Linear Regression

GENERAL ASSEMBLY

Kinds of ML

- **Supervised learning** - Given X, can we predict Y?
- **Unsupervised learning** - What patterns can we find in X? There is no Y.
- **Reinforcement learning** - How can a virtual agent maximize a reward.

Supervised learning

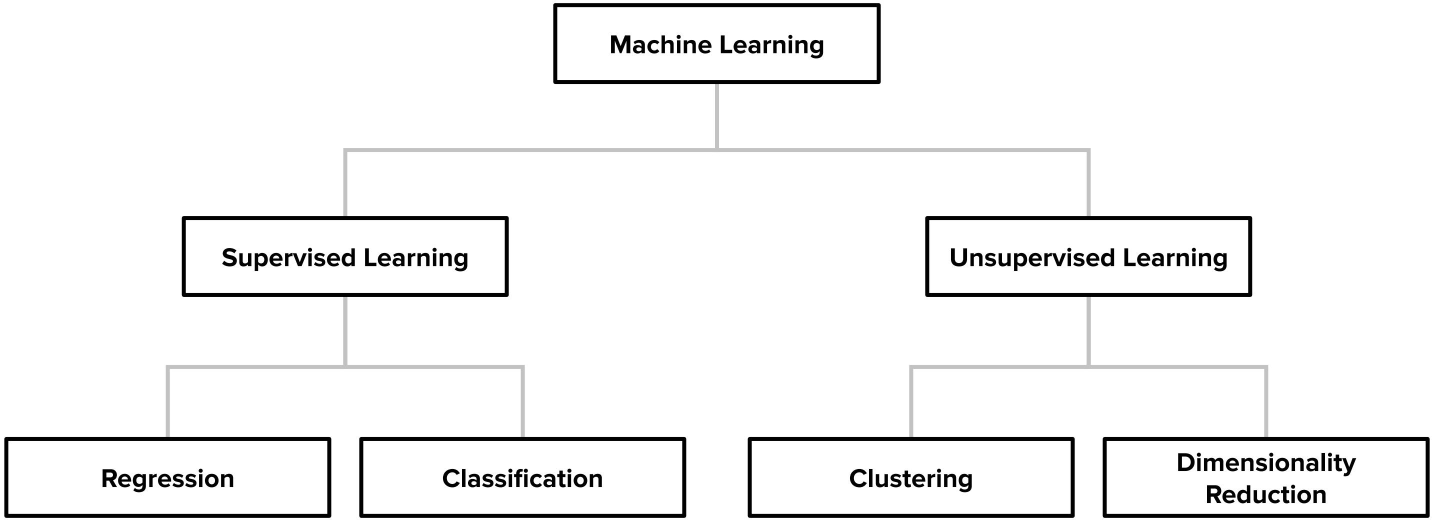
- Classification
- Regression

Two Kinds of Supervised Learning

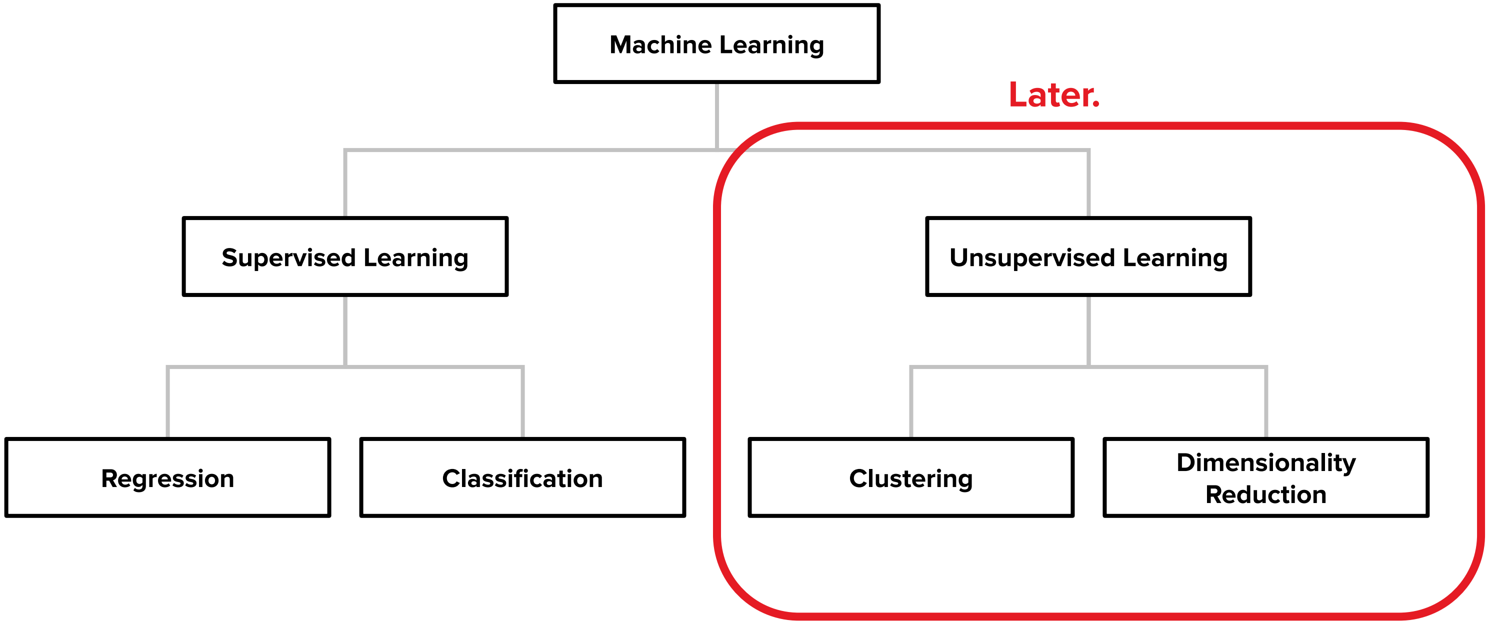
Regression: *y*-variable is a continuous number

- *Crop yields*
- *House sale price*
- *Other examples?*

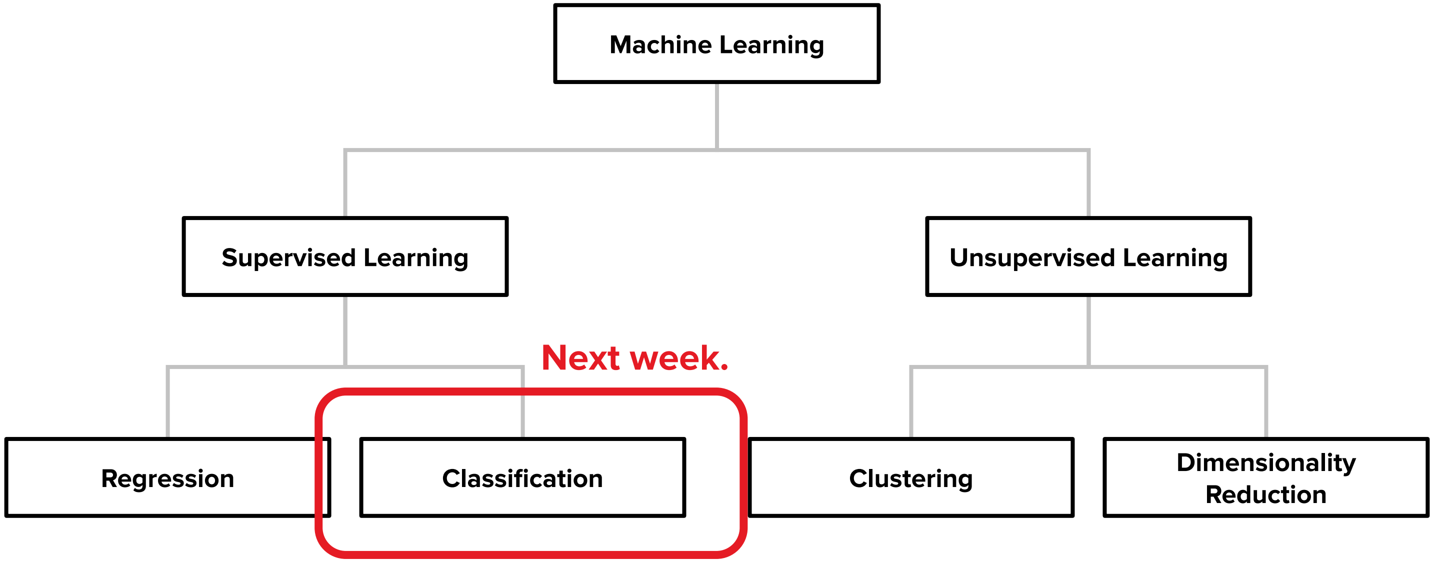
Roadmap of ML



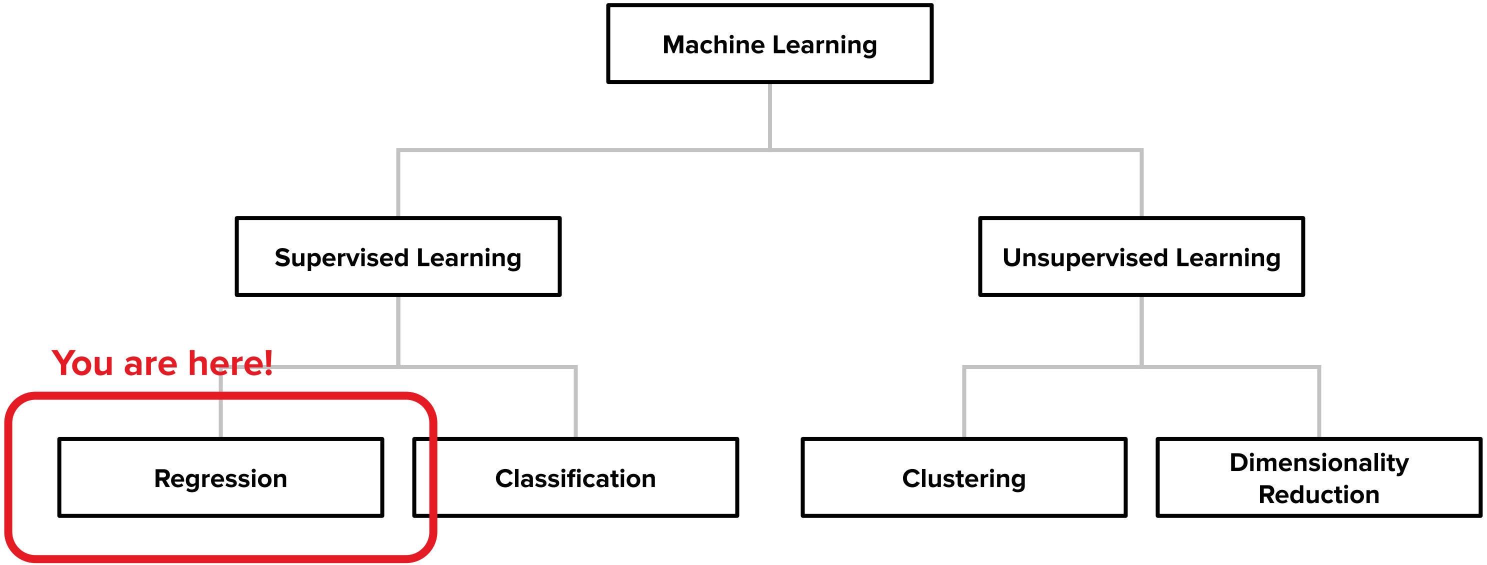
Roadmap of ML



Roadmap of ML

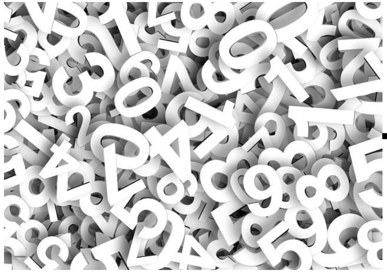


Roadmap of ML



Supervised Learning Transparency

Data



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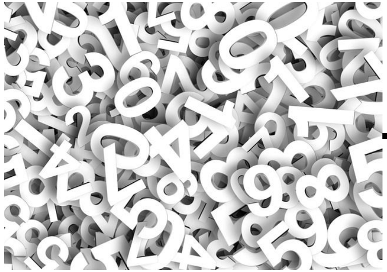
Profitable Results!

Supervised Learning Transparency

Data



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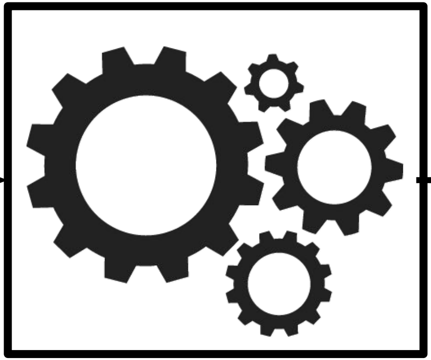


Profitable Results!

Let’s do it!

Linear Regression

Supervised, white-box, regression

Ordinary least squares linear regression (**OLS**)

Predict response variable (*y*) from at least one independent variable (*x*).

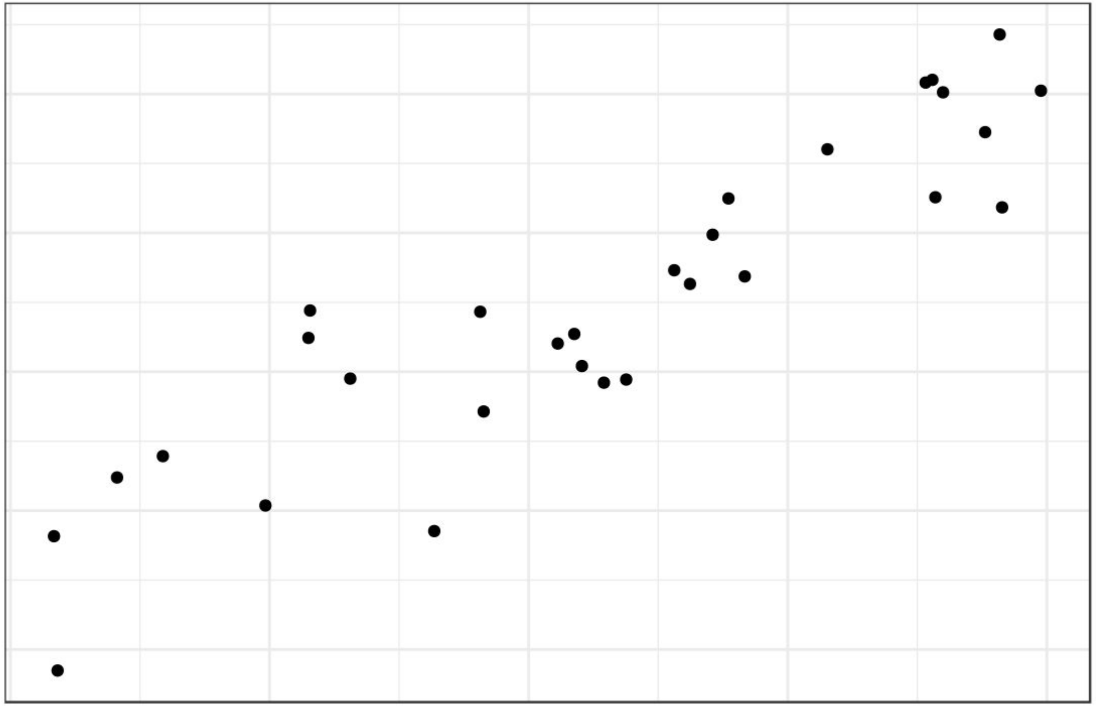
$$y = \beta_0 + \beta_1 x + \varepsilon$$

Ordinary least squares linear regression (**OLS**)

$$y = \beta_0 + \beta_1 x + \varepsilon$$

The game is “find the best betas.”

Graphically, this is:



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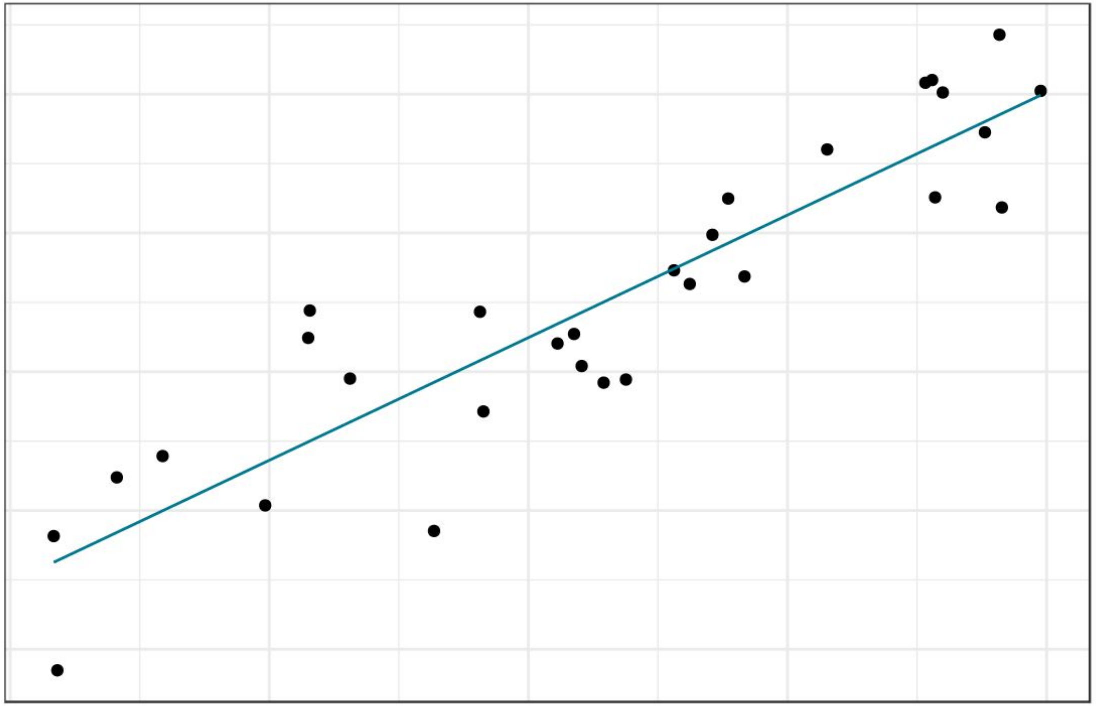

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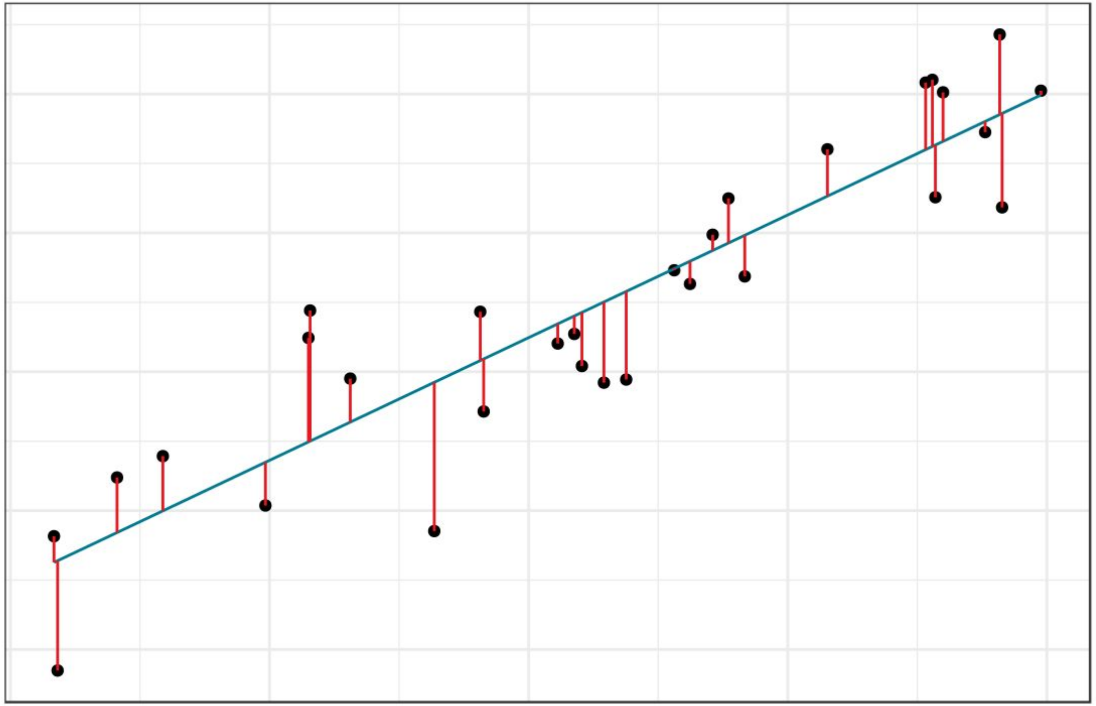
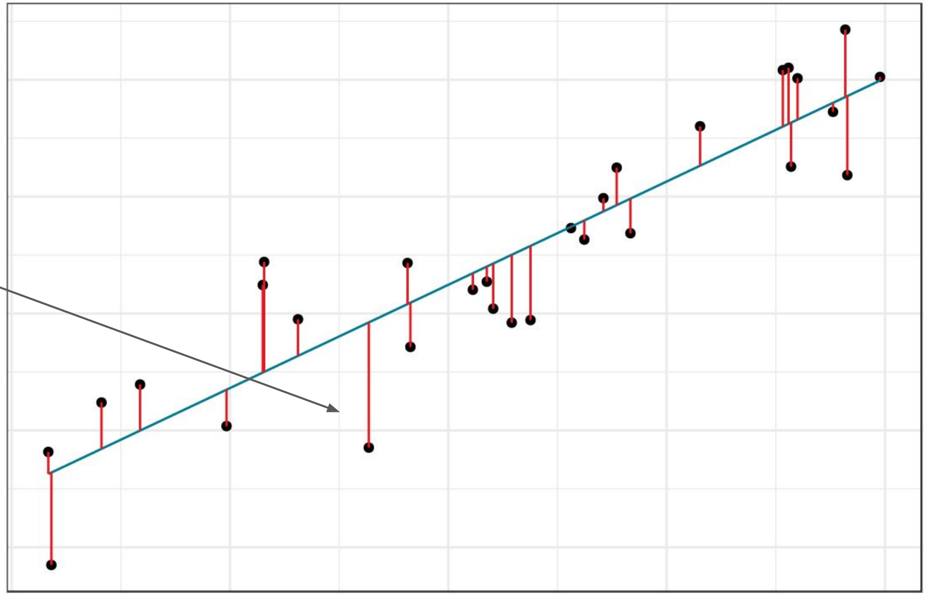


Graphically, this is:

The difference between the actual and the predicted value is called a **residual**.

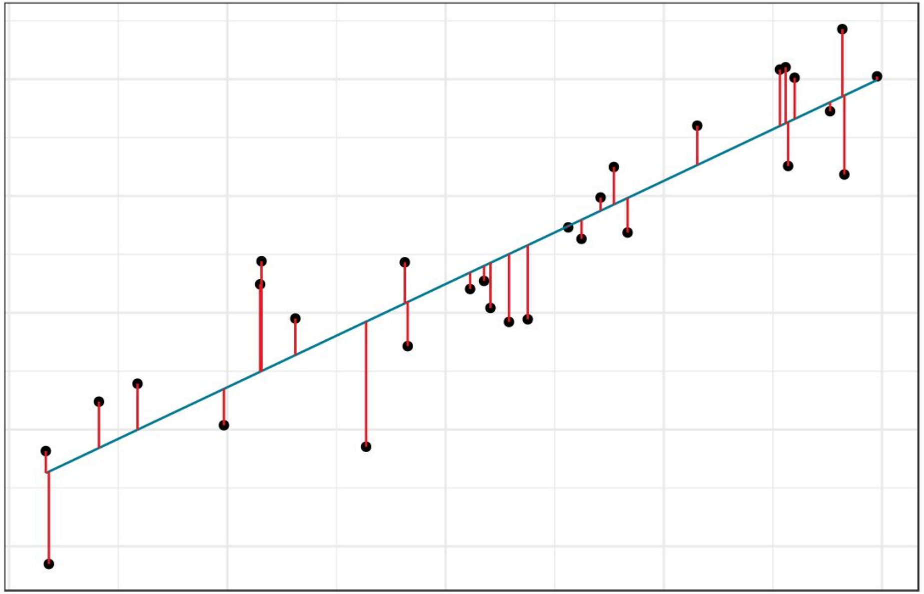
The line of “best fit” **minimizes the sum of the squared residuals**.



Graphically, this is:

Minimize the **sum of the squared residuals**

OLS: “ordinary least squares”



The Residual

A **residual** is the same as an error

$$e_i = y_i - \hat{y}_i$$

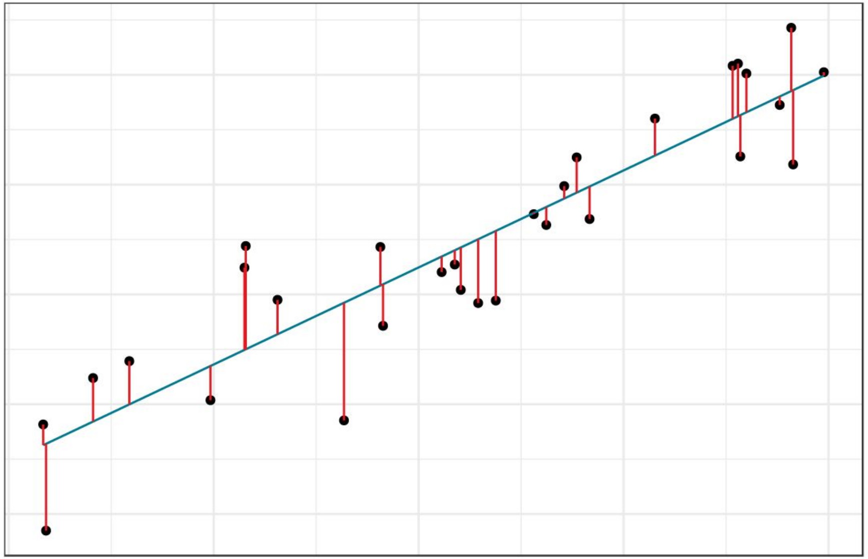
This measures how “off” a prediction was.

Sum of Squared Errors (SSE)

$$SSE = \sum (y_i - \hat{y}_i)^2 = \sum e_i^2$$

Fitting OLS models

This is the quantity we seek to **minimize** to find the best values for our betas!



Multiple Linear Regression

Supervised, white-box, regression

Multiple Linear Regression

Generally, you'll have more than one predictor variable.

Formula for a predicted y value:

$$\hat{\mathbf{y}} = \hat{\beta}_0 + \hat{\beta}_1 \mathbf{x}_1 + \hat{\beta}_2 \mathbf{x}_2 + \ldots \hat{\beta}_\mathbf{p} \mathbf{x}_\mathbf{p} + \varepsilon$$

Notation: Design Matrix

Mathematically speaking, every time we fit a model, we need a data matrix, sometimes called a **design matrix**:

$$X = \begin{bmatrix} 1 & x_{11} & x_{12} & \cdots & x_{1p} \\ 1 & x_{21} & x_{22} & \cdots & x_{2p} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & x_{n1} & x_{n2} & \cdots & x_{np} \end{bmatrix}$$

Notation: Design Matrix

$$X = \begin{bmatrix} 1 & x_{11} & x_{12} & \cdots & x_{1p} \\ \boxed{1 & x_{21} & x_{22} & \cdots & x_{2p}} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & x_{n1} & x_{n2} & \cdots & x_{np} \end{bmatrix}$$

Each **row** is an **observation.**

Notation: Design Matrix

$$X = \begin{bmatrix} 1 & x_{11} & \boxed{x_{12}} & \cdots & x_{1p} \\ 1 & x_{21} & x_{22} & \cdots & x_{2p} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & x_{n1} & x_{n2} & \cdots & x_{np} \end{bmatrix}$$

Each **column**
is a variable.

Notation: Design Matrix

$$X = \begin{bmatrix} 1 & x_{11} & x_{12} & \cdots & x_{1p} \\ 1 & x_{21} & x_{22} & \cdots & x_{2p} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & x_{n1} & x_{n2} & \cdots & x_{np} \end{bmatrix}$$

The first column is all 1s and corresponds to the **intercept**. (scikit-learn handles this automatically)

Multiple linear regression

Interpreting results

Now when you interpret a beta weight, you must use the caveat: “holding everything else constant”.

Multiple linear regression

Interpreting results example:

If beta1 is 5.0 then a 1 degree celsius increase in temperature is associated with a 5MW increase in energy usage, **holding everything else constant**.

Assumption

For inference to make sense:

There is no **M**ulticollinearity in the predictor variables.

Check for highly correlated variables using df.corr & seaborn. (Or better, VIF in statsmodels. Picks up patterns of correlation.).

Let’s do it!

The Residual

A **residual** is:

$$e_i = y_i - \hat{y}_i$$

How far “off” a prediction was.

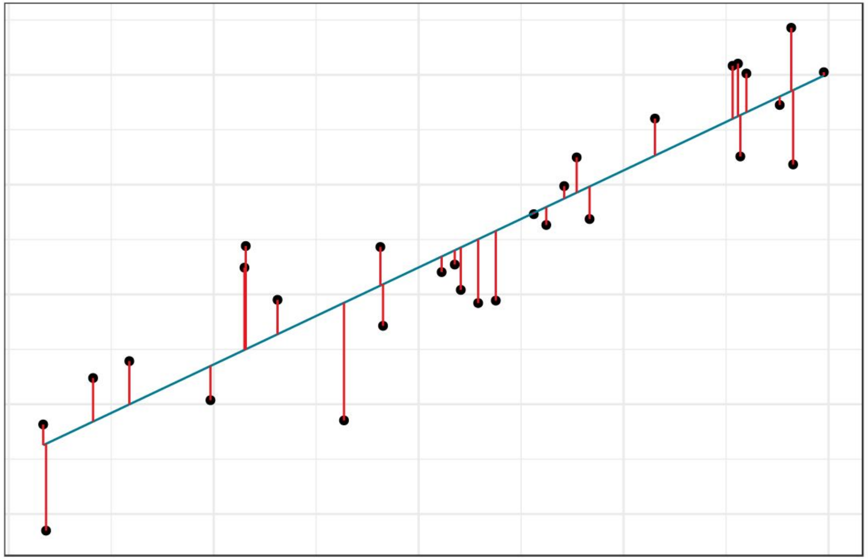
The error.

Sum of Squared Errors (SSE)

$$SSE = \sum (y_i - \hat{y}_i)^2 = \sum e_i^2$$

Fitting OLS models

This is the quantity we seek to **minimize** to find the best values for our betas!



LINE Assumptions

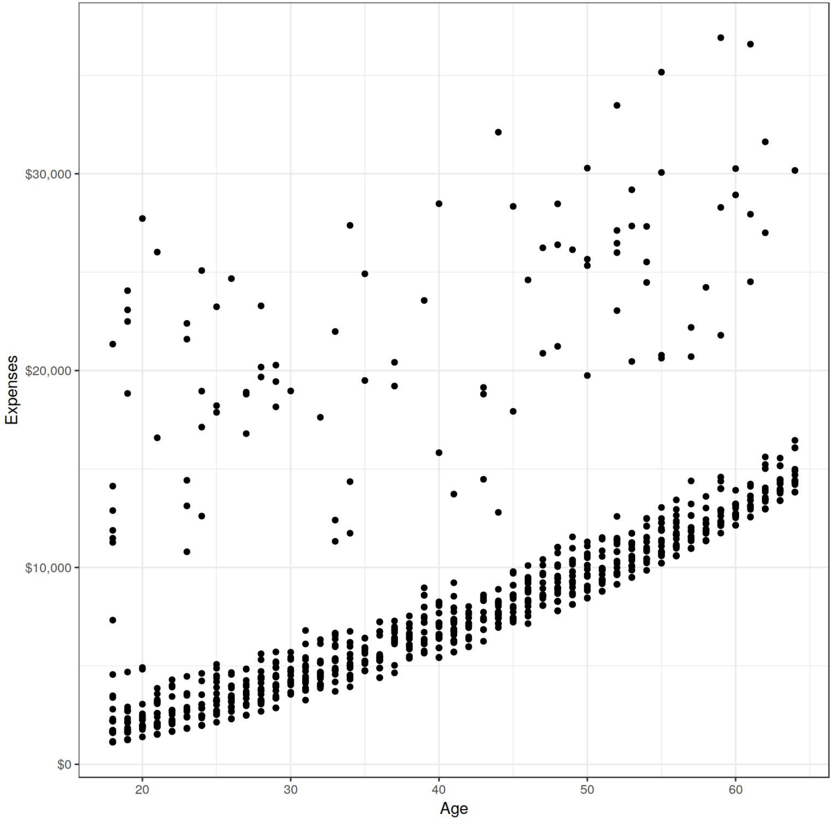
OLS Assumptions

OLS comes with some assumptions. Some matter for performance and some matter for inference.

OLS Regression Assumptions

- **L** - Linearity. Relationship between *x* and *y* should be approximately linear.
- **I** - Independence of errors. (Time series issue)
- **N** - Normality. Residuals should be approximately normally distributed.
- **E** - Equal variances, aka “**homoscedasticity**”. Residuals should have approximately equal variances.

L is for Linearity



L is for Linearity

We will learn how to transform X or y to try to make the relationship more linear.

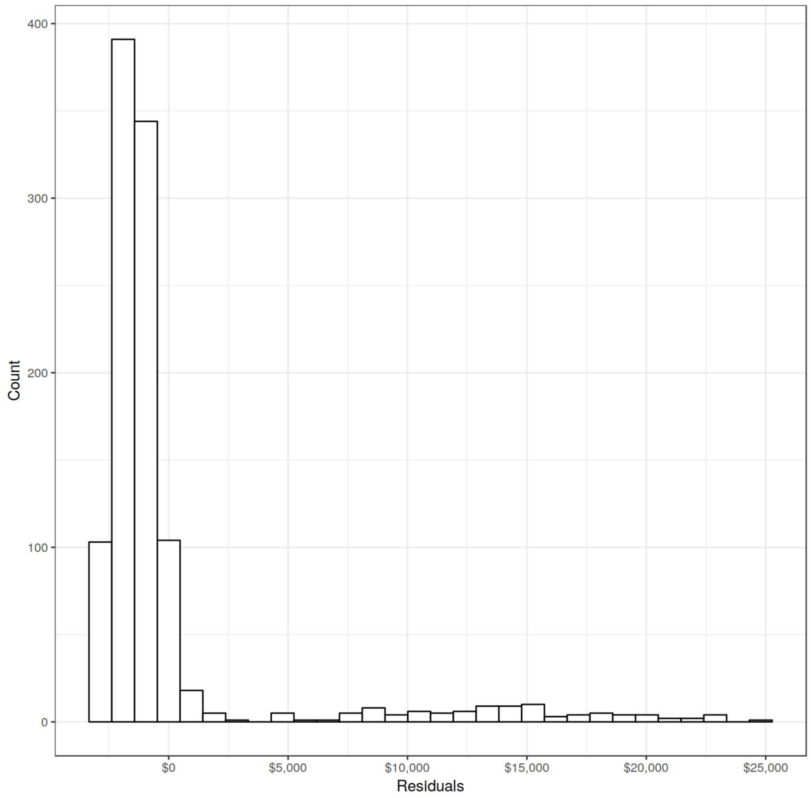
Or maybe another algorithm would be better.

I is for Independence

Are our samples independent from one another?

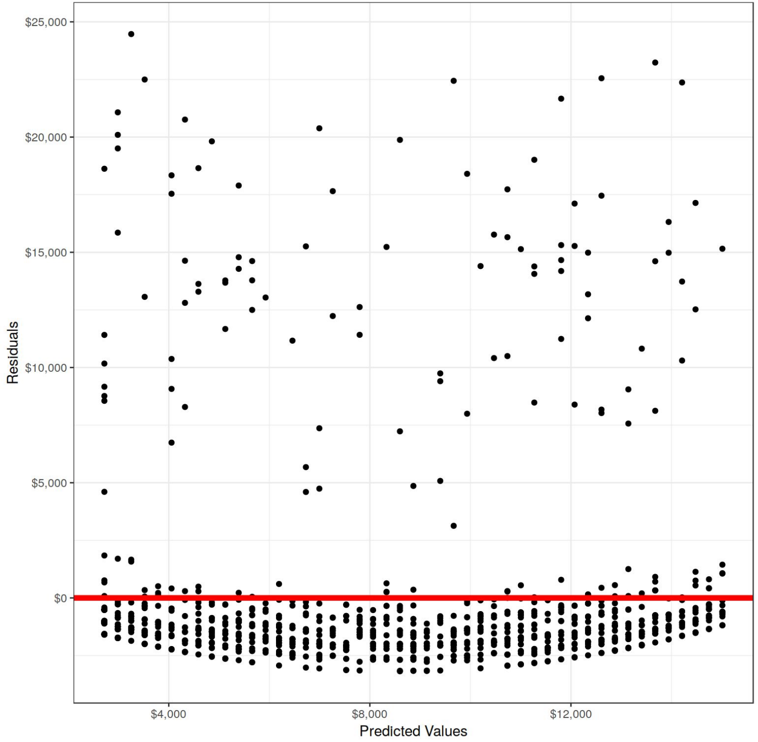
With time series, they usually are not independent. The previous row tells us something about the next row.

N is for Normality of errors



E is for Equal Variances

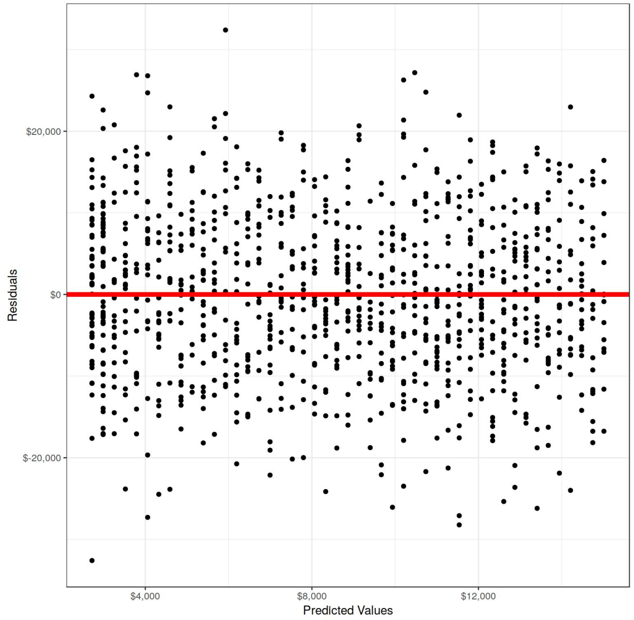
We can see a parabolic pattern in our residual plot.



E is for Equal Variance

We want to see **absolute randomness** in our residual plots. That is, no pattern whatsoever.

Here's an example of an ideal residual plot.



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What to do if our LINE assumptions are violated?

A common scenario:

- A slightly **curvilinear** relationship between *x* and *y*
- Very right-skew residuals
- Residuals that tend to spread out from right to left (a “fan shape”)

A log transformation often helps fix these issues

LINEM Assumptions

- **L** - Linear relationship
- **I** - Independent errors (time series issue, often)
- **N** - Normally distributed errors
- **E** - Equal variance of errors (homoscedasticity)

Add an M for multiple regression (more than 1 X)

- **M** - No multicollinearity (highly correlated predictors)

Let’s do it!