Compressor-Turbine Matching

The problem of matching turbine and compressor performance has great importance for jet engines, which must operate under conditions involving large variations in thrust, inlet pressure, and temperature, and flight Mach number. Matching the components of turbofan and turboprop engines involves similar considerations and procedures.

Essentially the matching problem is simple, though the computation can be length. The steady-state engine performance at each speed is determined by two conditions: continuity of flow and a power balance. The turbine mass flow must be the sum of the compressor mass flow and the fuel flow, minus compressor bleed flow. Also the power output of the turbine must be equal to that demanded by the compressor.

In principle, the matching computations could proceed as follows:

1. Select operating speed
2. Assume turbine inlet temperature
3. Assume compressor pressure ratio
4. Calculate compressor work per unit mass
5. Calculate turbine pressure ratio required to produce this work
6. Check to see if compressor mass flow plus fuel flow equals turbine mass flow; if not, assume a new value of compressor pressure ratio and repeat steps 4, 5, and 6 until continuity is satisfied.
7. Now calculate the pressure ratio across the jet nozzle from the pressure ratios across the diffuser, compressor, combustor, and turbine.
8. Calculate the area of jet nozzle outlet necessary to pass the turbine mass flow calculated in step 6 with pressure ratio calculated in step 7 and the stagnation temperature calculated. If the calculated area does not equal the actual exit area, assume a new value of turbine inlet temperature (step-2) and repeat the entire procedure.

The designer will try to match turbine and compressor so that the compressor is operating near its peak efficiency through the entire range of operation, as shown in the below figure, where the operating line (i.e., the locus of stead-state matching condition) runs through the centres of the islands defined by the constant-efficiency lines.

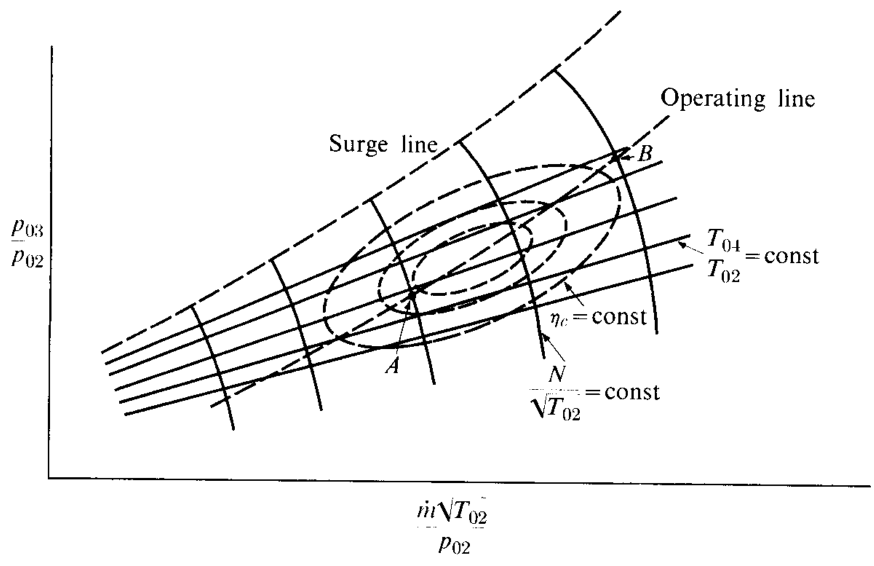
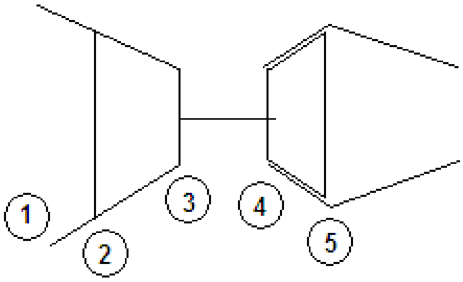


Fig Operating line on a compressor map



Compressor Work $= \dot{m}_a C_p \left(T_{03} - T_{02}\right)$

Turbine Work $= \left(\dot{m}_a + \dot{m}_f\right) C_p \left(T_{04} - T_{05}\right)$

W.K.T specific work of compressor
is equal to specific work of turbine

$$\frac{\text{Compressor Work}}{\text{mass flow rate}} = \frac{\text{Turbine Work}}{\text{mass flow rate}}$$

$$\frac{\dot{m}_a C_{pa}\left(T_{03} - T_{02}\right)}{\dot{m}_a} = \frac{\left(\dot{m}_a + \dot{m}_f\right) C_{pg}\left(T_{04} - T_{05}\right)}{\left(\dot{m}_a + \dot{m}_f\right)}$$

$$C_{pa}\left(T_{03} - T_{02}\right) = C_{pg}\left(T_{04} - T_{05}\right)$$

$$C_{pa}T_{02}\left(\frac{T_{03}}{T_{02}} - 1\right) = C_{pg}T_{04}\left(1 - \frac{T_{05}}{T_{04}}\right)$$

$$\frac{C_{pa}T_{02}}{\eta_c}\left[\left(\frac{P_{03}}{P_{02}}\right)^{(\gamma_a - 1)/\gamma_a} - 1\right] = \eta_t C_{pg}T_{04}\left[1 - \left(\frac{P_{05}}{P_{04}}\right)^{(\gamma_g - 1)/\gamma_g}\right]$$

the above equation is for ideal condition
For actual condition

$$\frac{C_{pa}T_{02}}{\eta_c}\left[\left(\frac{P_{03}}{P_{02}}\right)^{(\gamma_a - 1)/\gamma_a} - 1\right] = \eta_m \eta_t C_{pg}T_{04}\left[1 - \left(\frac{P_{05}}{P_{04}}\right)^{(\gamma_g - 1)/\gamma_g}\right]$$

where $\eta_m \rightarrow$ mechanical efficiency

Turbine Pressure Ratio

$$TPR = \frac{P_{05}}{P_{04}} = \left[1 - \frac{C_{pa}T_{02}}{\eta_c \eta_t C_{pg}T_{04}}\left[\left(\frac{P_{03}}{P_{02}}\right)^{(\gamma_a - 1)/\gamma_a} - 1\right]\right]^{\gamma_g/(\gamma_g - 1)}$$

Compressor Pressure Ratio

$$CPR = \frac{P_{03}}{P_{02}} = \left[1 + \frac{\eta_c \eta_t C_{pg}T_{04}}{C_{pa}T_{02}}\left[1 - \left(\frac{P_{05}}{P_{04}}\right)^{(\gamma_g - 1)/\gamma_g}\right]\right]^{\gamma_a/(\gamma_a - 1)}$$