2 QPSO intro

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The particles in PSO are ike particles in space of the quantum realm apperently

Usually:

PSO $\rightarrow$ $x_i^{t+1} = x_i^t + v_i^t$

In the quantum model of a PSO called here QPSO, the state of a particle is depicted by wave function $|\Psi(x,t)|^2$ (Schrödinger equation), instead of position and velocity. The dynamic behavior of the particle is widely divergent form that of that the particle in classical PSO systems in that the exact values of x_i and v_i cannot be determined simultaneously.

PSO algorithm is applied to the quantum space. The quantum space particle used wave function to describe

$$|\Psi|^2\, dx\, dy\, dz = Q dx\, dy\, dz$$

Among them, $|\Psi|^2$, is the square of the module of wave function, representing the probability density of particles in a position to appear.

Q is the probability density function and satisfies the normalization condition:

$$\boxed{\int_{-\infty}^{\infty} |\Psi|^2\, dx\, dy\, dz = \int_{-\infty}^{\infty} Q dx\, dy\, dz = 1}$$

Algorithm QPSO Description

Assume that

- n: No. of the particles
- D: Dimension (variables) associated with problem.
- $X_i = (x_{i1}, x_{i2}, \dots, x_{iD})$ location of the i^{th} particle.
- pbest - historical best location is $P_i = (p_{i1}, p_{i2}, \dots, p_{iD})$
- gbest - Global position is $P_g = (P_{g1}, P_{g2}, \dots, P_{gD})$

In quantum space, positions of particles after the particles get through stochastic simulation of Monte Carlo measurement:

$$x_{id} = p_{id} \pm \frac{L}{2}\ln\left(\frac{1}{u}\right) \; ; \quad d = 1,2,\ldots,D$$

Here, u is the random number uniformly in $[0,1]$. L is obtained by the particle's current position and pbest position, i.e.,

$P_i = (P_{i1}$

$$L = 2\beta \mid p_{id} - x_{id} \mid$$

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pbest positi

After simplifying all that we get this

Therefore, the update formula of QPSO optimization:

$$x_{id} = p_{id} \pm \beta \mid p_{id} - x_{id} \mid \ln\left(\frac{1}{u}\right)$$

In other words, with respect to iteration:

$$x_{id}^{t+1} = p_{id}^{t} \pm \beta \mid p_{id}^{t} - x_{id}^{t} \mid \ln\left(\frac{1}{u}\right)$$

Here, t is the iteration counter. β is the contraction expansion factor and is the only parameter of QPSO algorithm.

Only Beta here is not known this is called the shrikage factor,choosing this affects the overall performance of the algorithm

If the contraction factor is too large, the algorithm convergence has long search time and too slows time;

If it is too small, this can make the algorithm into a local optimal solution.

How to choose a good Beta

We make it adaptive based on the no of itereation we have done

$$\beta = (1 - 0.5) \cdot \frac{Maxiter - t}{Maxiter} + 0.5$$

Here,

"Maxiter" is the maximum number of iteration;

"t" is the current iteration.

$$\boxed{\beta = 0.5 + \frac{0.5(T-t)}{T}}$$

When t =0 Beta =1
When t = total no of iterations then Beta = 0.5

Variant 1(with delpa potential)

We use La in place of personal best

$$La_i = \frac{c_1 r_1 p_{id} + c_2 r_2 p_{gd}}{c_1 r_1 + c_2 r_2}$$

Pid -> pbest
Pgb -> gbest

⚙ The ϕ (phi) term — The Local Attractor

In QPSO, for each particle `i`, we compute a **local attractor point** `P_i` between its `pbest_i` and the global best `gbest`:

$$P_i = \phi \cdot pbest_i + (1-\phi) \cdot gbest$$

where

$$\phi \sim U(0,1)$$

(i.e., a random number between 0 and 1)

🔍 Why both φ and (1−φ)?

Because this creates a **random point on the line segment** between `pbest_i` and `gbest`.

- If `ϕ = 0.8`, then `P_i` is closer to the particle's own best (`pbest_i`).
- If `ϕ = 0.2`, then `P_i` is closer to the global best (`gbest`).

So:

- `ϕ` controls **local learning** (following its own best).
- `1-ϕ` controls **global learning** (following the swarm's best).

Where phi is defined loosely as :

ϕ

$$La_i = \frac{c_1 r_1 p_{id} + c_2 r_2 p_{gd}}{c_1 r_1 + c_2 r_2}$$

Therefore we make the updation factor as not just personal best but the weighted avergae of pbest and gbest

In the end the updation positition becomes this:
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In QDPSO, the position of particle x_i at iteration t is updated by employing the onte Carlo method

$$x_{id} = La_{id} \pm \beta \mid La_{id} - x_{id} \mid \ln\left(\frac{1}{u}\right)$$

i.e.,

$$\boxed{X_i = \begin{cases} La_i + \beta \mid La_i - X_i \mid \ln\left(\frac{1}{u}\right) & ; \; if\ rand(0,1) \geq 0.5 \\ La_i - \beta \mid La_i - X_i \mid \ln\left(\frac{1}{u}\right) & ; \quad otherwise \end{cases}}$$

Revised QPSO

In this version, a global point, denoted a mbest called as mean personal best position, is introduced to enhance the global searching ability of QPSO. The global point corresponding to the i^{th} iteration is compute

$$mbest = \frac{1}{n}\sum_{i=1}^{n} P_i(t)$$

$P_i \rightarrow$ pbest

The position of particle i at iteration t in RQPSO is updated a

$$x_{id}^{t+1} = La_i \pm \beta \mid mbest_d^t - x_{id}^t \mid \ln\left(\frac{1}{u}\right)$$

where $mbest_d^t$ is the mean personal best position of the population for the d^{th} dimension at the t^{th} iteration.

Guassion poisson QPSO

The l_{ai} updated as this:

In Gaussian quantum PSO (GQPSO), the parameters c_1, c_2 of La_i are modified by the following equation

$$La_i = \frac{c_1 \cdot \phi \cdot p_{id} + c_2 \cdot (1-\phi) \cdot p_{gd}}{c_1 + c_2}$$

Where, ϕ is random number are generated using the absolute value of the Gaussian probability distribution with mean $\boxed{\text{ZERO}}$ and Variance 1,

i.e., $\phi \sim abs(N(0,1))$

The position of particle i at iteration t in GQPSO is updated a

$$x_{id}^{t+1} = La_{id}^{t} \pm \beta \mid mbest_d^t - x_{id}^t \mid \ln\left(\frac{1}{G}\right)$$

where $G \sim abs(N(0,1))$

update position

$$X_i = \begin{cases} La_i + \beta \mid mbest - X_i \mid \ln\left(\frac{1}{G}\right) & ;\ if\ rand(0,1) \geq 0.5 \\ La_i - \beta \mid mbest - X_i \mid \ln\left(\frac{1}{G}\right) & ;\quad otherwise \end{cases}$$

Still there are drawbacks of this:

1. As particles are depenign on some particle p_{best} the diversity is lowered
2. The possible distirbution space of each particle gradully decreases during the algo

To mitigate tehse we can make it such that for each particle the going to the towards pbest or gbest is randomized

$$\boxed{attractor_i = u_i \cdot pbest_i + (1-u_i) \cdot pbest_b}$$

Where u_i is a random number with uniform distribution function over the interval [0,1].

Therefore, updated particle updated position in QPSO is

update

$$X_i = attractor_i \pm \beta \mid mbest - X_i \mid \ln\left(\frac{1}{u}\right)$$

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