tags: linear-two-stage-problems

Standard linear two-stage problems and dual of the second-stage

A standard linear two-stage problem has the following form:

$$\begin{aligned} \min_{x \in \mathbf{R}^n} \ & c^\top x + \mathbf{E}(Q(x,\xi)) \\ s.t. \ & Ax = b \\ & x \succeq 0 \end{aligned}$$

where $Q(x,\xi)$ is the optimal value of the second-stage problem:

$$\begin{aligned} \min_{y \in \mathbf{R}^m} \ & q^\top y \\ s.t. \ & Wy = h - Tx \\ & y \succeq 0 \end{aligned}$$

$\xi := (q,h,T,W)$ represents the data of the second-stage problem. Expectation is taken w.r.t. the distribution of ξ.
The dual of the second-stage linear programming is shown below:

$$\begin{aligned} \max_{\pi} \ & \pi^\top (h - Tx) \\ s.t. \ & W^\top \pi \preceq q \end{aligned}$$

State and prove the properties of $Q(\cdot,\xi)$

Properties are listed below. Please refer *Lectures on Stochastic Programming* for proofs.

1. For any given ξ, the function $Q(\cdot,\xi)$ is convex. Moreover, if the set $\{\pi : W^\top \pi \preceq\}$ is nonempty and second stage problem is feasible, then the function $Q(\cdot,\xi)$ is polyhedral.

2. Suppose that for given $x = x_0$ and $\xi \in \Xi$, the value $Q(x_0,\xi)$ is finite. Then $Q(\cdot,\xi)$ is subdifferentiable at x_0 and

$$\partial Q(x_0,\xi) = -T^\top \mathcal{D}(x_0,\xi)$$

where $\mathcal{D}(x,\xi) := \arg\ \max_{\pi \in \Pi(q)} \pi^\top (h - Tx)$ is the set of optimal solutions of the dual problem.

Definitions and conditions under which the two-stage problem has fixed recourse, complete recourse, relatively complete recourse, simple recourse

see textbook.