 10, 15, 20, 25

{

↑

(triggered 5 times)

of 5s in 3 5 0 3 is 3
me
unaided

Loop 1: size n

Loop 2: size \sqrt{n}

size of n	# of iterations (times $i \% \sqrt{n} = 0$)
1	1 (for $i=1$)
9	3 (for $i=3, 6, 9$)
25	5 (for $i=5, 10, 15, 20, 25$)
36	6 (for $i=6, 12, 18, 24, 30, 36$)

relationship is $\sqrt{n} = \# \text{ of iterations!}$

Loop 3:

each loop runs i^3 times

↑ greatest value of i is n , which would run n^3 times

i	# of iterations
3	$3^3 = 27$
6	$6^3 = 216$
9	$9^3 = 729$

(for size)
 $n=9$

of loops = \sqrt{n}

size of each loop = i^3 w/ $O(n^3)$

1st for loop
 $\theta(n + \sqrt{n} \cdot n^3)$

$\theta(n + n^{7/2})$

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c) for( int i=1; i <= n; i++) { // runs n times
    for( int k=1; k <= n; k++) { // runs n times
        if (A[i][k] == i) { // depends on data (worst case: runs one time!)
            for( int m=1; m <= n; m=m+m) { // runs n times!
                // O(1)
            }
        }
    }
}

```

$m \rightarrow m+m \rightarrow m+m+m \rightarrow \dots n$

$m \rightarrow 2m \rightarrow 3m \rightarrow \dots n$

$1 \rightarrow 2 \rightarrow 3 \rightarrow 4 \dots n$

$1 \quad 2 \quad 3 \quad 4 \dots X \text{ iterations}$

$X = n$

$\theta(n \cdot n + n)$

$\theta(n^2 + n)$


```

D) int f(int n){
    int* a = new int[10];
    int size = 10;
    for (int i = 0; i < n; i++) { // runs n times
        if (i == size) { // triggered by i = 10, 23, 49, 73
            int newsize = 3 * size / 2;
            int* b = new int[newsize];
            for (int j = 0; j < size; j++) b[j] = a[j];
            delete[] a;
            a = b;
            size = newsize;
        }
        a[i] = i * i;
    }
}

```

$\theta(K \cdot i)$
 runs i times
 # of times i is triggered
 $\theta(n + K(i))$
 # of times the 2nd for loop runs

ex) $n = 16$ (n must be greater than 10 in order for the if statement to be triggered)

```

for (i = 0; i < n) // runs 16 times
    if (i == size) // triggered by i = 10
        newsize = 30 / 2 = 15
        for (j = 0; j < size) // runs 10 times
            size = 15

```

```

    if (i == size) // triggered by i = 15
        newsize = 45 / 2 = 22
        for (j = 0; j < size) // runs 15 times
            size = 22

```

size of n	# of times if statement triggers	variable size takes on values:
i: 11-15	1	10, 15, 22, 33, 49, 73...
i: 16-33	2	
i: 34-49	3	
i: 50-73	4	