Contents

CHAPTER-1

THERMODYNAMICS OF AIR BREATHING PROPULSION SYSTEMS

Introduction-Thrust and efficiency-The Ramjet-Turbojet Engines-Turbofan Engines-Turboprop and turbo-shaft Engines-Typical Engine Performance-Engine-aircraft matching (introductory information)-Numerical problems.

INTRODUCTION

IDEAL BRAYTON CYCLE

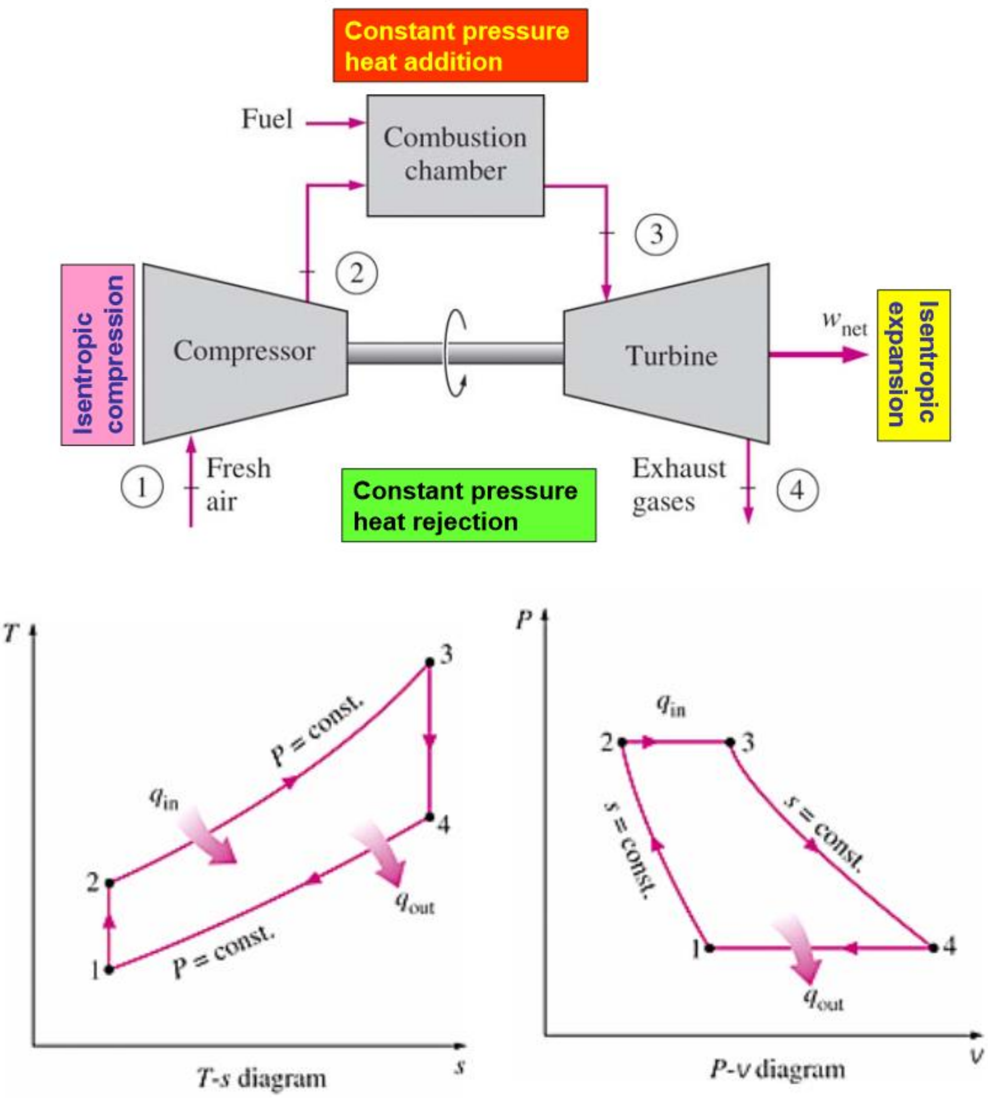


Figure 1 Ideal Brayton Cycle

The assumptions of ideal cycle are given as follows

i. Compression and expansion processes are reversible and adiabatic, i.e. isentropic.
ii. The change of kinetic energy of the working fluid between inlet and outlet of each component is negligible.
iii. There are no pressure losses in the inlet ducting, combustion chambers, heat exchangers, intercoolers, exhaust ducting, and ducts connecting the components.
iv. The working fluid has the same composition throughout the cycle and is a perfect gas with constant specific heats.
v. The mass flow of gas is constant throughout the cycle.
vi. Heat transfer in a heat exchanger (assumed counter flow) is 'complete,' so that in conjunction with (iv) and (v) the temperature rise on the cold side is the maximum possible and exactly equal to the temperature drop on the hot side.

Assumptions (iv) and (v) imply that the combustion chamber, in which fuel is introduced and burned, is considered as being replaced by a heater with the anexternal heat source. For this reason, as far as the calculation of performance of ideal cycles is concerned, it makes no difference whether one is thinking of them as 'open' or 'closed' cycles.

The ideal cycle for the simple gas turbine is the Joule or Brayton cycle, i.e. cycle 1234 in Fig-1the relevant steady flow energy equation is given by

$$h_1 + \frac{1}{2}C_1^{\ 2} + Q = h_2 + \frac{1}{2}C_2^{\ 2} + W$$

Where Q and Ware the heat and work transfers per unit mass flow.

Applying this to each component, we get

For Compressor

$$h_1 = h_2 + W_c$$
$$W_c = h_1 - h_2 = C_p\left(T_1 - T_2\right)$$

For combustion chamber

$$h_2 + Q_{c.c} = h_3$$
$$Q_{c.c} = h_3 - h_2$$
$$Q_{c.c} = C_p\left(T_3 - T_2\right)$$

For Turbine

$$h_3 = h_4 + W_T$$
$$W_T = h_3 - h_4 = C_p\left(T_3 - T_4\right)$$

The cycle efficiency

$$\eta = \frac{net\ work\ output}{heat\ \text{supplied}} = \frac{C_p\left(T_3 - T_4\right) - C_p\left(T_2 - T_1\right)}{C_p\left(T_3 - T_2\right)}$$
$$\eta = \frac{T_3 - T_4 - T_2 + T_1}{T_3 - T_2} = \frac{\left(T_3 - T_2\right) - \left(T_4 - T_1\right)}{\left(T_3 - T_2\right)}$$

Using the isentropic relation $\dfrac{T_2}{T_1} = r^{(\gamma-1)/\gamma} = \dfrac{T_3}{T_4}$

$$\eta = 1 - \frac{\left(T_4 - T_1\right)}{\left(T_3 - T_2\right)} = 1 - \frac{T_4\left(1 - \dfrac{T_1}{T_4}\right)}{T_3\left(1 - \dfrac{T_2}{T_3}\right)} = 1 - \frac{T_4}{T_3} = 1 - \left(\frac{1}{r}\right)^{(\gamma-1)/\gamma}$$

Where 'r' is the pressure ratio $\dfrac{P_2}{P_1} = r = \dfrac{P_3}{P_4}$

ACTUAL BRAYTON CYCLES

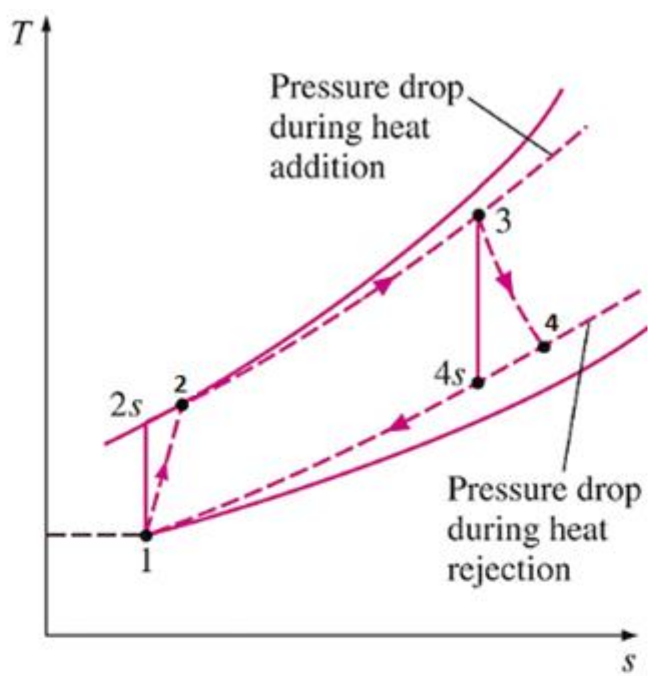


Figure 2 T-S diagram of Actual Brayton Cycle

i. The fluid velocities are high in turbo-machinery. So the change in kinetic energy between inlet and outlet of each component cannot necessarily be ignored. A further consequence is that the compression and expansion processes are irreversible adiabatic and therefore involve an increase in entropy.
ii. Fluid friction results in pressure losses in combustion chambers and heat exchangers, and also in the inlet and exhaust ducts.
iii. If a heat-exchanger is to be of an economic size, terminal temperature differences are inevitable; i.e. the compressed air cannot be heated to the temperature of the gas leaving the turbine.
iv. Slightly more work than that required for the compression process will be necessary to overcome bearing and 'windage' friction in the transmission between compressor and turbine, and to drive ancillary components such as fuel and oil pumps.
v. The values of C_p and γ of the working fluid vary throughout the cycle due to changes in temperature and, with internal combustion, due to changes in chemical composition.
vi. The definition of the efficiency of an ideal cycle is unambiguous, but this is not the case for an open cycle with internal combustion. Knowing the compressor delivery temperature, the composition of the fuel, and turbine inlet temperature required, a straightforward combustion calculation yields the fuel-to-air ratio necessary, and combustion efficiency can also be included to allow for incomplete combustion. Thus it will be possible to express the cycle performance unambiguously regarding fuel consumption per unit network output, i.e. regarding the specific fuel consumption. To convert this to cycle efficiency, it is necessary to adopt some convention for expressing the heating value of the fuel.
vii. It is necessary to account explicitly for the variation of mass flow through the engine.

Two very important parameters strongly influence the gas turbine properties. They are the isentropic and the polytropic efficiency.

Isentropic efficiency

In an ideal (isentropic) case, the enthalpy would rise from h_{01} to h_{02s}. However, in reality, it rises from h_{01} to h_{02}, which is a bigger increase. Similarly, in an ideal (isentropic) turbine, the enthalpy would decrease from h_{03} to h_{04s}. However, in reality, it decreases from h_{03} to h_{04}, which is a smaller decrease. This effect can be expressed in the isentropic efficiency. The efficiencies for compression and expansion are, respectively, given by

$$\eta_C = \frac{h_{02s} - h_{01}}{h_{02} - h_{01}} = \frac{C_{P_{air}}\left(T_{02s} - T_{01}\right)}{C_{P_{air}}\left(T_{02} - T_{01}\right)} = \frac{T_{02s} - T_{01}}{T_{02} - T_{01}}$$

$$\eta_T = \frac{h_{03} - h_{04}}{h_{03} - h_{04s}} = \frac{C_{P_{gas}}\left(T_{03} - T_{04}\right)}{C_{P_{gas}}\left(T_{03} - T_{04s}\right)} = \frac{T_{03} - T_{04}}{T_{03} - T_{04s}}$$

we may have trouble remembering which difference goes on top of the fraction, and which one goes below. In that case, just remember that we always have $\eta < 1$

By using the isentropic relations, we can rewrite the above equations.

$$\eta_C = \frac{\left(\frac{P_{02}}{P_{01}}\right)^{\frac{\gamma_{air}-1}{\gamma_{air}}} - 1}{\frac{T_{02}}{T_{02}} - 1} \text{ and } \eta_T = \frac{\frac{T_{04}}{T_{03}} - 1}{\left(\frac{P_{04}}{P_{03}}\right)^{\frac{\gamma_{gas}-1}{\gamma_{gas}}} - 1}$$

Polytropic efficiency

The compression process is divided into an infinite number of small steps. All these infinitely small steps have the same isentropic efficiency. This efficiency is known as the polytropic efficiency. The resulting process is also known as a polytropic process. This means that there is a polytropic exponent n, satisfying

$$\frac{T_0}{T_{0_{initial}}} = \left(\frac{P_0}{P_{0_{initial}}}\right)^{\frac{n_{air}}{n_{air}-1}}$$

The polytropic efficiencies for compression η_C and expansion η_T are now given by, respectively,

$$\eta_{poly,C} = \frac{\gamma_{air}-1}{\gamma_{air}} \frac{n_{air}}{n_{air}-1} = \frac{\ln\left(\frac{P_{02}}{P_{01}}\right)^{\frac{\gamma_{air}-1}{\gamma_{air}}}}{\ln\left(\frac{T_{02}}{T_{01}}\right)} \text{ and } \eta_{poly,T} = \frac{\gamma_{gas}}{\gamma_{gas}-1} \frac{n_{gas}-1}{n_{gas}} = \frac{\ln\left(\frac{T_{04}}{T_{03}}\right)}{\ln\left(\frac{P_{04}}{P_{03}}\right)^{\frac{\gamma_{gas}-1}{\gamma_{gas}}}}$$

The full compression/expansion process also has an isentropic efficiency. It is different from the polytrophic efficiency. In fact, the relation between the two is given by

$$\eta_C = \frac{\left(\frac{P_{02}}{P_{01}}\right)^{\frac{\gamma_{air}-1}{\gamma_{air}}} - 1}{\left(\frac{P_{02}}{P_{01}}\right)^{\frac{\gamma_{air}-1}{\eta_{poly,C}\gamma_{air}}} - 1} \text{ and } \eta_T = \frac{\left(\frac{P_{04}}{P_{03}}\right)^{\eta_{poly,T}\frac{\gamma_{gas}-1}{\gamma_{gas}}} - 1}{\left(\frac{P_{02}}{P_{01}}\right)^{\frac{\gamma_{gas}-1}{\gamma_{gas}}} - 1}$$

There are a few important rules to remember. For compression, the polytropic efficiency is higher than the isentropic efficiency. (So, $\eta_{\infty C} > \eta_C$) For expansion, the polytropic efficiency is lower than the isentropic efficiency. (So, $\eta_{\infty T} < \eta_T$) Finally, if the pressure ratio increases, then the difference between the two efficiencies increases.

Pressure Losses

Previously, we have assumed that no pressure losses occurred. This is, of course, not true.
Pressure losses occur at several places.The combustion chamber pressure loss is given by

$$\Delta P_b = P_{03} - P_{02}$$

Mechanical efficiency

Losses also occur due to internal friction in the system. These mechanical losses are joined in one term, being the transmission efficiency or mechanical efficiency η_m. It is given by

$$\eta_m = \frac{W_c}{W_t} = \frac{\dot{m}_a C_{pa}\left(T_{02r} - T_{01}\right)}{\left(\dot{m}_a + \dot{m}_f\right) C_{pg}\left(T_{03} - T_{04}\right)}$$

$$\Rightarrow T_{04r} = T_{03} - \frac{C_{pa}\left(T_{02} - T_{01}\right)}{\eta_m\left(1+f\right)C_{pg}}$$

Combustor efficiency

Ideally, we will have a full combustion of the fuel in the combustion chamber. In the ideal case, we would get the maximum heat out of it. This maximum heat is called the lower heating value LHV of the fuel, also known as the lower calorific value LCV. However, in reality, we have an incomplete combustion. This result in combustion products like carbon monoxide (CO) and unburned fuel.Next to this, heat may also escape. To take this into account, the combustor efficiency η_b is used. It is defined as

$$\dot{m}_a C_{pa} T_{02} + \dot{m}_f \eta_b Q_R = \left(\dot{m}_a + \dot{m}_f\right) C_{pg} T_{03}$$

$$\eta_b = \frac{\dot{m}_a\left(1+f\right)C_{pg}T_{03} - \dot{m}_a C_{pa} T_{02}}{\dot{m}_f Q_R}$$

$$\eta_b = \frac{\left(1+f\right)C_{pg}T_{03} - C_{pa}T_{02}}{f Q_R}$$

THRUST AND EFFICIENCY

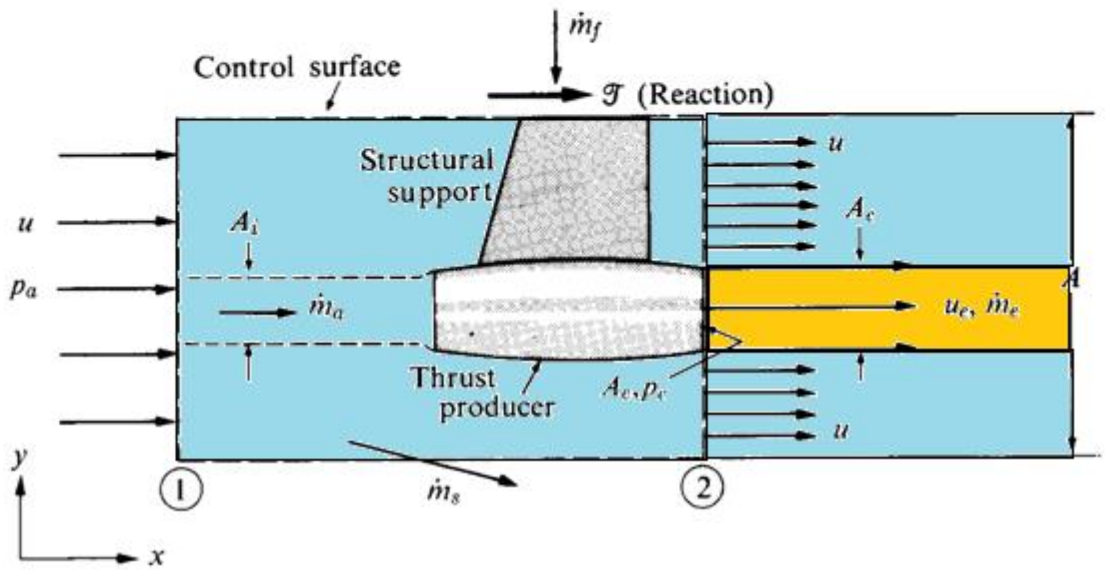


Figure 3 Thrust Producing Device

The generalized thrust producing device is shown in fig-3 as observed from a position stationary on the instrument. In fig-3 a control surface is specified that passes through the propellant outlet plane at $\boxed{2}$ and extends far upstream at $\boxed{1}$. The side surfaces of the control volume are parallel to the upstream flight velocity **u** and far removed from the thrust device. We assume that the thrust and conditions at all points within the control volume do not change with time. The reaction to the thrust **T** transmitted through the structural support is indicated in fig-3. The engine thrust is defined as the vector summation of all forces on the internal and external surfaces of engine and nacelle.

The thrust of the generalized thrust producer can be derived as

$$\sum F = \frac{d}{dt}\int_{CS} \rho u dV + \int_{CS} \rho u \left(u \bullet n\right) dA$$

Since the flow is steady, the control volume does not change with time

$$\sum F = \int_{CS} \rho u \left(u \bullet n\right) dA$$

Considering the component of force and moment flux in the x-direction only, we get

$$\sum F_x = \int_{CS} \rho u_x \left(u \bullet n\right) dA$$

With the assumption of reversible external flow, both the pressure and velocity may be assumed constant over the entire control surface, except over the exhaust area A_e of the engine.

If the exhaust velocity u_e is supersonic, the exhaust pressure may differ from the ambient pressure P_a. The net pressure force on the control surface is given by

$$The\ Net\ Pressure\ Force = \left(P_a - P_e\right)A_e$$

Adding up the forces on the control surface that act on the x-direction, we get

$$\sum F_x = \left(P_a - P_e\right)A_e + T_F$$

At station, the air that is drawn into the engine across the control surface through the capture area A_i at the rate $\dot{m}_a$ is given by

$$\dot{m}_a = \rho u A_i$$

Where ρ = the ambient density

u = the flight velocity.

The mass flux across the exit area is given by $\dot{m}_e = \rho_e u_e A_e$

$$\dot{m}_e = \dot{m}_a + \dot{m}_f$$

$$\Rightarrow \dot{m}_f = \rho_e u_e A_e - \rho u A_i$$

Now, if we consider the requirement of continuity for the control volume as a whole and assume that the fuel flow originates from outside the control volume

$$\frac{d}{dt}\int_{CV} \rho dV + \int_{CS} \rho\left(u \bullet n\right) dA = 0$$

Since the flow is steady, the control volume does not change with time

$$\int_{CS} \rho\left(u \bullet n\right) dA = 0$$

From the continuity equation,

Inlet mass flow rate = outlet mass flow rate

$$\dot{m}_f + \rho u A = \rho_e u_e A_e + \rho u\left(A - A_e\right) + \dot{m}_s$$

$$\dot{m}_s = \dot{m}_f + \rho u A_e - \rho_e u_e A_e$$

Substitute $\dot{m}_f$ in the above equation we get

$$\dot{m}_s = \rho_e u_e A_e - \rho u A_i + \rho u A_e - \rho_e u_e A_e$$

$$\dot{m}_s = \rho u \left(A_e - A_i \right)$$

The momentum carried out by the control volume is given by

$$\sum F = \int_{CS} \rho u \left(u \bullet n \right) dA = \dot{m}_e u_e + \dot{m}_s u + \rho u \left(A - A_e \right) u - \dot{m}_a u - \rho u \left(A - A_i \right) u$$

$$\sum F = \rho_e u_e A_e u_e + \rho u \left(A_e - A_i \right) u + \rho u \left(A - A_e \right) u - \rho u A u - \rho u \left(A - A_i \right) u$$

$$\sum F = \rho_e u^2{}_e A_e + \rho u^2 A_e - \rho u^2 A_i + \rho u^2 A - \rho u^2 A_e - \rho u^2 A_i - \rho u^2 A + \rho u^2 A_i$$

$$\sum F = \rho_e u^2{}_e A_e - \rho u^2 A_i$$

$$\sum F = \dot{m}_e u_e - \dot{m}_a u$$

Sub the above equation in momentum equation, we get

$$\sum F_x = \left(P_a - P_e \right) A_e + T_F$$

$$\dot{m}_e u_e - \dot{m}_a u = \left(P_a - P_e \right) A_e + T_F$$

$$T_F = \dot{m}_e u_e - \dot{m}_a u - \left(P_a - P_e \right) A_e$$

$$T_F = \dot{m}_e u_e - \dot{m}_a u + \left(P_e - P_a \right) A_e$$

$$T_F = \dot{m}_a \left(\left(1 + f \right) u_e - u \right) + \left(P_e - P_a \right) A_e$$

Where $f = \dfrac{\dot{m}_f}{\dot{m}_a}$ =fuel to air ratio

Turbojet Engine with Afterburner

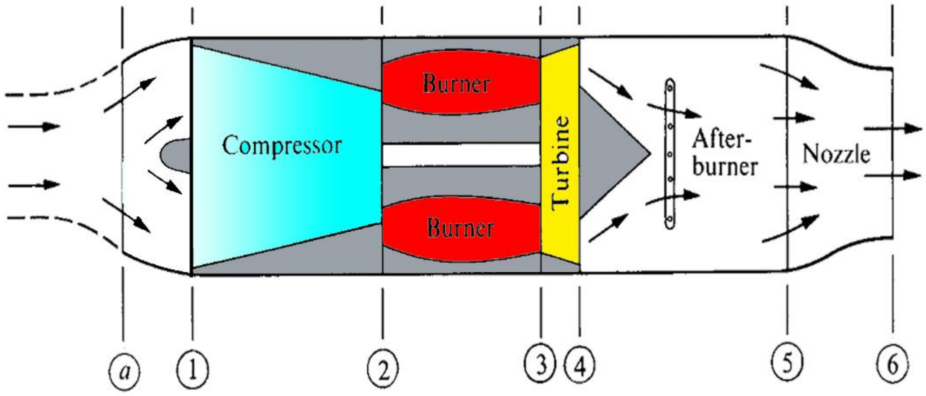


Figure 4 Schematic Diagram of Turbojet Engine with Afterburner

The internal arrangement of the turbojet engine is shown schematically in fig-4. In flowing through the machine, the air undergoes the following process:

$\boxed{a} - \boxed{1}$ From upstream, where the velocity of the air about the engine is the flight speed, the air is brought to the intake, usually with some acceleration or deceleration. The air velocity is decreased as the air is carried to the compressor inlet through the inlet diffuser and ducting system.

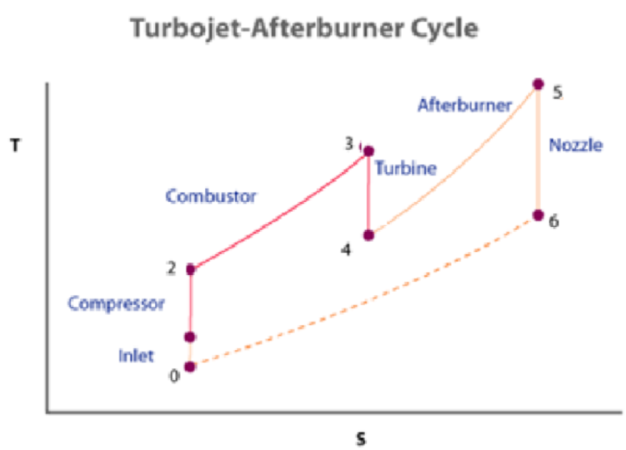
$\boxed{1} - \boxed{2}$ The air is compressed in a dynamic diffuser.

$\boxed{2} - \boxed{3}$ The air is "heated" by mixing and burning of fuel in the air.

$\boxed{3} - \boxed{4}$ The air is expanded through a turbine to obtain the power to drive the compressor.

$\boxed{4} - \boxed{5}$ The air may or may not be further "heated" by the addition and burning more fuel in an afterburner.

$\boxed{5} - \boxed{6}$ The air is accelerated and exhausted through the exhaust nozzle.



Turbojet without Afterburner

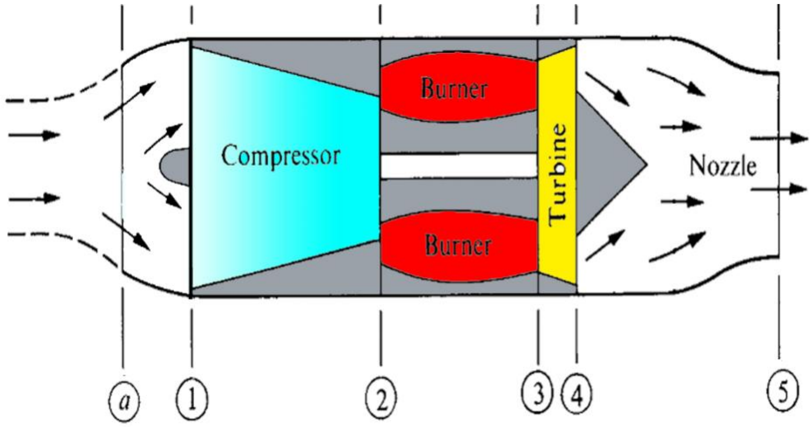


Figure 5Schematic Diagram of Turbojet Engine without Afterburner

The internal arrangement of the turbojet engine is shown schematically in fig-5. In flowing through the machine, the air undergoes the following process:

$\boxed{a} - \boxed{1}$ From upstream, where the velocity of the air about the engine is the flight speed, the air is brought to the intake, usually with some acceleration or deceleration. The air velocity is decreased as the air is carried to the compressor inlet through the inlet diffuser and ducting system.

$\boxed{1} - \boxed{2}$ The air is compressed in a dynamic diffuser.

$\boxed{2} - \boxed{3}$ The air is "heated" by mixing and burning of fuel in the air.

$\boxed{3} - \boxed{4}$ The air is expanded through a turbine to obtain the power to drive the compressor.

$\boxed{4} - \boxed{5}$ The air is accelerated and exhausted through the exhaust nozzle.

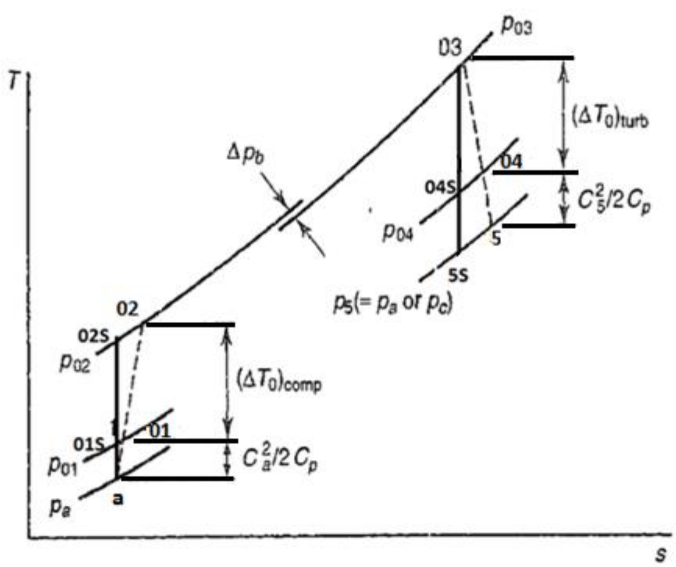


Figure 6 T-S Diagram of Turbojet engine without afterburner

Diffuser

Air intake aims at bringing large amounts of surrounding air into the engine. A tube-shaped inlet, like the one we would see on an airliner usually of cylindrical or conical design.Inlets come in many shapes and sizes depending on the aircraft.

$$\frac{T_{01}}{T_a} = \left[1 + \frac{\gamma_a - 1}{2} M_a{}^2\right]$$

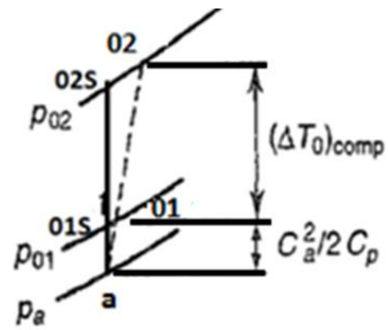
$$\Rightarrow T_{01} = T_a \left[1 + \frac{\gamma_a - 1}{2} M_a{}^2\right]$$

$$\eta_d = \frac{T_{01s} - T_a}{T_{01} - T_a}$$

$$\Rightarrow T_{01s} = T_a + \eta_d \left(T_{01} - T_a\right)$$

$$\frac{P_{01}}{P_a} = \left[\frac{T_{01s}}{T_a}\right]^{\frac{\gamma_a}{\gamma_a - 1}}$$

$$\Rightarrow P_{01} = P_a \left[\frac{T_{01s}}{T_a}\right]^{\frac{\gamma_a}{\gamma_a - 1}}$$



Compressor

The compressor rotates at very high speed, adding energy to the airflow and at the same time squeezing it into a smaller space. Compressing the air increases its pressure and temperature. The turbine drives the compressor.Compressors used in turbojet engines are mainly classified as axial flow compressor and centrifugal compressors.

Axial Flow Compressor

Axial compressors are rotating, airfoil based compressors in which the working fluid principally flows parallel to the axis of rotation. Axial compressors consist of a shaft that drives a central drum which has some annular airfoil rows attached. These rotate between a similar number of stationary airfoil rows connected to a stationary tubular casing. A pair of rotating and stationary airfoils is called a stage. The cross-sectional area between rotor drum and casing is reduced in the flow direction to maintain axial velocity as the fluid is compressed.

Centrifugal Compressor

Centrifugal compressors are rotating, airfoil based compressors in which the working fluid principally flows perpendicular to the axis of rotation. Centrifugal compressors consist of a shaft that drives an impeller which has a number of curved blades. The impeller rotates in a casing which is designed to convert the kinetic energy of the fluid into pressure energy before leaving the compressor.

$$P_{02} = CPR \times P_{01}$$

$$\frac{P_{02}}{P_{01}} = \left[\frac{T_{02s}}{T_{01}}\right]^{\frac{\gamma_a}{\gamma_a - 1}}$$

$$\Rightarrow T_{02s} = T_{01}\left[\frac{P_{02}}{P_{01}}\right]^{\frac{\gamma_a - 1}{\gamma_a}}$$

$$\eta_c = \frac{T_{02s} - T_{01}}{T_{02} - T_{01}}$$

$$\Rightarrow T_{02} = T_{01} + \frac{T_{02s} - T_{01}}{\eta_c}$$

Combustion chamber

In a turbojet, the air and fuel mixture passes unconfined through the combustion chamber. As the mixture burns its temperature increases dramatically.The combustion chamber is usually in the form of cans, which comprise the fuel injector and flame holder.

$$\dot{m}_a C_{pa} T_{02} + \dot{m}_f \eta_b Q_R = \left(\dot{m}_a + \dot{m}_f\right) C_{pg} T_{03}$$

$$\eta_b = \frac{\dot{m}_a (1+f) C_{pg} T_{03} - \dot{m}_a C_{pa} T_{02}}{\dot{m}_f Q_R}$$

$$\eta_b = \frac{(1+f) C_{pg} T_{03} - C_{pa} T_{02}}{f Q_R}$$

$$\Rightarrow f = \frac{(1+f) C_{pg} T_{03} - C_{pa} T_{02}}{\eta_b Q_R}$$

$$P_{03} = P_{02}(1 - loss\ percentage)$$

Mechanical Efficiency

Losses also occur due to internal friction in the system. These mechanical losses are joined in one term, being the transmission efficiency or mechanical efficiency η_m. It is given by

$$\eta_m = \frac{W_c}{W_t} = \frac{\dot{m}_a C_{pa} \left(T_{02} - T_{01}\right)}{\left(\dot{m}_a + \dot{m}_f\right) C_{pg} \left(T_{03} - T_{04}\right)}$$

$$\Rightarrow T_{04} = T_{03} - \frac{C_{pa} \left(T_{02} - T_{01}\right)}{\eta_m (1+f) C_{pg}}$$

Turbine

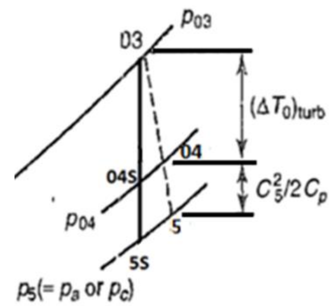
Hot gases leaving the combustor are allowed to expand through the turbine. Turbines are usually made up of high-temperature metals such as Inconel. The turbine's rotational energy is used primarily to drive the compressor And other accessories, like fuel, oil, and hydraulic pumps.In a turbojet, almost two-thirds of all the power generated by burning fuel is used by the compressor to compress the air for the engine.

$$\eta_t = \frac{T_{03} - T_{04}}{T_{03} - T_{04s}}$$

$$\Rightarrow T_{04s} = T_{03} - \frac{(T_{03} - T_{04})}{\eta_t}$$

$$\frac{P_{04}}{P_{03}} = \left[\frac{T_{04s}}{T_{03}}\right]^{\frac{\gamma_g}{\gamma_g - 1}}$$

$$\Rightarrow P_{04} = P_{03}\left[\frac{T_{04s}}{T_{03}}\right]^{\frac{\gamma_g}{\gamma_g - 1}}$$



At Nozzle

After the turbine, the gases are allowed to expand through the exhaust nozzle to atmospheric pressure, producing a high-velocity jet in the exhaust plume. In a convergent nozzle, the ducting narrows progressively to a throat.

Chocking Check

$$\eta_N = \frac{T_{04} - T_5}{T_{04} - T_{5s}}$$

$$\Rightarrow \frac{P_{04}}{P_c} = \frac{1}{\left[1 - \frac{1}{\eta_N}\left(\frac{\gamma_g - 1}{\gamma_g + 1}\right)\right]^{\frac{\gamma_g}{\gamma_g - 1}}}$$

if $\frac{P_{04}}{P_a} > \frac{P_{04}}{P_c}$ the nozzle is chocked.

so there is no flow possible after $\frac{P_{04}}{P_c}$

substitute $P_5 = P_c$ *if chocked*

$$\frac{T_{04}}{T_c} = \frac{T_{05}}{T_5} = \frac{\gamma_g + 1}{2}$$

$$\Rightarrow T_5 = T_c = T_{04}\left[\frac{2}{\gamma_g + 1}\right]$$

Advantages of Turbojet engine

- Very high power-to-weight ratio.
- Compact than most reciprocating engines of the same power rating.
- It has fewer moving parts than reciprocating engines.
- Low operating pressures.
- High operation speeds.
- Low lubricating oil cost and consumption.

Disadvantages of Turbojet engine

- Cost
- Longer startup than reciprocating engines
- Less responsive to changes in power demand compared to reciprocating engines.

Efficiency of turbojet engine

It is often convenient to break the overall efficiency into two parts: thermal efficiency and propulsive efficiency, where

$$\eta_{\text{thermal}} = \frac{\text{rate of production of kinetic energy}}{\text{fuel power}} = \frac{\left(\frac{\dot{m}_e u_e^2}{2} - \frac{\dot{m}_0 u_0^2}{2}\right)}{\dot{m}_f h}$$

$$\eta_{\text{propulsive}} = \frac{\text{propulsive power}}{\text{rate of production of propulsive kinetic energy}} = \frac{T u_0}{\left(\frac{\dot{m}_e u_e^2}{2} - \frac{\dot{m}_0 u_0^2}{2}\right)}$$

$$\eta_{propulsive} = \frac{\dot{m}u\left(u_e - u\right)}{\frac{\dot{m}}{2}\left(u_e^{\,2} - u^2\right)} = \frac{2u}{u + u_e} = \frac{2}{1 + \frac{u_e}{u}}$$

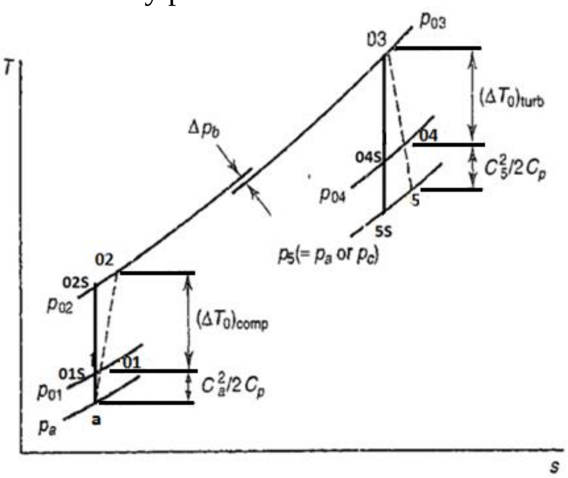
$$\eta_{\text{overall}} = \eta_{\text{thermal}}\eta_{\text{propulsive}}$$

Example 1.1

Determination of the specific thrust and SFC for a simple turbojet engine, having the following component performance at the design point at which the cruise speed and altitude are M=0.8 and 10,000m.

Compressor pressure ratio	=	8.0
Turbine inlet temperature	=	1200 K
Compressor efficiency, η_c	=	0.87
Turbine efficiency, η_t	=	0.90
Intake efficiency, η_i	=	0.93
Nozzle efficiency, η_N	=	0.95
Mechanical transmission efficiency, η_m	=	0.99
Combustion efficiency, η_b	=	0.98
Combustion pressure loss, ΔP_b	=	4% compressor delivery pressure.

Solution



From the ISA table at 10Km

$$P_a = 0.2650bar, T_a = 223.3K$$

At Diffuser

$$\frac{T_{01}}{T_a} = \left[1 + \frac{\gamma_a - 1}{2} M_a^{\ 2}\right]$$

$$\Rightarrow T_{01} = T_a \left[1 + \frac{\gamma_a - 1}{2} M_a^{\ 2}\right] = 251.9K$$

$$\eta_d = \frac{T_{01s} - T_a}{T_{01} - T_a}$$

$$\Rightarrow T_{01s} = T_a + \eta_d \left(T_{01} - T_a\right) = 249.898K$$

$$\frac{P_{01}}{P_a} = \left[\frac{T_{01s}}{T_a}\right]^{\frac{\gamma_a}{\gamma_a - 1}}$$

$$\Rightarrow P_{01} = P_a \left[\frac{T_{01s}}{T_a}\right]^{\frac{\gamma_a}{\gamma_a - 1}} = 0.393bar$$

At Compressor

$$P_{02} = CPR \times P_{01} = 3.144bar$$

$$\frac{P_{02}}{P_{01}} = \left[\frac{T_{02s}}{T_{01}}\right]^{\frac{\gamma_a}{\gamma_a - 1}}$$

$$\Rightarrow T_{02s} = T_{01} \left[\frac{P_{02}}{P_{01}}\right]^{\frac{\gamma_a - 1}{\gamma_a}} = 452.68K$$

$$\eta_c = \frac{T_{02s} - T_{01}}{T_{02} - T_{01}}$$

$$\Rightarrow T_{02} = T_{01} + \frac{T_{02s} - T_{01}}{\eta_c} = 486.8K$$

For combustion chamber

$$\dot{m}_a C_{pa} T_{02} + \dot{m}_f \eta_b Q_R = \left(\dot{m}_a + \dot{m}_f\right) C_{pg} T_{03}$$

$$\eta_b = \frac{\dot{m}_a \left(1+f\right) C_{pg} T_{03} - \dot{m}_a C_{pa} T_{02}}{\dot{m}_f Q_R}$$

$$\eta_b = \frac{\left(1+f\right) C_{pg} T_{03} - C_{pa} T_{02}}{f Q_R}$$

$$\Rightarrow f = \frac{\left(1+f\right) C_{pg} T_{03} - C_{pa} T_{02}}{\eta_b Q_R}$$

$$\Rightarrow f = 0.021$$

$$P_{03} = P_{02}(1 - 0.04) = 3.018bar$$

Mechanical Efficiency

$$\eta_m = \frac{W_c}{W_t} = \frac{\dot{m}_a C_{pa} \left(T_{02} - T_{01}\right)}{\left(\dot{m}_a + \dot{m}_f\right) C_{pg} \left(T_{03} - T_{04}\right)}$$

$$\Rightarrow T_{04} = T_{03} - \frac{C_{pa} \left(T_{02} - T_{01}\right)}{\eta_m \left(1+f\right) C_{pg}} = 996.55K$$

At Turbine

$$\eta_t = \frac{T_{03} - T_{04}}{T_{03} - T_{04s}}$$

$$\Rightarrow T_{04s} = T_{03} - \frac{\left(T_{03} - T_{04}\right)}{\eta_t} = 973.94K$$

$$\frac{P_{04}}{P_{03}} = \left[\frac{T_{04s}}{T_{03}}\right]^{\frac{\gamma_g}{\gamma_g - 1}}$$

$$\Rightarrow P_{04} = P_{03} \left[\frac{T_{04s}}{T_{03}}\right]^{\frac{\gamma_g}{\gamma_g - 1}} = 1.309bar$$

At Nozzle

Chocking Check

$$\eta_N = \frac{T_{04s} - T_5}{T_{04s} - T_{5s}}$$

$$\Rightarrow \frac{P_{04}}{P_c} = \frac{1}{\left[1 - (1/\eta_N)\left(\frac{\gamma_g - 1}{\gamma_g + 1}\right)\right]^{\frac{\gamma_g}{\gamma_g - 1}}} = 1.914$$

$$\frac{P_{04}}{P_a} = 4.845$$

since $\frac{P_{04}}{P_a} > \frac{P_{04}}{P_c}$ the nozzle is chocked.

so there is no flow possible after $\frac{P_{04}}{P_c}$

$$substitute\, P_5 = P_c = 0.617 bar$$

$$\frac{T_{04}}{T_c} = \frac{T_{05}}{T_5} = \frac{\gamma_g + 1}{2}$$

$$\Rightarrow T_5 = T_c = T_{04}\left[\frac{2}{\gamma_g + 1}\right] = 850.7K$$

$$\rho_5 = \frac{P_5}{RT_5} = 0.275 kg / m^3$$

$$C_5 = \sqrt{\gamma_g R T_5} = 570.5 m / s$$

$$\frac{A_5}{\dot{m}_a} = \frac{1+f}{\rho_5 C_5} = 0.006374 m^2 s / kg$$

Specific Thrust and SFC

$$F_s = (1+f)C_5 - C_1 + (P_5 - P_1)\frac{A_5}{\dot{m}_a} = 609 Ns / kg$$

$$SFC = \frac{f}{F_s} = 0.121 kg / Nh = 3.361 \times 10^{-5} kg / Ns$$

TURBOFAN ENGINE

For supersonic flight speeds, the overall efficiency of turbojets was outstanding. However, for high subsonic and transonic flight speeds (around 500-600 mph), the velocity of the exhaust gas jet was too high to obtain a good propulsive efficiency. Under these conditions, the bypass engine (also called turbofan) became a very attractive approach for improving the propulsive efficiency. In a turbofan engine, a fan of a diameter larger than the compressor is used to generate a mass flow higher than the core mass flow.

By pass ratio

Bypass ratio is defined as the ratio of the flow through the bypass duct (cold stream) to the flow at entry to the high-pressure compressor (hot stream), and it is given by

$$B = \frac{\dot{m}_c}{\dot{m}_h}$$

Low-Bypass/Mixed Turbofan Engine

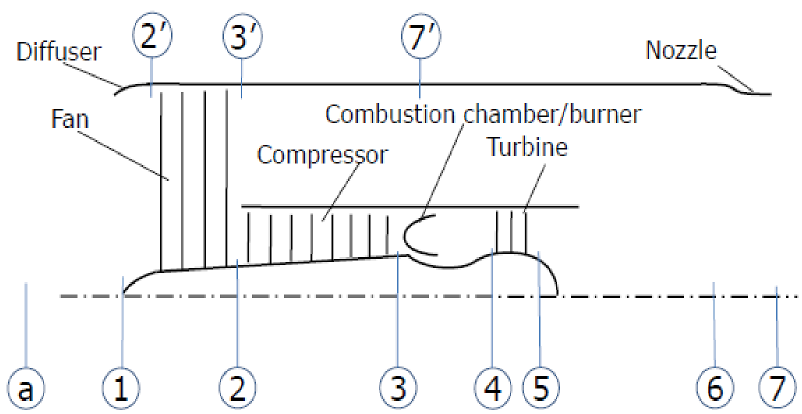


Figure 7 Schematic of Mixed Turbofan Engine

The different process in an unmixed turbofan cycle are given as follows

$\boxed{a}-\boxed{1}$ Air from far upstream is brought to the air intake/diffuser with some acceleration/deceleration.

$\boxed{1}-\boxed{2'}$ The air is decellerated as it passes through the diffuser.

$\boxed{2'}-\boxed{3'}$ The air is compressed in a fan.

$\boxed{2}-\boxed{3}$ The air is compressed in a axial/centrifugal type compressor.

$\boxed{3}-\boxed{4}$ The air is “heated” by mixing and burning of fuel with air in the combustion chamber.

$\boxed{4}-\boxed{5}$ The air is expanded through a turbine to obtain the power to drive the compressor.

$\boxed{5} - \boxed{6}$ The air may or may not be further "heated" by the addition and burning more fuel in an afterburner.

$\boxed{6} - \boxed{7}$ The air is accelerated and exhausted through the nozzle.

High-Bypass /Unmixed Turbofan Engine

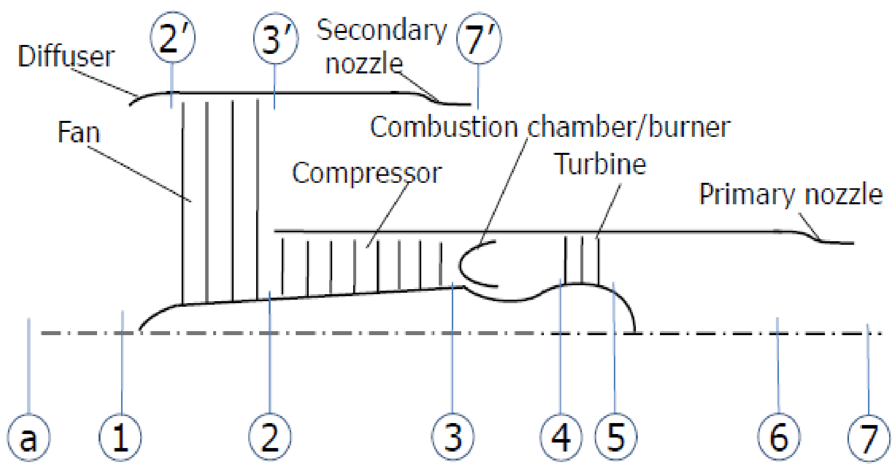


Figure 8 Schematic of Unmixed Turbofan Engine

The different process in an unmixed turbofan cycle are given as follows

$\boxed{a} - \boxed{1}$ Air from far upstream is brought to the air intake/diffuser with some acceleration/deceleration.

$\boxed{1} - \boxed{2'}$ The air is decellerated as it passes through the diffuser.

$\boxed{2'} - \boxed{3'}$ The air is compressed in a fan.

$\boxed{2} - \boxed{3}$ The air is compressed in a axial/centrifugal type compressor.

$\boxed{3} - \boxed{4}$ The air is "heated" by mixing and burning of fuel with air in the combustion chamber.

$\boxed{4} - \boxed{5}$ The air is expanded through a turbine to obtain the power to drive the compressor.

$\boxed{5} - \boxed{6}$ The air may or may not be further "heated" by the addition and burning more fuel in an afterburner.

$\boxed{6} - \boxed{7}$ The air is accelerated and exhausted through the primary nozzle.

$\boxed{3'} - \boxed{7'}$ The air in the bypass duct is accelerated and exhausted through the secondary nozzle.

Advantages of The High-Bypass-Ratio Turbofan Engine

1. High overall efficiency, resulting in a long flight range,

2. Strong increase in propulsive thrust at low flight speeds, which is important for takeoff, climbing, and efficient part-load operation,
3. Lower jet velocity, which leads to great noise reduction, and
4. Low fuel consumption, which reduces chemical emissions.

Example1.2

The following data apply to a twin-spool turbofan engine, with the fan driven by the LP turbine and the compressor by the HP turbine. Separate cold and hot nozzles are used.

Overall pressure ratio	25·0
Fan pressure ratio	1·65
Bypass ratio m_c/m_h	5·0
Turbine inlet temperature	1550 K
Fan, compressor and turbine *polytropic* efficiency	0·90
Isentropic efficiency of each propelling nozzle	0·95
Mechanical efficiency of each spool	0·99
Combustion pressure loss	1·50 bar
Total air mass flow	215 kg/s

It is required to find the thrust and *SFC* under sea level static conditions where the ambient pressure and temperature are 1·0 bar and 288 K.

solution

The values of $(n-1)/n$ for the polytropic compression and expansion are:

$$\text{for compression,}\quad \frac{n-1}{n}=\frac{1}{\eta_{\infty c}}\left(\frac{\gamma-1}{\gamma}\right)_a=\frac{1}{0{\cdot}9\times 3{\cdot}5}=0{\cdot}3175$$

$$\text{for expansion,}\quad \frac{n-1}{n}=\eta_{\infty t}\left(\frac{\gamma-1}{\gamma}\right)_g=\frac{0{\cdot}9}{4}=0{\cdot}225$$

Under static conditions $T_{01}=T_a$ and $p_{01}=p_a$

$$\frac{T_{02}}{T_{01}}=\left(\frac{p_{02}}{p_{01}}\right)^{(n-1)/n}\quad \text{yields } T_{02}=288\times 1{\cdot}65^{0{\cdot}3175}=337{\cdot}6\text{ K}$$

$$T_{02}-T_{01}=337{\cdot}6-288=49{\cdot}6\text{ K}$$

$$\frac{p_{03}}{p_{02}} = \frac{25 \cdot 0}{1 \cdot 65} = 15 \cdot 15$$

$$T_{03} = T_{02}\left(\frac{p_{03}}{p_{02}}\right)^{(n-1)/n} = 337 \cdot 6 \times 15 \cdot 15^{0 \cdot 3175} = 800 \cdot 1 \text{ K}$$

$$T_{03} - T_{02} = 800 \cdot 1 - 337 \cdot 6 = 462 \cdot 5 \text{ K}$$

The cold nozzle pressure ratio is

$$\frac{p_{02}}{p_a} = FPR = 1 \cdot 65$$

and the critical pressure ratio for this nozzle is

$$\frac{p_{02}}{p_c} = \frac{1}{\left[1 - \frac{1}{\eta_j}\left(\frac{\gamma - 1}{\gamma + 1}\right)\right]^{\gamma/(\gamma-1)}} = \frac{1}{\left[1 - \frac{1}{0 \cdot 95}\left(\frac{0 \cdot 4}{2 \cdot 4}\right)\right]^{3 \cdot 5}} = 1 \cdot 965$$

Thus the cold nozzle is not choking, so that $p_8 = p_a$ and the cold thrust F_c is given simply by

$$F_c = m_c C_8$$

The nozzle temperature drop, from equation (3.12), is

$$T_{02} - T_8 = \eta_j T_{02}\left[1 - \left(\frac{1}{p_{02}/p_a}\right)^{(\gamma-1)/\gamma}\right]$$

$$= 0 \cdot 95 \times 337 \cdot 6\left[1 - \left(\frac{1}{1 \cdot 65}\right)^{1/3 \cdot 5}\right] = 42 \cdot 8 \text{ K}$$

and hence

$$C_8 = [2c_p(T_{02} - T_8)]^{\frac{1}{2}} = (2 \times 1 \cdot 005 \times 42 \cdot 8 \times 1000)^{\frac{1}{2}} = 293 \cdot 2 \text{ m/s}$$

Since the bypass ratio B is 5·0,

$$m_c = \frac{mB}{B+1} = \frac{215 \times 5 \cdot 0}{6 \cdot 0} = 179 \cdot 2 \text{ kg/s}$$

$$F_c = 179 \cdot 2 \times 293 \cdot 2 = 52\,532 \text{ N}$$

Considering the work requirement of the HP rotor,

$$T_{04} - T_{05} = \frac{c_{pa}}{\eta_m c_{pg}}(T_{03} - T_{02}) = \frac{1 \cdot 005 \times 462 \cdot 5}{0 \cdot 99 \times 1 \cdot 148} = 409 \cdot 0 \text{ K}$$

and for the LP rotor

$$T_{05} - T_{06} = (B+1)\frac{c_{pa}}{\eta_m c_{pg}}(T_{02} - T_{01}) = \frac{6 \cdot 0 \times 1 \cdot 005 \times 49 \cdot 6}{0 \cdot 99 \times 1 \cdot 148} = 263 \cdot 2 \text{ K}$$

Hence

$$T_{05} = T_{04} - (T_{04} - T_{05}) = 1550 - 409{\cdot}0 = 1141{\cdot}0$$
$$T_{06} = T_{05} - (T_{05} - T_{06}) = 1141{\cdot}0 - 263{\cdot}2 = 877{\cdot}8$$

p_{06} may then be found as follows.

$$\frac{p_{04}}{p_{05}} = \left(\frac{T_{04}}{T_{05}}\right)^{n/(n-1)} = \left(\frac{1550}{1141{\cdot}0}\right)^{1/0{\cdot}225} = 3{\cdot}902$$
$$\frac{p_{05}}{p_{06}} = \left(\frac{T_{05}}{T_{06}}\right)^{n/(n-1)} = \left(\frac{1141{\cdot}0}{877{\cdot}8}\right)^{1/0{\cdot}225} = 3{\cdot}208$$
$$p_{04} = p_{03} - \Delta p_b = 25{\cdot}0 \times 1{\cdot}0 - 1{\cdot}50 = 23{\cdot}5 \text{ bar}$$
$$p_{06} = \frac{p_{04}}{(p_{04}/p_{05})(p_{05}/p_{06})} = \frac{23{\cdot}5}{3{\cdot}902 \times 3{\cdot}208} = 1{\cdot}878 \text{ bar}$$

Thus the hot nozzle pressure ratio is

$$\frac{p_{06}}{p_a} = 1{\cdot}878$$

while the critical pressure ratio is

$$\frac{p_{06}}{p_c} = \frac{1}{\left[1 - \dfrac{1}{0{\cdot}95}\left(\dfrac{0{\cdot}333}{2{\cdot}333}\right)\right]^4} = 1{\cdot}914$$

This nozzle is also unchoked, and hence $p_7 = p_a$.

$$T_{06} - T_7 = \eta_j T_{06}\left[1 - \left(\frac{1}{p_{06}/p_a}\right)^{(\gamma-1)/\gamma}\right]$$
$$= 0{\cdot}95 \times 877{\cdot}8\left[1 - \left(\frac{1}{1{\cdot}878}\right)^{\frac{1}{4}}\right] = 121{\cdot}6 \text{ K}$$
$$C_7 = [2c_p(T_{06} - T_7)]^{\frac{1}{2}} = [2 \times 1{\cdot}148 \times 121{\cdot}6 \times 1000]^{\frac{1}{2}}$$
$$= 528{\cdot}3 \text{ m/s}$$
$$m_h = \frac{m}{B+1} = \frac{215}{6{\cdot}0} = 35{\cdot}83 \text{ kg/s}$$
$$F_h = 35{\cdot}83 \times 528{\cdot}3 = 18\,931 \text{ N}$$

Thus the total thrust is

$$F_c + F_h = 52\,532 + 18\,931 = 71\,463 \text{ N or } 71{\cdot}5 \text{ kN}$$

$$m_f = 0{\cdot}0223 \times 35{\cdot}83 \times 3600 = 2876{\cdot}4 \text{ kg/h}$$

$$SFC = \frac{2876{\cdot}4}{71\,463} = 0{\cdot}0403 \text{ kg/h N}$$

Example 1.3

The Tomahawk is a long-range subsonic cruise missile powered by a small two-spool turbofan engine which has the following data:

Flight speed V_f = 247.22 m/s
Ambient temperature T_a = 275 K
Ambient pressure P_a = 0.79 bar
Thrust force F = 3.1 kN
Specific fuel consumption = 0.682 kg/kg-h
Bypass ratio = 1
Overall pressure ratio = 13.8
Fan pressure ratio = 2.1
Fan diameter= 0.305 m
Fuel heating value = 43,000 kJ/kg

Calculate air mass flow rate, fuel-to-air ratio, exhaust gas speed and engine maximum temperature.

Solution:
Air mass flow rate
Flight speed

$$V_f = 247.22\, m/s$$

$$A = \frac{\pi d^2}{4} = \frac{\pi \times 0.305^2}{4} = 0.07306\, m^2$$

$$\rho_a = \frac{P_a}{RT_a} = 1.001\, kg/m^3$$

$$\dot{m}_a = \rho_a V_f A = 18.08\, kg/s$$

$Since\ \beta = 1, then\ m_c = m_h = 9.04\, kg/s$

$$\frac{T}{\dot{m}_a} = 171.46 m/s$$

$TSFC = 0.682 \frac{kg}{kgfh} = \frac{0.682}{9.81 \times 3600} = 0.0000193 \frac{kg}{Ns} = 19.3 \frac{g}{kNs}$

Fuel-to-air ratio

$$TSFC = \frac{\dot{m}_f}{T} = \frac{\dot{m}_f/\dot{m}_h}{T/\dot{m}_h} = \frac{(\dot{m}_f/\dot{m}_h)}{(T/\dot{m}_a) \times (1+\beta)} = \frac{f}{(T/\dot{m}_a) \times (1+\beta)}$$

$$f = TSFC \times \left(\frac{T}{\dot{m}_a}\right)(1+\beta) = 0.0000193 \times 171.46 \times 2 = 0.006618$$

Exhaust gas speed

$$T = \left[\dot{m}_c + {}^c\dot{m}_h(1+f)^h\right]V_e - \dot{m}_a V_f$$

$$\frac{T}{\dot{m}_a} = \left[\frac{\beta}{1+\beta} + \frac{(1+f)}{1+\beta}\right]V_e - V_f = \left(\frac{\beta+1+f}{1+\beta}\right)V_e - V_f$$

$$V_e = \frac{(T/\dot{m}_a) + V_f}{\left(\frac{\beta+1+f}{1+\beta}\right)} = \frac{171.4622 + 247.22}{\left(\frac{2.006618}{2}\right)} = 417.3 \text{ m/s}$$

Engine maximum temperature is analyzed from cycle analysis.

Intake

$$M = \frac{V_f}{\sqrt{\gamma R T_a}} = 0.7437$$

$$P_{02} = P_a\left(1 + \frac{\gamma-1}{2}M^2\right)^{\frac{\gamma}{\gamma-1}} = 114.0522 \text{ kPa}$$

$$T_{02} = T_a\left(1 + \frac{\gamma-1}{2}M^2\right) = 305.42 \text{ K}$$

Fan

$$P_{03} = P_{02}\pi_f = 239.51 \text{ kPa}$$

$$T_{03} = T_{02}(\pi_f)^{\frac{\gamma-1}{\gamma}} = 377.62 \text{ K}$$

Compressor

$$P_{04} = P_{03}\pi_c = 1574 \text{ kPa}$$

$$T_{04} = T_{03}(\pi_c)^{\frac{\gamma-1}{\gamma}} = 645 \text{ K}$$

Combustion chamber

$$\dot{m}_f Q_R + \dot{m}_h C_{pc} T_{04} = (\dot{m}_h + \dot{m}_f) C_{ph} T_{05}$$

$$T_{05} = \frac{f Q_R + C_{pc} T_{04}}{(1+f) C_{ph}} = \frac{0.006618 \times 43000 + 1.005 \times 645.0}{1.006618 \times 1.148} = 807.2 \text{ K}$$

Maximum temperature is then $T_{0\max} = 807.2$ K.

TURBOPROP & TURBOSHAFT ENGINE

Turboprop and turboshafts usually have a free turbine or power turbine to drive the propeller or the main rotor blade(turboshafts). Stress limitation require that the large diameter propeller rotate at a lower rate and hence a speed reducer is required. Turboprops may also have a thrust component due to the jet exhaust in addition to the propeller thrust. Thus in turboprops, thrust consists of the propeller thrust and the nozzle thrust. The total thrust of a turboprop engine is equal to the sum of the nozzle thrust and the propeller thrust.

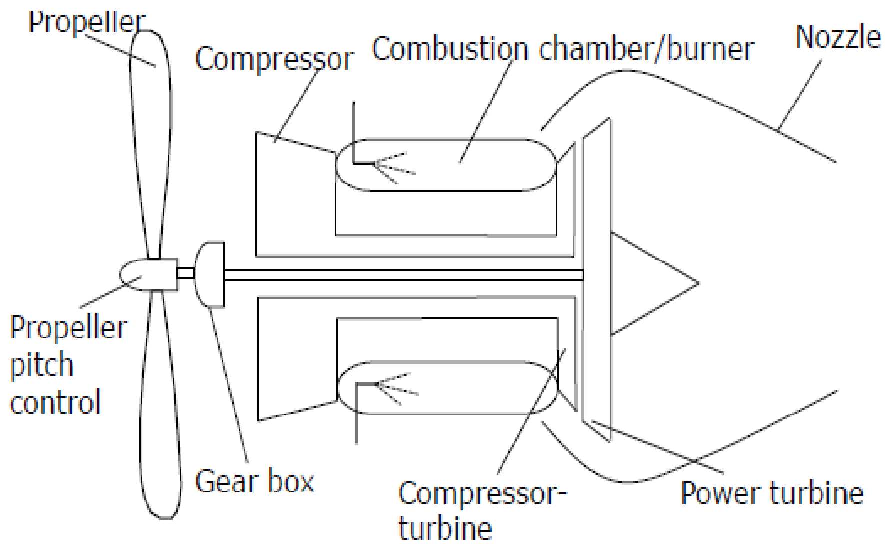


Figure 9 schematic of turboprop engine

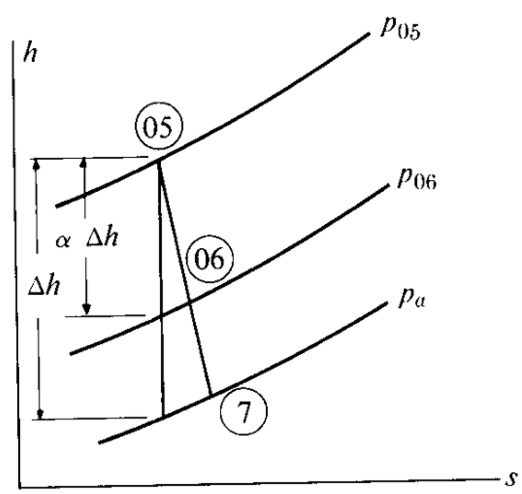


Figure 10 Enthalpy-entropy diagram for power turbine-exhaust nozzle analysis.

Turboprop engines may exert a significant part of their thrust by means of the hot-exhaust jet. Figure 10 indicates on an enthalpy-entropy diagram the thermodynamic path of the hot gases expanding through the power turbine and exhaust nozzle. It has been shown that there is an optimum hot-gas exhaust velocity that yields maximum thrust for a given gas generator and flight speed. Referring to Fig.10, let

Δh = enthalpy drop available in an ideal (isentropic) power turbine and exhaust nozzle,
α = fraction of Δh that would be used by an isentropic turbine having the actual stagnation pressure ratio,
η_{pt}, η_n = adiabatic efficiencies of the power turbine and exhaust nozzle, respectively,
η_g, η_{pr} = gear and propeller efficiencies, respectively.

The propeller thrust F_{pr} may be determined by considering the energy flux through the free-turbine and propeller shafts:

$$F_{pr} u = \eta_{pr}\eta_g\eta_{pt}\alpha\, \Delta h\, \dot{m}$$

$$F_{pr} = \frac{\eta_{pr}\eta_g\eta_{pt}\alpha\, \Delta h\, \dot{m}}{u}$$

where u is the flight velocity and

$\dot{m}$ is the hot-gas flow rate.

The exhaust nozzle thrust F_n may be written as

$$F_n = \dot{m}(u_e - u)$$

where the exhaust velocity is given by

$$u_e = \sqrt{2(1-\alpha)\eta_n\, \Delta h}$$

Thus the total thrust is given as follows

$$F = \frac{\eta_{pr}\eta_g\eta_{pt}\alpha\, \Delta h\, \dot{m}}{u} + \dot{m}(\sqrt{2(1-\alpha)\eta_n\, \Delta h} - u)$$

Example 1.4

Consider an aircraft engine flying at 200m/s with the inlet mass flow rate of 20kg/s. the propeller of the engine is having an efficiency of 0.8, which produces the thrust of 10000N, while the jet produces the thrust of 2000N. The power turbine and nozzles have the efficiencies of 0.88 and 0.92. Calculate the jet Thrust produced by the engine if the power turbine and propeller is removed.

Solution:

The Thrust power produced by the propeller is given by

$$FV_a = \eta_{pr}\eta_g\eta_{pt}\alpha\, \Delta h\, \dot{m}$$

$$\rightarrow \alpha\, \Delta h = \frac{FV_a}{\eta_{pr}\eta_g\eta_{pt}\, \dot{m}} = \frac{1000 \times 200}{0.8 \times 0.99 \times 0.88} = 142045.45\, J/kg$$

The nozzle thrust is given by

$$F_n = \dot{m}(V_e - V_a)$$

$$2000 = 20(V_e - 200)$$

$$V_e = 300 m/s$$

We know that

$$V_e = \sqrt{2(1-\alpha)\eta_n\, \Delta h}$$

$$300 = \sqrt{2(1-\alpha)0.92\, \Delta h}$$

$$(1-\alpha)\, \Delta h = 190958.49 J/kg$$

Since the power turbine and Propeller is removed the entire Δh occurs only in nozzle. So $\alpha = 0$

$$V_e = \sqrt{2\eta_n\, \Delta h} = \sqrt{2 \times 0.92 \times 190958.49} = 592.76 m/s$$

Thrust produced is $F_n = \dot{m}(V_e - V_a) = 20(592.76 - 200) = 7855.18 N$

RAMJET

Introduction

A RAMJET is a form of air-breathing jet engine that uses the engine's forward motion to compress incoming air without a rotary compressor. The ramjet obtains very high-pressure ratio of about 8 to 10 by ram compression which leads to design a jet engine without a mechanical compressor. A deceleration of the air from Mach number 3 at diffuser inlet to Mach number 0.3 in combustion chamber would cause the pressure of more than 30. Due to shock and other losses inevitable at such velocities, this entire pressure rise is not available, but still the achieved pressure rise is sufficient for the required combustion process. Thus the principle of ram pressure rise is used in the ramjet engines.

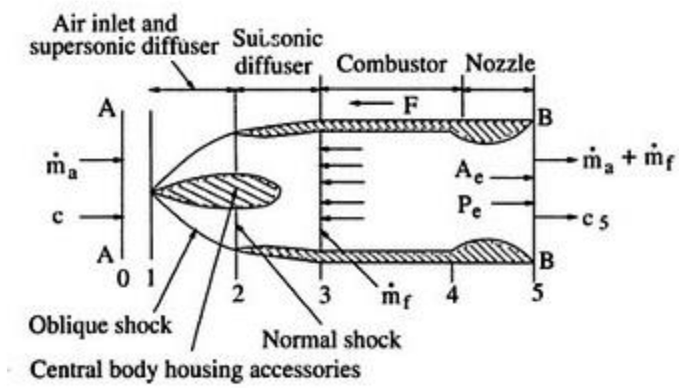


Figure 11 *RAMJET Engine*

Operating principle

A simplified sketch of the RAMJET engine is illustrated in fig 6. The engine consists of the following.

- Supersonic diffuser (1-2),
- Subsonic Diffuser (2-3),
- Combustion chamber (3-4),
- Nozzle (4-5).

Both supersonic and subsonic diffuser converts the kinetic energy of the entire air into pressure rise. This energy transformation is called ram effect and pressure rise due to this ram effect is known as the ram pressure

Air from the atmosphere enters the engine at a very high speed, and the velocity gets reduced first in the supersonic diffuser, thereby its static pressure increases. The air then enters the subsonic diffuser wherein it is compressed further. Afterwards, the air flows into the combustion chamber; suitable injectors inject the fuel and mixed with the unburnt air. The air is heated to a temperature of the order of 1500-2000K by the continuous combustion of fuel. The fresh supply of air from the diffuser builds up pressure at the diffuser end so that these gases are made to expand in the combustion chamber towards the tailpipe. Further, they are allowed to Further; they are authorized to expand in the exhaust nozzle section. The products will leave the engine with a speed exceeding that of the entering air. Because of the rate of increase in the momentum of the working fluid, a thrust is developed in the direction of flight.

Normally the air enters the engine with a supersonic speed which must be reduced to a subsonic value. This is necessary to prevent the blow out of the flame in the combustion chamber. The velocity must be small enough to make it possible to add the required quantity of fuel for stable combustion. Both theory and experiment indicates that the speed of the air entering the combustion chamber should not be higher than that corresponding to a local Mach number of 0.2 approximately.

The cycle pressure ratio or a ramjet engine depends on upon its flight velocity. The higher the flight velocity, the larger is the ram pressure, and consequently larger will be the thrust. This is true until a condition is reached where the discharge nozzle becomes chocked. After that, the nozzle operates with a constant Mach number of l at its throat. Therefore, a ramjet having fixed geometry is designed for a specific Mach number and altitude, and at the design point, will give the best performance.

Since the ramjet engine cannot operate under static conditions, as there will be no pressure rise in the diffuser, it is not self-propelling at zero flight velocity. To initiate its operation, the ramjet must be either launched from an airplane in flight or be given an initial velocity by some auxiliary means, such as launching rockets. Since the ramjet is an air-breathing engine, its maximum altitude is limited. Its field of operations is inherently in speed ranges above those of the other air-breathing engines. However, it has limited use in the high subsonic speed range. Its best performance capabilities in supersonic speed are in the range of Mach number between 2 and 5. The top speed is limited by the problem of cooling of the outer skin of the engine body at the high flight Mach numbers.

Thermodynamic cycle

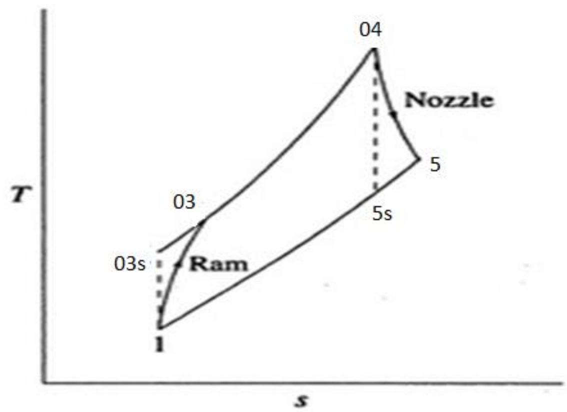


Figure 12 *T-S diagram of Ramjet*

The ramjet works on the thermodynamics cycle viz., the Brayton cycle. Fig 7 shows this cycle on a T-S diagram. The process in ideal cycle is given as follows

- 1→3s is isentropic ram compression in both the subsonic and supersonic diffuser
- 3→4 combustion of air-fuel mixture in combustion chamber
- 4→5s is the isentropic expansion in the nozzle

In actual or real cycle there will be losses due to shock, friction and mixing at the diffuser and losses in the nozzle. The process in actual cycle is given as follows

- 1→3 is ram compression in both the subsonic and supersonic diffuser
- 3→4 combustion of air-fuel mixture in combustion chamber
- 4→5 is the expansion in the nozzle

Since enlargement and compression are assumed isentropic, in an ideal cycle the stagnation pressure must remain constant, and the pressure ratio of ram compression and nozzle expansion ratio must be same.

$$\frac{P_{02}}{P_1} = \left(1 + \frac{\gamma - 1}{2} M_i^2\right)^{\frac{\gamma}{\gamma-1}}$$

$$\frac{P_{03}}{P_4} = \left(1 + \frac{\gamma - 1}{2} M_j^2\right)^{\frac{\gamma}{\gamma-1}}$$

Where M_1 and M_3 are the inlet and exhaust Mach number. We have,

$$\frac{P_{02}}{P_1} = \frac{P_{03}}{P_4}$$

Which gives

$$M_i = M_j$$

$$\frac{V_i}{\sqrt{\gamma R T_1}} = \frac{V_j}{\sqrt{\gamma R T_4}}$$

$$V_j = V_i \sqrt{\frac{T_4}{T_1}}$$

Knowing V_i and V_j, thrust can be calculated.

$$F = \dot{m}_a \left[(1+f) V_j - V_i \right]$$

Advantages

- Ramjet is very simple and does not have any moving part. It is very cheap to produce and requires less or no maintenance.
- Because a turbine is not used to drive the mechanical compressor, the maximum temperature which can be allowed in ramjet is very high, about $2000^\circ C$ as compared to about $900^\circ C$ in turbojets. This allows a greater thrust to be obtained by burning fuel at an air-fuel ratio of about 13:1, which gives higher temperature.
- The specific fuel consumption is better than other gas turbine power plants at high speed and high altitudes.
- Theoretically, there is no upper limit to the flight speed of ramjet.

Disadvantages

- Since the compression of air is obtained by its speed about the engine, the take-off thrust is zero, and it is not possible to start a ramjet without an external launching device
- The engine relies on the diffuser, and it 's hard to design a diffuser which will give good pressure recovery over a wide range of speeds.
- Due to high air speed, the combustion chamber requires the flame holder to stabilize the combustion.
- At very high temperature of about $2000^\circ C$ dissociation of products of combustion occurs which will reduce the efficiency of the plant if not recovered in nozzle during expansion.

Basic Characteristics and Applications

- Since the design is simple, it can be applicable for mass production at relatively low cost.
- It is independent of fuel technology, and a wide range of liquid and solid fuels can be used.
- Its fuel consumption is comparatively very large for its application in aircraft propulsion or missiles at low and moderate speeds.
- Its fuel consumption decreases with flight speed and approaches reasonable value when the flight Mach number is between 2 and 5, and therefore, it is suitable for propelling supersonic missiles.
- Due to its high thrust at high operational speed, it is widely used in high-speed military aircraft and missiles.
- Subsonic ramjets are used in target weapons, in conjunction with turbojets or rockets for getting the starting torque.

Example 1.5

A ramjet engine operates at M = 2 at an altitude of 6500 m. The diameter of the inlet diffuser at entry is 50 cm and the stagnation temperature at the nozzle entry is 1600 K. The calorific value of the fuel used is 40 MJ/kg. The properties of the combustion gases are same as those of air ($\gamma = 1.4$, R = 287 J/kg K). The velocity of air at the diffuser exit is negligible. Calculate (a) flight speed (b) air flow rate (c) diffuser pressure ratio (d) fuel to air ratio (e) nozzle pressure ratio (f) nozzle jet Mach number (e) propulsive efficiency (f) thrust. Assume the following values $\eta_D = 0.90, \eta_B = 0.98, \eta_N = 0.96$ stagnation pressure loss in the combustion chamber = $0.02P_{02}$.

Solution:

At h=6500m the properties of air are

$$T_1 = T_{sealevel} - \lambda h = 288.15 - (6.5 \times 6.5) = 245.90K$$

$$\frac{P_1}{P_{sealevel}} = \left[\frac{T_1}{T_{sealevel}}\right]^{g/R\lambda}$$

$$P_1 = P_{sealevel}\left[\frac{T_1}{T_{sealevel}}\right]^{g/R\lambda} = 1.01325 \times 10^5 \left[\frac{245.90}{288.15}\right]^{9.81\times1000/287\times6.5} = 0.440bar$$

$$\rho_1 = \rho_{sealevel}\left[\frac{T_1}{T_{sealevel}}\right]^{(g/R\lambda)-1} = 1.225\left[\frac{245.90}{288.15}\right]^{(9.81\times1000/287\times6.5)-1} = 0.624Kg/m^3$$

(a) flight speed

$$M_1 = \frac{V_1}{a_1}$$

$$V_1 = M_1 \times a_1 = 2 \times \sqrt{1.4 \times 287 \times 245.90} = 629m/s$$

(b) air flow rate

$$A_1 = \frac{\pi}{4}D_i^2 = \frac{\pi}{4}0.5^2 = 0.1963m^2$$

$$\dot{m}_a = \rho_1 V_1 A_1 = 0.624 \times 629 \times 0.1963$$

$$\dot{m}_a = 77.0469kg/s$$

(c) diffuser pressure ratio

$$\eta_D = \frac{T_{2S} - T_1}{T_2 - T_1} = \frac{T_{2S}/T_1 - 1}{T_2/T_1 - 1} = \frac{\left(P_2/P_1\right)^{(\gamma-1)/\gamma} - 1}{T_2/T_1 - 1}$$

since velocity is negligible in combustion chamber

$$T_{02} = T_2$$

$$\left(\frac{P_2}{P_1}\right)^{(\gamma-1)/\gamma} = \eta_D\left(\frac{T_2}{T_1} - 1\right) + 1 = 0.9\left(\left(\frac{T_{02}}{T_1}\right) - 1\right) + 1$$

$$\left(\frac{P_2}{P_1}\right)^{(\gamma-1)/\gamma} = 0.9\left(\left(1 + \frac{\gamma-1}{2}M_1^{\,2}\right) - 1\right) + 1$$

$$\left(\frac{P_2}{P_1}\right) = \left[0.9\left(\frac{1.4-1}{2}\times 2^2\right) + 1\right]^{\gamma/(\gamma-1)} = 6.6734$$

$$P_2 = 6.6734\times P_1 = 6.6734\times 0.440 = 2.9363 bar$$

(d) fuel-air ratio

$$\frac{T_{01}}{T_1} = 1 + \frac{\gamma-1}{2}M_1^{\,2} = 1 + \frac{1.4-1}{2}\times 2^2 = 1.8$$

$$T_{01} = T_{02} = 1.8\times 245.9 = 442.62K$$

$$\eta_B = \frac{\dot{m}_a\, C_p\left(\left(1+f\right)T_{03} - T_{02}\right)}{\dot{m}_f\, Q_f}$$

$$\Rightarrow f = \frac{1005\left(\left(1+f\right)1600 - 442.62\right)}{0.98\times 40\times 10^6}$$

$$f = \frac{0.02967}{0.9589} = 0.03094$$

(e) nozzle pressure ratio

$$P_{03} = P_{02} - 0.02P_{02} = 0.98P_{02}$$

$$P_{03} = 0.98\times 2.9363 = 2.8775 bar$$

$$\frac{P_{03}}{P_4} = \frac{P_3}{P_4} = 6.6597$$

(f) Nozzle jet Mach number

$$\frac{P_{03}}{P_4} = \left[1 + \frac{\gamma-1}{2}M_{4S}^2\right]^{\gamma/\gamma-1}$$

$$M_{4S}^2 = \left[\left(\frac{P_{03}}{P_4}\right)^{\gamma-1/\gamma} - 1\right]\times\frac{2}{\gamma-1} = \left[\left(6.6597\right)^{1.4-1/1.4} - 1\right]\times\frac{2}{1.4-1} = 3.5949$$

$$M_{4S} = 1.896$$

$$\frac{T_{04}}{T_{4S}} = \left[1 + \frac{\gamma - 1}{2} M_{4S}^2\right] = \left[1 + \frac{1.4 - 1}{2} 1.896^2\right] = 1.7189$$

$$T_{4S} = \frac{1600}{1.7189} = 930.82K$$

$$\eta_N = \frac{T_{04} - T_4}{T_{04} - T_{4S}}$$

$$T_4 = T_{04} - \eta_N \left(T_{04} - T_{4S}\right)$$

$$T_4 = 1600 - 0.96\left(1600 - 930.82\right)$$

$$T_4 = 957.58K$$

$$\frac{T_{04}}{T_4} = \left[1 + \frac{\gamma - 1}{2} M_4^2\right] \Rightarrow M_4 = \sqrt{\left[\left(\frac{T_{04}}{T_4}\right) - 1\right] \frac{2}{\gamma - 1}}$$

$$M_4 = \sqrt{\left[\left(\frac{1600}{957.58}\right) - 1\right] \frac{2}{1.4 - 1}} = 1.8315$$

(e) Propulsive efficiency

$$V_4 = M_4 \times a_4 = 1.8315 \times \sqrt{1.4 \times 287 \times 957.58} = 1136 m/s$$

$$\eta_P = \frac{2\left(V_1 / V_4\right)}{1 + \left(V_1 / V_4\right)} = \frac{2\left(629 / 1136\right)}{1 + \left(629 / 1136\right)} = 0.7127$$

(f) Thrust.

$$F = \dot{m}_a \left(\left(1 + f\right) V_4 - V_1\right)$$

$$F = 77.0469\left(\left(1 + 0.03094\right) 1136 - 629\right) = 41770.81N$$

$$F = 41.77KN$$