# Derivatives

## Basic Properties/Formulas/Rules

$\frac{d}{dx}(cf(x)) = cf'(x)$, $c$ is any constant. $(f(x) \pm g(x))' = f'(x) \pm g'(x)$

$\frac{d}{dx}(x^n) = nx^{n-1}$, $n$ is any number. $\frac{d}{dx}(c) = 0$, $c$ is any constant.

$(f\,g)' = f'\,g + f\,g'$ – **(Product Rule)** $\left(\frac{f}{g}\right)' = \frac{f'\,g - f\,g'}{g^2}$ – **(Quotient Rule)**

$\frac{d}{dx}(f(g(x))) = f'(g(x))\,g'(x)$ **(Chain Rule)**

$\frac{d}{dx}\left(\mathbf{e}^{g(x)}\right) = g'(x)\,\mathbf{e}^{g(x)}$ $\frac{d}{dx}(\ln g(x)) = \frac{g'(x)}{g(x)}$

## Common Derivatives

*Polynomials*

All

$\frac{d}{dx}(c) = 0$ $\frac{d}{dx}(x) = 1$ $\frac{d}{dx}(cx) = c$ $\frac{d}{dx}(x^n) = nx^{n-1}$ $\frac{d}{dx}(cx^n) = ncx^{n-1}$

*Trig Functions*

$\frac{d}{dx}(\sin x) = \cos x$ $\frac{d}{dx}(\cos x) = -\sin x$ $\frac{d}{dx}(\tan x) = \sec^2 x$

$\frac{d}{dx}(\sec x) = \sec x \tan x$ $\frac{d}{dx}(\csc x) = -\csc x \cot x$ $\frac{d}{dx}(\cot x) = -\csc^2 x$

*Inverse Trig Functions*

$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$ $\frac{d}{dx}(\cos^{-1} x) = -\frac{1}{\sqrt{1-x^2}}$ $\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$

$\frac{d}{dx}(\sec^{-1} x) = \frac{1}{|x|\sqrt{x^2-1}}$ $\frac{d}{dx}(\csc^{-1} x) = -\frac{1}{|x|\sqrt{x^2-1}}$ $\frac{d}{dx}(\cot^{-1} x) = -\frac{1}{1+x^2}$

*Exponential/Logarithm Functions*

$\frac{d}{dx}(a^x) = a^x \ln(a)$ $\frac{d}{dx}(\mathbf{e}^x) = \mathbf{e}^x$

$\frac{d}{dx}(\ln(x)) = \frac{1}{x},\ x > 0$ $\frac{d}{dx}(\ln|x|) = \frac{1}{x},\ x \neq 0$ $\frac{d}{dx}(\log_a(x)) = \frac{1}{x \ln a},\ x > 0$

*Hyperbolic Trig Functions*

$\frac{d}{dx}(\sinh x) = \cosh x$ $\frac{d}{dx}(\cosh x) = \sinh x$ $\frac{d}{dx}(\tanh x) = \operatorname{sech}^2 x$

$\frac{d}{dx}(\operatorname{sech} x) = -\operatorname{sech} x \tanh x$ $\frac{d}{dx}(\operatorname{csch} x) = -\operatorname{csch} x \coth x$ $\frac{d}{dx}(\coth x) = -\operatorname{csch}^2 x$

# Integrals

## Basic Properties/Formulas/Rules

$\int cf(x)\,dx = c\int f(x)\,dx$, $c$ is a constant. $\int f(x) \pm g(x)\,dx = \int f(x)\,dx \pm \int g(x)\,dx$

$\int_a^b f(x)\,dx = F(x)\Big|_a^b = F(b) - F(a)$ where $F(x) = \int f(x)\,dx$

$\int_a^b cf(x)\,dx = c\int_a^b f(x)\,dx$, $c$ is a constant. $\int_a^b f(x) \pm g(x)\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$

$\int_a^a f(x)\,dx = 0$ $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$ $\int_a^b c\,dx = c(b-a)$

If $f(x) \geq 0$ on $a \leq x \leq b$ then $\int_a^b f(x)\,dx \geq 0$

If $f(x) \geq g(x)$ on $a \leq x \leq b$ then $\int_a^b f(x)\,dx \geq \int_a^b g(x)\,dx$

## Common Integrals

*Polynomials* we often assume c=0

$\int dx = x + c$ $\int k\,dx = kx + c$ $\int x^n\,dx = \frac{1}{n+1}x^{n+1} + c,\ n \neq -1$

$\int \frac{1}{x}\,dx = \ln|x| + c$ $\int x^{-1}\,dx = \ln|x| + c$ $\int x^{-n}\,dx = \frac{1}{-n+1}x^{-n+1} + c,\ n \neq 1$

$\int \frac{1}{ax+b}\,dx = \frac{1}{a}\ln|ax+b| + c$ $\int x^{\frac{p}{q}}\,dx = \frac{1}{\frac{p}{q}+1}x^{\frac{p}{q}+1} + c = \frac{q}{p+q}x^{\frac{p+q}{q}} + c$

*Trig Functions*

$\int \cos u\,du = \sin u + c$ $\int \sin u\,du = -\cos u + c$ $\int \sec^2 u\,du = \tan u + c$

$\int \sec u \tan u\,du = \sec u + c$ $\int \csc u \cot u\,du = -\csc u + c$ $\int \csc^2 u\,du = -\cot u + c$

$\int \tan u\,du = \ln|\sec u| + c$ $\int \cot u\,du = \ln|\sin u| + c$

$\int \sec u\,du = \ln|\sec u + \tan u| + c$ $\int \sec^3 u\,du = \frac{1}{2}\left(\sec u \tan u + \ln|\sec u + \tan u|\right) + c$

$\int \csc u\,du = \ln|\csc u - \cot u| + c$ $\int \csc^3 u\,du = \frac{1}{2}\left(-\csc u \cot u + \ln|\csc u - \cot u|\right) + c$

*Exponential/Logarithm Functions*

$\int \mathbf{e}^u\,du = \mathbf{e}^u + c$ $\int a^u\,du = \frac{a^u}{\ln a} + c$ $\int \ln u\,du = u\ln(u) - u + c$

$\int \mathbf{e}^{au}\sin(bu)\,du = \frac{\mathbf{e}^{au}}{a^2+b^2}\left(a\sin(bu) - b\cos(bu)\right) + c$ $\int u\mathbf{e}^u\,du = (u-1)\mathbf{e}^u + c$

$\int \mathbf{e}^{au}\cos(bu)\,du = \frac{\mathbf{e}^{au}}{a^2+b^2}\left(a\cos(bu) + b\sin(bu)\right) + c$ $\int \frac{1}{u\ln u}\,du = \ln|\ln u| + c$

*Inverse Trig Functions*

$$\int \frac{1}{\sqrt{a^2-u^2}}\,du = \sin^{-1}\left(\frac{u}{a}\right)+c \qquad \int \sin^{-1}u\,du = u\sin^{-1}u+\sqrt{1-u^2}+c$$

$$\int \frac{1}{a^2+u^2}\,du = \frac{1}{a}\tan^{-1}\left(\frac{u}{a}\right)+c \qquad \int \tan^{-1}u\,du = u\tan^{-1}u-\frac{1}{2}\ln\left(1+u^2\right)+c$$

$$\int \frac{1}{u\sqrt{u^2-a^2}}\,du = \frac{1}{a}\sec^{-1}\left(\frac{u}{a}\right)+c \qquad \int \cos^{-1}u\,du = u\cos^{-1}u-\sqrt{1-u^2}+c$$

*Hyperbolic Trig Functions*

$$\int \sinh u\,du = \cosh u + c \qquad \int \cosh u\,du = \sinh u + c \qquad \int \operatorname{sech}^2 u\,du = \tanh u + c$$

$$\int \operatorname{sech}\tanh u\,du = -\operatorname{sech} u + c \qquad \int \operatorname{csch}\coth u\,du = -\operatorname{csch} u + c \qquad \int \operatorname{csch}^2 u\,du = -\coth u + c$$

$$\int \tanh u\,du = \ln\left(\cosh u\right)+c \qquad \int \operatorname{sech} u\,du = \tan^{-1}\left|\sinh u\right|+c$$

*Miscellaneous*

$$\int \frac{1}{a^2-u^2}\,du = \frac{1}{2a}\ln\left|\frac{u+a}{u-a}\right|+c \qquad \int \frac{1}{u^2-a^2}\,du = \frac{1}{2a}\ln\left|\frac{u-a}{u+a}\right|+c$$

$$\int \sqrt{a^2+u^2}\,du = \frac{u}{2}\sqrt{a^2+u^2}+\frac{a^2}{2}\ln\left|u+\sqrt{a^2+u^2}\right|+c$$

$$\int \sqrt{u^2-a^2}\,du = \frac{u}{2}\sqrt{u^2-a^2}-\frac{a^2}{2}\ln\left|u+\sqrt{u^2-a^2}\right|+c$$

$$\int \sqrt{a^2-u^2}\,du = \frac{u}{2}\sqrt{a^2-u^2}+\frac{a^2}{2}\sin^{-1}\left(\frac{u}{a}\right)+c$$

$$\int \sqrt{2au-u^2}\,du = \frac{u-a}{2}\sqrt{2au-u^2}+\frac{a^2}{2}\cos^{-1}\left(\frac{a-u}{a}\right)+c$$

**Standard Integration Techniques**

Note that all but the first one of these tend to be taught in a Calculus II class.

*u Substitution*

Given $\int_a^b f\left(g(x)\right)g'(x)\,dx$ then the substitution $u=g(x)$ will convert this into the integral, $\int_a^b f\left(g(x)\right)g'(x)\,dx = \int_{g(a)}^{g(b)} f(u)\,du$.

*Integration by Parts*

The standard formulas for integration by parts are,

$$\int u\,dv = uv - \int v\,du \qquad \int_a^b u\,dv = uv\Big|_a^b - \int_a^b v\,du$$

Choose $u$ and $dv$ and then compute $du$ by differentiating $u$ and compute $v$ by using the fact that $v=\int dv$.

$$\log(AB) = \log A + \log B$$

$$\log\left(\frac{A}{B}\right) = \log A - \log B$$

$$\log(A^c) = c \log A$$

$$X^{n-m} = \frac{X^n}{X^m}$$

$$X^{n+m} = X^n X^m$$

$$X^m Y^m = (XY)^m$$

$$\left(\frac{X}{Y}\right)^n = \frac{X^n}{Y^n}$$

$$(X^n)^m = X^{n*m}$$