UNIT-1 FUNDAMENTALS OF ROCKET PROPULSION

By
A.VinothKumar
Assistant Professor
Department of Aerospace Engineering
SRM University

UNIT-1 SYLLABUS

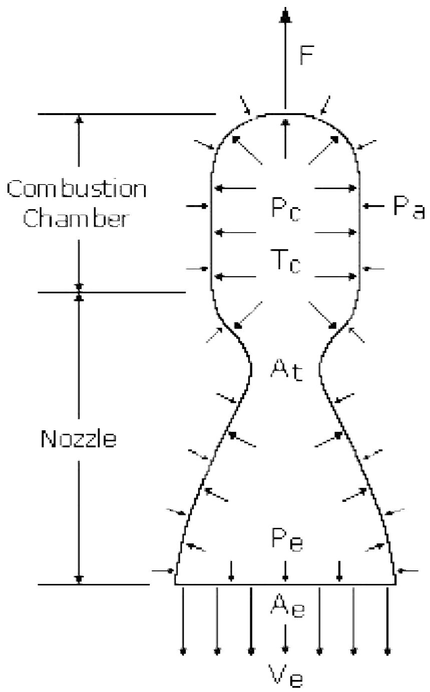
- History and evolution of rockets
- Rocket equation
- Definitions- Performance parameters
- Staging and Clustering
- Classification of rockets
- Rocket nozzle and performance
- Nozzle area ratio
- conical nozzle and contour nozzle
- Under and over expanded nozzles
- Flow separation in nozzles
- unconventional nozzles
- Mass flow rate, Characteristic velocity, Thrust coefficient, Efficiencies, Specific impulse
- Numerical problems.

History and evolution of rockets

- Archytas, 428 to 347 B.C.
- Hero Engine, c. A.D. 10 to 70
- Chinese Fire Arrows, A.D. 1232

Thrust

- *Thrust* is the force that propels a rocket or spacecraft and is measured in pounds, kilograms or Newtons.
- The thrust F is the resultant of the forces due to the pressures exerted on the inner and outer walls by the combustion gases and the surrounding atmosphere, taking the boundary between the inner and outer surfaces as the cross section of the exit of the nozzle.



$$F = \dot{m} V_j + (P_e - P_a) A_e$$

- A spacecraft's engine ejects mass at a rate of 30 kg/s with an exhaust velocity of 3,100 m/s. The pressure at the nozzle exit is 5 kPa and the exit area is 0.7 m². What is the thrust of the engine in a vacuum?

Given:

$\dot{m} = 30$ kg/s

$V_j = 3{,}100$ m/s

$A_e = 0.7$ m^2

$P_e = 5$ kPa $= 5{,}000$ N/m^2

$P_a = 0$

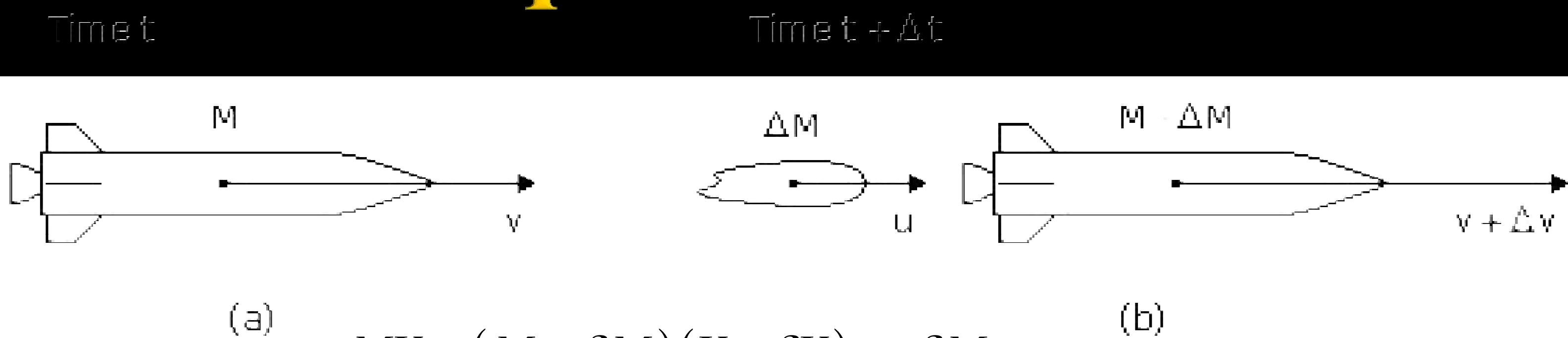
SOLUTION:

$$F = \dot{m} V_j + \left(P_e - P_a\right) A_e$$

$F = 30 \times 3{,}100 + (5{,}000 - 0) \times 0.7$

$F = 96{,}500$ N

Rocket Equation



$$MV = \left(M - \delta M\right)\left(V + \delta V\right) + u\delta M$$

$$MV = \left(M - \delta M\right)\left(V + \delta V\right) + \left(V + \delta V - V_J\right)\delta M$$

$$MV = MV + M\delta V - V\delta M - \delta M\delta V + V\delta M + \delta M\delta V - V_J\delta M$$

$$M\delta V - V_J\delta M = 0$$

$$\delta V = V_J \frac{\delta M}{M}$$

$$W.K.T \Rightarrow M = M_i - \dot{m}\,\tau$$

$$\delta M = \dot{m}\,\delta\tau$$

substituting the above equations

$$\delta V = V_J \frac{\dot{m}\,\delta\tau}{M_i - \dot{m}\,\tau}$$

Rocket Equation

$$\Delta V = \int_0^{\tau} \delta V = \int_0^{\tau} V_J \frac{\dot{m}\,\delta\tau}{M_i - \dot{m}\,\tau}$$

$$\Delta V = -V_J \ln\left(M_i - \dot{m}\,\tau \right)_0^{\tau_f}$$

$$\Delta V = -V_J \ln\left(\frac{M_f}{M_i} \right)$$

$$\Delta V = V_J \ln\left(\frac{M_i}{M_f} \right)$$

- A spacecraft's engine ejects mass at a rate of 30 kg/s with an exhaust velocity of 3,100 m/s. The pressure at the nozzle exit is 5 kPa and the exit area is 0.7 m^2 .it has an initial mass of 30,000 kg. What is the change in velocity if the spacecraft burns its engine for one minute?

Given:

M_i = 30,000 kg

$\dot{m}$ = 30 kg/s

V_j = 3,100 m/s

t = 60 s

SOLUTION

$$\Delta V = V_j \ln\left(\frac{M_i}{M_i - \dot{m}\,t}\right)$$

$$\Delta V = 3100 \times \ln\left(\frac{30000}{30000 - (30 \times 60)}\right)$$

$$\Delta V = 192 m/s$$

Definitions

- Mass Ratio:
  The ratio of final mass of the rocker after all the propellant has been consumed (M_f)to the initial mass before the rocket operation (M_i).

$$R_m = \frac{M_f}{M_i}$$

- Payload Mass Fraction $\alpha = \dfrac{M_u}{M_i}$
- Structural Mass Fraction $\beta = \dfrac{M_s}{M_i}$
- Propellant Mass Fraction $\gamma = \dfrac{M_P}{M_i}$

- A spacecraft's dry mass is 75,000 kg and the effective exhaust gas velocity of its main engine is 3,100 m/s. How much propellant must be carried if the propulsion system is to produce a total change in velocity of 700 m/s?

Given:

$Mf = 75{,}000$ kg

$V_j = 3{,}100$ m/s

^V = 700 m/s

SOLUTION

$Mi = Mf \times e^{(\wedge V / Vj)}$

$Mi = 75{,}000 \times e^{(700 / 3{,}100)}$

$Mi = 94{,}000$ kg

Propellant mass

$Mp = Mo - Mf$

$Mp = 94{,}000 - 75{,}000$

$Mp = 19{,}000$ kg

- A 5,000 kg spacecraft is in Earth orbit travelling at a velocity of 7,790 m/s. Its engine is burned to accelerate it to a velocity of 12,000 m/s placing it on an escape trajectory. The engine expels mass at a rate of 10 kg/s and an effective velocity of 3,000 m/s. Calculate the duration of the burn.

Given:

$Mi = 5{,}000$ kg

$\dot{m} = 10$ kg/s

$C = 3{,}000$ m/s

SOLUTION

$\hat{}V = 12{,}000 - 7{,}790 = 4{,}210$ m/s

$t = Mi / \dot{m} \times [\ 1 - 1 / e^{(\hat{}V / C)}\ ]$

$t = 5{,}000 / 10 \times [\ 1 - 1 / e^{(4{,}210 / 3{,}000)}\ ]$

$t = 377$ s

Performance Parameters of Rocket

- **Total Impulse**

The momentum associated with the efflux of propellants from the rocket to provide velocity to the rocket. The change in momentum is called impulse.

The *total impulse I* is the thrust force *F* (which can vary with time) integrated over the burning time *t*

$$I = \int_0^t F dt = Ft = \dot{m} V_J t = m V_J$$

- **Specific Impulse**

The total impulse delivered per unit weight of propellant.

$$I_{SP} = \frac{I}{w} = \frac{\dot{m} V_J t}{w} = \frac{F}{\dot{w}}$$

$$I_{SP} = \frac{\dot{m} V_J t}{w} = \frac{m V_J}{w} = \frac{m V_J}{m \times g} = \frac{V_J}{g}$$

Performance Parameters of Rocket

- **Thrust**
  The rate of change of impulse.

$$F = \frac{d}{dt} I = \frac{d}{dt}(mV_J) = \frac{dm}{dt} V_J = \dot{m} V_J$$

- **Impulse to mass ratio**
  The ratio of total impulse to the initial mass of the rocket.

$$\text{Impulse to mass ratio} = \frac{I}{M_i}$$

- **Thrust to mass ratio**
  The ratio of thrust to the initial mass of the rocket.

$$\text{Thrust to mass ratio} = \frac{F}{M_i}$$

- A rocket engine produces a thrust of 1,000 kN at sea level with a propellant flow rate of 400 kg/s. Calculate the specific impulse.

Given:

$F = 1{,}000{,}000\ \text{N}$

$\dot{m} = 400\ \text{kg/s}$

SOLUTION

$$Isp = F / (q \times g)$$

$$Isp = 1{,}000{,}000 / (400 \times 9.80665)$$

$$Isp = 255\ \text{s (sea level)}$$

Propulsive efficiency

- The ratio of the rate at which work is done by the rocket to the rate of energy supplied to it.

$$\eta_P = \frac{Vehicle\ power}{Vehicle\ power + residual\ kinetic\ jet\ power}$$

$$\eta_P = \frac{\dot{m} V V_J}{\dot{m} V V_J + \frac{1}{2}\dot{m}\left(V - V_J\right)^2}$$

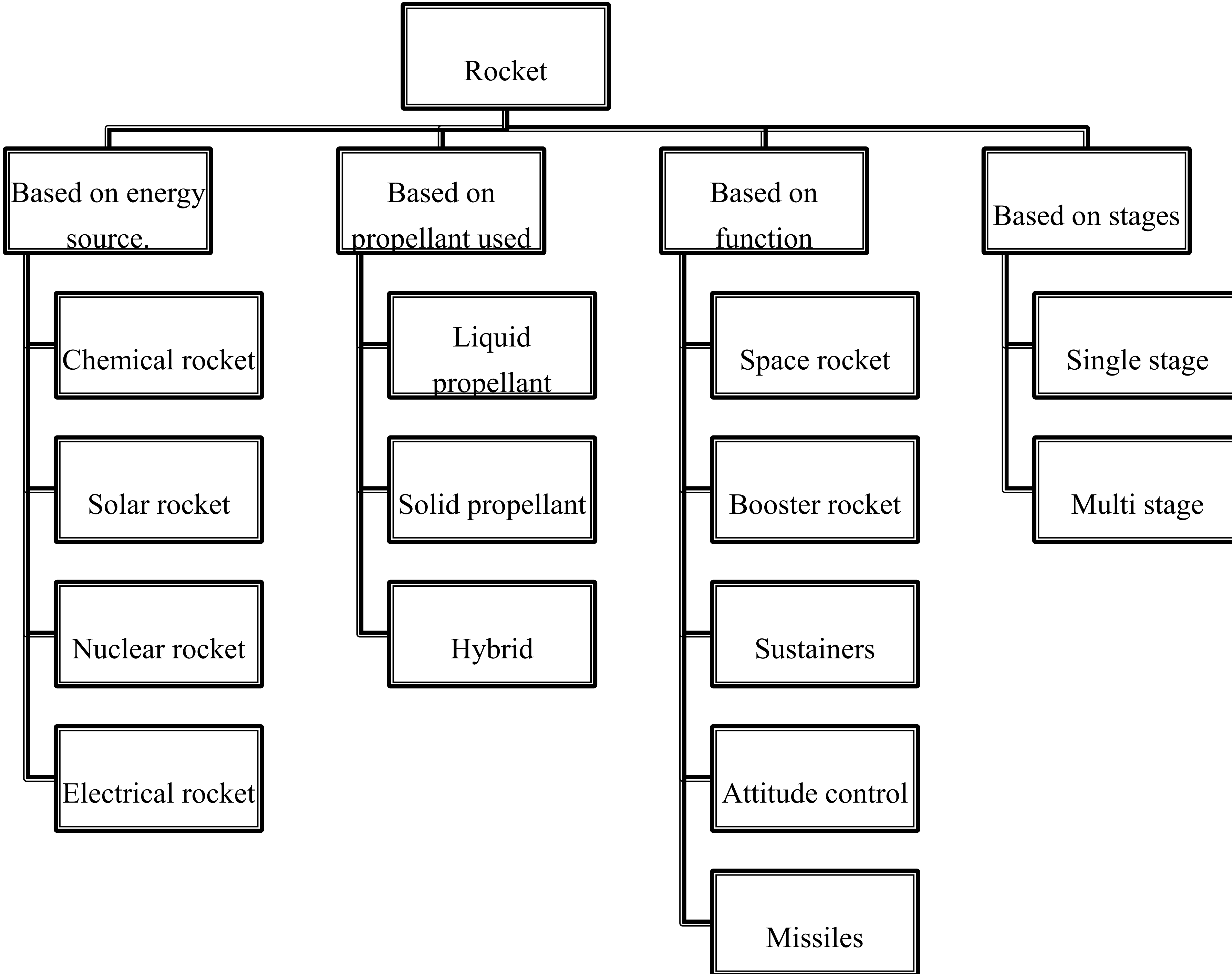
$$\eta_P = \frac{2VV_J}{V^2 + V_J^{\,2}} = \frac{2\frac{V}{V_J}}{1 + \left(\frac{V}{V_J}\right)^2}$$

Staging and Clustering

If series of rocket were put one on top of the other and operated separately in stages one after the other, the total velocity would be the sum of the ideal velocities provided by the operation of each stage of rocket.

$$\Delta V = \Delta V_1 + \Delta V_2 + \Delta V_3 + \ldots\ldots + \Delta V_n$$

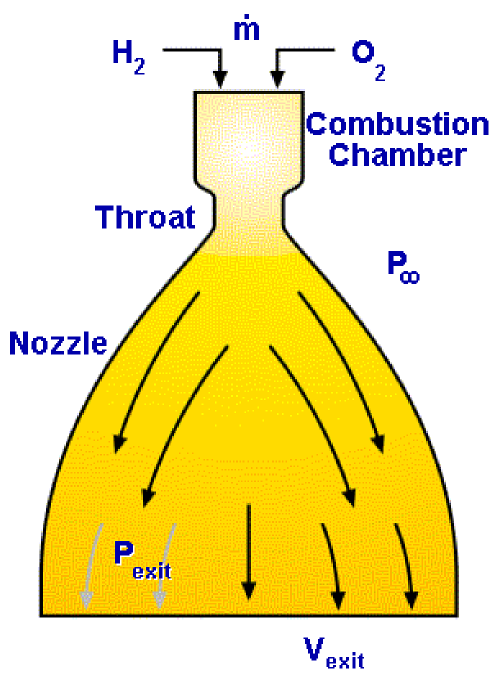
$$\Delta V = V_{J1} \ln\left(\frac{1}{Rm1}\right) + V_{J2} \ln\left(\frac{1}{Rm2}\right) + \ldots\ldots\ldots + V_{Jn} \ln\left(\frac{1}{Rmn}\right)$$


Rocket
Based on energy source.
Chemical rocket
Solar rocket
Nuclear rocket
Electrical rocket
Based on propellant used
Liquid propellant
Solid propellant
Hybrid
Based on function
Space rocket
Booster rocket
Sustainers
Attitude control
Missiles
Based on stages
Single stage
Multi stage

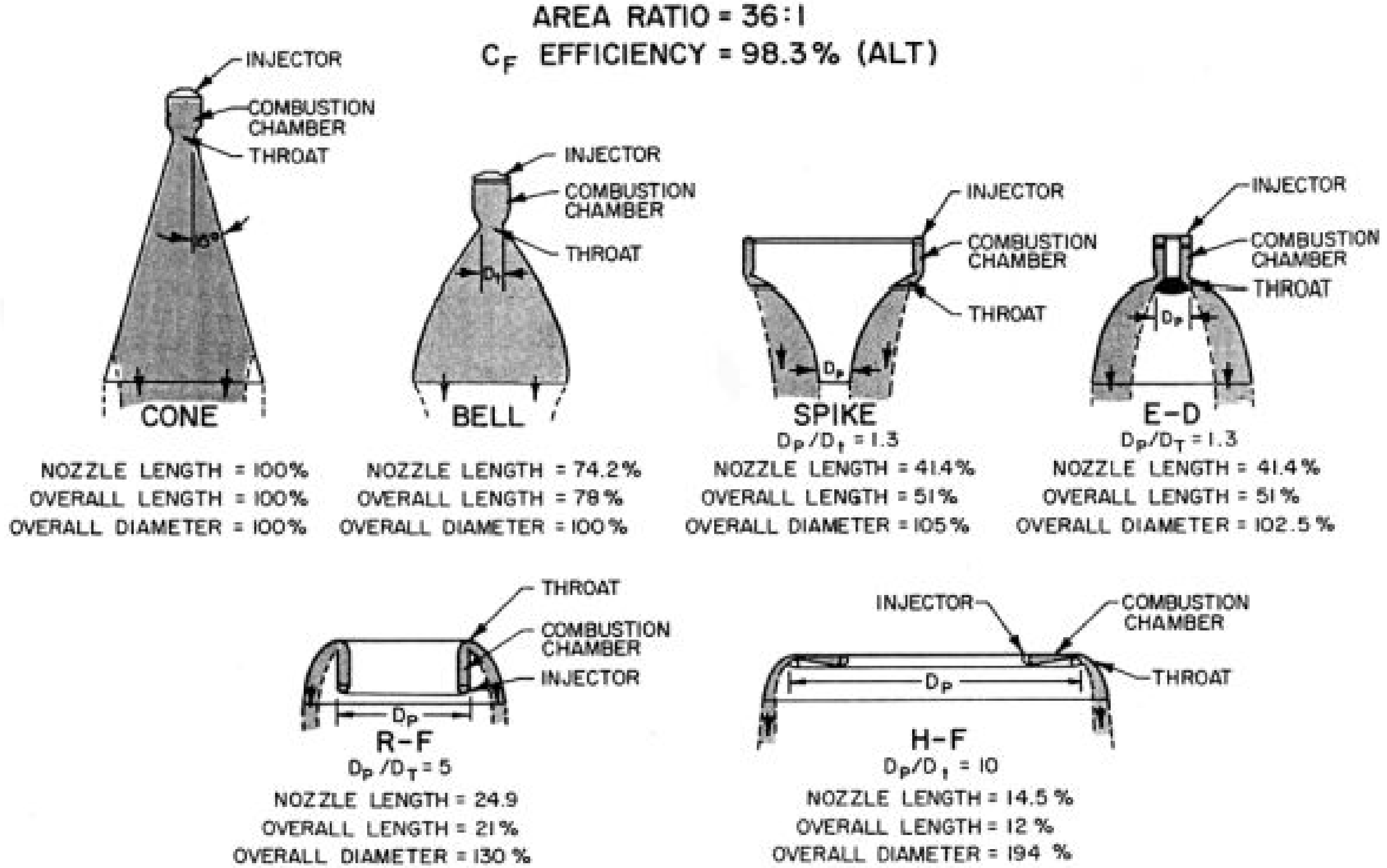
Rocket Nozzle

- Rocket nozzle is a device or a duct of smoothly varying cross sectional area to increase the velocity of a fluid at the expense of pressure.

$$F = \dot{m} V_j + \left(P_e - P_a \right) A_e$$



Types of nozzle



Nozzle Area Ratio

$$\dot{m} = \rho_t A_t V_t = \rho_e A_e V_e$$

$$\varepsilon = \frac{A_e}{A_t} = \frac{\rho_t}{\rho_e}\frac{V_t}{V_e}$$

$$\varepsilon = \frac{P_t}{P_e}\frac{T_e}{T_t}\frac{\sqrt{\gamma R T_t}}{\sqrt{\frac{2\gamma R T_c}{\gamma - 1}\left[1 - \left(\frac{P_e}{P_c}\right)^{\frac{\gamma-1}{\gamma}}\right]}}$$

$$\varepsilon = \frac{P_t}{P_c}\frac{P_c}{P_e}\frac{T_e}{T_c}\frac{T_c}{T_t}\frac{\sqrt{\frac{T_t}{T_c}}}{\sqrt{\frac{2}{\gamma - 1}\left[1 - \left(\frac{P_e}{P_c}\right)^{\frac{\gamma-1}{\gamma}}\right]}}$$

$$w.k.t\,\frac{T_t}{T_c}=\frac{2}{\gamma+1}$$

$$\frac{P_t}{P_c}=\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}}$$

$$\frac{T_e}{T_c}=\left(\frac{P_e}{P_c}\right)^{\frac{\gamma-1}{\gamma}}$$

$$\varepsilon=\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}}\frac{P_c}{P_e}\left(\frac{P_e}{P_c}\right)^{\frac{\gamma-1}{\gamma}}\left(\frac{2}{\gamma+1}\right)^{-1}\frac{\sqrt{\frac{2}{\gamma+1}}}{\sqrt{\frac{2}{\gamma-1}\left[1-\left(\frac{P_e}{P_c}\right)^{\frac{\gamma-1}{\gamma}}\right]}}$$

$$\varepsilon=\frac{\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}-\frac{1}{2}}\left(\frac{P_c}{P_e}\right)^{1-\frac{\gamma-1}{\gamma}}}{\sqrt{\frac{2}{\gamma-1}\left[1-\left(\frac{P_e}{P_c}\right)^{\frac{\gamma-1}{\gamma}}\right]}}=\frac{\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{2(\gamma-1)}}\left(\frac{P_c}{P_e}\right)^{\frac{1}{\gamma}}}{\sqrt{\frac{2}{\gamma-1}\left[1-\left(\frac{P_e}{P_c}\right)^{\frac{\gamma-1}{\gamma}}\right]}}$$

Mass Flow Rate and Characteristic Velocity

$$\dot{m} = \rho_t A_t V_t = \frac{\rho_t}{\rho_c}\rho_c A_t \sqrt{\gamma R T_t}$$

$$\dot{m} = \frac{P_t}{RT_t}\frac{RT_c}{P_c}\frac{P_c}{RT_c} A_t \sqrt{\gamma R T_t}$$

$$\dot{m} = \frac{P_t}{P_c}\frac{T_c}{T_t}\frac{P_c A_t}{\sqrt{RT_c}}\sqrt{\gamma}\sqrt{\frac{T_t}{T_c}}$$

$$\dot{m} = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}}\left(\frac{2}{\gamma+1}\right)^{-1}\frac{P_c A_t}{\sqrt{RT_c}}\sqrt{\gamma}\sqrt{\frac{2}{\gamma+1}}$$

$$\dot{m} = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}-1+\frac{1}{2}}\frac{P_c A_t}{\sqrt{RT_c}}\sqrt{\gamma}$$

$$\dot{m} = \sqrt{\gamma}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{2(\gamma-1)}}\frac{P_c A_t}{\sqrt{RT_c}}$$

$$\dot{m} = \Gamma\frac{P_c A_t}{\sqrt{RT_c}} = \frac{P_c A_t}{C^*}$$

$$where\, C^* = Characteristic\, Velocity$$

Thrust coefficient

$$F = \dot{m} V_j + \left(P_e - P_a\right) A_e$$

$$F = \sqrt{\gamma}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{2(\gamma-1)}} \frac{P_c A_t}{\sqrt{RT_c}} \sqrt{\frac{2\gamma RT_c}{\gamma-1}\left[1-\left(\frac{P_e}{P_c}\right)^{\frac{\gamma-1}{\gamma}}\right]} + \left(P_e - P_a\right) A_e$$

$$F = P_c A_t \left(\sqrt{\gamma}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{2(\gamma-1)}} \frac{1}{\sqrt{RT_c}} \sqrt{\frac{2\gamma RT_c}{\gamma-1}\left[1-\left(\frac{P_e}{P_c}\right)^{\frac{\gamma-1}{\gamma}}\right]} + \left(\frac{P_e}{P_c} - \frac{P_a}{P_c}\right)\frac{A_e}{A_t} \right)$$

$$C_F = \left(\sqrt{\gamma}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{2(\gamma-1)}} \frac{1}{\sqrt{RT_c}} \sqrt{\frac{2\gamma RT_c}{\gamma-1}\left[1-\left(\frac{P_e}{P_c}\right)^{\frac{\gamma-1}{\gamma}}\right]} + \left(\frac{P_e}{P_c} - \frac{P_a}{P_c}\right)\frac{A_e}{A_t} \right)$$

$$C^* = Characteristic\,Velocity = \frac{V_j}{C_F}$$

1. Hot gases are generated at a temperature of 2000K and a pressure of 15MPa in a rocket chamber. The molecular mass of the gas is 22KJ/Kmole and the specific heat ratio of the gas is 1.32. The gases are expanded to the ambient pressure of 0.1 Mpa in a C-D nozzle having a throat area of 0.1 m^2 . Calculate
    i. Exit jet velocity
    ii. Characteristic velocity
    iii. Ideal optimum thrust coefficient
    iv. Specific impulse and
    v. Thrust.

solution

i.Exit jet velocity

$$V_J = \sqrt{\frac{2\gamma RT_c}{\gamma-1}\left[1-\left(\frac{P_e}{P_c}\right)^{\frac{\gamma-1}{\gamma}}\right]} = \sqrt{\frac{2\gamma R_0 T_c}{Molecular\, mass\,(\gamma-1)}\left[1-\left(\frac{P_e}{P_c}\right)^{\frac{\gamma-1}{\gamma}}\right]}$$

$$V_J = \sqrt{\frac{2\times1.32\times8.314\times2000}{22\times(1.32-1)}\left[1-\left(\frac{0.1}{15}\right)^{\frac{1.32-1}{1.32}}\right]}$$

$$V_J = 2094 m/s$$

ii.Characteristic velocity

$$\Gamma = \sqrt{\gamma}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{2(\gamma-1)}} = 0.671$$

$$C^* = \frac{1}{\Gamma}\sqrt{RT_c} = \frac{1}{\Gamma}\sqrt{\frac{R_0 T_c}{Molecular\ mass}}$$

$$C^* = \frac{1}{0.671}\sqrt{\frac{8.314\times 2000}{22}} = 1296 m/s$$

$$C^0{}_F = \left(\sqrt{\gamma}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{2(\gamma-1)}} \frac{1}{\sqrt{RT_c}}\sqrt{\frac{2\gamma RT_c}{\gamma-1}\left[1-\left(\frac{P_e}{P_c}\right)^{\frac{\gamma-1}{\gamma}}\right]}\right) = 1.615$$

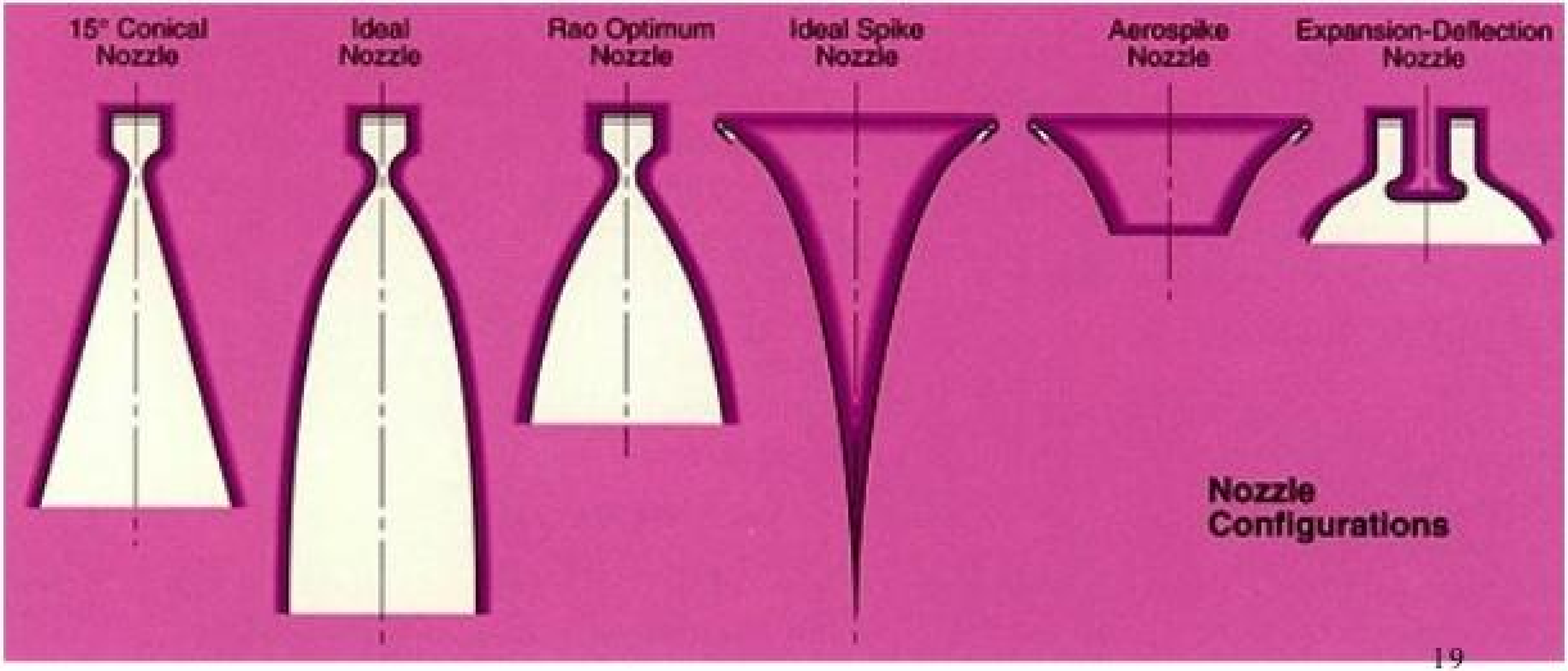
$$I_{sp} = \frac{V_J}{g} = \frac{2094}{9.81} = 213.45s$$

$$F = C^0{}_F P_c A_t = 1.615\times 15\times 10^6\times 0.1 = 2.42MN$$

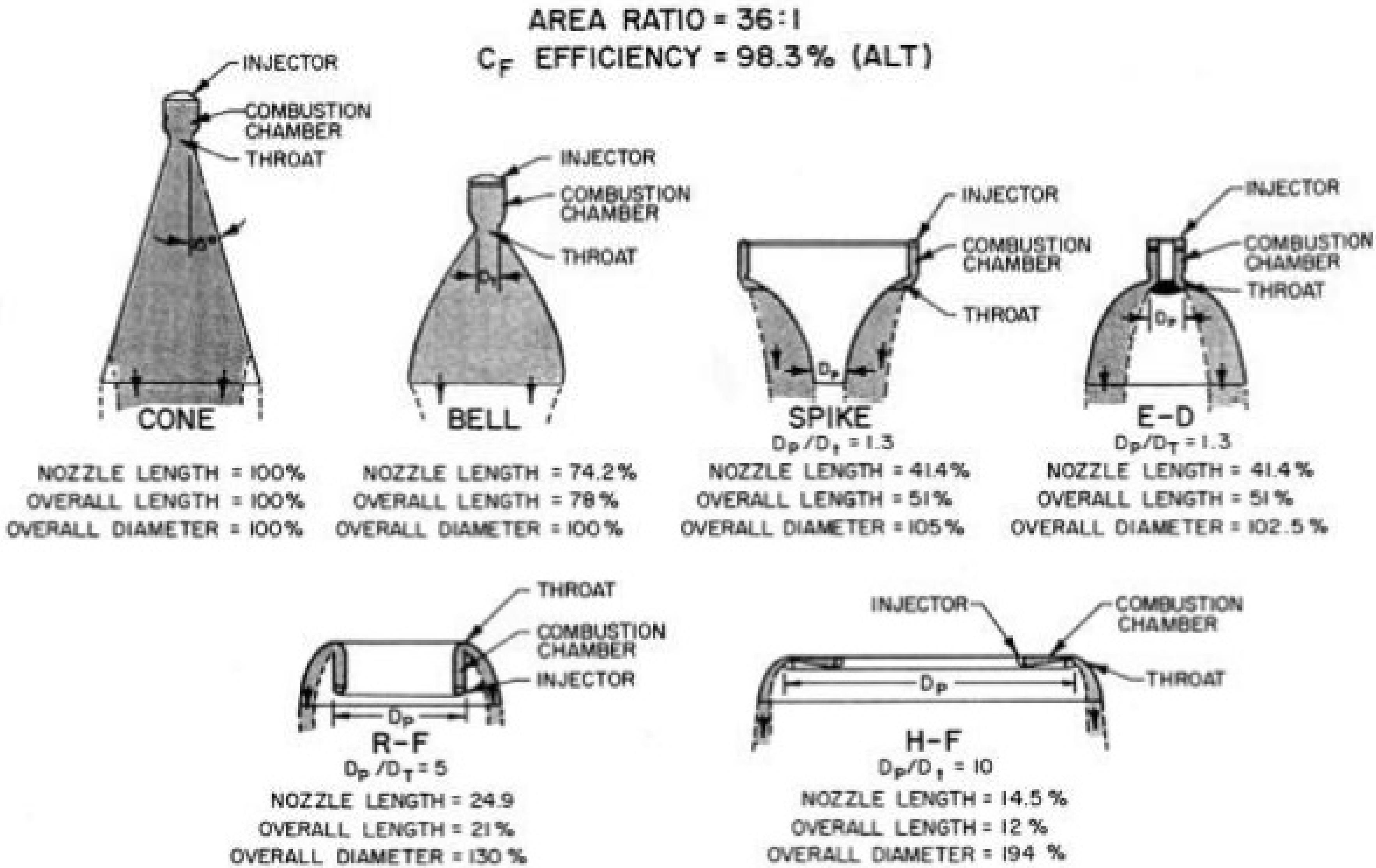
NOZZLE TYPES

3 primary groups of nozzle types

1. Cone (conical, linear)
2. Bell (contoured, shaped, classic converging-diverging)
3. Annular (spike, aerospike, plug, expansion, expansion-deflection)

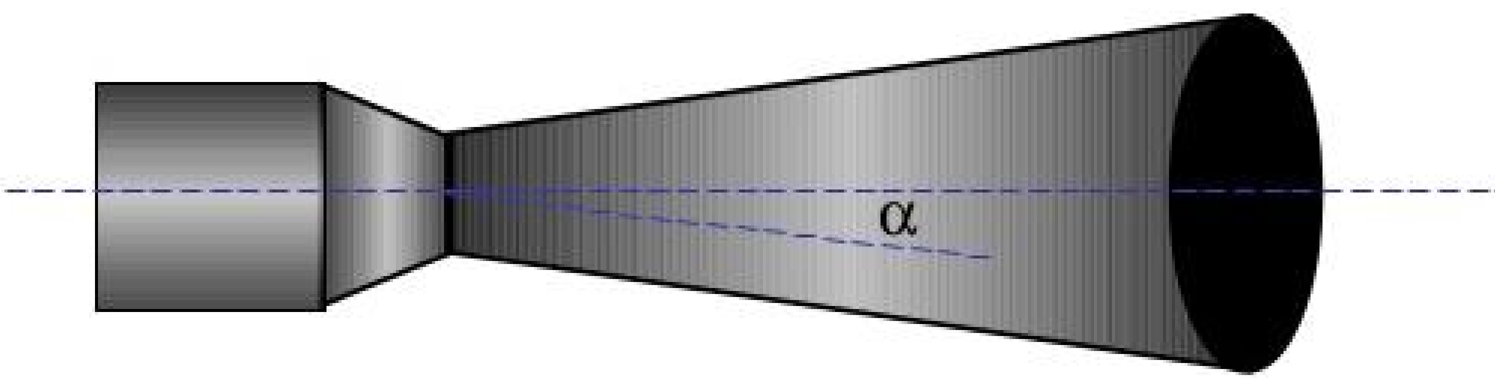


NOZZLE EXAMPLES



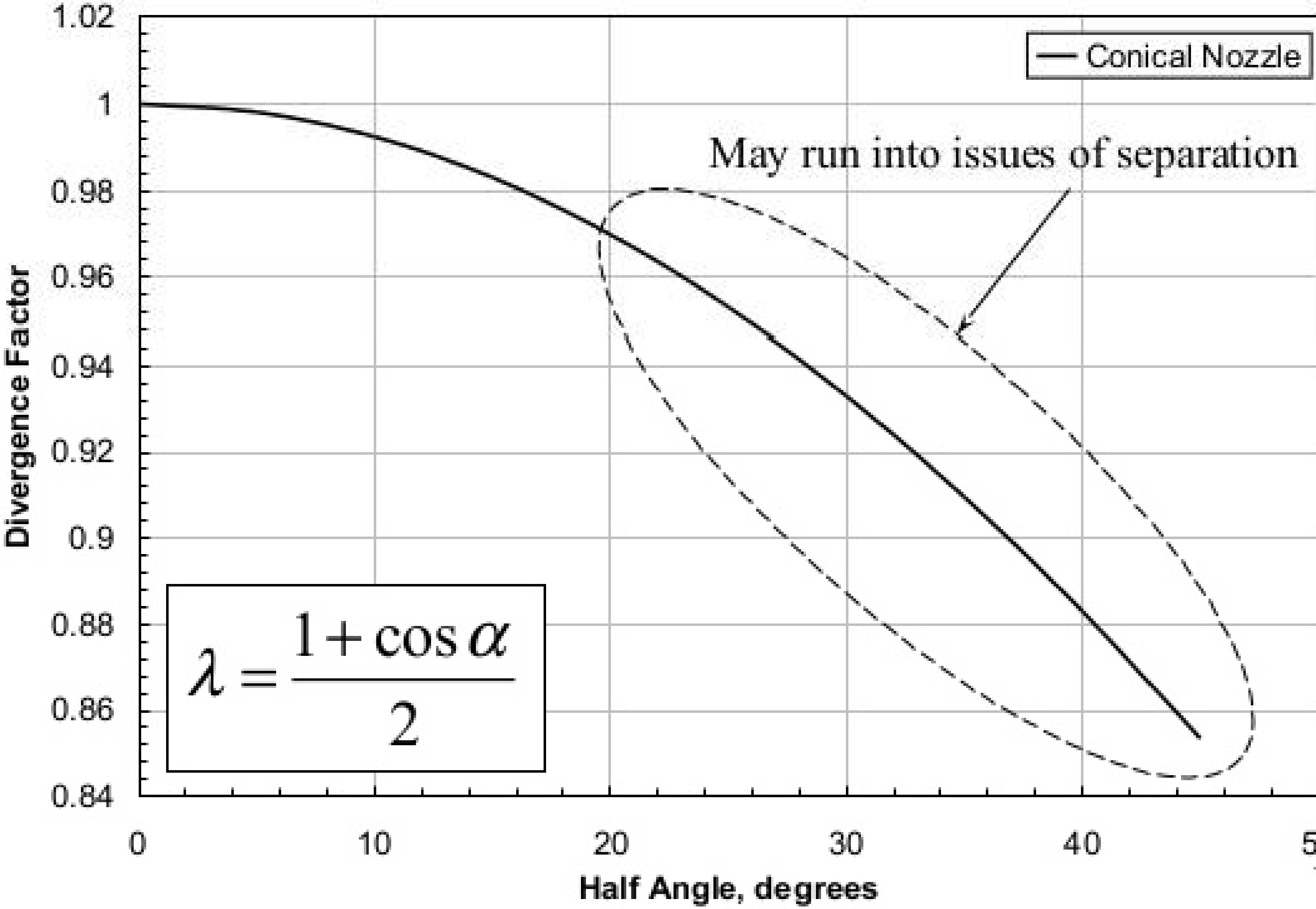
1. CONICAL NOZZLES

- Used in early rocket applications because of simplicity and ease of construction
- Cone gets its name from the fact that the walls diverge at a constant angle
- A small angle produces greater thrust, because it maximizes the axial component of exit velocity and produces a high specific impulse
- Penalty is longer and heavier nozzle that is more complex to build
- At the other extreme, size and weight are minimized by a large nozzle wall angle
  - Large angles reduce performance at low altitude because high ambient pressure causes overexpansion and flow separation
- Primary Metric of Characterization: Divergence Loss



CONICAL NOZZLES: SOME DETAILS

- Deviation of flow from axial (thrust direction) is called the divergence factor
- Longer Nozzle → Higher Thrust and Increased Weight



$$\lambda = \frac{1 + \cos\alpha}{2}$$

SIZING OF CONICAL NOZZLES

For a conical nozzle with half-angle α, length L, and diameter D^*, the ratio of exit to throat area is:

$$\frac{A_e}{A^*} = \left(\frac{D^* + 2L\tan\alpha}{D^*} \right)^2$$

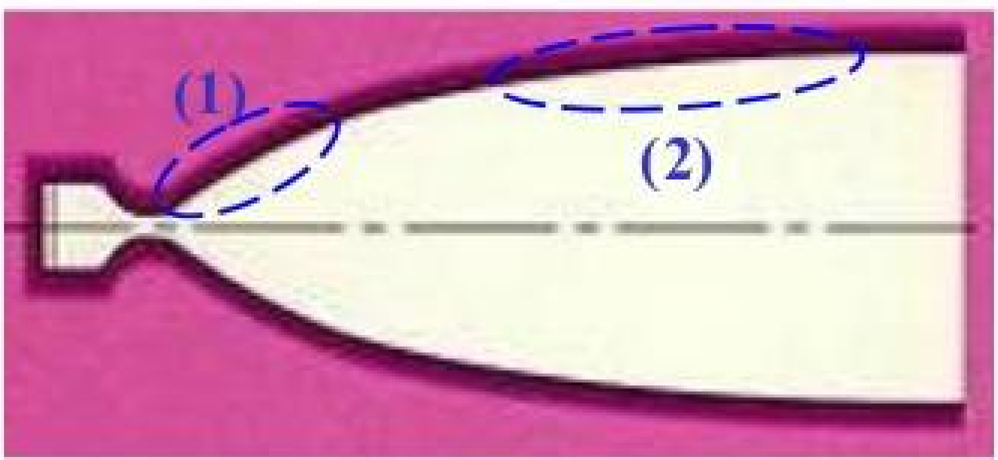
Solving for the Length of the nozzle knowing the area ratio, throat diameter and desired nozzle half angle

$$L = \frac{D^* \left(\sqrt{\frac{A_e}{A^*}} - 1 \right)}{2\tan\alpha}$$

- Throat diameter D^* is fixed by combustion chamber conditions and desired thrust
  - Nozzle length and mass are strongly dependent on α
- **Example:**
  - For area ratio of 100, $L/D^* = 7.8$ for 30º and 16.8 for 15º
  - Reducing α from 30º to 15º would more than double the mass of divergent portion of the nozzle

2. BELL NOZZLES

- Bell is most commonly used nozzle shape
- Offers significant advantages over conical nozzle, both in size and performance
- Bell consists of two sections
  - Near throat, nozzle diverges at relatively large angle, (1)
  - Degree of divergence tapers off further downstream
  - Near nozzle exit, divergence angle is very small ~2°-8°, (2)
  - Minimize weight / maximize performance ~10-25% shorter than conic
- Issue is to contour nozzle to avoid oblique shocks and maximize performance
- Remember: Shape only optimum at one altitude



3. ANNULAR NOZZLES

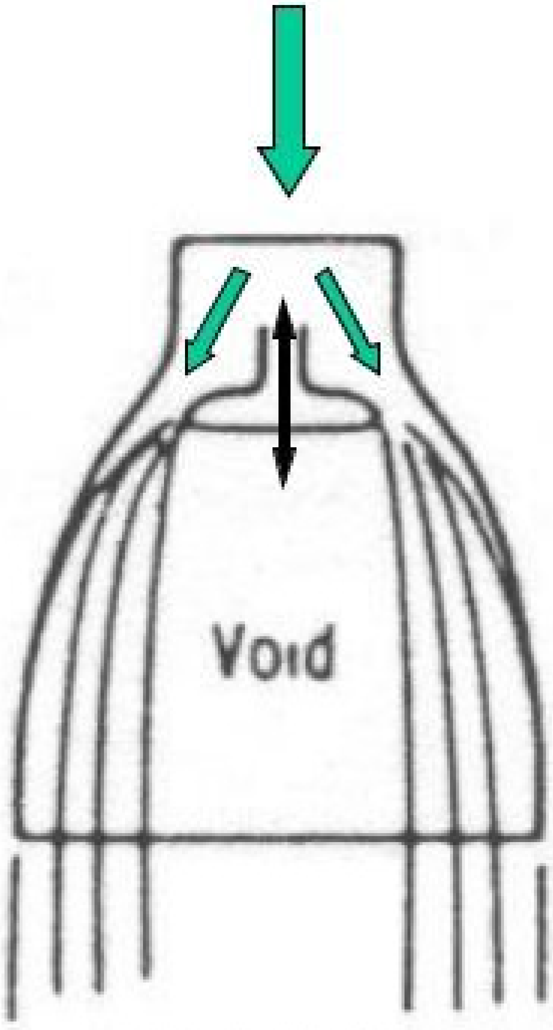
- Annular (plug or altitude-compensating) nozzle
  - Least employed due to greater complexity, actually be best in theory
  - Annular: combustion occurs along ring, or annulus, around base of nozzle
  - Plug: refers to centerbody that blocks flow from what would be center portion of traditional nozzle
  - Primary advantage: Altitude-compensating

- Expansion ratio: area of centerbody must be taken into account

$$\varepsilon = \frac{A_{exit} - A_{plug}}{A_{throat}}$$

- Another parameter annular diameter ratio, D_{plug} / D_{throat}
  - Ratio is used as a measure of nozzle geometry for comparison with other plug nozzle shapes

RADIAL OUT-FLOW NOZZLES



Flow clings to the walls

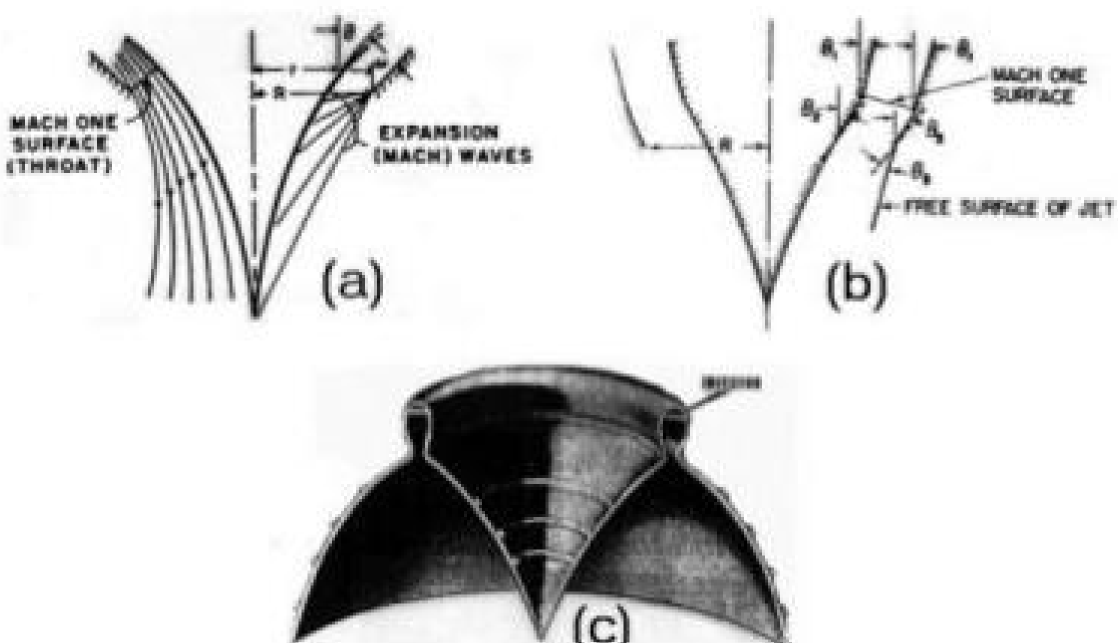
- Picture shows an example of an Expansion-Deflection (E-D) nozzle
- Expansion-deflection nozzle works much like a bell nozzle
- Exhaust gases forced into a converging throat before expanding in a bell-shaped nozzle
- Flow is deflected by a plug, or centerbody, that forces the gases away from center of nozzle and to stay attached to nozzle walls
- Centerbody position may move to optimize performance
- As altitude or back-pressure varies, flow is free to expand into 'void'
  - This expansion into void allows the nozzle to compensate for altitude
  - Pe adjusts to Pb within nozzle

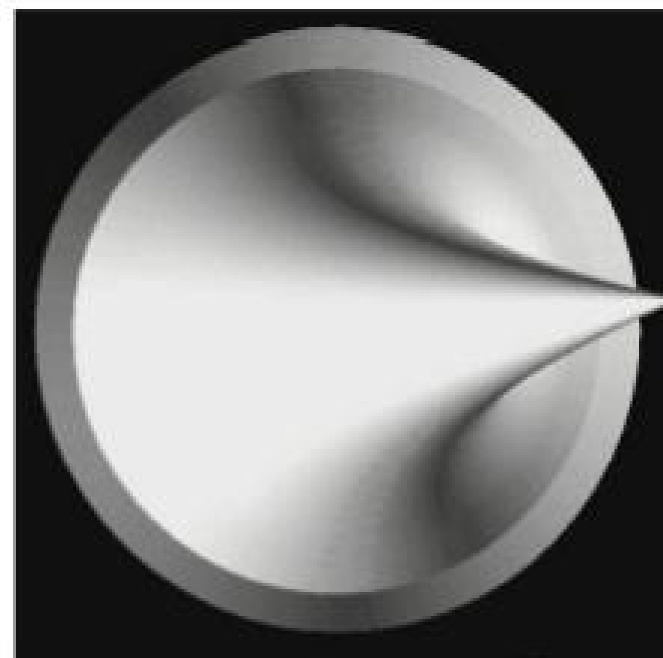
RADIAL OUT-FLOW NOZZLES: DETAILS

- Name of each of these nozzles indicates how it functions
- Expansion-deflection nozzle works much like a bell nozzle since the exhaust gases are forced into a converging throat region of low area before expanding in a bell-shaped nozzle. However, the flow is deflected by a plug, or centerbody, that forces the gases away from the center of the nozzle. Thus, the E-D is a radial out-flow nozzle.
- The reverse-flow nozzle gets its name because the fuel is injected from underneath, but the exhaust gases are rotated 180° thereby reversing their direction. Similarly, the fuel in the horizontal-flow nozzle is injected sideways, but the exhaust is rotated 90°.
- The E-D, has been one of the most studied forms of annular nozzles. While similar in nature to the bell nozzle, the most notable difference is the addition of a centerbody. As shown below, this "plug" may be located upstream of, downstream of, or in the throat, with each location resulting in better performance for a given set of operating conditions.
- The purpose of the centerbody is to force flow to remain attached to, to stick to nozzle walls
- This behavior is desirable at low altitudes because the atmospheric pressure is high and may be greater than pressure of exhaust gases. When this occurs, the exhaust is forced inward and no longer exerts force on the nozzle walls, so thrust is decreased and the rocket becomes less efficient. The centerbody, however, increases the pressure of the exhaust gases by squeezing the gases into a smaller area thereby virtually eliminating any loss in thrust at low altitude.

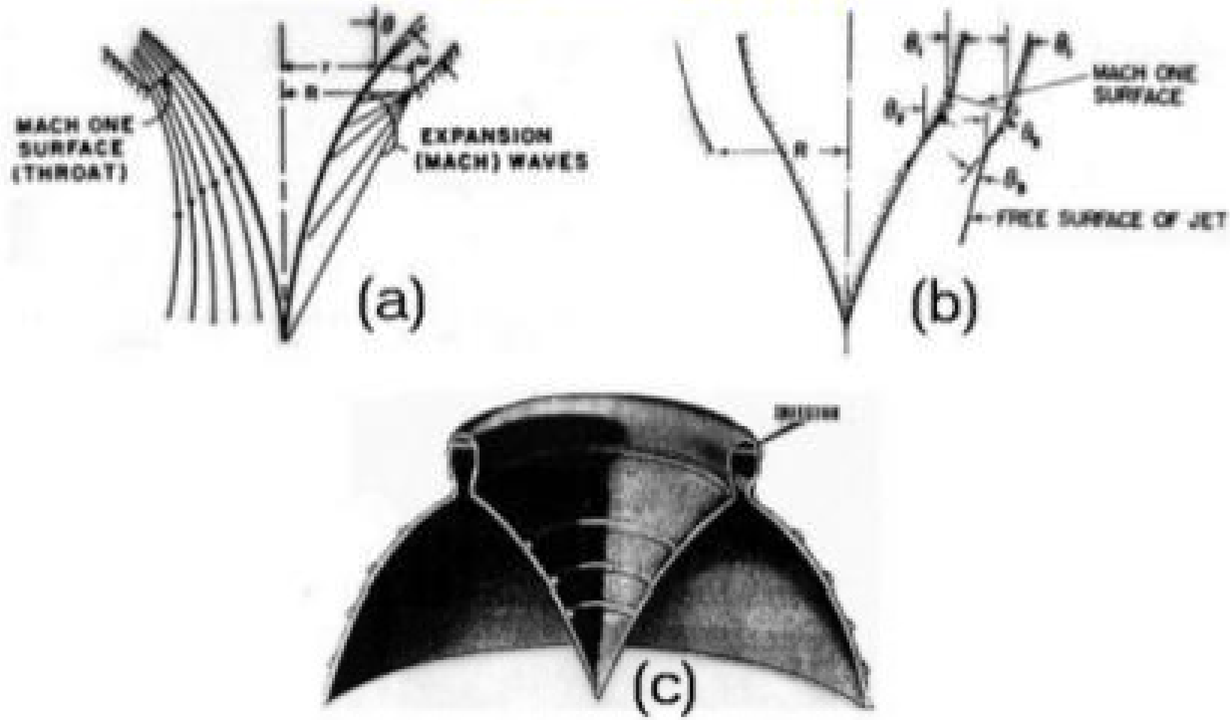
RADIAL IN-FLOW NOZZLES

- Often referred to as spike nozzles
  - Named for prominent spike centerbody
  - May be thought of as a bell turned inside out
  - Nozzle is only one of many possible spike configurations
    - (a) traditional curved spike with completely external supersonic expansion
    - (b) similar shape in which part of the expansion occurs internally
    - (c) design similar to E-D nozzle in which all expansion occurs internally




The ideal 100% length aerospike nozzle

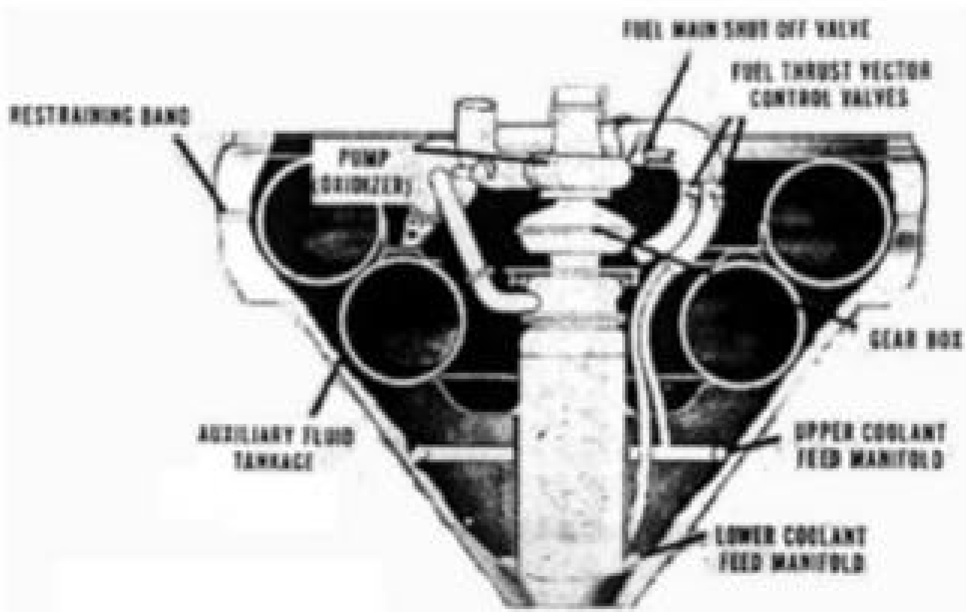
SPIKE NOZZLES

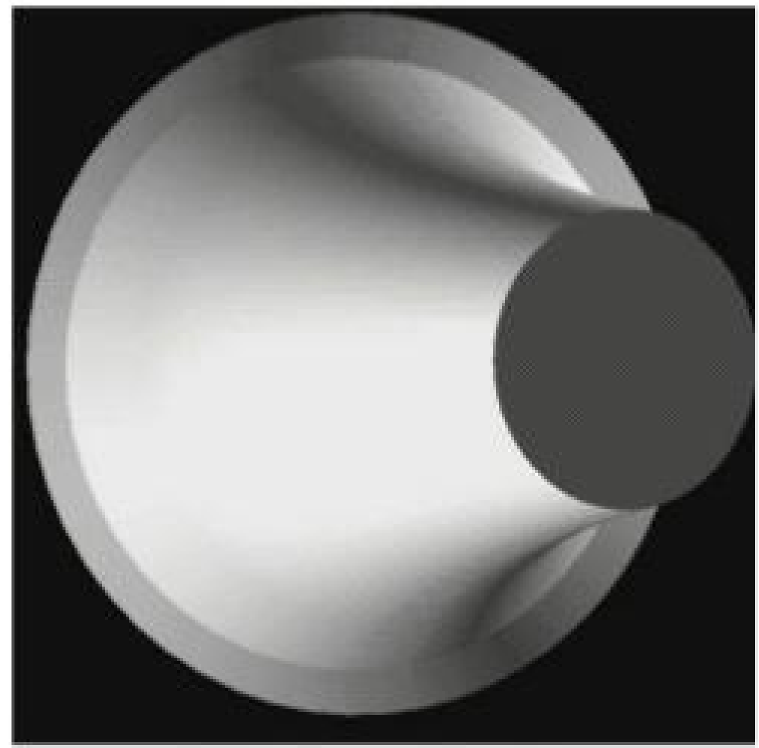


- Each of spike nozzles features a curved, pointed spike
  - Most ideal shape
- Spike shape allows exhaust gases to expand through isentropic process
- Nozzle efficiency is maximized and no energy is lost because of turbulent mixing
- Isentropic spike may be most efficient but tends to be prohibitively long and heavy
- Replace curve shape by shorter and easier to construct cone ~1% performance loss

AEROSPIKE NOZZLES

- Further subclass of radial in-flow family of spike nozzles is known as aerospike
- Go even further by removing pointed spike altogether and replace with a flat base
  - This configuration is known as a truncated spike

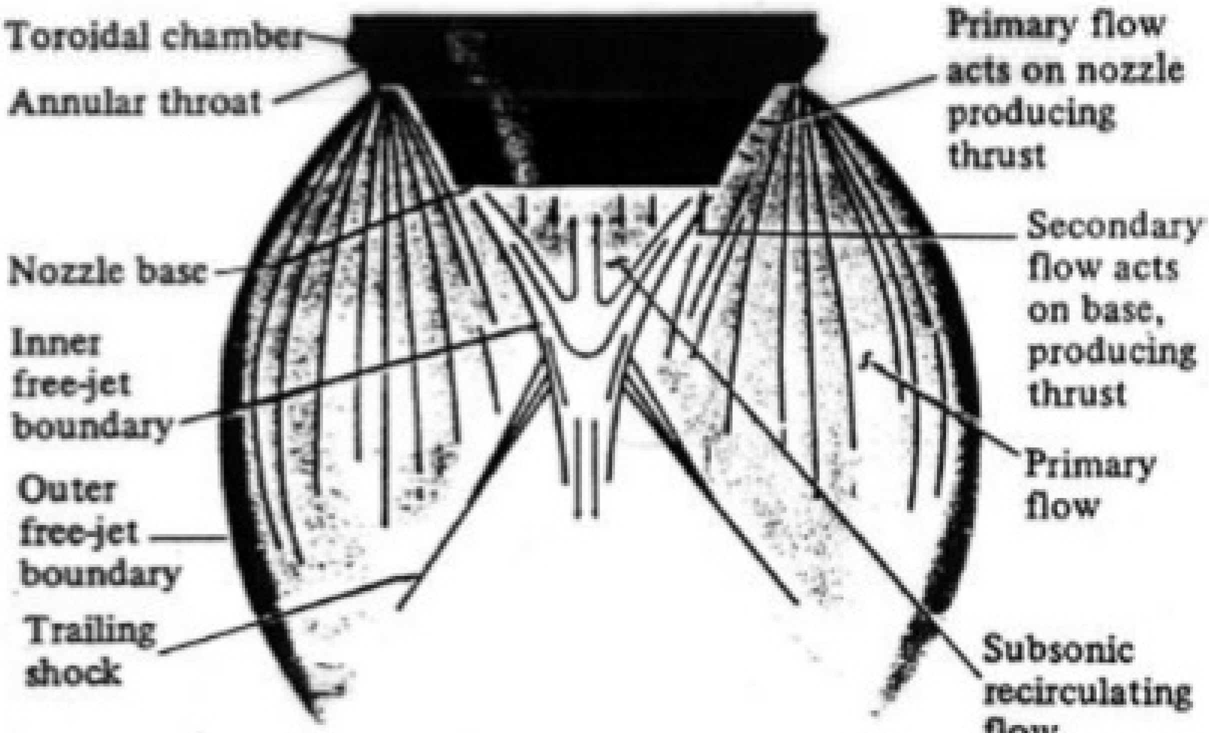




40% length aerospike nozzle

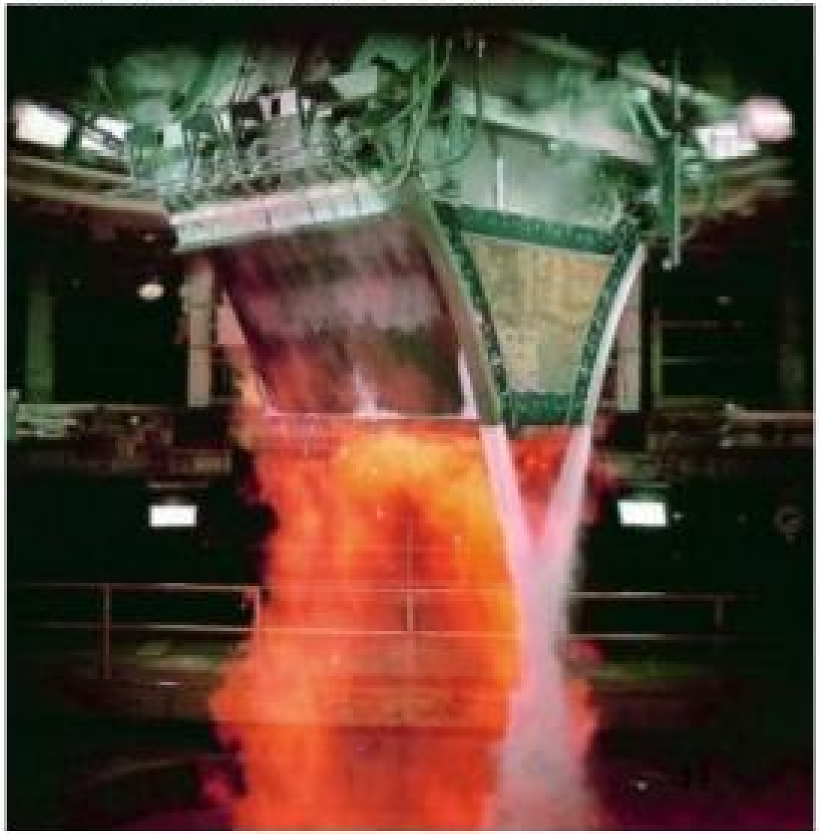
TRUNCATED AEROSPIKE NOZZLES

- Disadvantage of "flat" plug is turbulent wake forms aft of base at high altitudes resulting in high base drag and reduced efficiency
- Alleviated by introducing a "base bleed," or secondary subsonic flow
- Circulation of this secondary flow and its interaction with the engine exhaust creates an "aerodynamic spike" that behaves much like the ideal, isentropic spike
- Secondary flow re-circulates upward pushing on base to produce additional thrust
- It is this artificial aerodynamic spike for which the aerospike nozzle is named

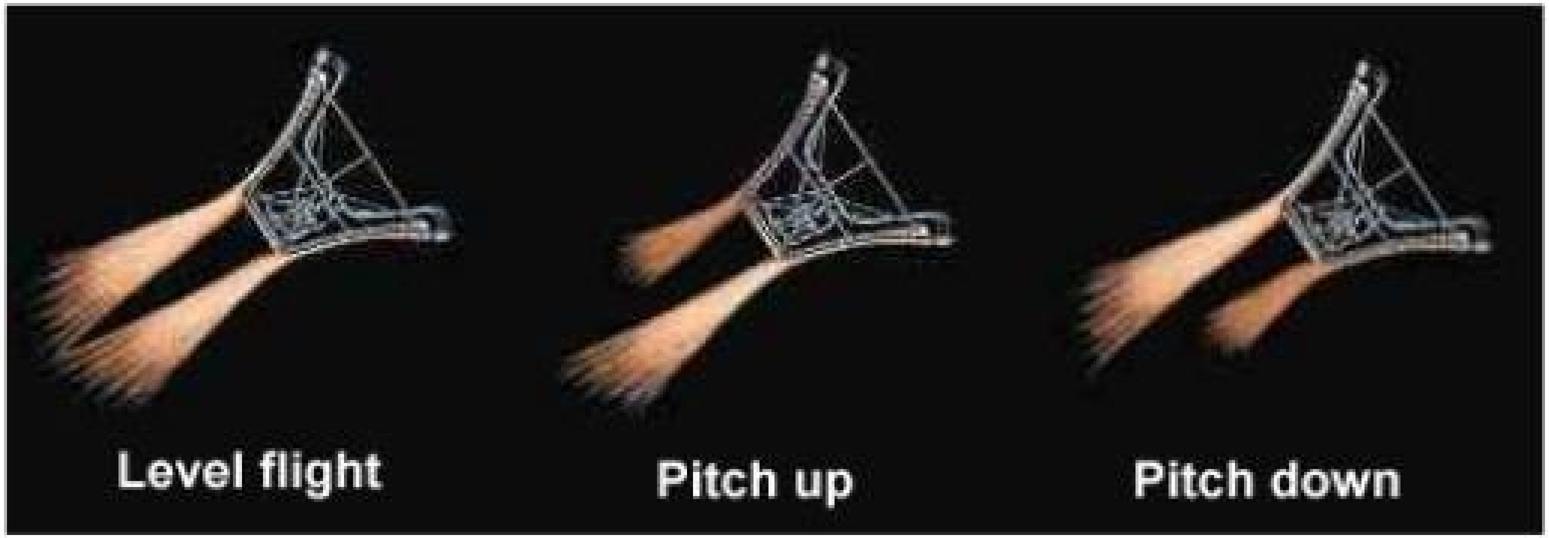


LINEAR AEROSPIKE NOZZLE

- Still another variation of aerospike nozzle is linear (instead of annular)
- Linear Aerospike pioneered by Rocketdyne (now division of Boeing) in 1970's
- Places combustion chambers in a line along two sides of nozzle
- Approach results in more versatile design
  - Use of lower-cost modular combustors
  - Modules can be combined in varying configurations depending on application.



LINEAR AEROSPIKE NOZZLE

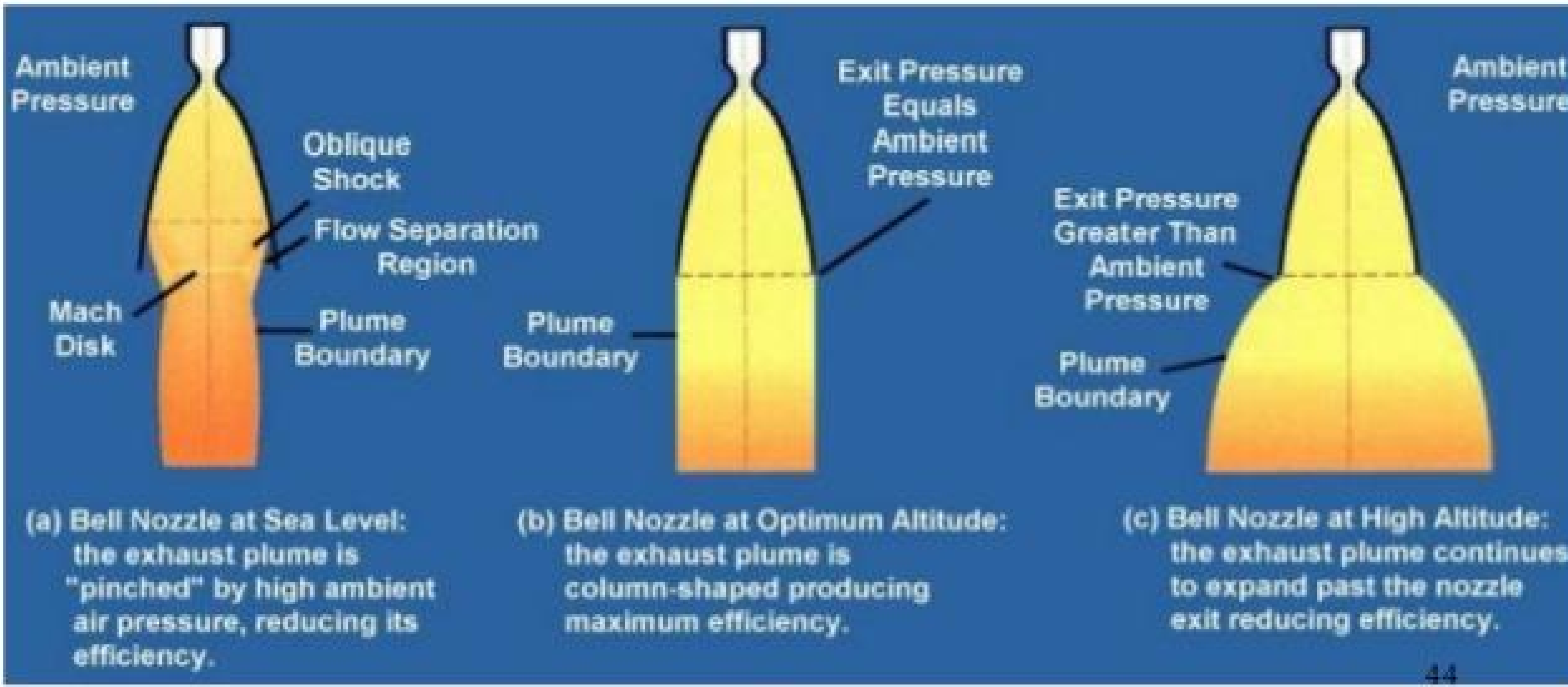


ALTITUDE BEHAVIOR: BELL

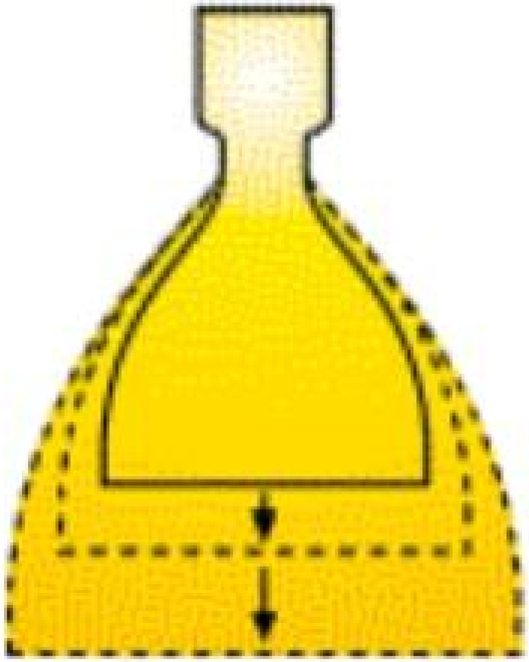
- Low Altitude
- OVER-Expanded
  - Pe < Pa
  - Do not expand beyond Pe=0.4 Pa

- Intermediate Altitude
- Ideally-Expanded
  - Pe = Pa

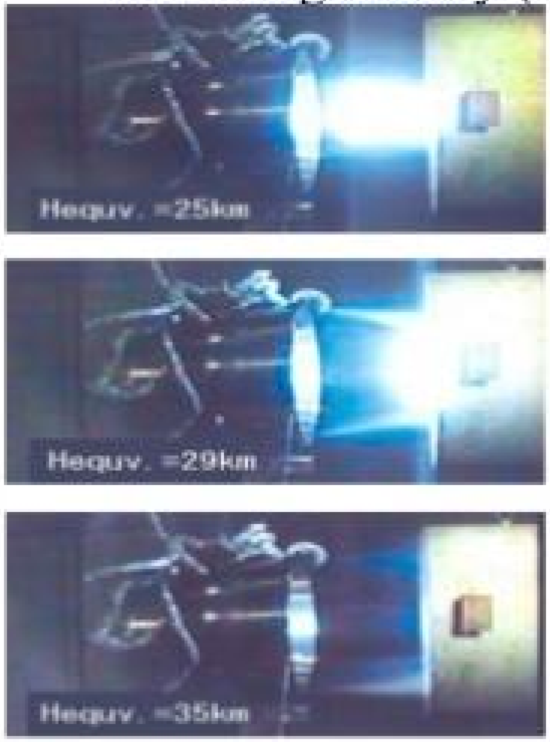
- High Altitude
- Under-Expanded
  - Pe > Pa



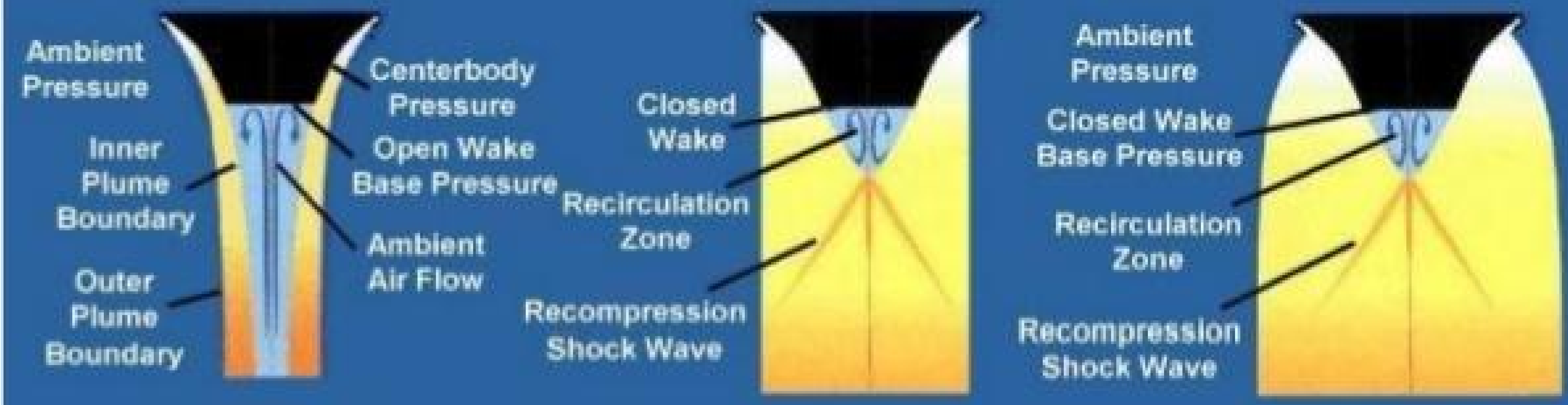
ALTITUDE COMPENSATION: BELL



- Ideal situation would be to have size of nozzle bell increase as altitude increases
- Altitude Adaptive Nozzles:
  - Dual-Bell Nozzle
  - Inserts, fixed and ejectable
  - Gas injection
  - Variable geometry (two-position)



ALTITUDE COMPENSATION: ANNULAR



(a) Aerospike at Sea Level: high ambient pressure forces the exhaust to remain close to the centerbody maintaining high efficiency.

(b) Aerospike at Optimum Altitude: the exhaust plume is column-shaped producing maximum efficiency.

(c) Aerospike at High Altitude: the exhaust plume is bound by shock waves that force it to remain column-shaped for high efficiency.

SUMMARY: SHAPED (BELL) NOZZLES

ADVANTAGES

- Structural Considerations
  - Essentially only hoop or tangential stresses which are easiest to design for
- Cooling
  - Fabricated with walls of simple tubular construction that enables cooling in a straightforward way
- Matching to combustion chamber
  - Relatively easy to match the combustor, which is most naturally a simple cylinder

DISADVANTAGES

- Over-Expansion Thrust Loss
- Flow instability when over-expanded
  - May lead to uncertainty or unsteadiness of the thrust direction and dangerous high frequency wobble

ANNULAR: ADVANTAGES

- Smaller nozzle
  - Truncated spike far smaller than typical bell nozzle for same performance
  - Spike can give greater performance for a given length
- Altitude compensation results in greater performance (no separation at over-expanded)
- Less risk of failure
  - Aerospike engine use simple gas generator cycle with a lower chamber pressure
  - Low pressures → reduced performance, high expansion ratio makes up for deficiency
- Lower vehicle drag
  - Aerospike nozzle fills base portion of vehicle thereby reducing base drag
- Modular combustion chambers
  - Linear aerospike engine is made up of small, easier to develop, less expensive thrusters
- Thrust Vectoring
  - Combustion chambers controlled individually
  - Vehicle maneuvered using differential thrust vectoring
  - Eliminates heavy gimbals and actuators used to vary direction of nozzles

ANNULAR: DISADVANTAGES

- Cooling
  - Central spike experiences far greater heat fluxes than does a bell nozzle
  - Addressed by truncating spike to reduce exposed area and by passing cold cryogenically-cooled fuel through spike
  - Secondary flow also helps cool centerbody.
- Manufacturing
  - Aerospike is more complex and difficult to manufacture than bell nozzle
  - More costly
- Flight experience
  - No aerospike engine has ever flown in a rocket application
  - Little flight design experience has been gained

Efficiencies

$$P_{\text{chem}} = \dot{m} Q_R J$$

$$\eta_{\text{comb}} P_{\text{chem}}$$

$$\eta_{\text{int}} = \frac{\text{kinetic power in jet}}{\text{available chemical power}} = \frac{\frac{1}{2}\dot{m}v^2}{\eta_{\text{comb}} P_{\text{chem}}}$$

$$\eta_P = \frac{\text{vehicle power}}{\text{vehicle power } + \text{ residual kinetic jet power}}$$

$$= \frac{Fu}{Fu + \frac{1}{2}(\dot{w}/g_0)(c-u)^2} = \frac{2u/c}{1+(u/c)^2}$$