4 Lattice Filters

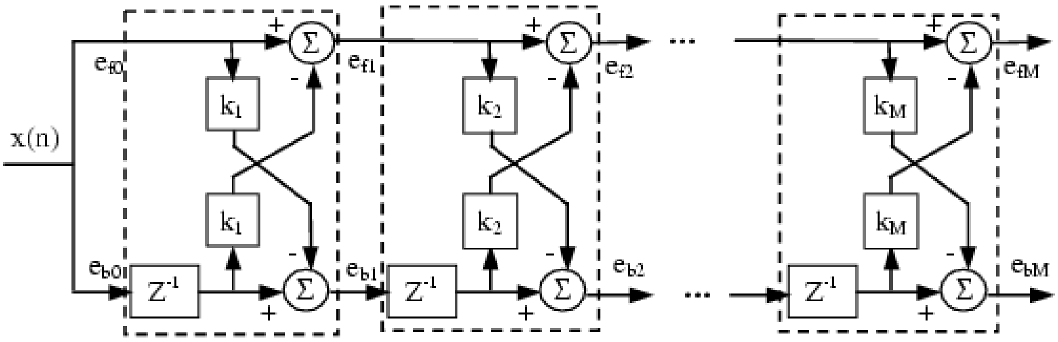

Figure 2.4: Block diagram of the lattice predictor.

Lattice structures are widely used in prediction applications. Fig. 2.4 shows the lattice predictor structure of order M. Stage $m+1$ of the lattice predictor has two inputs from the previous stage, namely the forward and backward prediction errors $e_{f_m}(n)$ and $e_{b_m}(n)$, respectively, and produces two outputs $e_{f_{m+1}}(n)$ and $e_{b_{m+1}}(n)$. The two outputs are given by the following order update equations

$$\begin{aligned} e_{f_{m+1}}(n) &= e_{f_m}(n) &- k_{m+1}e_{b_m}(n-1), \\ e_{b_{m+1}}(n) &= e_{b_m}(n-1) &- k_{m+1}e_{f_m}(n). \end{aligned}$$

The input of the first lattice stage has its forward and backward errors equal to the input signal

$$e_{f_0}(n) = e_{b_0}(n) = x(n).$$

The coefficient k_m of stage m is known as the partial correlation coefficient (PARCOR) or the reflection coefficient. The set of PARCOR coefficients for an M-stage lattice predictor are related to the coefficients of the transversal predictor of the same order (see Section 2.4.3). In fact the lattice and transversal predictors are equivalent. The Levinson-Durbin algorithm is an efficient procedure to calculate the transversal predictor coefficients $\mathbf{a}$ from the autocorrelation function of the input sequence and it also provides the PARCOR coefficients for the corresponding lattice predictor. The following properties are well known for lattice structures

- The PARCOR coefficients always satisfy the relation $|k_m| \leq 1$.
- The power of the forward prediction error $E[e^2_{f_m}(n)]$ and the backward prediction error $E[e^2_{b_m}(n)]$ of the same stage are equal.
- The backward prediction errors $e_{b_0}(n), e_{b_1}(n), \cdots, e_{b_M}(n)$ are uncorrelated with one another for any input sequence $x(n)$. This property is very important since it shows that the lattice predictor can be seen as an orthogonal transformation with the input signal samples $x(n), x(n-1), \cdots, x(n-M+1)$ as input and the uncorrelated (orthogonal) output as the backward error from the M-stages.
- The power of the prediction error decreases with increasing lattice order. The error power decrease is controlled by the PARCOR coefficients according to the relation $P_{m+1} = (1-k^2_{m+1})\, P_m$, where P_{m+1} is the power of the forward or backward prediction error at stage m. This indicates that the closer the value of k_{m+1} to unity the higher the contribution of stage m in reducing the prediction error. Usually the first few PARCOR coefficients have higher magnitude with the magnitude of the coefficients dropping to values close to zero for later stages.

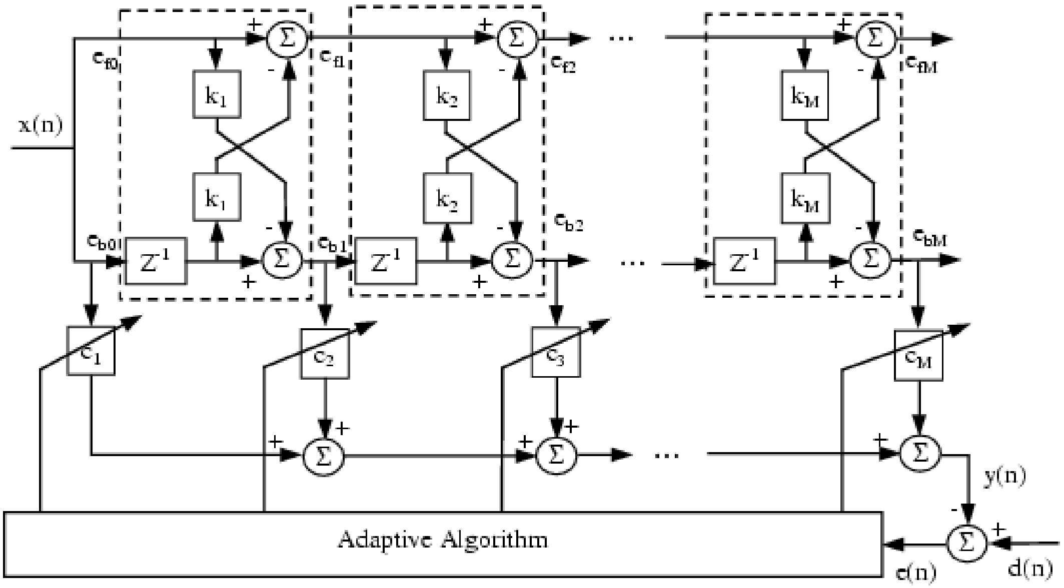

Figure 2.5: Block diagram of the joint process estimator.

Although the operation of lattice filters are usually described in the prediction context, the application of lattice filters is not limited to prediction applications. A traditional adaptive transversal filter can also be implemented using the lattice structure as shown in Fig. 2.5. The structure in Fig. 2.5 is known as the joint process estimator since it estimates a process $d(n)$ from another correlated process $x(n)$. The joint process estimator consists of two separate parts, the lattice predictor part and the linear combiner part. The lattice predictor part main function is to transform the input signal samples $x(n), x(n-1), \cdots, x(n-M+1)$ that might be well correlated to the uncorrelated backward prediction errors $e_{b_0}(n), e_{b_1}(n), \cdots, e_{b_M}(n)$. The linear combiner part calculates the equivalent transversal filter output according to the relationship

$$y(n) = \sum_{i=1}^{M} c_i \, e_{b_i}(n).$$

An adaptive joint process estimator adjusts both the PARCOR coefficients $k_i \; i = 1, 2, \cdots, M;$ and the linear combiner coefficients $c_i; \; i = 1, 2, \cdots, M$ simultaneously. The PARCOR coefficients are adjusted to minimize the forward and backward prediction error power and the linear combiner coefficients are adjusted to minimizes the square of error signal $e(n) = d(n) - x(n)$ as shown in Fig. 2.5.
