

# Adaptive Memetic Algorithm for the Vehicle Routing Problem with Time Windows

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## ABSTRACT

This paper presents an adaptive memetic algorithm (AMA) to minimize the total travel distance in the NP-hard vehicle routing problem with time windows (VRPTW). Although memetic algorithms (MAs) have been proven to be very efficient in solving the VRPTW, their main drawback is an unclear tuning of their numerous parameters. Here, we introduce the AMA in which the selection scheme and the population size are adjusted during the search. We propose a new adaptive selection scheme to balance the exploration and exploitation of the search space. An extensive experimental study confirms that the AMA outperforms a standard MA in terms of the convergence capabilities.

## Categories and Subject Descriptors

I.2.8 [Artificial Intelligence]: Problem Solving, Control Methods, and Search—*Heuristic methods*

## Keywords

Memetic algorithm; self-adaptation; vehicle routing problem with time windows

## 1. INTRODUCTION

The vehicle routing problem with time windows (VRPTW) is a well-known NP-hard discrete optimization problem. Its applications are of wide range, thus, a number of exact [3] and approximate algorithms to solve the VRPTW emerged over the years. The latter include various improvement and construction heuristics [2], ant colony algorithms, simulated annealing, genetic and memetic algorithms, and more [5].

The VRPTW is defined on the graph  $G$ , where each customer  $v_i$ ,  $i \in \{1, 2, \dots, C\}$  (and the depot  $v_0$ ) is given as a vertex, and each edge is a travel connection with the cost  $c_{i,j}$ ,  $i, j \in \{0, 1, \dots, C\}$ ,  $i \neq j$ . Customers define the demands  $d_i$ ,  $d_i \geq 0$ , and service times  $s_i$ . The time windows  $[e_i, l_i]$  are given for each  $v_i$  and the depot. Let  $K$  be a number of

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vehicles with a constant capacity  $Q$  in a solution  $\sigma$ . Then,  $\sigma$  is feasible if (i)  $Q$  is never exceeded, (ii) each  $v_i$  is served within  $[e_i, l_i]$  in exactly one route, (iii) every vehicle starts and finishes at  $v_0$  within  $[e_0, l_0]$ . The primary objective is to minimize  $K$ . Also, the total travel distance  $T = \sum_{i=1}^K T_i$  is to be minimized, where  $T_i$  is the distance of the  $i$ -th route.  $\sigma_A$  is of a higher quality than  $\sigma_B$  if  $(K(\sigma_A) < K(\sigma_B))$  or  $(K(\sigma_A) = K(\sigma_B) \text{ and } T(\sigma_A) < T(\sigma_B))$ .

The paper is organized as follows. The adaptive memetic algorithm is described in Section 2. The experimental study is reported in Section 3. Section 4 concludes the paper.

## 2. ADAPTIVE MEMETIC ALGORITHM

In the AMA, which extends our previous efforts [1, 6], a population of solutions evolves in time to decrease  $T$ . First,  $K$  is minimized by the guided ejection search [1], and a population of  $N_I$  feasible solutions (each containing  $K$  routes) is generated (Alg. 1, line 1). Then, according to a pre-selection scheme  $\mathcal{S}$ ,  $N$  pairs  $(\sigma_A, \sigma_B)$  are determined (line 4) and crossed-over to generate  $N_c$  children  $\sigma_c$  for each pair using the edge assembly crossover (EAX) (line 8). If  $\sigma_c$  is infeasible then it is repaired, and it undergoes the education and mutation procedures based on applying local search moves (line 9). The best child  $\sigma_c^b$  is determined for each  $(\sigma_A, \sigma_B)$  (line 10). Finally, the next population is formed (line 13).

The AMA parameters are adjusted during the search. First, we propose to incrementally increase the population size  $N$ , starting from the initial size  $N_I$  (with step  $\Delta N$ ). Also, we combine the AB-selection scheme, which proved to have high exploration capabilities [4, 6], with the scheme locally exploiting best individuals (termed *Local Exploitation Selection*, LES). In LES, the population is sorted and divided into  $\epsilon$  parts. Then,  $N/\epsilon$  pairs of parents are drawn and crossed-over for each part. The children form the next population of size  $N$ . Here, the elitist strategy is applied.

In the AMA,  $s$  indicates the number of consecutive generations for which the best solution  $\sigma^B$  was not improved.  $\mathcal{S}$  is switched to LES for better local exploitation once  $s$  exceeds  $S$ , i.e., the maximum steady state selection counter (line 19). If  $s$  surpasses  $P$  (the maximum steady state population counter), then  $\mathcal{S}$  is set back to the AB-selection, and  $N$  increases to explore new regions of the search space (line 21). Finally, the best solution is returned (line 26).

## 3. EXPERIMENTAL RESULTS

The AMA was implemented in C++ and run on an Intel i7 2.3 GHz (16 GB RAM) computer. It was tested on the

**Algorithm 1** Adaptive memetic algorithm (AMA).

```

1: Generate a population of  $N_I$  solutions with  $K$  routes;
2:  $done \leftarrow$  false;  $N \leftarrow N_I$ ;  $s \leftarrow 0$ ;  $T(\sigma_p^B) \leftarrow \infty$ ;  $\mathcal{S} \leftarrow AB$ ;
3: while not  $done$  do
4:   Determine  $N$  pairs  $(\sigma_A, \sigma_B)$ ;
5:   for all  $(\sigma_A, \sigma_B)$  do
6:      $\sigma_c^b \leftarrow \sigma_A$ ;
7:     for  $i \leftarrow 1$  to  $N_c$  do
8:        $\sigma_c \leftarrow \text{CrossoverAndRepair}(\sigma_A, \sigma_B)$ ;
9:        $\sigma_c \leftarrow \text{Educate}(\sigma_c)$ ;  $\sigma_c \leftarrow \text{Mutate}(\sigma_c)$ ;
10:       $\sigma_c^b \leftarrow \text{UpdateBestChild}(\sigma_c^b, \sigma_c)$ ;
11:    end for
12:  end for
13:  Form the next population of size  $N$  and update  $\sigma^B$ ;
14:  if  $T(\sigma^B) < T(\sigma_p^B)$  then
15:     $s \leftarrow 0$ ;
16:  else
17:     $s \leftarrow s + 1$ ;
18:    if ( $s > S$  and  $s < P$ ) then
19:       $\mathcal{S} \leftarrow \text{LES}$ ;
20:    else if  $s > P$  then
21:       $\mathcal{S} \leftarrow AB$ ;  $N \leftarrow N + \Delta N$ ;  $s \leftarrow 0$ ;
22:    end if
23:  end if
24:   $\sigma_p^B \leftarrow \sigma^B$ ;  $done \leftarrow \text{CheckStoppingCondition}()$ ;
25: end while
26: return best solution;

```

Gehring and Homberger's (GH) benchmark tests with 200 customers. GH tests are divided into six subclasses: C1, C2 (clustered customers), R1, R2 (random ones), RC1 and RC2 (both random and clustered). The subclasses C1, R1 and RC1 have smaller  $Q$  and shorter time windows than C2, R2 and RC2. There are 10 problem instances in each subclass.

Each test (out of 10) in each subclass was executed 5 times, and the best results (i.e., with the minimum  $T$ ) were averaged for each subclass. The AMA parameters were set as follows:  $N_I = \Delta N = 10$ ,  $N_c = 20$ ,  $\epsilon = 5$ ,  $S = 20$ ,  $P = S + 5$ . For the MA we used the AB-Selection and  $N = 20000/C = 100$ , as suggested in [5]. The maximum execution time of the AMA was limited to  $\tau = 5$  min.

The experimental results are given in Tab. 1. We compare the best travel distances obtained using the MA, the AMA without the selection scheme adaptation ( $S = P$ ) and the AMA, in various time points  $\tau_i$  ( $i$  stands for minutes). Also, we show the world's best known  $T$  averages (WB)<sup>1</sup>. In this study we obtained the world's best known  $K$  for each test.

It is easy to note that the initial best solutions (in  $\tau_0$ ) were of the highest quality in the MA, due to its large  $N$  – the probability of obtaining a well-fitted individual in the initial population was large. However, the MA, in which  $N$  is constant during the search, required much more time to converge to the better solutions compared with both versions of the AMA. They outperformed the MA in  $\tau < \tau_1$ . Here, the small populations were intensively exploited with LES and extended to include new individuals if necessary. The average population sizes  $N_1$  and  $N_5$  (in  $\tau_1$  and  $\tau_5$ ) prove that balancing the exploitation and exploration of the search space can be achieved by a smooth growth in  $N$ .

<sup>1</sup>See SINTEF website (ref.: March 18, 2014): <http://www.sintef.no/Projectweb/TOP/VRPTW/Homberger-benchmark>.

**Table 1: The best results averaged for each subclass.**

	C1	C2	R1	R2	RC1	RC2
MA	$\tau_0$	<b>2853.57</b>	<b>1856.97</b>	<b>3988.23</b>	<b>3068.85</b>	<b>3790.70</b>
	$\tau_1$	2784.95	1848.77	3876.20	3043.86	3607.92
	$\tau_5$	2720.07	1837.44	3721.14	2969.83	3432.39
(P)	$\tau_0$	2902.41	1870.89	4063.52	3102.48	3812.95
	$\tau_1$	<b>2720.06</b>	1834.78	3672.62	<b>2952.20</b>	3344.80
	$\tau_5$	2719.64	1832.59	3649.20	2937.43	3272.49
AMA	$\tau_1$	24	14	12	10	11
	$\tau_5$	60	34	30	18	29
	WB	2718.41	1831.59	3611.86	2929.41	3176.23
A	$\tau_0$	2880.22	1857.79	4043.66	3106.33	3842.79
	$\tau_1$	2720.23	<b>1834.37</b>	<b>3671.78</b>	2957.36	<b>3334.32</b>
	$\tau_5$	<b>2718.87</b>	<b>1831.95</b>	<b>3647.68</b>	<b>2936.61</b>	<b>3270.71</b>
A	$\tau_1$	15	11	13	10	11
	$\tau_5$	48	29	29	14	28
	WB	2718.41	1831.59	3611.86	2929.41	3176.23
WB						

## 4. CONCLUSIONS AND FUTURE WORK

We proposed an adaptive memetic algorithm for solving the VRPTW. The population size and the selection scheme are adjusted dynamically during the search in order to balance the exploitation and exploration of the search space. The experimental results proved its high convergence capabilities compared with the standard MA. Our ongoing research includes enhancing the AMA further, and incorporating it into our parallel algorithm.

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