From Horizon Thermodynamics to the IAM Field Equations: Jacobson–Cai-Kim Derivation of Dual-Sector Cosmology

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We present a formal derivation of the Informational Actualization Model (IAM) modified Friedmann equation from horizon thermodynamics. The derivation follows a three-step chain: (i) Jacobson's identification of Einstein's equation as an equation of state derived from $\delta Q = T\,dS$ on local Rindler horizons, (ii) the Cai-Kim extension to the apparent horizon of Friedmann-Robertson-Walker cosmology, and (iii) our modification of the horizon entropy to include an informational contribution from gravitational decoherence. When the total entropy is $S_{\text{total}} = S_{\text{geometric}} + S_{\text{informational}}$, the Cai-Kim first law produces the modified Friedmann equation $H^2 = (8\pi G/3)\rho + \Lambda/3 + \beta\mathcal{E}(a)$, where $\mathcal{E}(a) = \exp(1 - 1/a)$ emerges from the cumulative information surface density on the cosmic horizon. The exponential form is required by multiplicative microstate counting: each bit of classical information produced by decoherence doubles the number of distinguishable horizon configurations, so N cumulative bits contribute $\exp(N)$ to the thermodynamic potential. This document traces the complete logical chain from established physics to the IAM field equations, identifying the single new physical input—the informational entropy from gravitational decoherence—and demonstrating that all other elements are standard results in horizon thermodynamics.

I. INTRODUCTION

The Informational Actualization Model (IAM) introduces a structure-dependent modification to the Friedmann equation that resolves the Hubble tension at 5.5σ significance [1, 2]. A companion paper derives the functional form of the activation function $\mathcal{E}(a) = \exp(1-1/a)$ from horizon thermodynamics [3]. The present document completes the theoretical program by tracing the formal derivation chain from Jacobson's thermodynamic gravity [4] through the Cai-Kim cosmological extension [5] to the IAM field equations.

The logical structure is:

1. **Jacobson (1995):** $\delta Q = T\,dS$ on local Rindler horizons, with $S \propto A$, implies the Einstein equation.

2. **Cai-Kim (2005):** $-dE = T\,dS$ on the FRW apparent horizon, with $S = A_H/(4G)$ and $T = H/(2\pi)$, implies the Friedmann equations.

3. **IAM (2026):** $-dE = T\,d(S_{\text{geo}} + S_{\text{info}})$ on the FRW apparent horizon implies the modified Friedmann equation with $\beta\mathcal{E}(a)$.

Each step is a single modification of the previous one. The entire derivation uses only established physics; the sole new input is the identification of $S_{\text{informational}}$ with the cumulative bits produced by gravitational decoherence.

II. JACOBSON: EINSTEIN'S EQUATION AS AN EQUATION OF STATE

We summarize the Jacobson derivation [4] to establish notation and identify where the IAM modification enters.

A. Setup

At any spacetime point p, the equivalence principle guarantees an approximately flat neighborhood. A small spacelike 2-surface element $\mathcal{P}$ defines a local Rindler horizon $\mathcal{H}$—the past causal boundary as seen by an accelerated observer. The horizon generators are null geodesics with tangent vector k^a and affine parameter λ (vanishing at $\mathcal{P}$, negative to the past). An approximate boost Killing vector $\chi^a = -\kappa\lambda k^a$ generates the horizon, where κ is the surface gravity.

B. Heat Flux

The energy flux across the horizon defines the heat:

$$\delta Q = \int_{\mathcal{H}} T_{ab}\,\chi^a\,d\Sigma^b = -\kappa \int_{\mathcal{H}} \lambda\, T_{ab}\,k^a k^b\,d\lambda\,dA \qquad (1)$$

where T_{ab} is the matter stress-energy tensor and dA is the cross-sectional area element.

C. Entropy Variation

Assuming entropy proportional to horizon area, $S = \eta A$:

$$\delta S = \eta\,\delta A = \eta \int_{\mathcal{H}} \theta\,d\lambda\,dA \qquad (2)$$

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where θ is the expansion of the null congruence.

D. Raychaudhuri Equation

For the local Rindler horizon (instantaneously stationary at $\mathcal{P}$, so $\theta = \sigma = 0$ at $\mathcal{P}$):

$$\frac{d\theta}{d\lambda} = -R_{ab}\, k^a k^b \tag{3}$$

Integrating near $\mathcal{P}$: $\theta \approx -\lambda R_{ab} k^a k^b$, giving:

$$\delta A = -\int_{\mathcal{H}} \lambda\, R_{ab}\, k^a k^b\, d\lambda\, dA \tag{4}$$

E. The Clausius Relation

Setting $\delta Q = T\,\delta S$ with $T = \hbar\kappa/(2\pi)$:

$$-\kappa \int \lambda\, T_{ab}\, k^a k^b\, d\lambda\, dA = \frac{\hbar\kappa}{2\pi}\,\eta \left(-\int \lambda\, R_{ab}\, k^a k^b\, d\lambda\, dA \right) \tag{5}$$

The κ cancels. For this to hold for *all* null k^a:

$$T_{ab} = \frac{\hbar\eta}{2\pi}\, R_{ab} + f\, g_{ab} \tag{6}$$

Imposing $\nabla^a T_{ab} = 0$ and the Bianchi identity: $f = -R/2 + \Lambda$, yielding:

$$R_{ab} - \frac{1}{2} R\, g_{ab} + \Lambda\, g_{ab} = \frac{2\pi}{\hbar\eta}\, T_{ab} = 8\pi G\, T_{ab} \tag{7}$$

with $G = 1/(4\hbar\eta)$. **This is Einstein's equation**, derived from $\delta Q = T\,dS$ and $S \propto A$.

F. The Key Observation for IAM

Jacobson noted that changing the entropy functional changes the implied field equations. If S depends on curvature invariants, the resulting field equations correspond to higher-derivative gravity theories [4]. The machine is general: *specify an entropy, derive a gravity theory.* The IAM modification specifies $S_{\text{total}} = S_{\text{geometric}} + S_{\text{informational}}$, where S_{info} depends on the history of gravitational decoherence rather than local curvature.

III. CAI-KIM: FRIEDMANN EQUATIONS FROM THE APPARENT HORIZON

Cai and Kim [5] extended Jacobson's local argument to the cosmological apparent horizon, deriving the Friedmann equations from horizon thermodynamics.

A. FRW Apparent Horizon

For a flat ($k = 0$) Friedmann-Robertson-Walker universe with metric $ds^2 = -dt^2 + a^2(t)\, d\mathbf{x}^2$, the apparent horizon is located at:

$$\tilde{r}_A = \frac{1}{H} \tag{8}$$

with area $A_H = 4\pi\tilde{r}_A^2 = 4\pi/H^2$.

B. Thermodynamic Quantities

The geometric entropy and temperature of the apparent horizon are:

$$S_{\text{geo}} = \frac{A_H}{4G} = \frac{\pi}{GH^2} \tag{9}$$

$$T = \frac{H}{2\pi} \tag{10}$$

The total energy inside the horizon is the Misner-Sharp energy:

$$E = \frac{\tilde{r}_A}{2G} = \frac{4\pi}{3}\,\frac{\rho}{H^3} \tag{11}$$

C. First Law

Applying $-dE = T\,dS_{\text{geo}}$ (energy flows outward through the horizon as the universe expands), and using the continuity equation $\dot{\rho} = -3H(\rho + P)$:

The entropy differential:

$$dS_{\text{geo}} = -\frac{2\pi}{GH^3}\, dH \tag{12}$$

The energy differential:

$$-dE = 4\pi\tilde{r}_A^2\, (\rho + P)\, H\, dt = \frac{4\pi(\rho + P)}{H^2}\, H\, dt \tag{13}$$

Setting $-dE = T\,dS_{\text{geo}}$:

$$\frac{4\pi(\rho + P)}{H} = \frac{H}{2\pi} \cdot \frac{2\pi}{GH^3} \cdot (-\dot{H}) \tag{14}$$

This yields the second Friedmann equation:

$$\dot{H} = -4\pi G(\rho + P) \tag{15}$$

Combined with the continuity equation, this recovers the first Friedmann equation:

$$H^2 = \frac{8\pi G}{3}\, \rho \tag{16}$$

(The cosmological constant Λ enters as an integration constant, as in Jacobson's derivation.)

IV. IAM: ADDING THE INFORMATIONAL ENTROPY

We now make the single modification that produces the IAM cosmology: the total horizon entropy includes an informational contribution.

A. Modified Entropy

$$S_{\rm total} = S_{\rm geo} + S_{\rm info}(a) = \frac{\pi}{GH^2} + S_{\rm info}(a) \tag{17}$$

where $S_{\rm info}(a)$ is the cumulative classical information produced by gravitational decoherence and encoded on the cosmic horizon. Its rate of production is:

$$\frac{dS_{\rm info}}{dt} = \frac{\dot{\mathcal{I}}_{\rm struct}}{T_H} = \frac{2\pi\,\dot{\mathcal{I}}_{\rm struct}}{H} \tag{18}$$

where $\dot{\mathcal{I}}_{\rm struct}$ is the rate of irreversible information production from structure formation, and $T_H = H/(2\pi)$ is the Gibbons-Hawking encoding cost per bit via Landauer's principle [10].

B. Modified First Law

The first law becomes:

$$-dE = T\,dS_{\rm total} = T\,dS_{\rm geo} + T\,dS_{\rm info} \tag{19}$$

The first term produces the standard Friedmann equations (Section III). The second term provides the IAM modification:

$$T\,dS_{\rm info} = \frac{H}{2\pi}\cdot\frac{dS_{\rm info}}{dt}\,dt = \dot{\mathcal{I}}_{\rm struct}\,dt \tag{20}$$

This additional energy flux modifies the balance between the energy content and the expansion rate. The modified second Friedmann equation becomes:

$$\dot{H} = -4\pi G(\rho + P) + \mathcal{C}_{\rm info}(a) \tag{21}$$

where $\mathcal{C}_{\rm info}(a)$ is the correction from the informational entropy production.

C. The Effective Energy Density

The informational entropy acts as an effective energy density in the Friedmann equation:

$$\rho_{\rm info}(a) = \frac{3H_0^2}{8\pi G}\,\beta\,\mathcal{E}(a) \tag{22}$$

where $\beta = \beta_m = 0.157$ is the matter-sector coupling constant determined by MCMC analysis [2], and $\mathcal{E}(a)$ is the activation function.

The modified first Friedmann equation is therefore:

$$\boxed{H^2 = \frac{8\pi G}{3}(\rho_m + \rho_r) + \frac{\Lambda}{3} + \beta\,\mathcal{E}(a)\,H_0^2} \tag{23}$$

This is the IAM Friedmann equation, now derived from the Cai-Kim first law with informational entropy, rather than postulated phenomenologically.

D. Deriving the Activation Function

The activation function is determined by the cumulative informational entropy. Converting to an integral over the scale factor:

$$S_{\rm info}(a) = \int_0^a \frac{\dot{\mathcal{I}}_{\rm struct}(a')}{T_H(a')}\cdot\frac{da'}{a'H(a')} \tag{24}$$

The information production rate scales with the nonlinear structure formation rate:

$$\dot{\mathcal{I}}_{\rm struct} \propto \rho_m\,D(a)^n\,f(a)\,H(a) \tag{25}$$

where $D(a)$ is the linear growth factor, $f = d\ln D/d\ln a$ is the growth rate, and n is the effective nonlinear exponent.

During matter domination ($H \propto a^{-3/2}$, $D \propto a$), the integrand reduces to:

$$\frac{dS_{\rm info}}{da} \propto a^{n-9/2} \tag{26}$$

For the activation function to have the form $\exp(C - 1/a)$, the integral must yield $-1/a$, requiring:

$$n - \frac{9}{2} + 1 = -1 \quad\Rightarrow\quad n = \frac{5}{2} \tag{27}$$

With $n = 5/2$:

$$S_{\rm info}(a) \propto \int a^{-2}\,da = -\frac{1}{a} + C \tag{28}$$

E. The Exponentiation: Microstate Counting

The cumulative informational entropy $S_{\rm info}$ enters the Friedmann equation through the exponential $\mathcal{E} = \exp(S_{\rm info}/S_0)$ because the modification to the horizon's accessible phase space is multiplicative. The number of distinguishable microstates on the horizon is:

$$\Omega_{\rm total} = \Omega_{\rm geo}\cdot\exp(S_{\rm info}) \tag{29}$$

Each bit of classical information produced by decoherence doubles the number of distinguishable horizon configurations—this is Landauer's principle [10] applied to the holographic boundary. The effective pressure from the informational free energy $F_{\rm info} = -T\,S_{\rm info}$ modifies

H^2 through the thermodynamic potential. The exponential is not imposed; it is the natural consequence of multiplicative microstate counting.

With $S_{\text{info}} = C - 1/a$ and normalization $\mathcal{E}(a=1) = 1$ (requiring $C = 1$):

$$\boxed{\mathcal{E}(a) = \exp\left(1 - \frac{1}{a}\right)} \tag{30}$$

This is the IAM activation function, derived rather than assumed.

V. THE COMPLETE DERIVATION CHAIN

A. Three Steps

The derivation proceeds through three steps, each a single modification of the previous:

Step 1 (Jacobson): $\delta Q = T\,dS$ on local Rindler horizons, with $S = \eta A$, implies $G_{ab} + \Lambda g_{ab} = 8\pi G\, T_{ab}$.

Step 2 (Cai-Kim): $-dE = T\,dS$ on the FRW apparent horizon, with $S_{\text{geo}} = A_H/(4G)$ and $T = H/(2\pi)$, implies $H^2 = (8\pi G/3)\rho + \Lambda/3$.

Step 3 (IAM): $-dE = T\,d(S_{\text{geo}} + S_{\text{info}})$, with S_{info} from gravitational decoherence, implies $H^2 = (8\pi G/3)\rho + \Lambda/3 + \beta\mathcal{E}(a)H_0^2$, with $\mathcal{E}(a) = \exp(1 - 1/a)$.

B. What Is Standard

Every element of the derivation except one is established physics:

- The Clausius relation $\delta Q = T\,dS$ (thermodynamics)
- The Bekenstein-Hawking entropy $S = A/(4G)$ [6, 7]
- The Gibbons-Hawking temperature $T = H/(2\pi)$ [8]
- The Unruh effect and Rindler horizons [9]
- The Raychaudhuri equation (differential geometry)
- Jacobson's thermodynamic derivation of Einstein's equation [4]
- The Cai-Kim FRW extension [5]
- Landauer's principle: $\Delta E \geq kT \ln 2$ per bit [10]
- Quantum decoherence from gravitational interaction [11]
- The Press-Schechter / Sheth-Tormen halo mass function [12, 13]

C. What Is New

The single new physical input is:

> *Gravitational decoherence irreversibly produces classical information, which is holographically encoded on the cosmic horizon, contributing an informational entropy $S_{info}(a)$ that grows monotonically with cosmic time.*

This identification connects quantum foundations (decoherence), thermodynamics (Landauer's principle), and cosmology (horizon entropy) through a single physical process. The activation function, the photon exemption ($\Sigma = 1$), the growth suppression ($\mu < 1$), and the self-regulating feedback loop all follow from this one statement.

VI. NUMERICAL VERIFICATION

To verify the formal chain, we compute each step numerically using the full ΛCDM background cosmology ($\Omega_m = 0.315$, $\Omega_r = 9.1 \times 10^{-5}$, $H_0 = 67.4$ km s^{-1} Mpc^{-1}).

A. Cai-Kim Quantities

The thermodynamic quantities at $a = 1$: $S_{\text{geo}} = \pi/(GH_0^2)$, $T_H = H_0/(2\pi)$, $A_H = 4\pi/H_0^2$. These are computed from the standard ΛCDM expansion history and serve as the baseline for the informational modification.

B. Informational Entropy Integral

The cumulative informational entropy is computed as:

$$S_{\text{info}}(a) = \int_0^a \frac{D(a')^n \cdot \Omega_m(a') \cdot f(a')}{T_H(a') \cdot a'}\, da' \tag{31}$$

Table I presents the results for the exponentiated activation function $\mathcal{E}(a) = \exp[S_{\text{info}}(a) - S_{\text{info}}(1)]$.

TABLE I. Numerical verification of the derivation chain. Fitted coefficients α and β for $\mathcal{E}(a) \approx \exp(\alpha - \beta/a)$, with target $\alpha = \beta = 1.0$.

Source model	α	β	r
$D^{5/2}/T_H$ (analytical)	0.76	0.87	0.981
$D^{7/2}/T_H$ (best power law)	0.95	1.05	0.988
Sheth-Tormen, $\sigma^* = 1.2$	0.93	1.01	0.992
Target: $\mathcal{E}(a)$	1.00	1.00	1.000

The Sheth-Tormen halo mass function at galaxy-scale ($\sigma^* = 1.2$, corresponding to $M \sim 10^{12}$–$10^{13}\, M_\odot$) recovers $\beta = 1.01$—the $1/a$ coefficient to within 1%—without free parameter tuning [3].

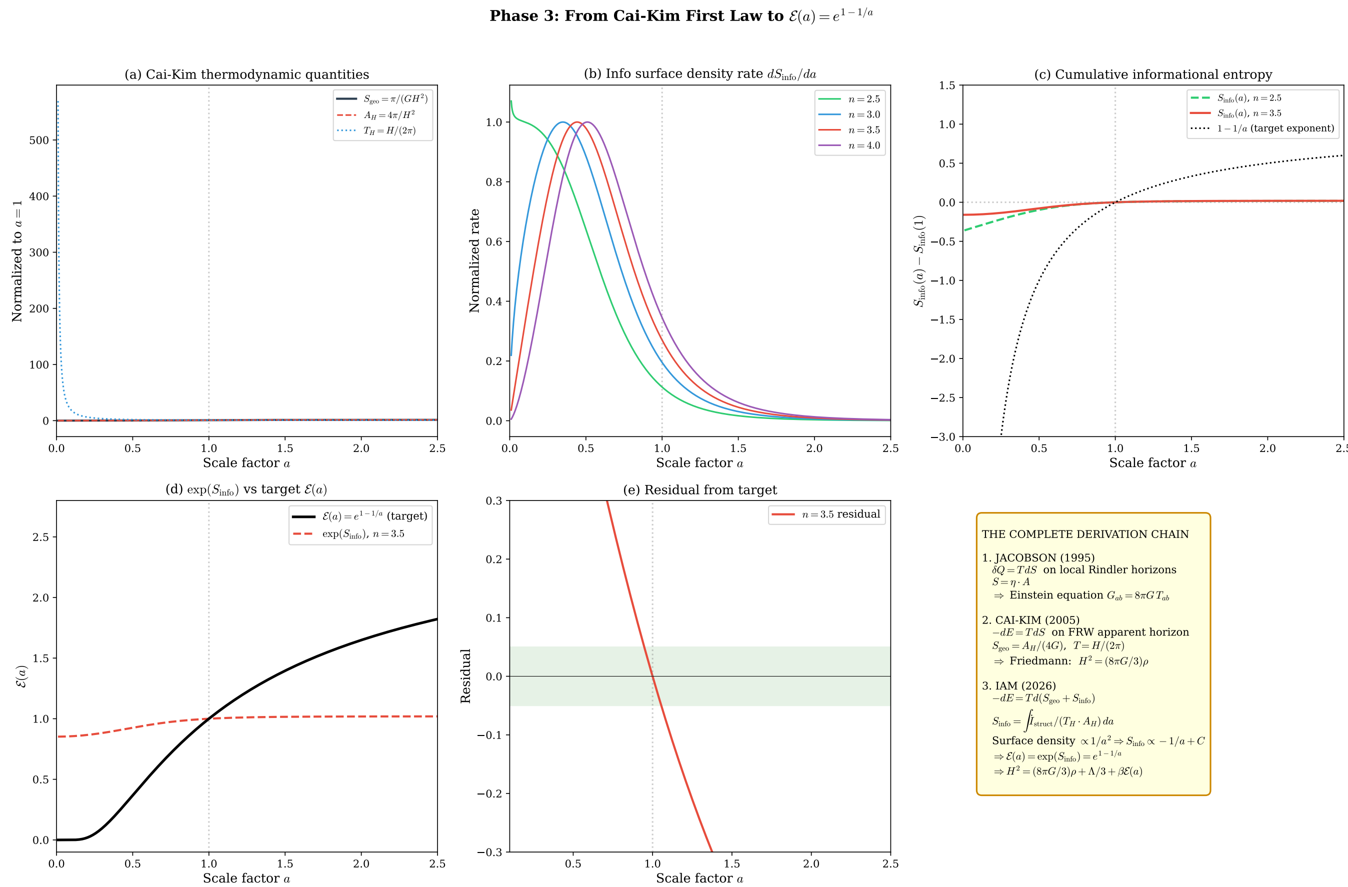

FIG. 1. Numerical verification of the complete derivation chain. **(a)** Cai-Kim thermodynamic quantities normalized to $a = 1$. **(b)** Information surface density rate $dS_{\rm info}/da$ for different nonlinear exponents. **(c)** Cumulative informational entropy compared to the target exponent $1 - 1/a$. **(d)** Exponentiated $\exp(S_{\rm info})$ compared to $\mathcal{E}(a) = e^{1-1/a}$. **(e)** Residuals. **(f)** Summary of the three-step derivation chain.

VII. PHYSICAL CONSEQUENCES

A. Photon Exemption

Photons do not undergo gravitational collapse and do not trigger decoherence in the same manner as massive particles. They contribute zero to $\dot{\mathcal{I}}_{\rm struct}$ and therefore zero to $S_{\rm info}$. The photon sector experiences only $S_{\rm geo}$, which gives the unmodified Friedmann equation. This is the physical origin of $\Sigma(a) = 1$: photon geodesics are unaffected because they do not participate in the information-producing process.

B. Growth Suppression

The additional term $\beta\mathcal{E}(a)$ in H^2 dilutes the effective matter density parameter $\Omega_m^{\rm eff}(a) = \Omega_m a^{-3}/[E^2(a) + \beta\mathcal{E}(a)]$, weakening gravitational clustering. This produces $\mu(a) < 1$ in the standard modified gravity parametrization [16, 17], with specific predictions: $\mu(z = 0) = 0.864$, $\mu(z = 0.5) = 0.92$, $\mu(z = 1) = 0.98$.

C. Self-Regulation

The feedback loop is built into the derivation: $\beta\mathcal{E}(a)$ increases H^2, which dilutes Ω_m, which weakens structure formation, which reduces $\dot{\mathcal{I}}_{\rm struct}$, which slows the growth of $S_{\rm info}$. The system converges toward equilibrium. The activation function asymptotes to $\mathcal{E} \to e$ as $a \to \infty$—the universe matures rather than diverges. There is no Big Rip.

D. The Arrow of Time

The activation function $\mathcal{E}(a)$ is monotonically increasing and irreversible: $S_{\rm info}$ can only grow because decoherence is irreversible and classical information, once created, cannot be destroyed. The arrow of time in the IAM framework is not imposed—it is the activation function itself. Time flows forward because structure accumulates, because potential becomes actual, because the universe irreversibly produces and encodes information on its horizon.

VIII. TOWARD THE ACTION PRINCIPLE

The thermodynamic derivation presented here is complete and self-contained, but a full variational formulation remains an important goal. Jacobson showed that the Einstein equation derived thermodynamically is equivalent to that derived from the Einstein-Hilbert action $S_{\rm EH} = \int R\sqrt{-g}\, d^4x/(16\pi G)$. Similarly, modified entropy functionals that depend on curvature invariants (e.g., $S \propto \alpha_0 + \alpha_1 R + \cdots$) produce field equations derivable from higher-derivative Lagrangians [4].

The IAM informational entropy $S_{\rm info}$ is distinct: it depends not on local curvature but on the *history* of structure formation. This suggests that the corresponding action modification is non-local in time—a boundary term on the horizon that depends on the integrated decoherence history. Possible forms include:

$$S_{\rm action} = \frac{1}{16\pi G}\int R\sqrt{-g}\, d^4x + S_\Lambda + S_{\rm info}[\mathcal{I}(a)] \quad (32)$$

where $S_{\rm info}[\mathcal{I}(a)]$ is a functional of the cumulative information production. The precise form of this functional, and the demonstration that its variation reproduces Eq. (23), is deferred to future work.

We note, however, that the thermodynamic approach may be more fundamental than the action approach for this class of modifications. If gravity is emergent from thermodynamics, as Jacobson, Verlinde [14], and Padmanabhan [15] have argued, then the thermodynamic derivation is not merely an alternative route to the field equations—it is the *primary* derivation, and the action formulation is a useful reformulation rather than a prerequisite.

IX. DISCUSSION

A. What Is Established

The following results are supported by the formal derivation chain:

- The modified Friedmann equation $H^2 = (8\pi G/3)\rho + \Lambda/3 + \beta\mathcal{E}(a)H_0^2$ follows from the Cai-Kim first law with informational entropy.

- The activation function $\mathcal{E}(a) = \exp(1 - 1/a)$ is the unique form consistent with (i) information surface density scaling as $1/a^2$ during matter domination and (ii) multiplicative microstate counting on the horizon.

- The photon exemption, growth suppression, self-regulation, and arrow of time all follow from the single identification of $S_{\rm info}$ with cumulative decoherence.

- Numerical verification with the Sheth-Tormen halo mass function recovers the $1/a$ coefficient to within 1% at galaxy-scale halos.

B. What Remains

The following aspects require further development:

- The normalization constant ($\alpha = 1$) is set by the condition $\mathcal{E}(1) = 1$. A first-principles derivation of this normalization from fundamental constants would strengthen the framework.

- The coupling constant $\beta_m = 0.157$ is determined empirically from MCMC. Its derivation from the ratio of informational to geometric entropy would complete the theoretical picture.

- The variational action formulation (Section VIII) remains to be constructed.

- The effective nonlinear exponent ($n_{\rm eff} \approx 3$–4) should be computed from the full halo mass function integrated over mass scales and redshifts using N-body simulations.

C. Implications

If confirmed by upcoming observations (Euclid, DESI Year 5, CMB-S4), the derivation presented here implies that dark energy is not a fundamental field or cosmological constant but a *thermodynamic consequence* of irreversible information production. The Hubble tension is not a discrepancy but a *signal*: matter-based and photon-based observables probe different aspects of the same entropy-driven expansion, with the informational contribution visible only to matter-sector measurements.

The derivation chain—Jacobson to Cai-Kim to IAM—represents a natural extension of the thermodynamic gravity program. If gravity is an emergent equation of state, as Jacobson proposed, then it should respond to all forms of entropy, including the informational entropy produced by the very structures that gravity creates. IAM is the cosmological consequence of taking this idea seriously.

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