Velocity triangles and power output By Elementary theory

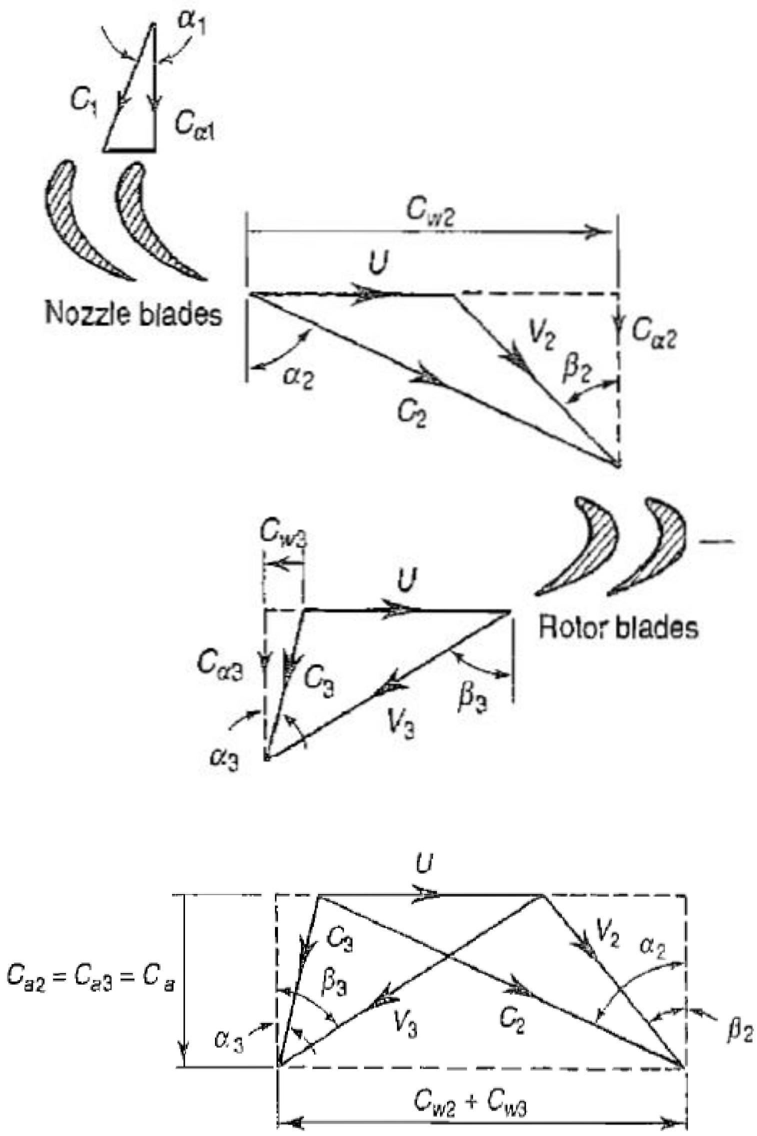

Fig1.5 Velocity Triangle

Figure 1.5 shows the velocity triangles of axial flow turbine stage and the nomenclature employed. The gas enters the row of nozzle blades with a static pressure P_l, temperature T_1 and velocityC_1, which is expanded to P_2, T_2 and leaves with an increased velocity C_2 at an angle α_2. The rotor blade inlet angle will be chosen to suit the direction β_2 of the gas velocity V_2 relative to the blade at inlet. β_2 and V_2 are found by vectorial subtraction of the blade speed U from the absolute velocity C_2. After being deflected and expanded in the rotor blade passages, the gas leaves at P_3, T_3 with relative velocity V_3 at angle β_3. Vectorial addition of U yields the magnitude and direction of the gas velocity at exit from the stage, C_3 and α_3. α_3 is known as the swirl angle.

In a single-stage turbine C_l will be axial, i.e. α_1= 0 and C_1 = C_{al}. In a multi-stage turbine, C_1 and α_l will probably be equal to C_3 and α_3 so that the same blade shapes can be used in successive stages: it is then sometimes called a repeating stage. Because the blade speed U increases with increasing radius, the shape of the velocity triangles varies from root to tip of the blade. We shall assume in this section that we are talking about conditions at the mean diameter of the annulus, and that this represents an average picture of what happens to the total mass flow m as it passes through the stage. This approach is valid when the ratio of the tip radius to the root radius is low, i.e. for short blades, but for long blades it is essential to account for three dimensional effects.

(C_{w2} + C_{w3}) represents the change in whirl (or tangential) component of momentum per unit mass flow which produces the useful torque. The change in axial component (C_{a2}-C_{a3}) produces an axial thrust on the rotor which may supplement or offset the pressure thrust arising from the pressure drop (P_2 - P_3). In a gas turbine the net thrust on the turbine rotor will be partially balanced by the thrust on the compressor rotor, so easing the design of the thrust bearing. In what follows we shall largely restrict our attention to designs in which the axial flow velocity C_a is constant through the rotor. This will imply an annulus flared as in Fig. 7.1 to accommodate the decrease in density as the gas expands through the stage. With this restriction, when the velocity triangles are superimposed in the usual way we have the velocity diagram for the stage shown in Fig. 1.5

From the first triangle

$$U = C_{W2} - C_{a2}\tan\beta_2 = C_{a2}\tan\alpha_2 - C_{a2}\tan\beta_2$$

From the second triangle

$$U = C_{a3}\tan\beta_3 - C_{W3} = C_{a3}\tan\beta_3 - C_{a3}\tan\alpha_3$$

Assuming that axial velocity is constant throughout the stage

$$C_{a1} = C_{a2} = C_{a3} = C_a \text{ and } C_1 = C_3$$

$$U = C_{a2}\tan\alpha_2 - C_{a2}\tan\beta_2 = C_a(\tan\alpha_2 - \tan\beta_2)$$

$$U = C_{a3}\tan\beta_3 - C_{a3}\tan\alpha_3 = C_a(\tan\beta_3 - \tan\alpha_3)$$

$$\Rightarrow \mathrm{U} = C_a(\tan\alpha_2 - \tan\beta_2) = C_a(\tan\beta_3 - \tan\alpha_3)$$

$$\Rightarrow \frac{U}{C_a} = \tan\alpha_2 - \tan\beta_2 = \tan\beta_3 - \tan\alpha_3$$

$$\Rightarrow \tan\alpha_2 + \tan\alpha_3 = \tan\beta_2 + \tan\beta_3$$

Applying the principle of angular momentum to the rotor, the stage work output per unit mass flow is.

$$W_S = U\Delta C_W = U(\mathrm{C_{W2}} + \mathrm{C_{W3}})$$

$$W_S = U\left(C_{a2}\tan\alpha_2 + C_{a3}\tan\alpha_3\right)$$

$$W_S = UC_a\left(\tan\alpha_2 + \tan\alpha_3\right)$$

$$W_S = UC_a\left(\tan\beta_2 + \tan\beta_3\right)$$

The power output of a single stage axial flow gas turbine is given by

$$W = \dot{m}UC_a\left(\tan\beta_2 + \tan\beta_3\right)$$

$$W = \dot{m}UC_a\left(\tan\alpha_2 + \tan\alpha_3\right)$$

T-S diagram

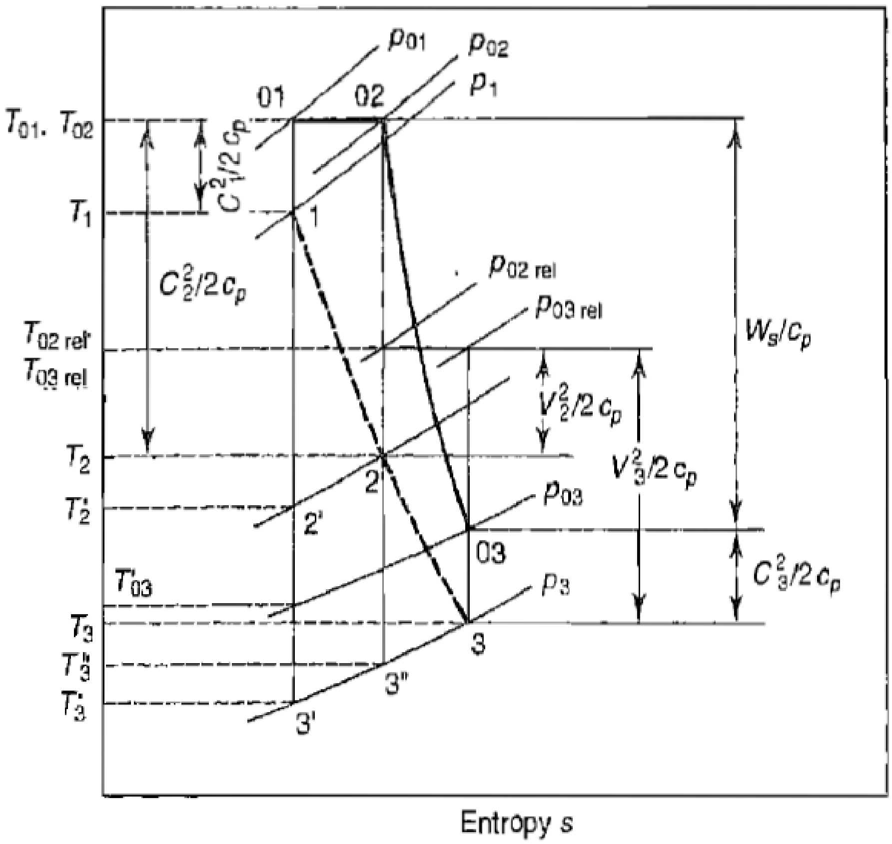

Fig 1.6 T-S diagram of gas turbine stage

The temperature entropy diagram for the flow through a general turbine stage is shown in fig1.6. Both static and stagnation values of temperature and entropies are indicated at various stations.

The relation between static and stagnation temperatures (enthalpies) are given by using energy equation from the first law of thermodynamics.

$$Q = W + [E_2 - E_1] \qquad \rightarrow (1)$$

For application in turbomachinery, the energy terms like internal energy, gravitational potential energy and kinetic energy are included.

$$E = U + mgz + \frac{1}{2}mC^2 \qquad \rightarrow (2)$$

$$dE = dU + mg.dz + \frac{1}{2}md(C^2) \qquad \rightarrow (3)$$

$$E_2 - E_1 = [U_2 - U_1] + mg[z_2 - z_1] + \frac{1}{2}m\left[C_2^2 - C_1^2\right] \qquad \rightarrow (4)$$

Sub equation (4) in equation (1)

$$Q = W + [U_2 - U_1] + mg[z_2 - z_1] + \frac{1}{2}m\left[C_2^2 - C_1^2\right] \qquad \rightarrow (5)$$

From steady flow process through turbomachinery, the work term contains shaft work and flow work

$$W = W_{shaft} + (P_2V_2 - P_1V_1) \qquad \rightarrow (6)$$

Sub equation (6) in equation (5)

$$Q = W_{shaft} + (P_2V_2 - P_1V_1) + \left[U_2 - U_1\right] + mg\left[z_2 - z_1\right] + \frac{1}{2}m\left[C_2^2 - C_1^2\right]$$

$$Q = W_{shaft} + \left[U_2 + P_2V_2\right] - \left[U_1 + P_1V_1\right] + mg\left[z_2 - z_1\right] + \frac{1}{2}m\left[C_2^2 - C_1^2\right]$$

We know that enthalpy H=U+PV

$$Q = W_{shaft} + \left[H_2 - H_1\right] + mg\left[z_2 - z_1\right] + \frac{1}{2}m\left[C_2^2 - C_1^2\right]$$

Rearranging the above equation

$$H_1 + mgz_1 + \frac{1}{2}mC_1^2 + Q = H_2 + mgz_2 + \frac{1}{2}mC_2^2 + W_{shaft}$$

In terms of specific quantities

$$h_1 + gz_1 + \frac{1}{2}C_1^2 + q = h_2 + gz_2 + \frac{1}{2}C_2^2 + w_{shaft} \qquad \rightarrow (7)$$

Equation (7) is the steady flow energy equation for a control volume/open system

Since most of the turbo machines are adiabatic machines, there is no heat transfer (q=0). In these machines the change in potential energy is negligible as compared to change in enthalpy $\left(h_2 - h_1\right)$ and kinetic energy $\frac{1}{2}\left[C_2^2 - C_1^2\right]$

$$h_1 + \frac{1}{2}C_1^2 + q = h_2 + \frac{1}{2}C_2^2 + w_{shaft}$$

The shaft work is given by

$$w_{shaft} = \left[h_1 + \frac{1}{2}C_1^2\right] - \left[h_2 + \frac{1}{2}C_2^2\right]$$

If the entry and exit velocity are small or the difference between them is negligible the shaft work is given by the difference between static enthalpies two states.

$$w_{shaft} = \left[h_1 - h_2\right]$$

In the stator/nozzle the shaft work is absent and the flow is almost adiabatic.

$$h_1 + \frac{1}{2}C_1^2 = h_2 + \frac{1}{2}C_2^2 = \text{constant}$$

The stagnation enthalpy is given by

$$h_0 = h + \frac{1}{2}C^2$$

$$\frac{h_0}{h} = \left[1 + \frac{1}{2h}C^2\right]$$

$$\frac{T_0}{T} = \left[1 + \frac{C^2}{2C_pT}\right]$$

$$W.K.T \quad C_p = \frac{\gamma R}{\gamma - 1}$$

$$\frac{T_0}{T} = \left[1 + \frac{(\gamma - 1)C^2}{2\gamma \mathrm{R} T}\right]$$

$$\frac{T_0}{T} = \left[1 + \frac{(\gamma - 1)C^2}{2a^2}\right]$$

$$\frac{T_0}{T} = \left[1 + \frac{(\gamma - 1)}{2} M^2\right]$$

The temperature loss coefficient in the stator/nozzle is given by

$$\lambda_N = \frac{h_2 - h_{2S}}{\frac{1}{2}C_2^2} = \frac{2C_p}{C_2^2}(T_2 - T_{2S}) = \frac{T_2 - T_{2S}}{C_2^2 / 2C_p}$$

The pressure loss coefficient in the nozzle is given by

$$Y_N = \frac{P_{01} - P_{02}}{\frac{1}{2}\rho C_2^2}$$

The temperature loss coefficient in the rotor is given by

$$\lambda_R = \frac{h_3 - h_{3S}}{\frac{1}{2}V_3^2} = \frac{2C_p}{V_3^2}(T_3 - T_{3S}) = \frac{T_3 - T_{3S}}{V_3^2 / 2C_p}$$

The pressure loss coefficient in the rotor is given by

$$Y_R = \frac{P_{02rel} - P_{03rel}}{\frac{1}{2}\rho V_3^2}$$

Degree of reaction

Degree of reaction is defined as the ratio of enthalpy drop in the rotor to that of the enthalpy drop in the stage.

$$\Lambda = \frac{Enthalpy\,drop\,in\,Rotor}{Enthalpy\,drop\,in\,stage}$$

$$\Lambda = \frac{h_2 - h_3}{h_1 - h_3} = \frac{T_2 - T_3}{T_1 - T_3}$$

Assuming that axial velocity is constant throughout the stage

$$C_{a1} = C_{a2} = C_{a3} = C_a \text{ and } C_1 = C_3$$

$$C_p(T_1 - T_3) = C_p(T_{01} - T_{03}) = UC_a(\tan\ \beta_2 + \tan\ \beta_3)$$

$$C_p(T_2 - T_3) = \frac{1}{2}(V_3{}^2 - V_2{}^2)$$

$$= \frac{1}{2}\left[\left(\frac{C_a}{\cos\ \beta_3}\right)^2 - \left(\frac{C_a}{\cos\ \beta_2}\right)^2\right]$$

$$= \frac{1}{2}C_a{}^2(\sec^2\beta_3 - \sec^2\beta_2)$$

$$= \frac{1}{2}C_a{}^2[(\tan^2\beta_3 + 1) - (\tan^2\beta_2 + 1)]$$

$$C_p(T_2 - T_3) = \frac{1}{2}C_a{}^2(\tan^2\beta_3 - \tan^2\beta_2)$$

$$C_p(T_2 - T_3) = \frac{1}{2}C_a{}^2[(\tan\beta_3 + \tan\beta_2)(\tan\beta_3 - \tan\beta_2)]$$

$$\Lambda = \frac{1}{2}C_a{}^2 \frac{[(\tan\beta_3 + \tan\beta_2)(\tan\beta_3 - \tan\beta_2)]}{UC_a(\tan\ \beta_2 + \tan\ \beta_3)}$$

$$\Lambda = \frac{Ca}{2U}(\tan\beta_3 - \tan\beta_2)$$

Degree of reaction in terms of gas angles

$$\frac{U}{C_a} = \tan\alpha_2 - \tan\beta_2 \Rightarrow \tan\beta_2 = \tan\alpha_2 - \frac{U}{C_a}$$

$$\frac{U}{C_a} = \tan\beta_3 - \tan\alpha_3 \Rightarrow \tan\beta_3 = \tan\alpha_3 + \frac{U}{C_a}$$

$$\Lambda = \frac{Ca}{2U}(\tan\beta_3 - \tan\beta_2)$$

$$\Lambda = \frac{Ca}{2U}\left[\tan\alpha_3 + \frac{U}{C_a} - \left(\tan\alpha_2 - \frac{U}{C_a}\right)\right]$$

$$\Lambda = \frac{Ca}{2U}\left[\tan\alpha_3 + \frac{U}{C_a} - \tan\alpha_2 + \frac{U}{C_a}\right]$$

$$\Lambda = \frac{Ca}{2U}\left[\tan\alpha_3 - \tan\alpha_2 + \frac{2U}{C_a}\right]$$

$$\Lambda = 1 + \frac{Ca}{2U}[\tan\alpha_3 - \tan\alpha_2]$$

Blade-loading coefficient

The blade-loading coefficient is used to express work capacity of the stage.
It is defined as the ratio of the specific work of the stage to the square of the blade velocity

$$\psi = \frac{W_s}{U^2} = \frac{C_p \Delta T_{os}}{U^2} = \frac{C_a}{U}(\tan\beta_2 + \tan\beta_3)$$

Flow coefficient

The flow coefficient is defined as the ratio of the inlet velocity C_a to the blade speed U

$$\phi = \frac{C_a}{U}$$

The blade angles in terms of $\psi, \phi, and\ \Delta$

$$\psi = \phi(\tan\beta_2 + \tan\beta_3)$$

$$\Lambda = \frac{\phi}{2}(\tan\beta_3 - \tan\beta_2)$$

$adding$ and sub above eqn we get

$$\tan\beta_3 = \frac{1}{2\phi}(\psi + 2\Lambda)$$

$$\tan\beta_2 = \frac{1}{2\phi}(\psi - 2\Lambda)$$

we know that

$$\frac{U}{C_a} = \tan\alpha_2 - \tan\beta_2 = \tan\beta_3 - \tan\alpha_3$$

$$\tan\alpha_3 = \frac{1}{2\phi}(\psi + 2\Lambda) - \frac{1}{\phi}$$

$$\tan\alpha_3 = \frac{1}{2\phi}(\psi + 2\Lambda - 2)$$

$$\tan\alpha_2 = \frac{1}{2\phi}(\psi - 2\Lambda) + \frac{1}{\phi}$$

$$\tan\alpha_2 = \frac{1}{2\phi}(\psi - 2\Lambda + 2)$$

Let us consider 50% reaction at mean radius

$$\Lambda = 0.5$$

$$\frac{1}{\phi} = \tan\beta_3 - \tan\beta_2$$

$$\Rightarrow \beta_3 = \alpha_2 \text{ and } \beta_2 = \alpha_3$$

$$consider\ C_1 = C_3 \ \Rightarrow\ \alpha_1 = \alpha_3 = \beta_2$$

$$\psi = 4\phi\tan\beta_3 - 2 = 4\phi\tan\alpha_2 - 2$$

$$\psi = 4\phi\tan\beta_2 + 2 = 4\phi\tan\alpha_3 + 2$$