Free vortex theory

It was pointed out earlier that the shape of the velocity triangles must vary from root to tip of the blade because the blade speed U increases with radius. Another reason is that the whirl component in the flow at outlet from the nozzles causes the static pressure and temperature to vary across the annulus. With a uniform pressure at inlet, or at least with a much smaller variation because the whirl component is smaller, it is clear that the pressure drop across the nozzle will vary giving rise to a corresponding variation in efflux velocity C_2. Twisted blading designed to take account of the changing gas angles is called vortex blading.

Fig. 1.6 refers to a single-stage turbine with axial inlet velocity and no swirl at outlet, the whirl component at inlet and outlet of a repeating stage will be small compared with C_{W2} the reaction will therefore still increase from root to tip, if somewhat less markedly.

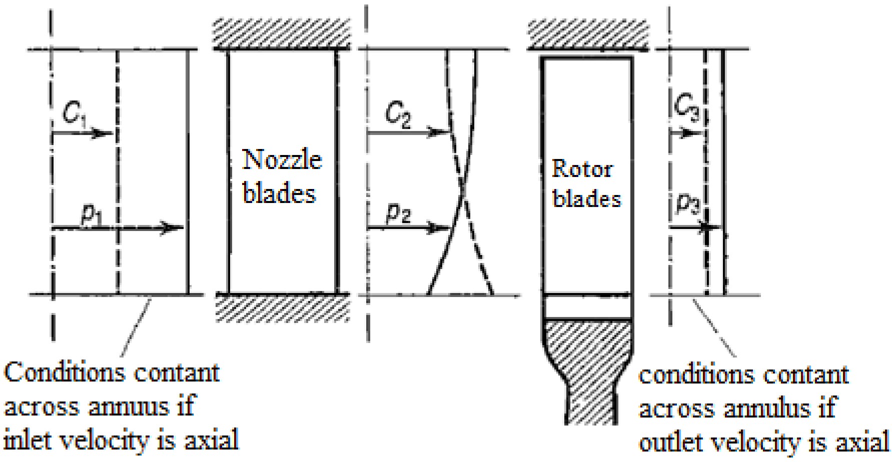

Fig 1.7 single-stage axial flow gas turbine

$$W_s = U(C_{w2} + C_{w3})$$
$$W_s = \omega(C_{w2}r + C_{w3}r) = \text{constant}$$
$$\delta m = \rho_2 2\pi \delta r C_{a2}$$
$$m = 2\pi C_{a2}\int_{r_r}^{r_t} \rho_2 r dr$$
$$h_0 = h + \frac{C^2}{2} = h + (C_a{}^2 + C_w{}^2)$$

the variation of enthalpy with radius is given as

$$\frac{dh_0}{dr} = \frac{dh}{dr} + C_a\frac{dC_a}{dr} + C_w\frac{dC_w}{dr}$$

from the thermodynamics relation

$$Tds = dh - dp/\rho$$
$$\frac{dh}{dr} = T\frac{ds}{dr} + ds\frac{dT}{dr} + \frac{1}{\rho}\frac{dp}{dr} - \frac{1}{\rho^2}\frac{d\rho}{dr}dp$$

neglecting the second order terms from the equation we get

$$\frac{dh}{dr} = T\frac{ds}{dr} + \frac{1}{\rho}\frac{dp}{dr}$$

substituting for $\frac{dh}{dr}$ we get

$$\Rightarrow \frac{dh_0}{dr} = T\frac{ds}{dr} + \frac{1}{\rho}\frac{dp}{dr} + C_a\frac{dC_a}{dr} + C_w\frac{dC_w}{dr}$$

we know that $\frac{1}{\rho}\frac{dp}{dr} = \frac{C_w{}^2}{r}$

$$\Rightarrow \frac{dh_0}{dr} = T\frac{ds}{dr} + \frac{C_w{}^2}{r} + C_a\frac{dC_a}{dr} + C_w\frac{dC_w}{dr}$$

$$\Rightarrow \frac{dh_0}{dr} = \frac{C_w{}^2}{r} + C_a\frac{dC_a}{dr} + C_w\frac{dC_w}{dr}$$

the above equation is reffered as vortex energy equation

since the enthalpy is constant $\frac{dh_0}{dr} = 0$

$$\frac{C_w{}^2}{r} + C_a\frac{dC_a}{dr} + C_w\frac{dC_w}{dr} = 0$$

since the C_a is constant $\frac{dC_a}{dr} = 0$

$$\frac{C_w{}^2}{r} + C_w\frac{dC_w}{dr} = 0$$

$$\frac{C_w{}^2}{r} = -C_w\frac{dC_w}{dr}$$

$$\frac{dr}{r} = -\frac{dC_w}{C_w}$$

integrating the above equation we get

$$C_w r = \text{constant}$$

from the velocity triangle

$$C_w r = rC_a \tan\alpha_2 = \text{constant}$$

now α_2 at any radius r isrelated to α_{2m} at the mean radius r_m by

$$\tan\alpha_2 = \left(\frac{r_m}{r}\right)_2 \tan\alpha_{2m}$$

$$\tan\alpha_3 = \left(\frac{r_m}{r}\right)_3 \tan\alpha_{3m}$$

simillarly the blade angles can be derived by

$$\tan\beta_2 = \tan\alpha_2 - \frac{U}{C_{a2}}$$

$$\tan\beta_2 = \left(\frac{r_m}{r}\right)_2 \tan\alpha_{2m} - \left(\frac{r}{r_m}\right)_2 \frac{U}{C_{a2}}$$

$$\tan\beta_3 = \left(\frac{r_m}{r}\right)_3 \tan\alpha_{3m} - \left(\frac{r}{r_m}\right)_3 \frac{U}{C_{a3}}$$