(330 pts total):

## Section 1 – Short answer

1. (10 pts) What is the formula for Pearson's product moment correlation (i.e. the "normal" parametric correlation $r$)?

2.(25 pts)

(a) (5 pts) What kind of response variables are modeled by logistic regression.

(b) (10 pts) Write down the equation(s) that describes a logistic regression model. Be sure to include a complete description of the model.

(c) (10 pts) List three reasons why logistic regression is preferred over ordinary linear regression in cases where it is more appropriate.

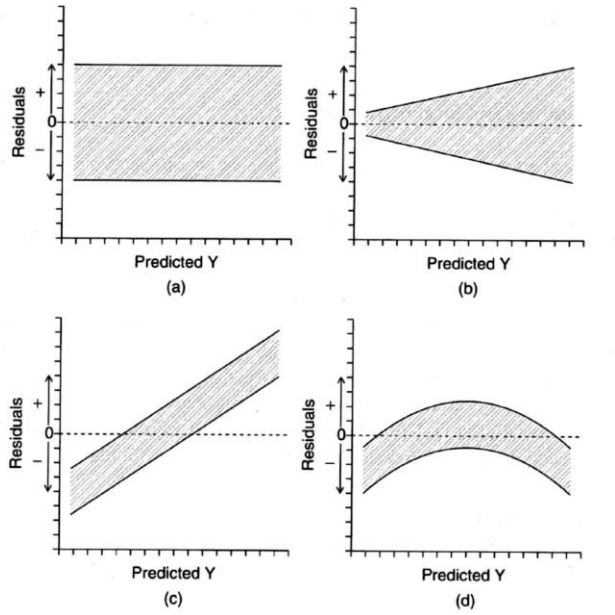
3. (25 pts)

(a) (5 pts) Define multicollinearity (in words).

(b) (10 pts) Why is multicollinearity a concern for linear regression? (In other words, what is its impact on the model fit?)

(c) (10 pts) Name one strategy for fitting a model in which two covariates are highly collinear.

4. (20 pts) Below are four plots depicting the residuals of a linear model plotted as a function of $\hat{Y}$.

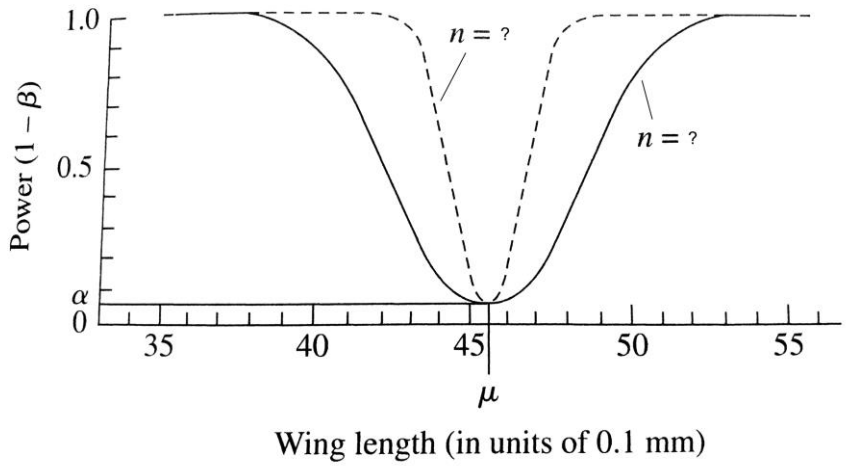



Which panel might be better modelled by a generalized linear model (GLM)? What model would you suggest, and why would this suggested model be more appropriate?

5. (15 pts) Consider a study of fish length in which fish are sampled in each of four ponds. Two potential modeling approaches are (1) a one-way ANOVA, and (2) a series of pairwise t-tests. Briefly describe two reasons why these methods may yield difference inference regarding the true difference in fish lengths among ponds.

6. (5 pts) Two variables can be uncorrelated (in the sense of having $\rho = 0$) but not be independent, while two variables that are independent are necessarily uncorrelated. Draw a scatterplot of two variables that have $\rho = 0$ that are not independent.

7. (10 pts)




The figure above (a "power curve") plots the power of a one-sample t-test as a function of wing length for a fly population with null hypothesis $H_0$: $wing\ length\ =\ \mu$. The values along the x-axis can be thought of many possible alternative values, $H_A$, and the values along the y-axis represents the probability of rejecting $H_0$ if $H_A$ is true. One of the power curves is for an experiment with 5 flies, and the other curve is for an experiment with 35 flies.

Which curve (dashed or solid) represents the power curve for the experiment with 5 flies and how do you know?

8. (18 points) The figure on the following page illustrates the possible outcomes for a hypothetical experiment testing for the effects of substrate and predation treatment on barnacle recruitment. Each symbol represents a different treatment combination mean. Predation treatments are indicated by the x-axis label and substrate treatments are indicated by the different shades of gray (black=granite; light gray = slate; darker gray = cement). (The lines connect predation levels for each substrate, and each of the three lines represents a different substrate.) The partial error bars represent + or – 2 standard error and should be assumed equal across all three substrates. (Most of the error bars have been left off the plot for clarity. The exact size of the error bars is inconsequential to the question.)

For each panel (A-F), decide whether the main effect of predation, the main effect of substrate, and the interaction of predation and substrate is significant ($p<0.05$) or not significant.

Panel A
Predation: significant / not significant (circle one)
Substrate: significant / not significant (circle one)
Predation x Substrate: significant / not significant (circle one)

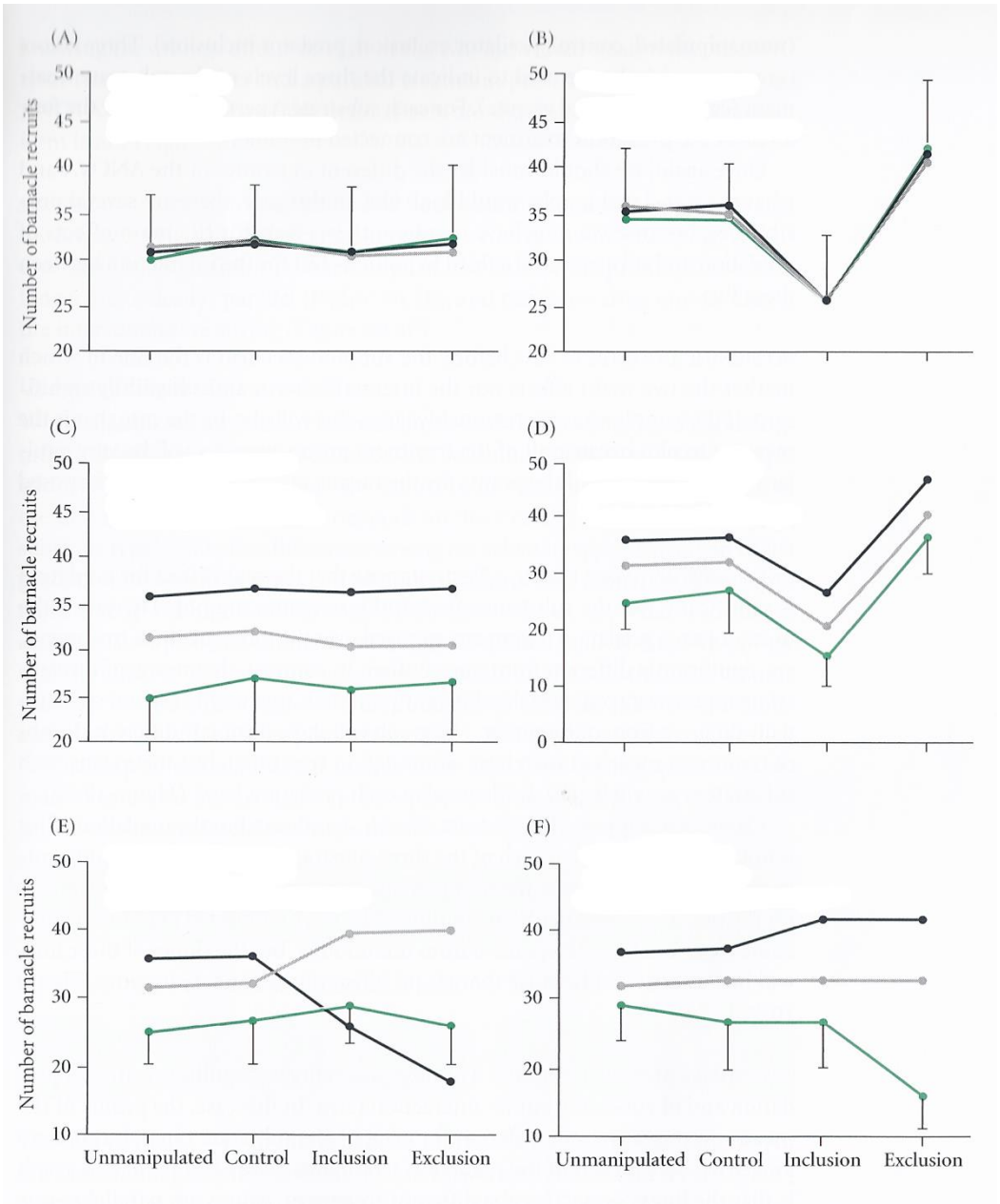
Panel B
Predation: significant / not significant (circle one)
Substrate: significant / not significant (circle one)
Predation x Substrate: significant / not significant (circle one)

Panel C
Predation: significant / not significant (circle one)
Substrate: significant / not significant (circle one)
Predation x Substrate: significant / not significant (circle one)

Panel D
Predation: significant / not significant (circle one)
Substrate: significant / not significant (circle one)
Predation x Substrate: significant / not significant (circle one)

Panel E
Predation: significant / not significant (circle one)
Substrate: significant / not significant (circle one)
Predation x Substrate: significant / not significant (circle one)

Panel F
Predation: significant / not significant (circle one)
Substrate: significant / not significant (circle one)
Predation x Substrate: significant / not significant (circle one)


(A)
(B)
(C)
(D)
(E)
(F)
Number of barnacle recruits
Unmanipulated
Control
Inclusion
Exclusion

9. (27 pts)

a) (10 pts) What is the likelihood ratio test (LRT)? (In other words, what is the test statistic and what is its distribution under the null hypothesis?)

b) (7 pts) What is the null hypothesis for the LRT?

c) (10 pts) Name one advantage of the LRT over Information Theoretic Criteria such as AIC.

## Section 2 – Long answer

10. (110 pts) The following data are from a (fictitious) study of 160 frogs investigating the relationship between abnormal limb development and two factors thought to influence amphibian abnormalities: (1) the presence of pesticides, and (2) high levels of UV-B due to ozone depletion. The table below indicates the number of cases of frog abnormalities in each combination of factors, and the number in parenthesis represents the total number of frogs in each combination of factors.

| Cases of abnormalities | | High UV-B | |
|---|---|---|---|
| | | No | Yes |
| Pesticides | No | 2 (32) | 16 (28) |
| | Yes | 36 (47) | 44 (53) |

(Note: Part I and Part II are unrelated to one another [except for sharing the same basic dataset above], so if you get stuck on Part I, you should move on to Part II and Part III. Your answer in Part I will not affect your answer in Parts II or III. Part IIe is required for Part III.)

## Part I:

(40 pts) Calculate the $1 - \alpha$ confidence interval on the **difference** in the probability of an anomaly between the No Pesticide frogs and the Yes Pesticide frogs. Since I am not concerned here with UV status, this is the **marginal** probability of an anomaly in the two Pesticide categories. Hint: There are three steps involved: (1) First you need to figure out the sampling distribution for the true probability of an anomaly in the No Pesticide category. Then you need to do the same for the Yes Pesticide category. (2) Then you need to figure out the distribution for their difference. (3) Then you need to turn that into a $1 - \alpha$ confidence interval.

(left blank for work)

## Part II:

The goal is to model the probability that a frog has abnormalities, denoted by θ, conditional on the frog's pesticide status (P) and UV status (U). Let $I_{\{A\}}$ be an indicator variable for event $A$, so that $I_{\{A\}} = 1$ means that event $A$ occurred and $I_{\{A\}} = 0$ means that event $A$ did not occur.

Suppose the following 'cell-means' model is fit to these data

$$Y \sim Bern(\theta)$$

$$log\left(\frac{\theta}{1-\theta}\right) = \alpha_1 I_{\{P=No,U=No\}} + \alpha_2 I_{\{P=Yes,U=No\}} + \alpha_3 I_{\{P=No,U=Yes\}} + \alpha_4 I_{\{P=Yes,U=Yes\}}$$

a) (15 pts) Use the data to estimate $\widehat{\alpha_1}$, $\widehat{\alpha_2}$, $\widehat{\alpha_3}$, and $\widehat{\alpha_4}$. Since you don't have a calculator, you will only be able to express your answers as fractions or unsimplified mathematical expressions.

b) (5 pts) What is the (implied) null hypothesis being tested for $\alpha_2$?

c) (5 pts) What is the correct interpretation of $\widehat{\alpha_4}$?

Now suppose you re-parameterize this model to use an effects coding

$$Y \sim Bern(\theta)$$

$$log\left(\frac{\theta}{1-\theta}\right) = \beta_1 + \beta_2 I_{\{P=Yes\}} + \beta_3 I_{\{U=Yes\}} + \beta_4 I_{\{P=Yes\}} \times I_{\{U=Yes\}}$$

d) (15 pts) Use the data to estimate $\widehat{\beta_1}$, $\widehat{\beta_2}$, $\widehat{\beta_3}$, and $\widehat{\beta_4}$. Since you don't have a calculator, you will only be able to express your answers as fractions or unsimplified mathematical expressions.

e) (5 pts) What is the (implied) null hypothesis being tested for $\beta_2$?

f) (5 pts) What is the correct interpretation of $\widehat{\beta_1}$?

## Part III:

(20 pts) Briefly describe how you would use a permutation-based approach to test the null hypothesis from Part IIe.

11. (35 pts)

The table below is from Reddy et al. (2016) *No evidence for contagious yawning in lemurs*. In this experiment, 28 lemurs from two species were used in three experiments (S1, S2, and E2). Here we will focus on Group, Species, and Sex.

**Table 1** Lemur subjects in all experiments

| Group | Subject | Species | Sex | Age (years) | S1 | S2 | E2 |
|---|---|---|---|---|---|---|---|
| 1 | Pyxis | *V v. rubra* | F | 17 | X | X | X |
| 1 | Hunter | *V v. rubra* | M | 16 | X | X | X |
| 1 | Scorpius | *V v. rubra* | M | 5 | X | X | X |
| 1 | Aries | *V v. rubra* | M | 5 | X | X | X |
| 1 | Esther | *V v. rubra* | F | 3 | X | X | X |
| 1 | Orion | *V v. rubra* | M | 3 | | | X |
| 1 | Phoebe | *V v. rubra* | F | 3 | X | X | X |
| 2 | Carina | *V v. rubra* | F | 8 | X | X | X |
| 2 | Alphard | *V v. rubra* | M | 23 | X | | |
| 2 | Avior | *V v. rubra* | M | 4 | | | X |
| 2 | Hydra | *V v. rubra* | F | 4 | X | | |
| 2 | Lyra | *V v. rubra* | F | 4 | X | X | X |
| 2 | Pandora | *V v. rubra* | F | <1 | | X | X |
| 2 | Cordelia | *V v. rubra* | F | <1 | | X | X |
| 3 | Schroeder | *L. catta* | F | 20 | X | X | X |
| 3 | Edelweiss | *L. catta* | F | 2 | X | | |
| 3 | Liesl | *L. catta* | F | 4 | X | X | X |
| 3 | Aracus | *L. catta* | M | 21 | X | X | X |
| 3 | Johan | *L. catta* | M | 2 | | | X |
| 3 | Rolfe | *L. catta* | M | 1 | X | X | X |
| 3 | Brigitta | *L. catta* | F | 1 | | X | X |
| 3 | Gretl | *L. catta* | F | <1 | | X | X |
| 4 | Sprite | *L. catta* | F | 11 | | X | X |
| 4 | Ginger | *L. catta* | F | 6 | X | X | X |
| 4 | Randy | *L. catta* | M | 6 | X | X | X |
| 4 | Schweppes | *L. catta* | M | 2 | | | X |
| 4 | Sobe | *L. catta* | F | 1 | | X | X |
| 4 | Sarsaparilla | *L. catta* | F | 1 | | X | X |
| 4 | Crystal light | *L. catta* | F | 2 | X | | |
| 4 | Canada dry | *L. catta* | F | 1 | X | X | X |
| 4 | Izze | *L. catta* | F | <1 | | X | X |
| 4 | Jones | *L. catta* | M | <1 | | X | X |
| 4 | Stewart | *L. catta* | M | <1 | | X | X |

a) (10 pts) Draw a diagram that illustrates the relationship between these three factors, specifically illustrating which factors are crossed and which are nested.

b) (5 pts) Is Group best considered as a fixed effect or a random effect?

c) (5 pts) Is Species best considered as a fixed effect or a random effect?

d) (5 pts) Is Sex best considered as a fixed effect or a random effect?

e) (5 pts) If the authors had wanted to use a one-way ANOVA to model yawning as a function of Group, how many degrees of freedom would they have for the "Within Group" sum-of-squares calculation?

f) (5 pts) Given which animals were available for each of the experiments (S1, S2, and E2), do you have any concerns about the experimental design?

## Section 3 – Essay (30 pts)

12. (30 pts) In *Oikos*, a paper was published entitled "Ecologists should not use statistical significance tests to interpret simulation model results". Specifically, White et al. (2013) argue that

> "[s]imulation models are widely used to represent the dynamics of ecological systems. A common question with such models is how changes to a parameter value or functional form in the model alter the results. Some authors have chosen to answer that question using frequentist statistical hypothesis tests (e.g., ANOVA). This is inappropriate for two reasons."

12a. What are those two reasons?

(To frame the situation more precisely, assume that you have a model [could be mathematical, statistical, or even mechanistic] that takes a parameter as an input, and you want to know whether changing the value of the input parameter leads to significantly different output from the model. Why can we not use NHST to answer that question?)

12b. What is an appropriate way to use simulations for understanding the sensitivity of a model to changes in its parameter values?