Lecture-1

Computational Model

Mathematical Abstractions which help us analyze the efficiency of a program

→ RAM model
- One of the most common model used for basic binary computation devices.

★ Time Complexity

```
sum=0                  — 1 time
for i= 0...n-1         — 2n (i++ & i<n for each cycle)
    sum+= a(i)         — 3n (load a(i), a(i)+sum, store in sum)
print (sum)            — 1
```

time complexity = $1 + 2n + 3n + 1$
$= 2 + 5n$

→ Constant is not the property of algorithm but property of how we write code.

$\therefore O(n)$

$\exists\ n_0, c \quad \forall n \geq n_0 : f(n) \leq c \cdot g(n)$ — Eq (1)

↳ $2+5n \Rightarrow f(n) = 2+5n$
$g(n) = n$ such that $f(n) = O(g(n))$ — Eq (2)

Thus $n_0 = 2$
$c = 6$
↳ $2+5n \leq 6n$ Based on Eq (1)
$2 \leq n$

$O(n)$ → Upper bound of complexity or the worst case complexity

$\Omega(n)$ → Lower bound of complexity or the best case complexity.

If $O(n)$ and $\Omega(n)$ are same then $\Theta(n)$ ↓ Asymptotic Notation

Practice Q

```
for i = 0 ... n-1   — n
    for j = 0 ... n-1   — n
        ...
    ...
```

$O(n) = n \times n = n^2$

Practice Q

```
i = 0          — 1
while i * i < n   — √n
    i++        — 1
```

$O(n) = \sqrt{n}$

Practice Que

```
i=1
while i<n
    i*=2
```

After K iterations $i = 2^K$

$$2^K \geq n$$
$$K \geq \log_2 n$$

$O(\log_2 n) \Rightarrow O(\log n)$

base only denotes the slope of logarithmic curve and is also a constant.

For Recursion

```
def : f(n)
    if n=0   — 1
        return
    else   f(n-1) — n-1
```

↳ $\underbrace{f(n) \to f(n-1) \to f(n-2) \ldots}_{n}$

$\therefore O(n)$

Practice Q.

```
def : f(n)
  if  n = 0
      return
  else  f(n/2)
```

↳ $f(n) \to f(n/2) \to f(n/4) \to \dots f(0)$

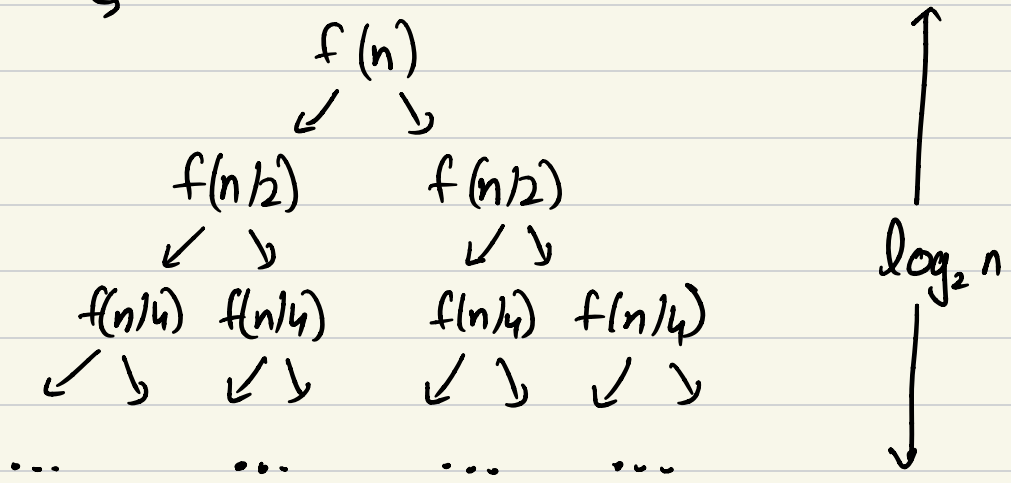
$\log_2 n$

$O(\log n)$

Practice Q

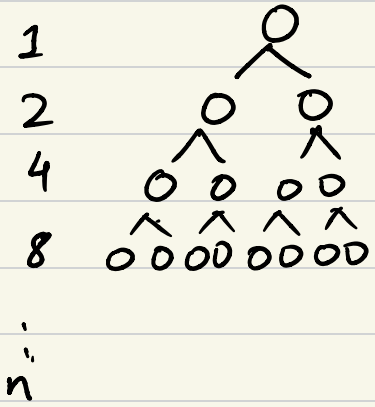
```
def : f(n)
  if  n = 0   - 1
      return
  f(n/2)
  f(n/2)
```

↳



↳ 1 × no. of nodes

no. of nodes →



no. of nodes =
$1 + 2 + 4 \ldots + 2^H$
$= 2^{H+1} - 1$

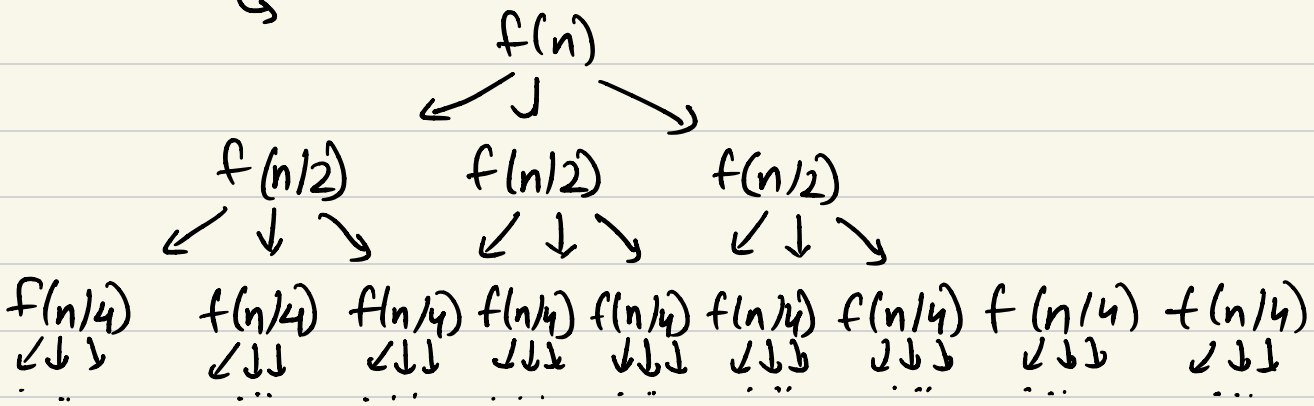
$O(2^H)$

$\Rightarrow 2^{height} = 2^{\log_2 n} = n$

$O(n)$

Practice Q

```
def : f(n)
    if n=0
       return
    f(n/2)
    f(n/2)
    f(n/2)
```



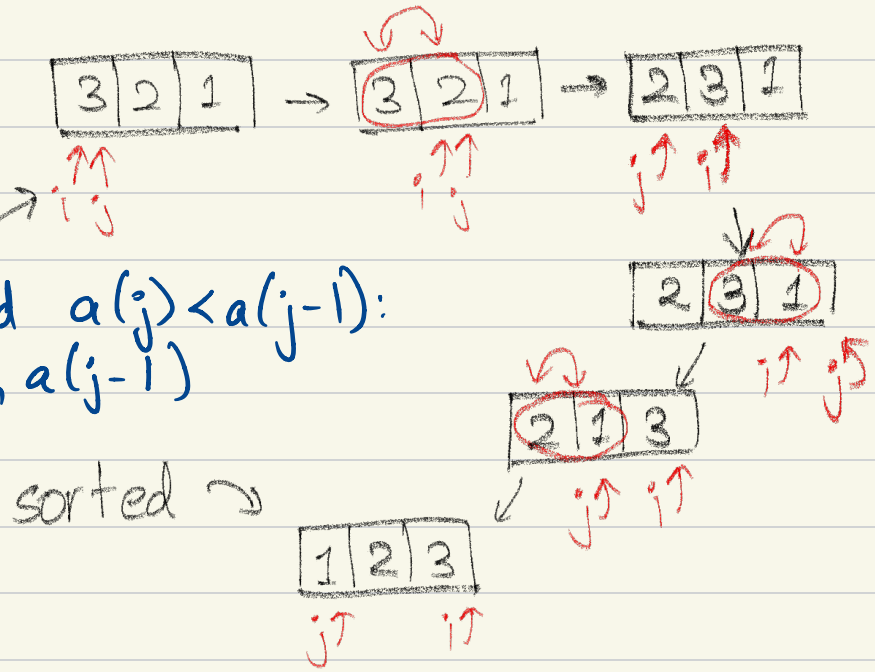
no. of nodes = $1 + 3 + 9 + \ldots + 3^H$
$= 3^{H+1} - 1$
$= 3^H$

$$\Rightarrow O(n^{\log_2 3^{\log n}})$$

Sorting Algorithms

↳ Insertion Sort

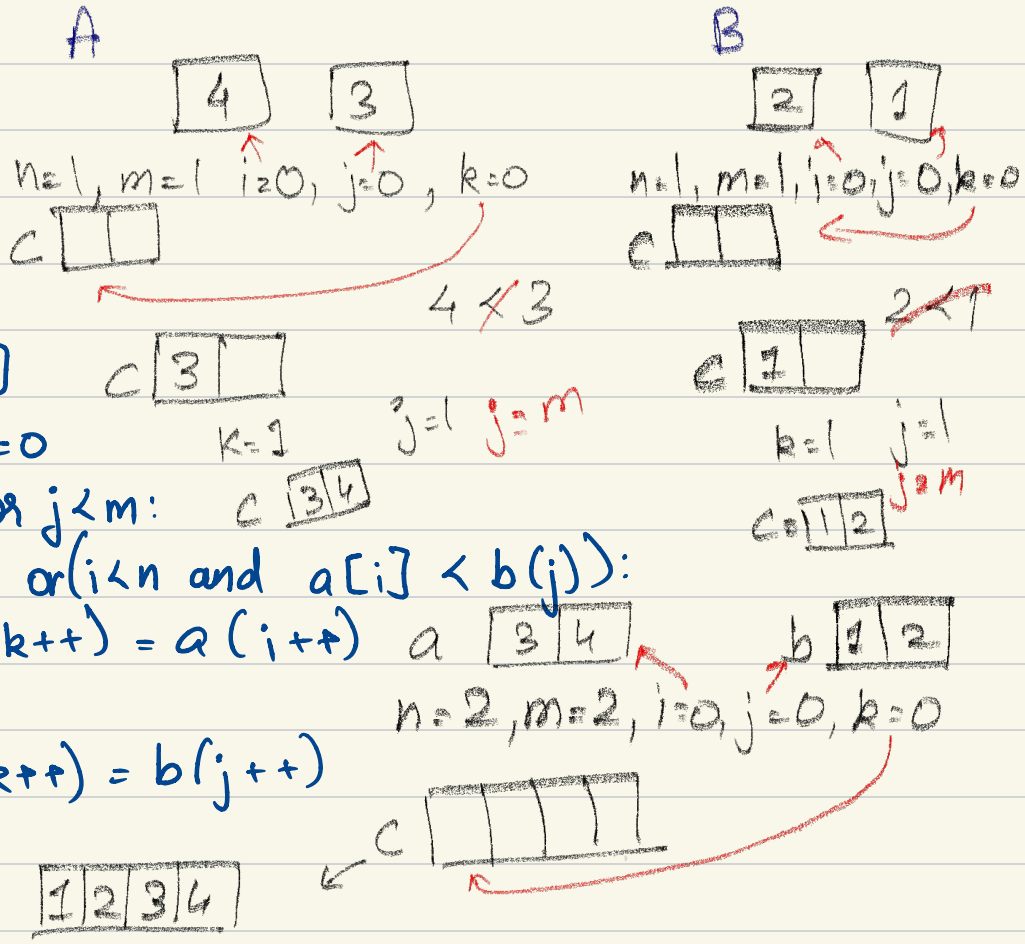
```
for i=0...n-1
    j = i
    while j>0 and a(j)<a(j-1):
        swap(a(j), a(j-1))
        j--
```



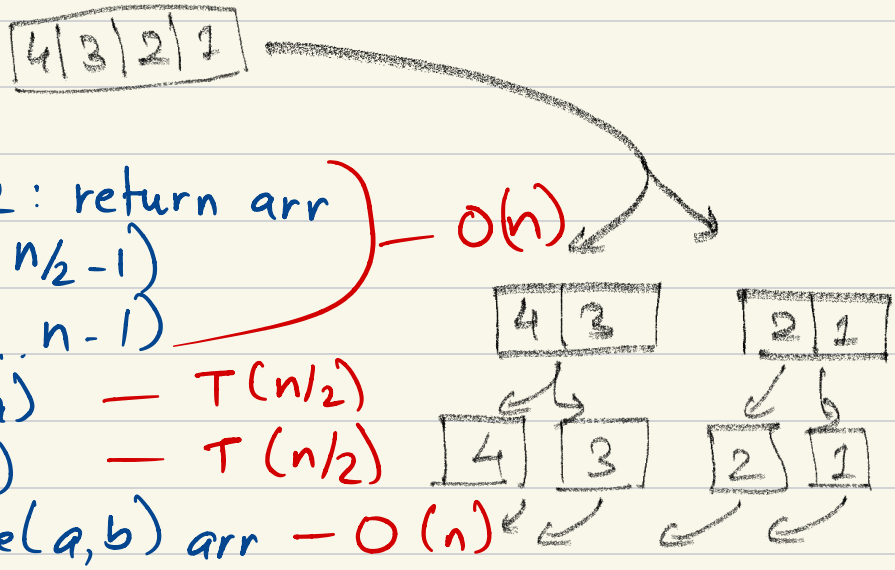
Best case - $O(n)$
Worst case - $O(n^2)$

↳ Merge Sort

```
merge(a, b):
    n = len(a)
    m = len(b)
    int c[n+m]
    i=0, j=0, k=0
    while i<n or j<m:
        if j=m or(i<n and a[i] < b(j)):
            c(k++) = a(i++)
        else
            c(k++) = b(j++)
    return c
```



```
sort (arr):
    if len(arr) < 2 : return arr          } — O(n)
    a = arr( 0 ... n/2 - 1)
    b = arr (n/2 ... n - 1)
    a = sort (a)   — T(n/2)
    b = sort (b)   — T(n/2)
    return merge(a,b) arr — O(n)
```



$$T(n) = 2 \cdot T(n/2) + c \cdot n$$

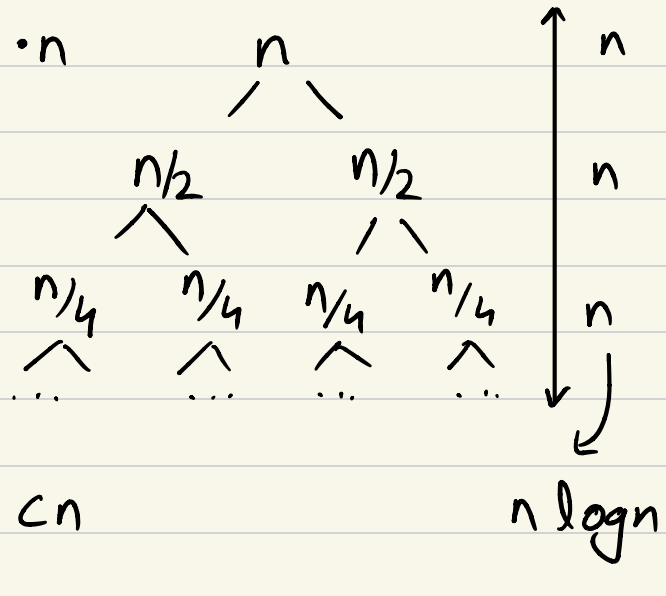
$$T(n) \leq c\, n \log n$$

$$T(n) \leq 2 \cdot c \frac{n}{2} \log \frac{n}{2} + cn$$

$$\leq cn(\log n - 1) + cn$$

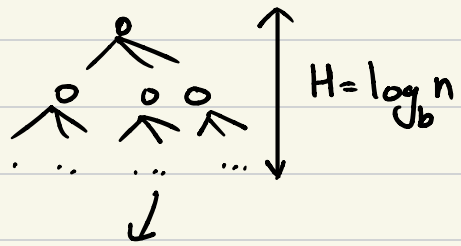
$$T(n) \leq c\, n \log n$$

$$O(n \log n)$$

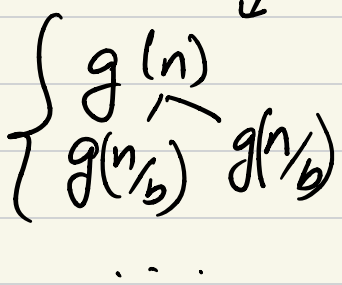


Master's Theorem

$$T(n) = a \cdot T\left(\frac{n}{b}\right) + f(n)$$



$$a^{\log_b n} \quad \text{vs} \quad n^{\log_b a}$$



$f(n) = O(n^{\log_b a - \varepsilon})$ a

$\Downarrow$

$T(n) = O(n^{\log_b a})$

$f(n) = O(n^{\log_b a + \varepsilon})$

$\Downarrow$

$T(n) = O(n^{\log_b a + \varepsilon})$

$f(n) = O(n^{\log_b a})$

$\Downarrow$

$T(n) = O(n^{\log_b a} \cdot \log n)$