Homework 1 – Knight’s Tour

EECS 592 – Introduction to Artificial Intelligence

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Contents

1 Description of the Strategy

1.1 General Description of the Searching Algorithm

To search for closed knight's tour, iterative depth-first search algorithm (DFS) is implemented. However, the memory and time requirement of DFS is very huge. Several heuristic algorithms are applied to optimize the searching process.

Staring from the root node ($(2, 3)$ in this homework), iterative DFS algorithm pushes all its neighbors into a stack according to a specific order determined by different heuristics. Then, it iteratively `peek()` the top element in the stack, add it to our current path and push all its valid neighbors into the stack. Once we reach a stage where the current node does not have any valid neighbors and closed knight's tour has not been found, we start backtracking. To backtrack from the current node, `remove()` this node from the path, `pop()` it from the top of the stack and start next iteration by peeking the top of the stack. If the next element on top of the stack is the same as the last element in the path, we should directly `pop()` this node from stack and `remove()` from the path because all its neighbors have been explored and no closed knight's tour has been found. This process is executed iterative until the stack is clear.

1.2 Sorting of Different Strategy

The general framework of finding closed Knights' tour has been briefly described above. One of the most important steps in the above algorithm is the ordering of neighbor nodes in stack. Several heuristics has been given in the task sheet. They are briefly described below.

0. Order neighbors of the current node (the last one in path) randomly. (Blind search is performed)
1. Order the neighbors according to their fixed degree.
   This strategy favors nodes living near the edges and corners then nodes in the center of the board.
2. Order the neighbors according to their dynamic degree.
   This strategy firstly visits nodes with less degrees. In this way, the possibility of reaching a dead end is minimized.
3. Backup when more than one neighbor of the current node has dynamic degree 1.
   If there are two neighbors having dynamic degree 1 and one of them is added to path, then Knight has to visit the other neighbor to form a closed tour eventually. But it will reach a dead end because the dynamic degree of the node will be 0. Applying this strategy can avoid lots of dead ends and thus enhance the efficiency and effectiveness of the searching process.

4. Prioritize neighbor with dynamic degree 1 and discard its siblings.
   This is because paths containing its siblings rather then the 1 degree neighbor will reach a dead end in the end. Suppose the algorithm chooses a neighbor with dynamic degree larger than 1. The path has to reach the 1 degree node in order to form a tour. But once this node is added to path, a dead end is reached. This strategy can also avoid lots of dead ends and thus will yield many correct results.
5. Combine $2, 3, 4$.
   When strategy 3 and 4 are combined, they will improve the searching together rather than fighting with each other because this two heuristics are applied to two different situations.

1.3 An Extra Strategy 6

After doing many experiments, choosing to visit nodes lying along the edges and at corners first seems to be a very good strategy because the alternative, visiting nodes in the center of the board first, tends to isolate those nodes at corner. Those nodes are very hard to be accessed (especially when most of the nodes at center are visited). Based on this idea, we can favor those neighbors at corners when they have the same dynamic degree (in strategy 2). To characterize this, we can measure the manhattan distance between the candidate neighbors and the four corners and choose the one with the smallest distance to any of the four corners.

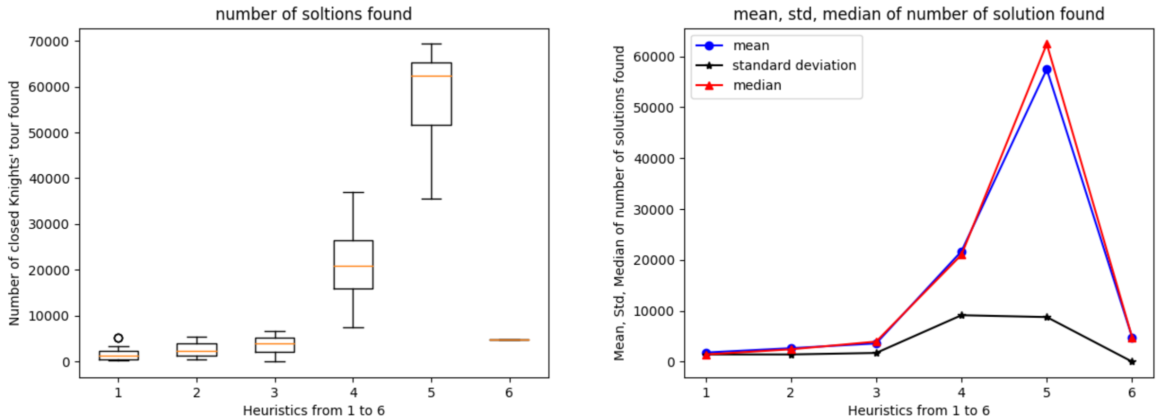
2 Experiment Summary

2.1 Statistics of each strategy

Note:

- Statistics of strategy 0 which almost couldn't find any solutions in each run are not included in the following discussion.
- Strategy 6 which is an extra strategy is compared with strategy 1 to 5.
- All the strategies are run for 1000000 states. So, the statistics of number of moves are not shown below.

2.1.1 Number of solution found



(a) Number of solutions found for each heuristic (box plot)

(b) Number of solutions found for each heuristic (Mean, Standard Deviation and Median)

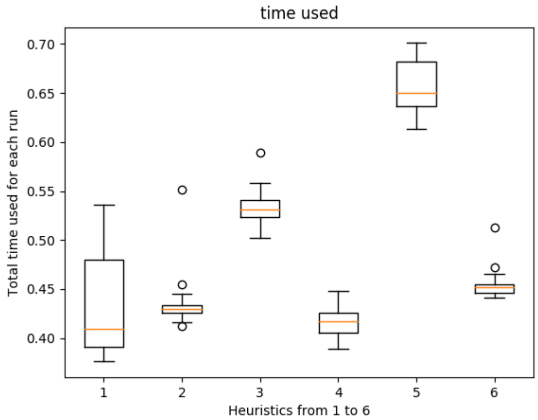
Figure 1: *Statistics of number of solutions found*

Table 1: *Statistics of number of solutions found*

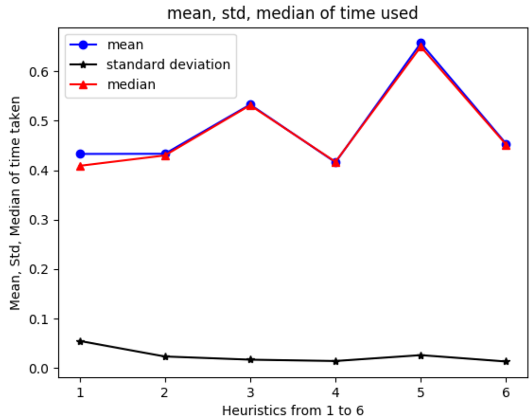
statistics	strategy 1	strategy 2	strategy 3	strategy 4	strategy 5	strategy 6
Mean	1791.5	2649.3	3591	21705	57496.2	4796
Std	1426.05	1421.61	1716.08	9131.67	8768.17	0
Median	1418	2428.5	3944	21019	62474.5	4796

From the Figure (1), we can see that the number of closed tour solutions given by strategy 1 to 5 increases slowly at first and then boosts later. This kind of behavior has not surprised me. Strategy 0 which applies blind search strategy almost cannot find a single closed Knights' Tour. All the informed search (from strategy 1 to strategy 5) are better than strategy 0 because all of the heuristics applied are effective. Strategy 1 which only relies on the fixed degree gets less results on average compared to strategy $3, 4, 5$ because it cannot avoid dead ends as strategy $3, 4, 5$ do. The extra strategy 6 always gives the same ordering each time because it will break the tie by favoring the square with the smallest manhattan distance to 4 corners. The heuristic applied by strategy 6 helps by walk through nodes in corners first. However, strategy 6 cannot avoid many dead ends as strategy 4 and 5. That's why statistics in table show that strategy 6 is better than strategy 1 to 3 but not as good as strategy 4 and 5. The detailed analysis of each strategy is given in section 1.2.

2.1.2 Time used for each run



(a) Time used for each heuristic



(b) Time used for each heuristic (Mean, Standard Deviation and Median)

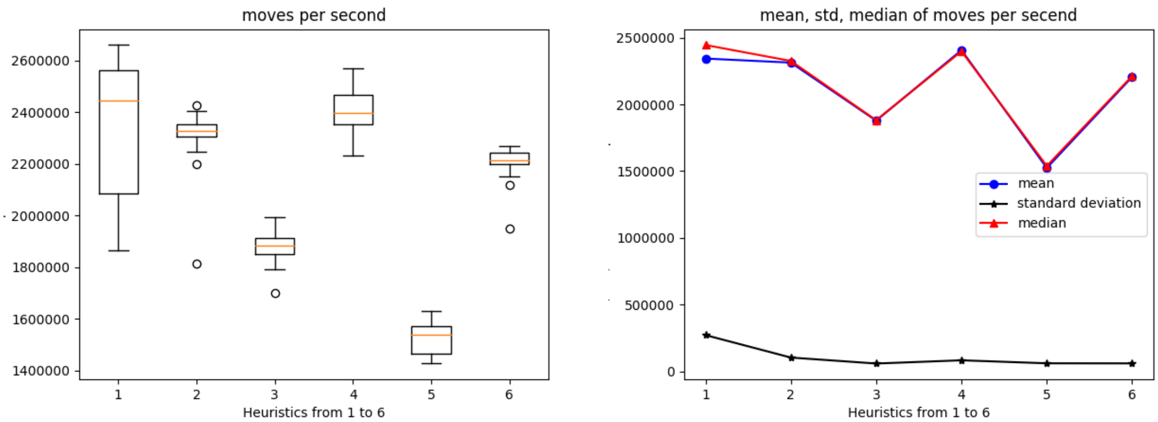
Figure 2: *Statistics of time used for each run*

Table 2: *Statistics of time used for each run*

statistics	strategy 1	strategy 2	strategy 3	strategy 4	strategy 5	strategy 6
Mean	0.433033	0.433333	0.532933	0.416	0.657367	0.453733
Std	0.0548698	0.0233971	0.0169311	0.014215	0.0261374	0.0132059
Median	0.409	0.43	0.5315	0.417	0.65	0.452

From figure (2), we can see that strategy 5 are the slowest.The reason of this is combining strategy 2 to 4 requires more time to query and update the dynamic degree of the neighbors. To the contrary, methods based on fix degree generally requires less time to run because getting the fix degree of a neighbor in graph only needs constant time. Strategy 4 is faster than strategy 3 and 5 because it does not need to do as much backups as strategy 3 and 5 would do.

2.1.3 Moves per second



(a) Moves per second for each heuristic

(b) Moves per second for each heuristic (Mean, Standard Deviation and Median)

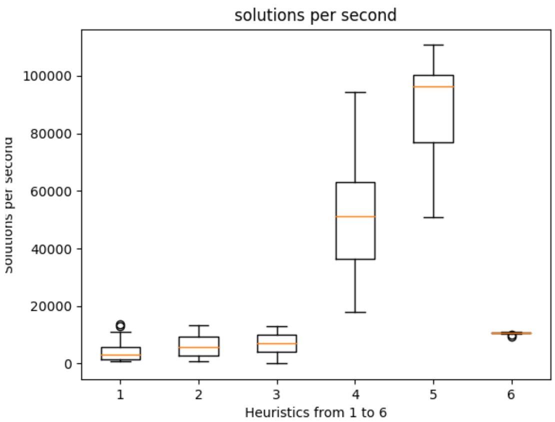
Figure 3: *Statistics of moves per second for each run*

Table 3: *Statistics of time used for each run*

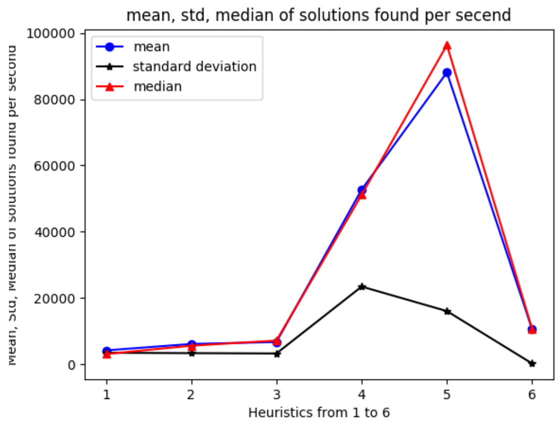
statistics	strategy 1	strategy 2	strategy 3	strategy 4	strategy 5	strategy 6
Mean	2.34356e+06	2.31323e+06	1.87825e+06	2.40666e+06	1.5236e+06	2.20566e+06
Std	271223	102873	58136.7	82292.1	59844.1	59298.3
Median	2.44512e+06	2.32558e+06	1.88147e+06	2.39808e+06	1.53846e+06	2.21239e+06

From figure (3), we can see that by running $1000,000$ states in total, strategy 1 has the highest number of moves in one second. This is reasonable in the sense that strategy add nodes to path without too much thoughts compared to strategy 5. The smaller the number of moves are made in one second, the larger the number of backtracks are made. This explains why strategy 3 and 5 move less (because once multiple neighbors having dynamic degree 1 are detected, they start to backtrack). Strategy 4 which directly discard siblings of neighbors with dynamic degree 1 does not need to do many backtracks and therefore moves a lot in each second. But those moves are "smarter" compared to strategy 1 because they can help to avoid lots of dead ends. (details are explained in section 1.2)

2.1.4 Solutions per second



(a) Solutions per second for each heuristic



(b) Solutions per second for each heuristic (Mean, Standard Deviation and Median)

Figure 4: *Statistics of solutions per second for each run*

Table 4: *Statistics of solutions per second for each run*

statistics	strategy 1	strategy 2	strategy 3	strategy 4	strategy 5	strategy 6
Mean	4240.31	6180.35	6774.16	52717.4	88051.6	10578.4
Std	3525.62	3412.95	3317.89	23481.2	16094.2	284.395
Median	3135.54	5660.18	7202.01	51193.4	96327.7	10610.6

Figure (4) records the statistics of the number of solutions of each heuristics get per second. Not surprisingly, strategy 5 can give the most solutions per second because it can effectively avoid many dead end path and increase the searching efficiency. Strategy 6 is better than strategy 1 to 3 by making reasonable choice along neighbors having the same dynamic degree but lagged behind strategy 4 and 5 for not being able to avoid dead ends.