Numerical Problems

Illustrative Example 1.1 Consider an impulse gas turbine in which gas enters at pressure = 5.2 bar and leaves at 1.03 bar. The turbine inlet temperature is 1000 K and isentropic efficiency of the turbine is 0.88. If mass flow rate of air is 28 kg/s, nozzle angle at outlet is 57°, and absolute velocity of gas at inlet is 140 m/s, determine the gas velocity at nozzle outlet, whirl component at rotor inlet and turbine work output. Take, $\gamma = 1.33$, and $C_{pg} = 1.147$ kJ/kgK .

Solution

From isentropic $p-T$ relation for expansion process

$$\frac{T'_{02}}{T_{01}} = \left(\frac{p_{02}}{p_{01}}\right)^{(\gamma-1)/\gamma}$$

or $$T'_{02} = T_{01}\left(\frac{p_{02}}{p_{01}}\right)^{(\gamma-1)/\gamma} = 1000\left(\frac{1.03}{5.2}\right)^{(0.248)} = 669\ \text{K}$$

Using isentropic efficiency of turbine

$$T_{02} = T_{01} - \eta_t\left(T_{01} - T'_{02}\right) = 1000 - 0.88(1000 - 669)$$

$$= 708.72\ \text{K}$$

Using steady-flow energy equation

$$\frac{1}{2}\left(C_2^2 - C_1^2\right) = C_p(T_{01} - T_{02})$$

Therefore, $C_2 = \sqrt{[(2)(1147)(1000 - 708.72) + 19600]} = 829.33$ m/s

From velocity triangle, velocity of whirl at rotor inlet

$$C_{w2} = 829.33 \sin 57° = 695.5 \quad \text{m/s}$$

Turbine work output is given by

$$W_t = mC_{pg}(T_{01} - T_{02}) = (28)(1.147)(1000 - 708.72)$$

$$= 9354.8\ \text{kW}$$

Design Example 1.2 In a single-stage gas turbine, gas enters and leaves in axial direction. The nozzle efflux angle is 68°, the stagnation temperature and stagnation pressure at stage inlet are 800°C and 4 bar, respectively. The exhaust static pressure is 1 bar, total-to-static efficiency is 0.85, and mean blade speed is 480 m/s, determine (1) the work done, (2) the axial velocity which is constant through the stage, (3) the total-to-total efficiency, and (4) the degree of reaction. Assume $\gamma = 1.33$, and $C_{pg} = 1.147$ kJ/kgK.

Solution

(1) The specific work output

$$\begin{aligned} W &= C_{pg}(T_{01} - T_{03}) \\ &= \eta_{ts} C_{pg} T_{01}\left[1 - (1/4)^{0.33/1.33}\right] \\ &= (0.85)(1.147)(1073)\left[1 - (0.25)^{0.248}\right] = 304.42\ \text{kJ/kg} \end{aligned}$$

(2) Since $\alpha_1 = 0$, $\alpha_3 = 0$, $C_{w1} = 0$ and specific work output is given by

$$W = UC_{w2} \quad \text{or} \quad C_{w2} = \frac{W}{U} = \frac{304.42 \times 1000}{480} = 634.21\ \text{m/s}$$

From velocity triangle

$$\sin \alpha_2 = \frac{C_{w2}}{C_2}$$

or

$$C_2 = \frac{C_{w2}}{\sin \alpha_2} = \frac{634.21}{\sin 68^\circ} = 684\ \text{m/s}$$

Axial velocity is given by

$$Ca_2 = 684 \cos 68^\circ = 256.23\ \text{m/s}$$

(3) Total-to-total efficiency, η_{tt}, is

$$\begin{aligned} \eta_{tt} &= \frac{T_{01} - T_{03}}{T_{01} - T'_{03}} \\ &= \frac{w_s}{T_{01} - \left(T_3 + \frac{C_3^2}{2C_{pg}}\right)} = \frac{w_s}{\frac{w_s}{\eta_{ts}} - \frac{C_3^2}{2C_{pg}}} \\ &= \frac{304.42}{\frac{304.42}{0.85} - \frac{(256.23)^2}{2 \times 1147}} = 92.4\% \end{aligned}$$

(4) The degree of reaction

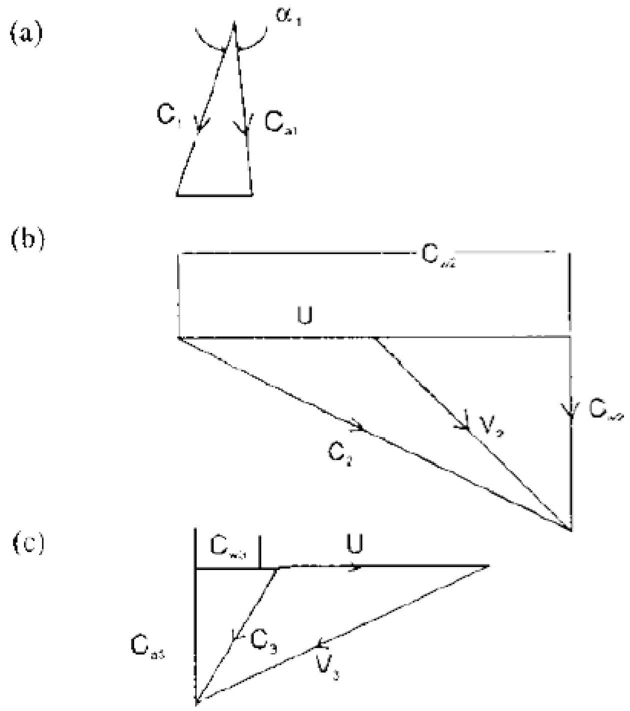
$$\Lambda = \frac{Ca}{2U}(\tan\beta_3 - \tan\beta_2)$$

$$= \left(\frac{Ca}{2U}\times\frac{U}{Ca}\right) - \left(\frac{Ca}{2U}\tan\alpha_2\right) + \left(\frac{U}{Ca}\times\frac{Ca}{2U}\right)$$

(from velocity triangle)

$$\Lambda = 1 - \frac{Ca}{2U}\tan\alpha_2 = 1 - \frac{256.23}{(2)(480)}\tan 68^\circ = 33.94\%$$

Design Example 1.3 In a single-stage axial flow gas turbine gas enters at stagnation temperature of 1100 K and stagnation pressure of 5 bar. Axial velocity is constant through the stage and equal to 250 m/s. Mean blade speed is 350 m/s. Mass flow rate of gas is 15 kg/s and assume equal inlet and outlet velocities. Nozzle efflux angle is 63°, stage exit swirl angle equal to 9°. Determine the rotor-blade gas angles, degree of reaction, and power output.



Solution

$$Ca_1 = Ca_2 = Ca_3 = Ca = 250 \text{ m/s}$$

From velocity triangle (b)

$$C_2 = \frac{Ca_2}{\cos\alpha_2} = \frac{250}{\cos 63^\circ} = 550.67 \text{ m/s}$$

From figure (c)

$$C_3 = \frac{Ca_3}{\cos\alpha_3} = \frac{250}{\cos 9^\circ} = 253 \text{ m/s}$$

$$C_{w3} = Ca_3 \tan\alpha_3 = 250\tan 9^\circ = 39.596 \text{ m/s}$$

$$\tan\beta_3 = \frac{U + C_{w3}}{Ca_3} = \frac{350 + 39.596}{250} = 1.5584$$

i.e., $\beta_3 = 57.31^\circ$

From figure (b)

$$C_{w2} = Ca_2 \tan\alpha_2 = 250\tan 63^\circ = 490.65 \text{ m/s}$$

and

$$\tan\beta_2 = \frac{C_{w2} - U}{Ca_2} = \frac{490.65 - 350}{250} = 0.5626$$

$$\therefore \beta_2 = 29^\circ 21'$$

Power output

$$W = mUCa(\tan\beta_2 + \tan\beta_3)$$

$$= (15)(350)(250)(0.5626 + 1.5584)/1000$$

$$= 2784 \text{ kW}$$

The degree of reaction is given by

$$\Lambda = \frac{Ca}{2U}(\tan\beta_3 - \tan\beta_2)$$

$$= \frac{250}{2 \times 350}(1.5584 - 0.5626)$$

$$= 35.56\%$$

Design Example.1.4 Calculate the nozzle throat area for the same data as in the precious question, assuming nozzle loss coefficient, $T_N = 0.05$. Take $\gamma = 1.333$, and $C_{pg} = 1.147$ kJ/kgK.

Solution

Nozzle throat area, $A = m/\rho_2 Ca_2$

and $\rho_2 = \dfrac{p_2}{RT_2}$

$$T_2 = T_{02} - \frac{C_2^2}{2C_p} = 1100 - \frac{(550.67)^2}{(2)(1.147)(1000)} \quad (T_{01} = T_{02})$$

i.e., $T_2 = 967.81\,\mathrm{K}$

From nozzle loss coefficient

$$T_2' = T_2 - \lambda_N \frac{C_2^2}{2C_p} = 967.81 - \frac{0.05 \times (550.67)^2}{(2)(1.147)(1000)} = 961.2\,\mathrm{K}$$

Using isentropic p–T relation for nozzle expansion

$$p_2 = p_{01} / \left(T_{01}/T_2'\right)^{\gamma/(\gamma-1)} = 5/(1100/961.2)^4 = 2.915 \text{ bar}$$

Critical pressure ratio

$$p_{01}/p_c = \left(\frac{\gamma+1}{2}\right)^{\gamma/(\gamma-1)} = \left(\frac{2.333}{2}\right)^4 = 1.852$$

or $\quad p_{01}/p_2 = 5/2.915 = 1.715$

Since $\frac{p_{01}}{p_2} < \frac{p_{01}}{p_c}$, and therefore nozzle is unchoked.
Hence nozzle gas velocity at nozzle exit

$$C_2 = \sqrt{[2C_{pg}(T_{01} - T_2)]}$$
$$= \sqrt{[(2)(1.147)(1000)(1100 - 967.81)]} = 550.68 \text{ m/s}$$

Therefore, nozzle throat area

$$A = \frac{m}{\rho_2 C_2}, \text{ and } \rho_2 = \frac{p_2}{RT_2} = \frac{(2.915)(10^2)}{(0.287)(967.81)} = 1.05 \text{ kg/m}^3$$

Thus

$$A = \frac{15}{(1.05)(550.68)} = 0.026 \text{ m}^2$$

Design Example.1.5 In a single-stage turbine, gas enters and leaves the turbine axially. Inlet stagnation temperature is 1000 K, and pressure ratio is 1.8 bar. Gas leaving the stage with velocity 270 m/s and blade speed at root is 290 m/s. Stage isentropic efficiency is 0.85 and degree of reaction is zero. Find the nozzle efflux angle and blade inlet angle at the root radius.

Solution

Since $\Lambda = 0$, therefore

$$\Lambda = \frac{T_2 - T_3}{T_1 - T_3},$$

hence

$$T_2 = T_3$$

From isentropic p–T relation for expansion

$$T'_{03} = \frac{T_{01}}{\left(p_{01}/p_{03}\right)^{(\gamma-1)/\gamma}} = \frac{1000}{(1.8)^{0.249}} = 863.558\ \text{K}$$

Using turbine efficiency

$$T_{03} = T_{01} - \eta_t\left(T_{01} - T'_{03}\right)$$

$$= 1000 - 0.85(1000 - 863.558) = 884\ \text{K}$$

In order to find static temperature at turbine outlet, using static and stagnation temperature relation

$$T_3 = T_{03} - \frac{C_3^2}{2C_{pg}} = 884 - \frac{270^2}{(2)(1.147)(1000)} = 852\ \text{K} = T_2$$

Dynamic temperature

$$\frac{C_2^2}{2C_{pg}} = 1000 - T_2 = 1000 - 852 = 148\ \text{K}$$

$$C_2 = \sqrt{[(2)(1.147)(148)(1000)]} = 582.677\ \text{m/s}$$

Since, $C_{pg}\Delta T_{os} = U(C_{w3} + C_{w2}) = UC_{w2}\ \ (C_{w3} = 0)$

Therefore, $C_{w2} = \dfrac{(1.147)(1000)(1000 - 884)}{290} = 458.8\ \text{m/s}$

From velocity triangle

$$\sin\alpha_2 = \frac{C_{w2}}{C_2} = \frac{458.8}{582.677} = 0.787$$

That is, $\alpha_2 = 51°54'$

$$\tan\beta_2 = \frac{C_{w2} - U}{Ca_2} = \frac{458.8 - 290}{C_2 \cos\alpha_2}$$
$$= \frac{458.8 - 290}{582.677 \cos 51.90°} = 0.47$$

i.e., $\beta_2 = 25°9'$

Design Example 1.6 In a single-stage axial flow gas turbine, gas enters the turbine at a stagnation temperature and pressure of 1150 K and 8 bar, respectively. Isentropic efficiency of stage is equal to 0.88, mean blade speed is 300 m/s, and rotational speed is 240 rps. The gas leaves the stage with velocity 390 m/s. Assuming inlet and outlet velocities are same and axial, find the blade height at the outlet conditions when the mass flow of gas is 34 kg/s, and temperature drop in the stage is 145 K.

Solution

Annulus area A is given by

$$A = 2\pi r_m h$$

where h = blade height

r_m = mean radius

As we have to find the blade height from the outlet conditions, in this case annulus area is A_3.

$$\therefore \quad h = \frac{A_3}{2\pi r_m}$$
$$U_m = \pi D_m N$$

or $D_m = \dfrac{(U_m)}{\pi N} = \dfrac{300}{(\pi)(240)} = 0.398$

i.e., $r_m = 0.199$ m

Temperature drop in the stage is given by

$$T_{01} - T_{03} = 145 \text{ K}$$

Hence $T_{03} = 1150 - 145 = 1005$ K

$$T_3 = T_{03} - \frac{C_3^2}{2C_{pg}} = 1005 - \frac{390^2}{(2)(1.147)(1000)} = 938.697 \text{ K}$$

Using turbine efficiency to find isentropic temperature drop

$$T'_{03} = 1150 - \frac{145}{0.88} = 985.23 \text{ K}$$

Using isentropic $p-T$ relation for expansion process

$$p_{03} = \frac{p_{01}}{(T_{01}/T'_{03})^{\gamma/(\gamma-1)}} = \frac{8}{(1150/985.23)^4} = \frac{8}{1.856}$$

i.e., $p_{03} = 4.31$ bar

Also from isentropic relation

$$p_3 = \frac{p_{03}}{(T'_{03}/T_3)^{\gamma/(\gamma-1)}} = \frac{4.31}{(985.23/938.697)^4} = \frac{4.31}{1.214} = 3.55 \text{ bar}$$

$$\rho_3 = \frac{p_3}{RT_3} = \frac{(3.55)(100)}{(0.287)(938.697)} = 1.32 \text{ kg/m}^3$$

$$A_3 = \frac{m}{\rho_3 Ca_3} = \frac{34}{(1.32)(390)} = 0.066 \text{ m}^2$$

Finally,

$$h = \frac{A_3}{2\pi r_m} = \frac{0.066}{(2\pi)(0.199)} = 0.053 \text{ m}$$

Design Example 1.7 The following data refer to a single-stage axial flow gas turbine with convergent nozzle:

Inlet stagnation temperature, T_{01}	1100 K
Inlet stagnation pressure, p_{01}	4 bar
Pressure ratio, p_{01}/p_{03}	1.9
Stagnation temperature drop	145 K
Mean blade speed	345 m/s
Mass flow, m	24 kg/s
Rotational speed	14, 500 rpm
Flow coefficient, Φ	0.75
Angle of gas leaving the stage	12°

$C_{pg} = 1147$ J/kg K, $\gamma = 1.333$, $\lambda_N = 0.05$

Assuming the axial velocity remains constant and the gas velocity at inlet and outlet are the same, determine the following quantities at the mean radius:

(1) The blade loading coefficient and degree of reaction
(2) The gas angles
(3) The nozzle throat area

Solution

(1) $$\Psi = \frac{C_{pg}(T_{01} - T_{03})}{U^2} = \frac{(1147)(145)}{345^2} = 1.4$$

Using velocity diagram

$$U/Ca = \tan\beta_3 - \tan\alpha_3$$

or $$\tan\beta_3 = \frac{1}{\Phi} + \tan\alpha_3$$

$$= \frac{1}{0.75} + \tan 12^\circ$$

$$\beta_3 = 57.1^\circ$$

From Equations (7.14) and (7.15), we have

$$\Psi = \Phi\left(\tan\beta_2 + \tan\beta_3\right)$$

and

$$\Lambda = \frac{\Phi}{2}\left(\tan\beta_3 - \tan\beta_2\right)$$

From which

$$\tan\beta_3 = \frac{1}{2\Phi}(\Psi + 2\Lambda)$$

Therefore

$$\tan 57.1^\circ = \frac{1}{2 \times 0.75}(1.4 + 2\Lambda)$$

Hence

$$\Lambda = 0.4595$$

(2) $$\tan\beta_2 = \frac{1}{2\Phi}(\Psi - 2\Lambda)$$

$$= \frac{1}{2 \times 0.75}(1.4 - [2][0.459])$$

$$\beta_2 = 17.8^\circ$$

$$\tan\alpha_2 = \tan\beta_2 + \frac{1}{\Phi}$$

$$= \tan 17.8^\circ + \frac{1}{0.75} = 0.321 + 1.33 = 1.654$$

$$\alpha_2 = 58.8^\circ$$

(3) $$Ca_1 = U\Phi$$

$$= (345)(0.75) = 258.75 \text{ m/s}$$

$$C_2 = \frac{Ca_1}{\cos\alpha_2} = \frac{258.75}{\cos 58.8^\circ} = 499.49 \text{ m/s}$$

$$T_{02} - T_2 = \frac{C_2^2}{2C_p} = \frac{499.49^2}{(2)(1147)} = 108.76\,\text{K}$$

$$T_2 - T_{2s} = \frac{(T_N)(499.49^2)}{(2)(1147)} = \frac{(0.05)(499.49^2)}{(2)(1147)} = 5.438\,\text{K}$$

$$T_{2s} = T_2 - 5.438$$

$$T_2 = 1100 - 108.76 = 991.24\,\text{K}$$

$$T_{2s} = 991.24 - 5.438 = 985.8\,\text{K}$$

$$\frac{p_{01}}{p_2} = \left(\frac{T_{01}}{T_{2s}}\right)^{\gamma/(\gamma-1)}$$

$$p_2 = 4 \times \left(\frac{985.8}{1100}\right)^4 = 2.58$$

$$\rho_2 = \frac{p_2}{RT_2} = \frac{(2.58)(100)}{(0.287)(991.24)} = 0.911\,\text{kg/m}^3$$

(4) $$\text{Nozzle throat area} = \frac{m}{\rho_1 C_1} = \frac{24}{(0.907)(499.49)} = 0.053\,\text{m}^2$$

$$A_1 = \frac{m}{\rho_1 Ca_1} = \frac{24}{(0.907)(258.75)} = 0.102\,\text{m}^2$$

Design Example 1.8 A single-stage axial flow gas turbine with equal stage inlet and outlet velocities has the following design data based on the mean diameter:

Mass flow	20 kg/s
Inlet temperature, T_{01}	1150K
Inlet pressure	4 bar
Axial flow velocity constant through the stage	255 m/s
Blade speed, U	345 m/s
Nozzle efflux angle, α_2	60°
Gas-stage exit angle	12°

Calculate (1) the rotor-blade gas angles, (2) the degree of reaction, blade-loading coefficient, and power output and (3) the total nozzle throat area if the throat is situated at the nozzle outlet and the nozzle loss coefficient is 0.05.

Solution

(1) From the velocity triangles

$$C_{w2} = Ca \tan \alpha_2$$
$$= 255 \tan 60° = 441.67 \text{ m/s}$$
$$C_{w3} = Ca \tan \alpha_3 = 255 \tan 12° = 55.2 \text{ m/s}$$
$$V_{w2} = C_{w2} - U = 441.67 - 345 = 96.67 \text{ m/s}$$
$$\beta_2 = \tan^{-1}\frac{V_{w2}}{Ca} = \tan^{-1}\frac{96.67}{255} = 20.8°$$

Also $V_{w3} = C_{w3} + U = 345 + 55.2 = 400.2$ m/s

$$\therefore \ \beta_3 = \tan^{-1}\frac{V_{w3}}{Ca} = \tan^{-1}\frac{400.2}{255} = 57.5°$$

(2) $$\Lambda = \frac{\Phi}{2}\left(\tan\beta_3 - \tan\beta_2\right)$$
$$= \frac{255}{2 \times 345}(\tan 57.5° - \tan 20.8°) = 0.44$$
$$\Psi = \frac{Ca}{U}\left(\tan\beta_2 + \tan\beta_3\right)$$
$$= \frac{255}{345}(\tan 20.8° + \tan 57.5°) = 1.44$$

Power $W = mU(C_{w2} + C_{w3})$

$$= (20)(345)(441.67 + 54.2) = 3421.5\,\text{kW}$$

(3) $$\lambda_N = \frac{C_p\left(T_2 - T_2'\right)}{\frac{1}{2}C_2^2},\ C_2 = Ca\ \sec\alpha_2 = 255\sec 60° = 510 \text{ m/s}$$

or $$T_2 - T_2' = \frac{(0.05)(0.5)(510^2)}{1147} = 5.67$$
$$T_2 = T_{02} - \frac{C_2^2}{2C_p} = 1150 - \frac{510^2}{(2)(1147)} = 1036.6\,\text{K}$$
$$T_2' = 1036.6 - 5.67 = 1030.93\,\text{K}$$

$$\frac{p_{01}}{p_2} = \left(\frac{T_{01}}{T_2}\right)^{\gamma/(\gamma-1)} = \left(\frac{1150}{1030.93}\right)^4 = 1.548$$

$$p_2 = \frac{4}{1.548} = 2.584\,\text{bar}$$

$$\rho_2 = \frac{p_2}{RT_2} = \frac{2.584 \times 100}{0.287 \times 1036.6} = 0.869\,\text{kg/m}^3$$

$$m = \rho_2 A_2 C_2$$

$$A_2 = \frac{20}{0.869 \times 510} = 0.045\,\text{m}^2$$