Hospital Location Optimization

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Introduction

Montreal was founded on May 17, 1642. Back then, there wasn't adequate technology to mathematically optimize urban planning and layout. This is no longer the case, and we can input complex optimization functions with multiple constraints to suit the needs of the population. Thus, modern urban planning techniques are much more efficient at allocating its resources. One of the key aspects of modern society is ensuring access to health care. Health care is a universally valued commodity and making it easily accessible to a mass population with high quality is one of the many requirements for a successful society. As institutions that provide health care, hospitals are key landmarks in urban planning. Whether it is planning for a new city or looking to remodel/rebuild a city, the government will have to make the construction and access to hospitals one of its main priorities.

When determining the locations for the development of new medical facilities, they will have to take into consideration the level of medical assistance required per age categories, the feasibility of access, and the number of patients visiting a hospital (i.e., the expected demand). Having a large number of hospitals would be better in serving the citizens, however, it is expensive to build hospitals and the necessary facilities it requires. Therefore, there must be a restriction on the budget availability, number of hospitals built, and the hospital capacity. In this project, we will develop a location optimization model to integrate these key factors and help decision-makers identify the most strategic location to build the hospitals. We will use Montreal as an example for this project with the assumption that there are no current hospitals in the city and the government needs to build the first few hospitals based on the needs of current population demographic and the available provincial budget.

1. Symptoms: Problem Description and Formulation

1.1. Problem Description and Scope

The basic optimization model was developed by quantifying the expected level of medical assistance required for different age groups to maximize the accessibility of healthcare within Montreal based on the demographics of the different sectors within the city and subject to hospital related logistical constraints (ex. budget, minimum number of locations, etc.). These different sectors are broken down by the outward section of the postal codes (i.e., the first three digits of the postal code before the space in the middle); within Montreal, there are a total of 116 outward codes (see Appendix 1). The age groups are broken down into the four following groups: 0 - 14-year-olds, 15 - 64-year-olds, 65 - 84-year-olds, 85+ year-olds.

1.2. Variables and Parameters Description

Capital letters are used for aggregated parameters such as overall budget and total population; lowercase letters are used for decision variables; Greek alphabets are used for demographic and hospital related parameters.

1.2.1 Parameters

- B: Overall budget for the construction of hospitals in Montreal
- γ: Fixed hospital cost, millions of dollars per hospital

- α: Increase in hospital costs with one additional bed (variable cost), millions of dollars per bed
- $\mathrm{P_i}$: Total population that live inside outward code i
- φ_{ij}: population of age group i that live inside outward code j
- δ_{ij}: distance in kilometres between outward code i and outward code j
- μ_{i}: expected percentage of population (in decimal) requiring frequent medical assistance within age group i

1.2.2 Decision Variables

- $\mathrm{x_i}$: A binary variable that indicates whether a hospital gets built in outward code i. If yes, $\mathrm{x_i} = 1$; otherwise, $\mathrm{x_i} = 0$
- $\mathrm{b_i}$: The number of beds allocated to hospital i
- $\mathrm{y_{ij}}$: A binary variable that indicates whether a hospital in outward code j is the allocated hospital for people living in outward code i. If yes, $\mathrm{y_{ij}} = 1$; otherwise, $\mathrm{y_{ij}} = 0$

In early 2022, the government pledged to invest approximately $9 billion over the next five years in upgrading medical data processing systems and improving hospital infrastructures (Bruemmer, 2022). Assuming these funds get reallocated to building new hospitals in Montreal, we set the overall budget for the construction of hospitals (B) to $9 billion. According to Assets America, the construction costs for large hospitals with more than 500 beds can range from $800 million to $1 billion. The average hospital construction costs can rise by $1 million for one unit increase in the number of beds (Gerardi, 2022). Thus, we set the fixed hospital cost (γ) as $500 million and the variable cost (α) as $1 million.

For our optimization problem, we decided to specially prioritize the accessibility of hospitals for children and senior citizens because they are the most vulnerable age groups. Thus, the expected percentage of medical assistance level required (μ_{i}) was set as 0.6, 0.3, 0.7 and, 0.8 for 0 - 14-year-olds, 15-64-year-olds, 65 - 84-year-olds, and 85+ year-olds respectively.

1.3. Main Model Formulation

The objective function looks to accomplish two purposes simultaneously: its primary objective is the maximization of health care accessibility based on the demographics in Montreal and as a by-product of this, it also seeks to maximize the number of hospitals that can be built within the city.

Objective:

$$\max \sum_{\mathrm{i,j}} \mathrm{x_i} \cdot \varphi_{\mathrm{ij}} \cdot \mu_{\mathrm{j}} + \mathrm{y_{ij}} \cdot \varphi_{\mathrm{ij}} \cdot \mu_{\mathrm{j}}$$

$$\text{where } \mathrm{i} = \text{outward code};\ \mathrm{j} = \text{age group}$$

Subject to:

$$\textstyle\sum_{\mathrm{i}}(\mathrm{x_i} \cdot \gamma + \mathrm{b_i} \cdot \alpha) \leq \mathrm{B} \qquad \mathrm{i} = \{1...\mathrm{n}\},\ \mathrm{n} = 116 \tag{1.1}$$

$$\textstyle\sum_{\mathrm{i}}(\mathrm{x_i}) \geq 12 \qquad \mathrm{i} = \{1...\mathrm{n}\},\ \mathrm{n} = 116 \tag{1.2}$$

$$\mathrm{b_i} \leq \mathrm{x_i} \cdot \mathrm{M} \qquad \mathrm{i} = \{1...\mathrm{n}\},\ \mathrm{n} = 116;\ \mathrm{M} = 10000 \tag{1.3}$$

$$\sum_j(y_{ij}) = 1 \qquad i = \{1...n\};\ j = \{1...n\},\ n = 116 \qquad (1.4)$$

$$\sum_j(y_{ij} \cdot \delta_{ij}) \leq 10 \qquad i = \{1...n\};\ j = \{1...n\},\ n = 116 \qquad (1.5)$$

$$b_j \geq \sum_i\left(y_{ij} \cdot 0.0002 \cdot P_i\right) + x_j \cdot 0.002 \cdot P_j$$

$$i = \{1...n\};\ j = \{1...n\},\ n = 116 \qquad (1.6)$$

$$x_i, y_{ij} \in \{0,1\} \qquad \text{(Binary)}$$

$$b_i \in \mathbb{N}_0 \qquad \text{(Integrality)}$$

i) Constraint 1.1 enforces the budget constraint. It takes the sum of the fixed costs and the variable costs associated with each hospital that gets constructed and ensures that it does not exceed B = $9 billion.

ii) Constraint 1.2 ensures that at least twelve hospitals get constructed. We felt that with a city the size of Montreal, it would not make sense if it had fewer than twelve hospitals.

iii) Constraint 1.3 ensures that the beds are only allocated to the locations where hospitals are being constructed. If there are no hospitals in location i, x_i would equal 0 and thus, $x_i \cdot M = 0$. This would then enforce b_i to also equal 0. If there is a hospital in location i, x_i would equal 1 and thus, $x_i \cdot M = 10000$. This would then enforce b_i to be any value from 0 to 10000.

iv) Constraint 1.4 enforces that the people from location i are only designated to one hospital. This is to ensure that the hospitals are allocated in an equitable manner.

v) Constraint 1.5 enforces that each hospital is within at least 10 km of all the outward codes that it is allocated to.

vi) Constraint 1.6 seeks to ensure that the number of beds per hospital is proportional to the population. It enforces that each hospital can serve at any given time at least 0.2% of the total population in its native outward code plus 0.02% of the combined population of all the outward codes that it is allocated to.

1.4. Data scraping and problem boundaries

To implement the problem, we defined two key matrices and a few constraints.

Since our objective is to maximize accessibility to hospitals for populations in need, two matrices were used in our model: the priority matrix, named Medical Assistance Level Matrix, and the Population Distribution Matrix. For the Medical Assistance Level Matrix, we looked for the most vulnerable age clusters. Reports and medical news such as the Canadian Medical Association and statscan mention the increase of the weight of the elder part of the population in Quebec in the last two years. Furthermore, we also investigated the age distribution of COVID-19 hospitalization and victims. Quebec Ministère de la Santé et des Services Sociaux and other medical reports show that senior patients were the most affected and vulnerable to the pandemic. For example, we collected the data from CIRANO's 2020 report Demographic Profile of COVID-19 Cases, Fatalities, Hospitalizations and Recoveries Across Canadian Provinces and found that 60-80 and 80+ years old categories were the most concerned by the pandemic.

age clusters	0-19	20-29	30-39	40-49	50-59	60-69	70-79	80+	Total
	12,5	25	50	100	200	250	400	900	female
	6,25	12,5	50	125	250	350	450	550	male
	18,75	37,5	100	225	450	600	850	1450	3731,25
relative proportion	0,50%	1,01%	2,68%	6,03%	12,06%	16,08%	22,78%	38,86%	1
age clusters	0-19	20-59				60-79		80+	Total
consolidated proportion	0,50%	21,78%				38,86%		38,86%	100,00%

Table 1: Age & gender distribution of the proportion of COVID-19 confirmed positive cases (2020, CIRANO)

To constitute the Population Distribution Matrix, the information from Statistics Canada Census data was collected using BeautifulSoup, a web scraping library in Python. The population distribution information for each outward code of Montreal was categorized into age buckets – 0 to 14 years, 15 to 64 years, 65-84 years, and 85 years and above.

Furthermore, distance between two combinations of outward code was collected using Selenium package in python, a web scraping library intended for dynamic web pages. The information is stored in a matrix from where a position in the matrix P_{ij} = Distance between Row Index i and Column index j, where the rows and columns store all the outward codes.

In terms of constraints, we looked for realistic cost estimations while keeping the problem feasible. To get an order of magnitude for the total budget, we looked at health budgets in Quebec and found that 9 billion dollars would be allocated to the health system in 2022 (sources: Montreal Gazette, TD). We also used 1 million dollars as the estimated cost per bed (2022, https://proest.com/construction/cost-estimates/hospitals/).

2. Operating Room: Numerical Implementation

2.1. Implementation

This mathematical optimization model which has 13688 decision variables and 1398 constraints, has 360 non-binding constraints and 1038 binding constraints. One of the major non-binding constraints is budget with a slack value of 3,000K dollar. The optimization model explored 590 nodes (10041 simplex iterations) in 1.69 seconds (0.74 work units).

2.2. Results

Therefore based on the 0 and 1s given for the decision variable Xi, 1 indicates that a hospital has to be built at that location. Therefore, the model presents to built hospitals in the following locations ith the said amount of beds. 15 hospital are built at the following locations.

Outward codes that will have hospitals

	postal_code	place_name	No. of beds
0	H1B	Montreal East	89
1	H1E	Rivière-Des-Prairies Southwest	127
2	H1G	Montreal North North	102
3	H1H	Montreal North South	79
4	H1K	Anjou East	86
5	H1Z	Saint-Michel West	186
6	H4E	Ville Émard	83
7	H4L	Saint-Laurent Inner Northeast	124
8	H8N	LaSalle Northwest	148
9	H7C	Saint-Vincent-de-Paul	32
10	H7L	Sainte-Rose	94
11	H7N	Laval-des-Rapides	90
12	H7W	Chomedey South	121
13	H9H	Sainte-Geneviève	67
14	H9W	Beaconsfield	72

Table 2: Outward codes that will have hospital with the bed number as given.

Using the values from the Aux[i,j] variable, we can find out people from which outward codes will visit the hospital at which outward code.

	Outward codes	Hospital
0	H1A	H1B
1	H1B	H1B
2	H1C	H1B
3	H1E	H1B
4	H1G	H1E
5	H1H	H1E
6	H1J	H1B
7	H1K	H1B
8	H1L	H1B
9	H1M	H1B
10	H1N	H1B
11	H1P	H1E
12	H1R	H1E
13	H1S	H1E
14	H1T	H1Z

Table 3 : People in Outward codes visit its respective hospitals

The smallest distance between 2 hospitals is 2.28 kms while the farthest distance is 23.52 kms.

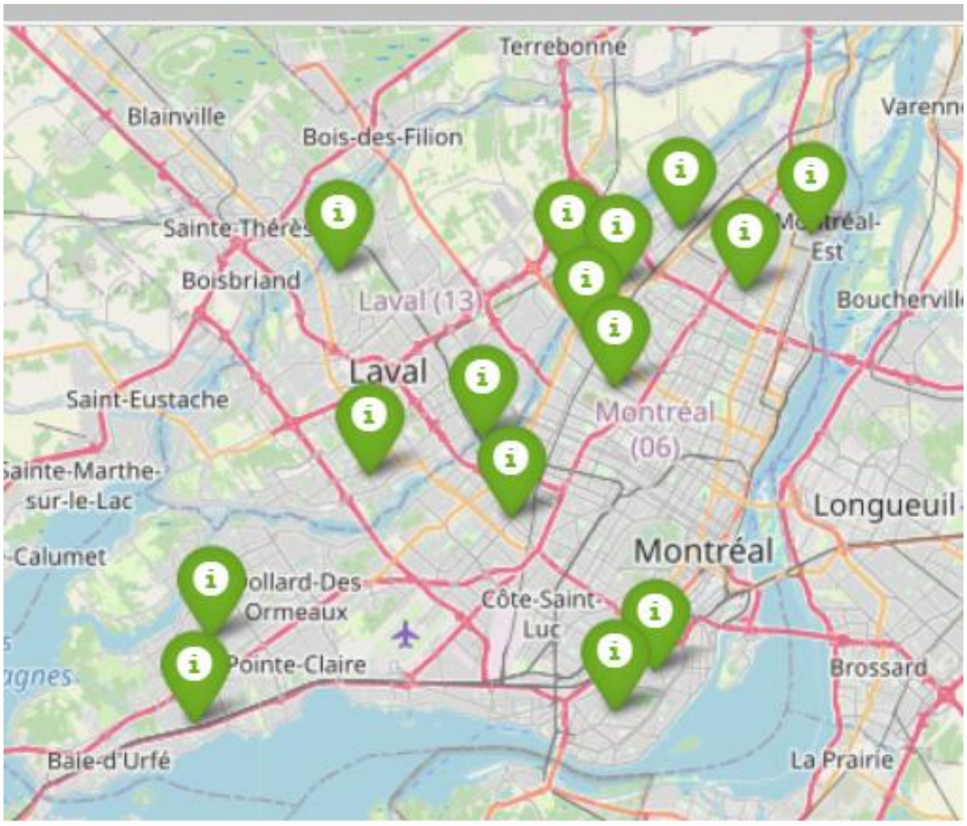

Figure 1 : Visualization of the Outward codes selected by the model for hospital building

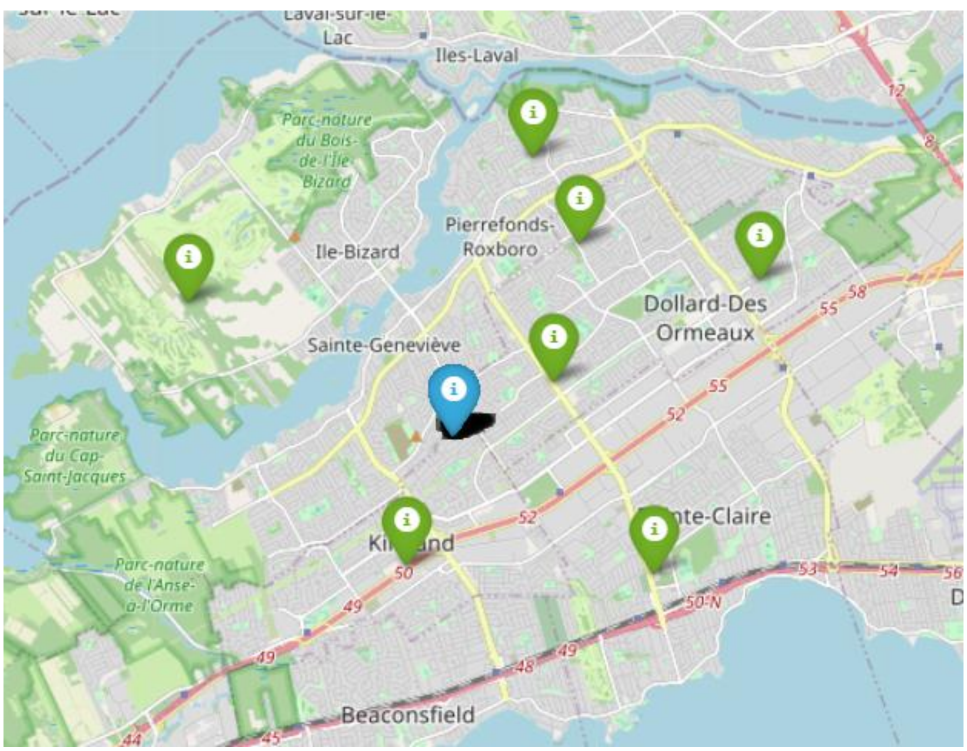

Figure 2: Visualization of the Hospitals that people who belong to outward code with no hospitals (in green) can visit hospital (in blue)

Figure 3: Map represents hospital is built at outward code H1B – Montreal East (blue pin) and people from outward codes H1A, H1C, H1E, etc(green pin)

3. Prescription: Problem Extensions

3.1. Additional Parameters

Our project gives the optimal location for hospitals in the city. We added the bed allocation decision variable to make it more insightful, adapting the number of beds to the population dependent on each hospital. To go further, we could consider two types of beds: normal beds, and intensive care unit beds.

An additional feature that could be added is the hospital type. There are four types of hospitals: emergency hospitals, smart hospitals (equipped with advanced technologies), small hospitals (designed for short stays, with eight to ten beds), and micro hospitals. To investigate this option, we would need to know more precisely the fixed and variable costs for each type and create one decision variable per hospital type. It would also affect the minimum and maximum number of beds depending on the type.

Besides, our problem does not consider every possible center of costs that would add up to variable costs. Licensing, land acquisition, material and labor, equipment, heating, ventilation, air conditioning, plumbing, and electricity are part of the costs that hospitals must include in their business plan (see Appendix 2). The fixed and variable costs we used could be split into many sub-costs, and some of them could be defined in terms of price per square meter. In that case, the surface required would become a decision variable and be subject to the cost constraint as well as a surface constraint depending on the surface available.

3.2. Business Applications

This type of location optimization based on the target audience can be applied to a good many different industries. Indeed, it is an optimization solution that could interest administrative and governmental functions such as police stations based on criminality rates of neighborhood, or transportations services based on the population characteristics (wealth-wise, age-wise, propensity to taking private or public transportations) such as metro stations, bike stations (bixi), taxi stations, etc. It could also easily apply in the case of deployment of vaccination centers through the city to face a pandemic as it was the case with COVID-19 recently.

Location optimization also applies to the retail industry. In terms of marketing, brands want to address a specific audience named segment. Each segment has a persona that embodies the main characteristics of consumers from this audience. Therefore, retail companies will want to locate their shops near their ideal customers, by adapting the coefficients of the priority matrix. For example, a brand like Decathlon will want to target a young and dynamic population.

The same logic applies to restaurant chains and fast food. The example of some already implemented chains can illustrate this logic: fast-food chains will want to be near teenagers and young adults, usually downtown or around schools and universities; while fast-food opened until late will want to move close to where the night life is (A&W, McDonald's). Another example could be coffee chains such as Starbucks that favor locations around universities, libraries and business districts. Finally, having an optimized location strategy based on the level of importance of population segments not only allows the business to increase sales but also increase loyalty and understand/influence consumers' habits.

Conclusion & Recommendations

According to our optimization model, the optimal number of hospitals to be constructed, which prioritized the maximization of the health care accessibility based on the demographics, was fifteen. Downtowns are often referred to as a city's commercial, cultural, and geographic heart. This usually results in a large population density within the area. In a study by Statistics Canada, it was reported that downtown areas of large cities also often attract many older seniors because they have easier access to more services and amenities (StatCan, 2022). In our medical assistance level per age cluster, seniors were the age group more likely in need of medical facilities. Downtown Montreal consists of the following outer codes: H2Z, H3A, H3B, H3G, and H3H. Based on our results, this means that citizens living in downtown Montreal have access to either hospitals in H4E or H4L. These hospitals have 115 and 134 beds respectively, making them the fifth and second largest hospitals respectively; thus, increasing the likelihood of having the capacity to serve its patients satisfactorily.

In our project, we optimized the location of the hospitals based on the demographic data collected from the census. We were assuming that there are no current hospitals in the city, but the population remains as it is. However, in a real-world problem, this would rarely be the case. If a new city was being built, the population would grow as the city gets more developed. Citizens would settle based on the initial city designs and further urban planning would have to be adjusted based on the settlement of the population.

If the plan was regarding an already existing city, then one would have to account for the already existing landmarks (in this case, hospitals). Thus, while our model provides a great starting point for a location-based optimization problem, one would have to explore many more factors before this model can be integrated into a real-world scenario.

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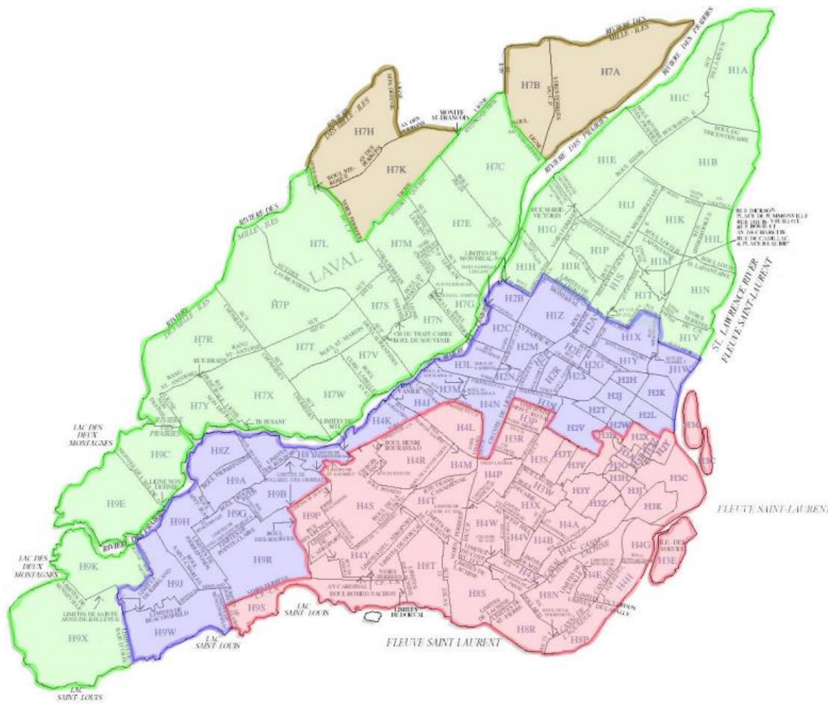
Appendix



Appendix 1: Map of Montral based on the outward codes

COSTS	COST PER SQUARE FOOT
Labor	$234
Contractor fees	$58
Architectural fees	$26
Subtotal	$318

MATERIAL COSTS	COST PER SQUARE METER
Concrete	$22.45
Masonry	$14.86
Metals	$23.52
Wood and Plastics	$32.52
Thermal and Moisture	$25.32
Openings	$8.90
Equipment	$13.34

Appendix 2: Cost per Square Meter in Hospitals

Source : https://proest.com/construction/cost-estimates/hospitals/