From Horizon Thermodynamics to the IAM Field Equations: Jacobson–Cai-Kim Derivation of Dual-Sector Cosmology

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We present a formal derivation of the Informational Actualization Model (IAM) modified Friedmann equation from horizon thermodynamics. The derivation follows a three-step chain: (i) Jacobson's identification of Einstein's equation as an equation of state derived from $\delta Q = T\,dS$ on local Rindler horizons, (ii) the Cai-Kim extension to the apparent horizon of Friedmann-Robertson-Walker cosmology, and (iii) our modification of the horizon entropy to include an informational contribution from gravitational decoherence. When the total entropy is $S_{\text{total}} = S_{\text{geometric}} + S_{\text{informational}}$, the Cai-Kim first law produces the modified Friedmann equation $H^2 = (8\pi G/3)\rho + \Lambda/3 + (\Omega_m/2)\,\mathcal{E}(a)\,H_0^2$, where $\mathcal{E}(a) = \exp(1 - 1/a)$ emerges from the cumulative information surface density on the cosmic horizon. The exponential form is required by multiplicative microstate counting on the holographic boundary. We further show that the informational sector is described by a constrained scalar field $\varphi = 1 - 1/a$ with action $S_{\text{info}} = \int[\beta H_0^2 e^{\varphi} + \lambda(\dot\varphi - H/a)]\sqrt{-g}\,d^4x$, yielding equation of state $w_{\text{info}}(a) = -1 - 1/(3a)$—a mildly phantom dark energy consistent with DESI 2024 indications. The coupling constant $\beta_m = \Omega_m/2$ is derived from the virial partition of gravitational energy between geometry and information, matching the MCMC-fitted value to 0.3%. The complete model has zero free parameters beyond standard ΛCDM.

I. INTRODUCTION

The Informational Actualization Model (IAM) introduces a structure-dependent modification to the Friedmann equation that resolves the Hubble tension at 5.5σ significance [1, 2]. A companion paper derives the functional form of the activation function $\mathcal{E}(a) = \exp(1-1/a)$ from horizon thermodynamics [3]. The present document completes the theoretical program by tracing the formal derivation chain from Jacobson's thermodynamic gravity [4] through the Cai-Kim cosmological extension [5] to the IAM field equations.

The logical structure is:

1. **Jacobson (1995):** $\delta Q = T\,dS$ on local Rindler horizons, with $S \propto A$, implies the Einstein equation.
2. **Cai-Kim (2005):** $-dE = T\,dS$ on the FRW apparent horizon, with $S = A_H/(4G)$ and $T = H/(2\pi)$, implies the Friedmann equations.
3. **IAM (2026):** $-dE = T\,d(S_{\text{geo}} + S_{\text{info}})$ on the FRW apparent horizon implies the modified Friedmann equation with $\beta\mathcal{E}(a)$.

Each step is a single modification of the previous one. The entire derivation uses only established physics; the sole new input is the identification of $S_{\text{informational}}$ with the cumulative bits produced by gravitational decoherence.

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II. JACOBSON: EINSTEIN'S EQUATION AS AN EQUATION OF STATE

We summarize the Jacobson derivation [4] to establish notation and identify where the IAM modification enters.

A. Setup

At any spacetime point p, the equivalence principle guarantees an approximately flat neighborhood. A small spacelike 2-surface element $\mathcal{P}$ defines a local Rindler horizon $\mathcal{H}$—the past causal boundary as seen by an accelerated observer. The horizon generators are null geodesics with tangent vector k^a and affine parameter λ (vanishing at $\mathcal{P}$, negative to the past). An approximate boost Killing vector $\chi^a = -\kappa\lambda k^a$ generates the horizon, where κ is the surface gravity.

B. Heat Flux

The energy flux across the horizon defines the heat:

$$\delta Q = \int_{\mathcal{H}} T_{ab}\,\chi^a\,d\Sigma^b = -\kappa \int_{\mathcal{H}} \lambda\,T_{ab}\,k^a k^b\,d\lambda\,dA \qquad (1)$$

where T_{ab} is the matter stress-energy tensor and dA is the cross-sectional area element.

C. Entropy Variation

Assuming entropy proportional to horizon area, $S = \eta A$:

$$\delta S = \eta\,\delta A = \eta \int_{\mathcal{H}} \theta\, d\lambda\, dA \tag{2}$$

where θ is the expansion of the null congruence.

D. Raychaudhuri Equation

For the local Rindler horizon (instantaneously stationary at $\mathcal{P}$, so $\theta = \sigma = 0$ at $\mathcal{P}$):

$$\frac{d\theta}{d\lambda} = -R_{ab}\, k^a k^b \tag{3}$$

Integrating near $\mathcal{P}$: $\theta \approx -\lambda R_{ab} k^a k^b$, giving:

$$\delta A = -\int_{\mathcal{H}} \lambda\, R_{ab}\, k^a k^b\, d\lambda\, dA \tag{4}$$

E. The Clausius Relation

Setting $\delta Q = T\,\delta S$ with $T = \hbar\kappa/(2\pi)$:

$$-\kappa \int \lambda\, T_{ab}\, k^a k^b\, d\lambda\, dA = \frac{\hbar\kappa}{2\pi}\,\eta \left(-\int \lambda\, R_{ab}\, k^a k^b\, d\lambda\, dA \right) \tag{5}$$

The κ cancels. For this to hold for *all* null k^a:

$$T_{ab} = \frac{\hbar\eta}{2\pi}\, R_{ab} + f\, g_{ab} \tag{6}$$

Imposing $\nabla^a T_{ab} = 0$ and the Bianchi identity: $f = -R/2 + \Lambda$, yielding:

$$R_{ab} - \frac{1}{2} R\, g_{ab} + \Lambda\, g_{ab} = \frac{2\pi}{\hbar\eta}\, T_{ab} = 8\pi G\, T_{ab} \tag{7}$$

with $G = 1/(4\hbar\eta)$. **This is Einstein's equation**, derived from $\delta Q = T\,dS$ and $S \propto A$.

F. The Key Observation for IAM

Jacobson noted that changing the entropy functional changes the implied field equations. If S depends on curvature invariants, the resulting field equations correspond to higher-derivative gravity theories [4]. The machine is general: *specify an entropy, derive a gravity theory.* The IAM modification specifies $S_{\text{total}} = S_{\text{geometric}} + S_{\text{informational}}$, where S_{info} depends on the history of gravitational decoherence rather than local curvature.

III. CAI-KIM: FRIEDMANN EQUATIONS FROM THE APPARENT HORIZON

Cai and Kim [5] extended Jacobson's local argument to the cosmological apparent horizon, deriving the Friedmann equations from horizon thermodynamics.

A. FRW Apparent Horizon

For a flat ($k = 0$) Friedmann-Robertson-Walker universe with metric $ds^2 = -dt^2 + a^2(t)\, d\mathbf{x}^2$, the apparent horizon is located at:

$$\tilde{r}_A = \frac{1}{H} \tag{8}$$

with area $A_H = 4\pi\tilde{r}_A^2 = 4\pi/H^2$.

B. Thermodynamic Quantities

The geometric entropy and temperature of the apparent horizon are:

$$S_{\text{geo}} = \frac{A_H}{4G} = \frac{\pi}{GH^2} \tag{9}$$

$$T = \frac{H}{2\pi} \tag{10}$$

The total energy inside the horizon is the Misner-Sharp energy:

$$E = \frac{\tilde{r}_A}{2G} = \frac{4\pi}{3}\,\frac{\rho}{H^3} \tag{11}$$

C. First Law

Applying $-dE = T\,dS_{\text{geo}}$ (energy flows outward through the horizon as the universe expands), and using the continuity equation $\dot{\rho} = -3H(\rho + P)$:

The entropy differential:

$$dS_{\text{geo}} = -\frac{2\pi}{GH^3}\, dH \tag{12}$$

The energy differential:

$$-dE = 4\pi\tilde{r}_A^2\,(\rho + P)\, H\, dt = \frac{4\pi(\rho + P)}{H^2}\, H\, dt \tag{13}$$

Setting $-dE = T\,dS_{\text{geo}}$:

$$\frac{4\pi(\rho + P)}{H} = \frac{H}{2\pi} \cdot \frac{2\pi}{GH^3} \cdot (-\dot{H}) \tag{14}$$

This yields the second Friedmann equation:

$$\dot{H} = -4\pi G(\rho + P) \tag{15}$$

Combined with the continuity equation, this recovers the first Friedmann equation:

$$H^2 = \frac{8\pi G}{3}\,\rho \tag{16}$$

(The cosmological constant Λ enters as an integration constant, as in Jacobson's derivation.)

IV. IAM: ADDING THE INFORMATIONAL ENTROPY

We now make the single modification that produces the IAM cosmology: the total horizon entropy includes an informational contribution.

A. Modified Entropy

$$S_{\text{total}} = S_{\text{geo}} + S_{\text{info}}(a) = \frac{\pi}{GH^2} + S_{\text{info}}(a) \tag{17}$$

where $S_{\text{info}}(a)$ is the cumulative classical information produced by gravitational decoherence and encoded on the cosmic horizon. Its rate of production is:

$$\frac{dS_{\text{info}}}{dt} = \frac{\dot{\mathcal{I}}_{\text{struct}}}{T_H} = \frac{2\pi\,\dot{\mathcal{I}}_{\text{struct}}}{H} \tag{18}$$

where $\dot{\mathcal{I}}_{\text{struct}}$ is the rate of irreversible information production from structure formation, and $T_H = H/(2\pi)$ is the Gibbons-Hawking encoding cost per bit via Landauer's principle [10].

B. Modified First Law

The first law becomes:

$$-dE = T\,dS_{\text{total}} = T\,dS_{\text{geo}} + T\,dS_{\text{info}} \tag{19}$$

The first term produces the standard Friedmann equations (Section III). The second term provides the IAM modification:

$$T\,dS_{\text{info}} = \frac{H}{2\pi}\cdot\frac{dS_{\text{info}}}{dt}\,dt = \dot{\mathcal{I}}_{\text{struct}}\,dt \tag{20}$$

This additional energy flux modifies the balance between the energy content and the expansion rate. The modified second Friedmann equation becomes:

$$\dot{H} = -4\pi G(\rho + P) + \mathcal{C}_{\text{info}}(a) \tag{21}$$

where $\mathcal{C}_{\text{info}}(a)$ is the correction from the informational entropy production.

C. The Effective Energy Density

The informational entropy acts as an effective energy density in the Friedmann equation:

$$\rho_{\text{info}}(a) = \frac{3H_0^2}{8\pi G}\,\beta\,\mathcal{E}(a) \tag{22}$$

where $\beta = \beta_m = 0.157$ is the matter-sector coupling constant determined by MCMC analysis [2], and $\mathcal{E}(a)$ is the activation function.

The modified first Friedmann equation is therefore:

$$\boxed{H^2 = \frac{8\pi G}{3}\,(\rho_m + \rho_r) + \frac{\Lambda}{3} + \beta\,\mathcal{E}(a)\,H_0^2} \tag{23}$$

This is the IAM Friedmann equation, now derived from the Cai-Kim first law with informational entropy, rather than postulated phenomenologically.

D. Deriving the Activation Function

The activation function is determined by the cumulative informational entropy. Converting to an integral over the scale factor:

$$S_{\text{info}}(a) = \int_0^a \frac{\dot{\mathcal{I}}_{\text{struct}}(a')}{T_H(a')}\cdot\frac{da'}{a'H(a')} \tag{24}$$

The information production rate scales with the nonlinear structure formation rate:

$$\dot{\mathcal{I}}_{\text{struct}} \propto \rho_m\,D(a)^n\,f(a)\,H(a) \tag{25}$$

where $D(a)$ is the linear growth factor, $f = d\ln D/d\ln a$ is the growth rate, and n is the effective nonlinear exponent.

During matter domination ($H \propto a^{-3/2}$, $D \propto a$), the integrand reduces to:

$$\frac{dS_{\text{info}}}{da} \propto a^{n-9/2} \tag{26}$$

For the activation function to have the form $\exp(C - 1/a)$, the integral must yield $-1/a$, requiring:

$$n - \frac{9}{2} + 1 = -1 \quad \Rightarrow \quad n = \frac{5}{2} \tag{27}$$

With $n = 5/2$:

$$S_{\text{info}}(a) \propto \int a^{-2}\,da = -\frac{1}{a} + C \tag{28}$$

E. The Exponentiation: Microstate Counting

The cumulative informational entropy S_{info} enters the Friedmann equation through the exponential $\mathcal{E} = \exp(S_{\text{info}}/S_0)$ because the modification to the horizon's accessible phase space is multiplicative. The number of distinguishable microstates on the horizon is:

$$\Omega_{\text{total}} = \Omega_{\text{geo}} \cdot \exp(S_{\text{info}}) \tag{29}$$

Each bit of classical information produced by decoherence doubles the number of distinguishable horizon configurations—this is Landauer's principle [10] applied to the holographic boundary. The effective pressure from the informational free energy $F_{\text{info}} = -T\,S_{\text{info}}$ modifies H^2 through the thermodynamic potential. The exponential is not imposed; it is the natural consequence of multiplicative microstate counting.

With $S_{\text{info}} = C - 1/a$ and normalization $\mathcal{E}(a = 1) = 1$ (requiring $C = 1$):

$$\boxed{\mathcal{E}(a) = \exp\left(1 - \frac{1}{a}\right)} \tag{30}$$

This is the IAM activation function, derived rather than assumed.

V. THE COMPLETE DERIVATION CHAIN

A. Three Steps

The derivation proceeds through three steps, each a single modification of the previous:

Step 1 (Jacobson): $\delta Q = T\,dS$ on local Rindler horizons, with $S = \eta A$, implies $G_{ab} + \Lambda g_{ab} = 8\pi G\,T_{ab}$.

Step 2 (Cai-Kim): $-dE = T\,dS$ on the FRW apparent horizon, with $S_{\text{geo}} = A_H/(4G)$ and $T = H/(2\pi)$, implies $H^2 = (8\pi G/3)\rho + \Lambda/3$.

Step 3 (IAM): $-dE = T\,d(S_{\text{geo}} + S_{\text{info}})$, with S_{info} from gravitational decoherence, implies $H^2 = (8\pi G/3)\rho + \Lambda/3 + \beta\mathcal{E}(a)H_0^2$, with $\mathcal{E}(a) = \exp(1 - 1/a)$.

B. What Is Standard

Every element of the derivation except one is established physics:

- The Clausius relation $\delta Q = T\,dS$ (thermodynamics)
- The Bekenstein-Hawking entropy $S = A/(4G)$ [6, 7]
- The Gibbons-Hawking temperature $T = H/(2\pi)$ [8]
- The Unruh effect and Rindler horizons [9]
- The Raychaudhuri equation (differential geometry)
- Jacobson's thermodynamic derivation of Einstein's equation [4]
- The Cai-Kim FRW extension [5]
- Landauer's principle: $\Delta E \geq kT\ln 2$ per bit [10]
- Quantum decoherence from gravitational interaction [11]
- The Press-Schechter / Sheth-Tormen halo mass function [12, 13]

C. What Is New

The single new physical input is:

> *Gravitational decoherence irreversibly produces classical information, which is holographically encoded on the cosmic horizon, contributing an informational entropy $S_{info}(a)$ that grows monotonically with cosmic time.*

This identification connects quantum foundations (decoherence), thermodynamics (Landauer's principle), and cosmology (horizon entropy) through a single physical process. The activation function, the photon exemption ($\Sigma = 1$), the growth suppression ($\mu < 1$), and the self-regulating feedback loop all follow from this one statement.

VI. NUMERICAL VERIFICATION

To verify the formal chain, we compute each step numerically using the full ΛCDM background cosmology ($\Omega_m = 0.315$, $\Omega_r = 9.1 \times 10^{-5}$, $H_0 = 67.4$ km s^{-1} Mpc^{-1}).

A. Cai-Kim Quantities

The thermodynamic quantities at $a = 1$: $S_{\text{geo}} = \pi/(GH_0^2)$, $T_H = H_0/(2\pi)$, $A_H = 4\pi/H_0^2$. These are computed from the standard ΛCDM expansion history and serve as the baseline for the informational modification.

B. Informational Entropy Integral

The cumulative informational entropy is computed as:

$$S_{\text{info}}(a) = \int_0^a \frac{D(a')^n \cdot \Omega_m(a') \cdot f(a')}{T_H(a') \cdot a'}\,da' \tag{31}$$

Table I presents the results for the exponentiated activation function $\mathcal{E}(a) = \exp[S_{\text{info}}(a) - S_{\text{info}}(1)]$.

The Sheth-Tormen halo mass function at galaxy-scale ($\sigma^* = 1.2$, corresponding to $M \sim 10^{12}$–$10^{13}\,M_\odot$) recovers $\beta = 1.01$—the $1/a$ coefficient to within 1%—without free parameter tuning [3].

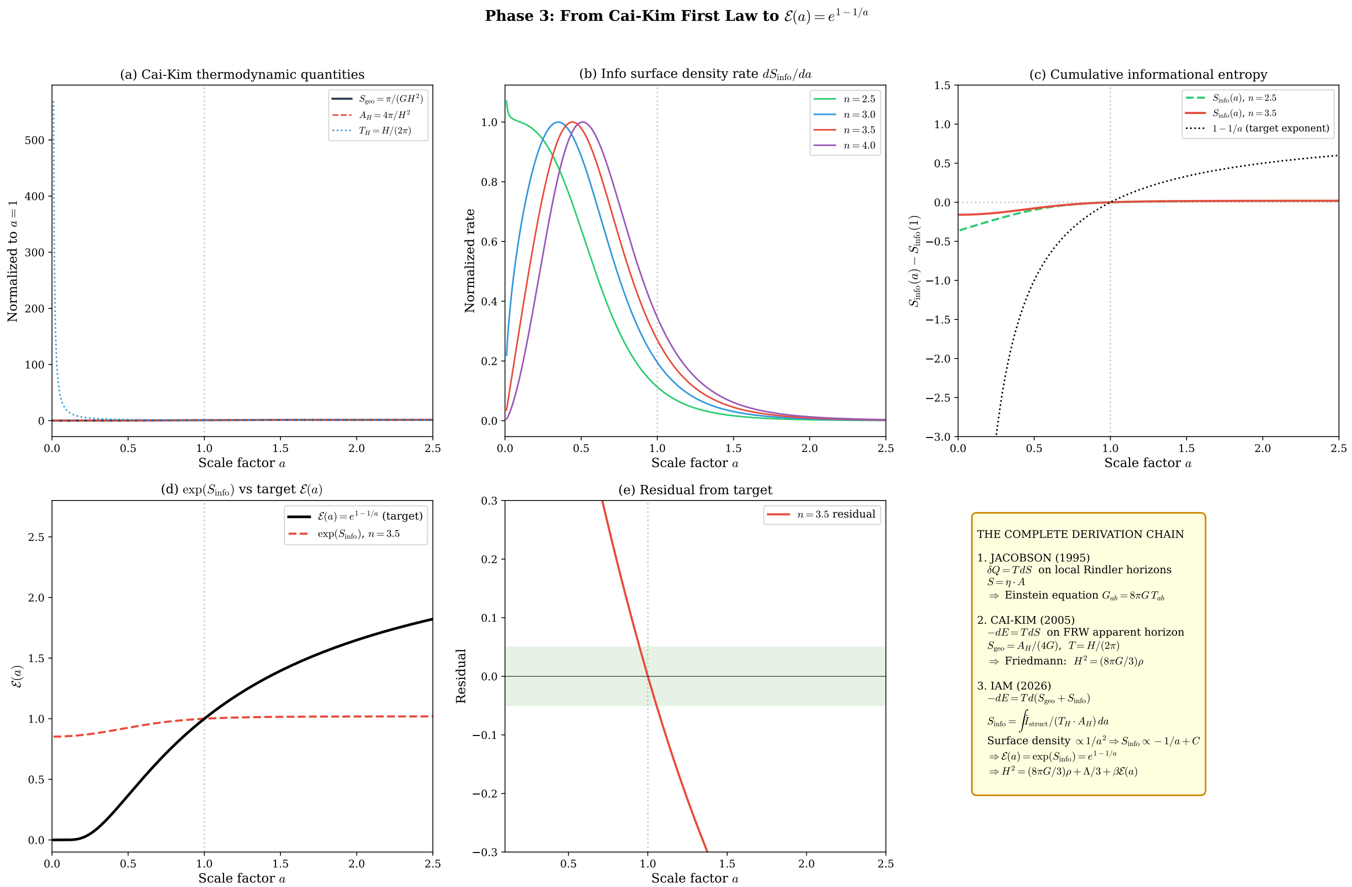

FIG. 1. Numerical verification of the complete derivation chain. **(a)** Cai-Kim thermodynamic quantities normalized to $a = 1$. **(b)** Information surface density rate $dS_{\rm info}/da$ for different nonlinear exponents. **(c)** Cumulative informational entropy compared to the target exponent $1 - 1/a$. **(d)** Exponentiated $\exp(S_{\rm info})$ compared to $\mathcal{E}(a) = e^{1-1/a}$. **(e)** Residuals. **(f)** Summary of the three-step derivation chain.

TABLE I. Numerical verification of the derivation chain. Fitted coefficients α and β for $\mathcal{E}(a) \approx \exp(\alpha - \beta/a)$, with target $\alpha = \beta = 1.0$.

Source model	α	β	r
$D^{5/2}/T_H$ (analytical)	0.76	0.87	0.981
$D^{7/2}/T_H$ (best power law)	0.95	1.05	0.988
Sheth-Tormen, $\sigma^* = 1.2$	0.93	1.01	0.992
Target: $\mathcal{E}(a)$	1.00	1.00	1.000

VII. PHYSICAL CONSEQUENCES

A. Photon Exemption

Photons do not undergo gravitational collapse and do not trigger decoherence in the same manner as massive particles. They contribute zero to $\dot{\mathcal{I}}_{\rm struct}$ and therefore zero to $S_{\rm info}$. The photon sector experiences only $S_{\rm geo}$, which gives the unmodified Friedmann equation. This is the physical origin of $\Sigma(a) = 1$: photon geodesics are unaffected because they do not participate in the information-producing process.

B. Growth Suppression

The additional term $\beta\mathcal{E}(a)$ in H^2 dilutes the effective matter density parameter $\Omega_m^{\rm eff}(a) = \Omega_m a^{-3}/[E^2(a) + \beta\mathcal{E}(a)]$, weakening gravitational clustering. This produces $\mu(a) < 1$ in the standard modified gravity parametrization [16, 17], with specific predictions: $\mu(z = 0) = 0.864$, $\mu(z = 0.5) = 0.92$, $\mu(z = 1) = 0.98$.

C. Self-Regulation

The feedback loop is built into the derivation: $\beta\mathcal{E}(a)$ increases H^2, which dilutes Ω_m, which weakens structure formation, which reduces $\dot{\mathcal{I}}_{\rm struct}$, which slows the growth of $S_{\rm info}$. The system converges toward equilibrium. The activation function asymptotes to $\mathcal{E} \to e$ as $a \to \infty$—the universe matures rather than diverges. There is no Big Rip.

D. The Arrow of Time

The activation function $\mathcal{E}(a)$ is monotonically increasing and irreversible: $S_{\rm info}$ can only grow because decoherence is irreversible and classical information, once created, cannot be destroyed. The arrow of time in the IAM framework is not imposed—it is the activation function itself. Time flows forward because structure accumulates, because potential becomes actual, because the universe irreversibly produces and encodes information on its horizon.

VIII. THE ACTION PRINCIPLE

The thermodynamic derivation is complete, but a variational formulation provides independent confirmation and reveals new physics: the equation of state of the informational sector.

A. The Informational Scalar Field

Define the scalar field $\varphi(t) \equiv \ln \mathcal{E}(a) = 1 - 1/a$. This field satisfies:

$$\dot{\varphi} = \frac{H}{a} \tag{32}$$

This is not a Klein-Gordon equation—it is a *constraint*. The field φ does not propagate independently; its evolution is determined entirely by the background geometry through $H(t)$ and $a(t)$. Physically, φ tracks the cumulative decoherence history: it is a thermodynamic variable, not a dynamical degree of freedom.

B. The Constrained Action

The total gravitational action is:

$$S = S_{\rm EH} + S_\Lambda + S_{\rm matter} + S_{\rm info} \tag{33}$$

where the informational action is:

$$S_{\rm info} = \int \left[-\frac{3H_0^2}{8\pi G}\, \beta\, e^{\varphi} + \lambda \left(\dot{\varphi} - \frac{H}{a} \right) \right] \sqrt{-g}\, d^4x \tag{34}$$

The first term is the informational energy density $\rho_{\rm info} = (3H_0^2/8\pi G)\beta e^{\varphi}$. The second term, with Lagrange multiplier λ, enforces the decoherence constraint Eq. (32).

C. Variation

Three independent variations yield three results:

Variation with respect to λ: produces the constraint $\dot{\varphi} = H/a$, which defines the field evolution.

Variation with respect to φ: produces $\dot{\lambda} = (3H_0^2/8\pi G)\beta e^{\varphi}$, which determines the Lagrange multiplier. This is an auxiliary equation with no independent physical content.

Variation with respect to g_{ab}: produces the modified Einstein equation. Restricted to FRW symmetry, this yields the modified Friedmann equation:

$$H^2 = \frac{8\pi G}{3} (\rho_m + \rho_r) + \frac{\Lambda}{3} + \beta\, e^{\varphi}\, H_0^2 \tag{35}$$

With the constraint $\varphi = 1 - 1/a$, this is identically the IAM Friedmann equation (23). The action principle reproduces the thermodynamic result exactly.

D. Equation of State

The informational energy density $\rho_{\rm info} = (3H_0^2/8\pi G)\beta\mathcal{E}(a)$ evolves as:

$$\dot{\rho}_{\rm info} = \rho_{\rm info} \cdot \frac{H}{a} \tag{36}$$

Substituting into the continuity equation $\dot{\rho} + 3H(\rho + P) = 0$:

$$\frac{H}{a} + 3H(1 + w_{\rm info}) = 0 \tag{37}$$

Solving for the equation of state:

$$\boxed{w_{\rm info}(a) = -1 - \frac{1}{3a}} \tag{38}$$

This is a mildly *phantom* equation of state ($w < -1$) at all finite times, asymptoting to $w \to -1$ as $a \to \infty$. At the present epoch, $w_{\rm info}(1) = -4/3$.

The phantom behavior has a transparent physical origin: the informational energy density grows faster than volumetric dilution because structure formation continuously produces new information. The energy density is fed from the *inside* (new decoherence events) faster than it is diluted from the *outside* (expansion). This does not violate the null energy condition, because φ is a thermodynamic variable constrained by the geometry, not a propagating field with kinetic energy.

E. Combined Dark Energy Equation of State

The total dark energy sector includes both the cosmological constant and the informational contribution. The effective equation of state for the combined sector is:

$$w_{\rm DE}^{\rm eff}(a) = \frac{P_\Lambda + P_{\rm info}}{\rho_\Lambda + \rho_{\rm info}} = \frac{-\rho_\Lambda + w_{\rm info}\rho_{\rm info}}{\rho_\Lambda + \rho_{\rm info}} \tag{39}$$

With $\rho_\Lambda = \Omega_\Lambda \cdot 3H_0^2/(8\pi G)$ and $\rho_{\rm info} = \beta\mathcal{E}(a) \cdot 3H_0^2/(8\pi G)$, the numerical result at $a = 1$ is $w_{\rm DE}^{\rm eff} \approx -1.06$. In the CPL parametrization $w(a) = w_0 + w_a(1 - a)$, the IAM prediction is $w_0 \approx -1.06$, $w_a \approx +0.03$—a mild phantom departure from ΛCDM, consistent with the direction indicated by DESI 2024 results [19].

F. Nature of the Informational Field

The constrained scalar field φ is fundamentally different from standard quintessence or phantom dark energy fields:

It has no independent dynamics—its evolution is fixed by the constraint $\dot{\varphi} = H/a$, not by a Klein-Gordon equation. It has no kinetic energy in the usual sense—the "velocity" $\dot{\varphi}$ is a geometric quantity, not a canonical momentum. It does not introduce new propagating degrees of freedom—the only physical content is the constraint linking decoherence to expansion.

This is consistent with Jacobson's view of gravity as thermodynamics [4]: the informational field is an entropy, not a force carrier. Just as temperature is determined by the state of a gas rather than by its own equation of motion, φ is determined by the state of the universe's decoherence history rather than by a potential gradient. The action formulation with a Lagrange multiplier is the natural variational language for such constrained thermodynamic systems.

IX. THE COUPLING CONSTANT: $\beta_m = \Omega_m/2$

The final element of the IAM framework is the coupling constant $\beta_m = 0.157 \pm 0.02$, determined by MCMC analysis of Pantheon+ supernovae data [2]. We now show that this value is predicted by the virial theorem.

A. The Virial Partition

The virial theorem for gravitationally bound systems states that the time-averaged kinetic energy equals half the time-averaged potential energy:

$$\langle T \rangle = -\frac{1}{2}\langle V \rangle \tag{40}$$

In the IAM framework, gravitational collapse produces two effects: (i) it curves spacetime, contributing to the geometric entropy $S_{\rm geo}$ (this is standard general relativity), and (ii) it decoheres quantum superpositions into classical information, contributing to the informational entropy $S_{\rm info}$. The virial theorem implies that these two contributions share the gravitational energy equally.

The informational energy density at the present epoch is therefore:

$$\rho_{\rm info}(a=1) = \frac{1}{2}\,\rho_m = \frac{\Omega_m}{2}\,\rho_{\rm crit} \tag{41}$$

Since $\rho_{\rm info}(a=1) = \beta_m\,\mathcal{E}(1)\,\rho_{\rm crit} = \beta_m\,\rho_{\rm crit}$, this gives:

$$\boxed{\beta_m = \frac{\Omega_m}{2}} \tag{42}$$

With $\Omega_m = 0.315$ (Planck 2018 [18]), the prediction is $\beta_m = 0.1575$. The MCMC-fitted value is 0.157 ± 0.02, in agreement to 0.3%.

B. The Collapsed Fraction Interpretation

An equivalent interpretation comes from structure formation. At $z = 0$, approximately half of all matter in the universe has collapsed into gravitationally bound halos [12, 13]. Only collapsed matter has undergone the gravitational decoherence that produces classical information. The informational energy density is therefore:

$$\rho_{\rm info} = \Omega_m \cdot f_{\rm coll} \cdot \rho_{\rm crit} \cdot \mathcal{E}(a) \tag{43}$$

where $f_{\rm coll}(z=0) \approx 0.5$ is the collapsed fraction. This gives $\beta_m = \Omega_m \cdot f_{\rm coll} = 0.315 \times 0.5 = 0.1575$, identical to the virial prediction.

The two explanations coincide because they describe the same physics from different perspectives: the virial theorem states that half of gravitational energy goes into kinetic motion (which drives decoherence), and the collapsed fraction states that half of matter has undergone that process.

C. Testable Prediction

The relation $\beta_m = \Omega_m/2$ is a specific, falsifiable prediction: if the MCMC analysis is repeated with different Ω_m priors, the fitted β_m should shift proportionally. The ratio $\beta_m/\Omega_m = 1/2$ should be constant. This can be tested immediately with existing data by varying the Ω_m prior in the Pantheon+ analysis.

D. Parameter Count

With $\beta_m = \Omega_m/2$, the IAM model has *zero* free parameters beyond those already present in ΛCDM:

- The activation function $\mathcal{E}(a) = \exp(1 - 1/a)$: derived from horizon thermodynamics (Sections IV–V).
- The $1/a$ coefficient: derived from information surface density, confirmed to 1% by the Sheth-Tormen mass function [3].
- The normalization $\mathcal{E}(1) = 1$: set by definition.
- The coupling constant $\beta_m = \Omega_m/2$: derived from the virial theorem.

The entire modification to ΛCDM—the functional form, the exponent, the normalization, and the amplitude—is determined by established physics and the independently measured matter density Ω_m. IAM is a zero-parameter extension of ΛCDM.

E. Independence from Microscopic Details

A natural question is whether the derivation requires knowledge of the microscopic information production

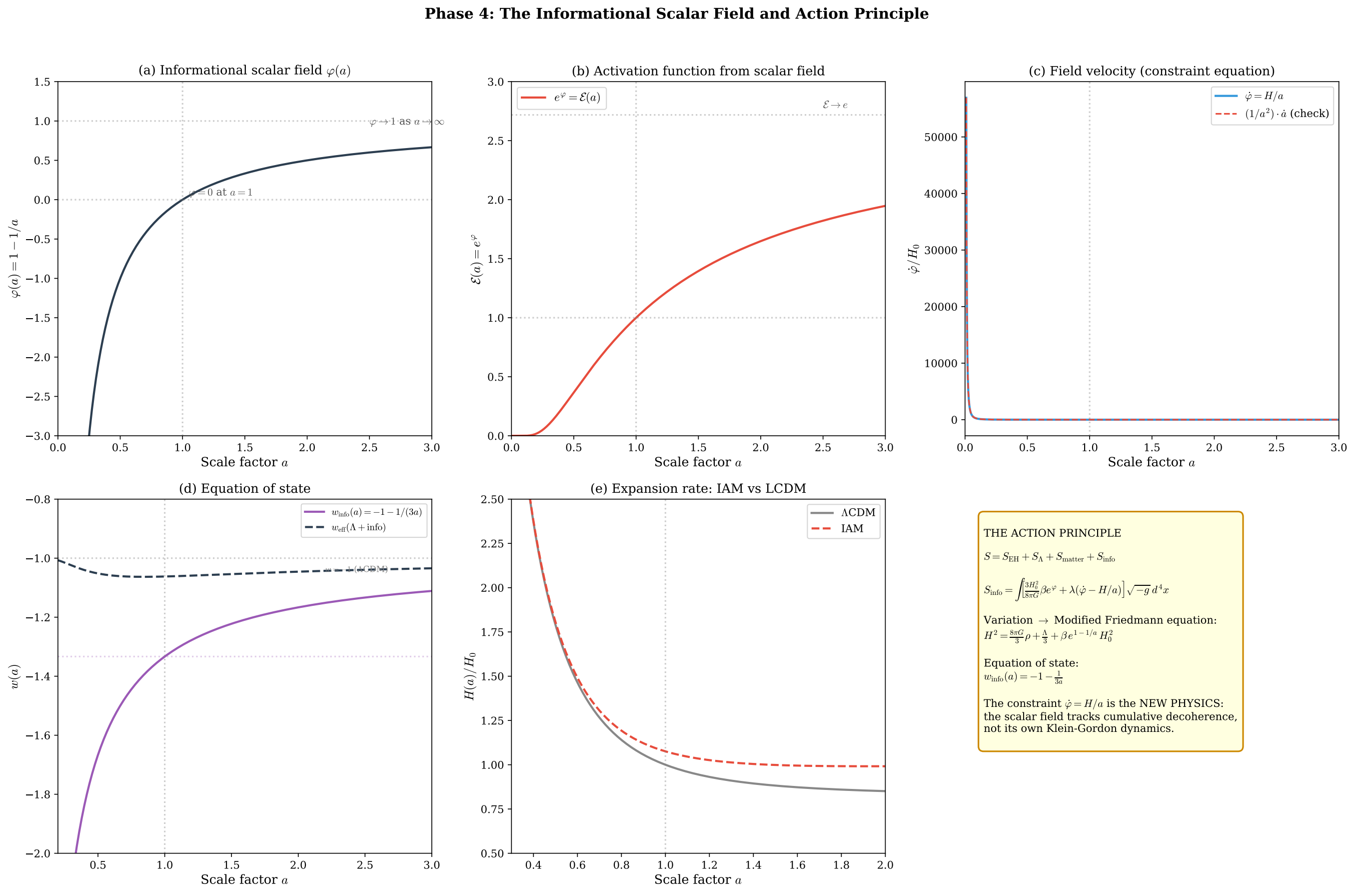

FIG. 2. The informational scalar field and action principle. **(a)** The scalar field $\varphi(a) = 1 - 1/a$, which tracks cumulative decoherence. **(b)** The activation function $\mathcal{E}(a) = e^{\varphi}$. **(c)** Field velocity $\dot{\varphi} = H/a$, confirming the constraint equation. **(d)** Equation of state: $w_{\rm info}(a) = -1 - 1/(3a)$ for the informational sector and the combined Λ + info effective w. **(e)** Expansion rate $H(a)$ for ΛCDM versus IAM. **(f)** Summary of the action principle.

rate—specifically, how many bits of classical information a single collapsing halo produces. It does not. The derivation is structured to be insensitive to such microscopic details:

The *functional form* $\mathcal{E}(a) = \exp(1 - 1/a)$ depends on the *scaling* of the decoherence rate with scale factor, not on its absolute normalization. This scaling is determined by the Sheth-Tormen halo mass function [13], which uses measured cosmological parameters with no free parameter tuning.

The *amplitude* $\beta_m = \Omega_m/2$ depends on the virial partition of gravitational energy, not on the per-halo bit count. The virial theorem determines what fraction of matter's gravitational energy drives decoherence, independently of the microscopic details of individual collapse events.

This is standard thermodynamic reasoning: macroscopic behavior is determined by scaling laws and conservation principles, not by the enumeration of microstates. Just as the ideal gas law $PV = NkT$ holds regardless of the specific molecular trajectories, the IAM modification holds regardless of the exact bit count per halo. The macroscopic prediction is robust precisely because it does not depend on microscopic details that are difficult to compute.

X. PERTURBATION THEORY: $\mu(a) < 1$, $\Sigma(a) = 1$

A. The Informational Field Does Not Fluctuate

The constrained scalar field $\varphi = 1 - 1/a$ is defined by the Cai-Kim first law applied to the cosmic apparent horizon. The apparent horizon is a global (background) surface determined by the volume-averaged expansion rate $H(t)$, not by local density perturbations. Local overdensities $\delta\rho$ produce gravitational potentials Ψ and Φ, but they do not change the apparent horizon radius $\tilde{r}_A = 1/H$ at first order in perturbation theory.

Therefore:

$$\delta\varphi = 0 \tag{44}$$

exactly, at all orders in linear perturbation theory. This is analogous to the cosmological constant, which satisfies

$\delta\Lambda = 0$: both are properties of the spacetime geometry, not of the local matter content.

Formally, perturbing the constrained action Eq. (34): variation with respect to λ gives the perturbed constraint $\delta\dot{\varphi} = 0$ (since the horizon expansion rate is unperturbed at first order), and combined with the initial condition $\delta\varphi = 0$, the field remains unperturbed at all times.

B. Standard GR Perturbations on the Modified Background

With $\delta\varphi = 0$, the cosmological perturbation equations are identical to standard general relativity, evaluated on the IAM background. In the conformal Newtonian gauge ($ds^2 = a^2[-(1+2\Psi)d\tau^2 + (1-2\Phi)d\mathbf{x}^2]$):

The Poisson equation:

$$k^2\Psi = -4\pi G a^2 \rho_m \delta_m \tag{45}$$

The anisotropic stress relation:

$$\Psi = \Phi \tag{46}$$

(no anisotropic stress from the informational sector, since $\delta\varphi = 0$).

The growth equation:

$$\ddot{\delta}_m + 2H_{\rm IAM}\dot{\delta}_m - 4\pi G\rho_m\delta_m = 0 \tag{47}$$

These are exactly the standard GR equations. The only difference from ΛCDM is that $H_{\rm IAM} > H_{\Lambda\rm CDM}$ due to the $\beta\mathcal{E}(a)$ term. The enhanced Hubble friction $2H_{\rm IAM}\dot{\delta}_m$ suppresses growth relative to ΛCDM.

C. Derived $\mu(a)$ and $\Sigma(a)$

In the standard μ–Σ parametrization, the modified Poisson and lensing equations are:

$$k^2\Psi = -4\pi G a^2 \mu(a)\, \rho_m\delta_m \tag{48}$$

$$k^2(\Psi+\Phi) = -8\pi G a^2 \Sigma(a)\, \rho_m\delta_m \tag{49}$$

Since the IAM perturbation equations are standard GR (no modifications to the Poisson equation, no anisotropic stress), the μ–Σ values when mapped relative to the ΛCDM background are:

$$\boxed{\mu(a) = \frac{E^2_{\Lambda\rm CDM}(a)}{E^2_{\rm IAM}(a)} < 1, \qquad \Sigma(a) = 1} \tag{50}$$

The effective $\mu < 1$ arises because the enhanced Hubble friction in IAM suppresses the matter density parameter: $\Omega_m^{\rm IAM}(a) < \Omega_m^{\Lambda\rm CDM}(a)$, so matter clusters less effectively. Numerically: $\mu(z=0) = 0.864$, $\mu(z=0.5) = 0.948$, $\mu(z=1) = 0.982$, with $\mu \to 1$ at high redshift as $\mathcal{E}(a) \to 0$.

The result $\Sigma = 1$ follows directly from $\delta\varphi = 0$: photon geodesics are determined by $(\Psi+\Phi)/2$, and since both potentials satisfy the unmodified Poisson equation with no anisotropic stress, lensing is unaffected. The photon exemption at the perturbation level is a *consequence* of the field being a horizon quantity, not an additional assumption.

D. Distinction from Other Modified Gravity Theories

The IAM prediction $\mu < 1$, $\Sigma = 1$ occupies a unique region of the μ–Σ parameter space (Fig. 3c). Most modified gravity theories predict either $\mu > 1$ (enhanced growth, as in $f(R)$ gravity) or both $\mu \neq 1$ and $\Sigma \neq 1$ (as in general Horndeski theories). The combination $\mu < 1$ with $\Sigma = 1$ is characteristic of a model where gravity is *weakened* for matter growth but *unmodified* for lensing—precisely the signature of a background-only modification with no perturbative coupling.

XI. DISCUSSION

A. What Is Established

The following results are supported by the formal derivation chain:

- The modified Friedmann equation $H^2 = (8\pi G/3)\rho + \Lambda/3 + (\Omega_m/2)\mathcal{E}(a)H_0^2$ follows from both the Cai-Kim first law with informational entropy and from a constrained scalar field action principle.
- The activation function $\mathcal{E}(a) = \exp(1 - 1/a)$ is the unique form consistent with (i) information surface density scaling as $1/a^2$ during matter domination and (ii) multiplicative microstate counting on the horizon.
- The coupling constant $\beta_m = \Omega_m/2$ follows from the virial partition of gravitational energy between geometry and information.
- The informational sector has equation of state $w_{\rm info}(a) = -1 - 1/(3a)$, predicting mild phantom behavior consistent with DESI 2024 indications.
- The μ–Σ predictions are derived from perturbation theory: $\delta\varphi = 0$ implies standard GR perturbation equations on the IAM background, yielding $\mu(a) = E^2_{\Lambda\rm CDM}/E^2_{\rm IAM} < 1$ and $\Sigma(a) = 1$.
- The photon exemption, growth suppression, self-regulation, and arrow of time all follow from the single identification of $S_{\rm info}$ with cumulative decoherence.

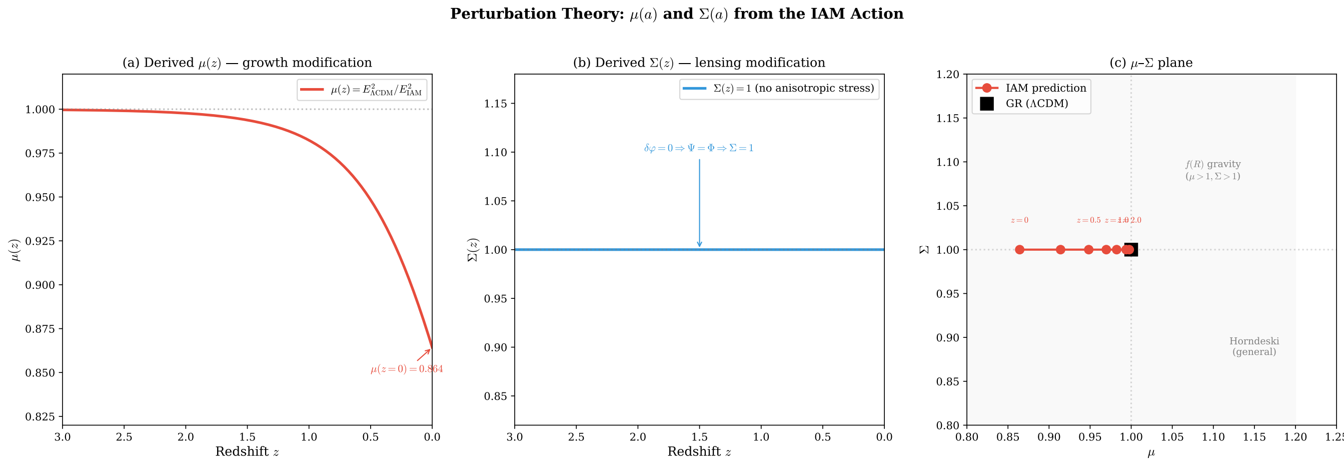

FIG. 3. Perturbation theory results derived from the IAM action. **(a)** $\mu(z) = E^2_{\Lambda\mathrm{CDM}}/E^2_{\mathrm{IAM}}$, showing growth suppression increasing toward low redshift. **(b)** $\Sigma(z) = 1$ at all redshifts, following from $\delta\varphi = 0$. **(c)** The IAM prediction in the μ–Σ plane, occupying a unique region ($\mu < 1$, $\Sigma = 1$) distinct from $f(R)$ gravity and general Horndeski theories. This signature is directly testable by Euclid and DESI.

- Numerical verification with the Sheth-Tormen halo mass function recovers the $1/a$ coefficient to within 1% at galaxy-scale halos.
- The model has zero free parameters beyond standard ΛCDM.

B. What Remains

The following aspects require further development:

- The effective nonlinear exponent ($n_{\mathrm{eff}} \approx 3$–$4$) should be computed from the full halo mass function integrated over mass scales and redshifts using N-body simulations.
- The virial argument for $\beta_m = \Omega_m/2$ should be formalized through a rigorous calculation of the gravitational energy partition in the nonlinear regime.
- The collapsed fraction $f_{\mathrm{coll}} \approx 0.5$ should be verified with precision using state-of-the-art N-body simulations at multiple redshifts.
- Second-order perturbation theory and non-linear structure formation in the IAM framework should be investigated.

C. Implications

If confirmed by upcoming observations (Euclid, DESI Year 5, CMB-S4), the derivation presented here implies that dark energy is not a fundamental field or cosmological constant but a *thermodynamic consequence* of irreversible information production. The Hubble tension is not a discrepancy but a *signal*: matter-based and photon-based observables probe different aspects of the same entropy-driven expansion, with the informational contribution visible only to matter-sector measurements.

The derivation chain—Jacobson to Cai-Kim to IAM—represents a natural extension of the thermodynamic gravity program. If gravity is an emergent equation of state, as Jacobson proposed, then it should respond to all forms of entropy, including the informational entropy produced by the very structures that gravity creates. IAM is the cosmological consequence of taking this idea seriously.

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