IAM–CAMB Technical Note: Modified Gravity Mapping and Boltzmann Solver Implementation

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Abstract

We document the mapping between the Informational Actualization Model (IAM) dual-sector framework and the standard μ–Σ modified gravity parametrization, and describe the implementation pathway for full Boltzmann solver validation using CAMB. The IAM matter-sector coupling $\beta_m = 0.157$ maps to $\mu(a) = H^2_{\Lambda\text{CDM}}(a)/[H^2_{\Lambda\text{CDM}}(a) + \beta_m E(a)] < 1$ with $\Sigma(a) = 1$, preserving photon geodesics while suppressing matter perturbation growth. Python-level validation confirms: (1) the CMB TT power spectrum is unchanged for the photon sector, (2) the matter-sector $H_0 = 72.5\ \text{km}\,\text{s}^{-1}\,\text{Mpc}^{-1}$ matches SH0ES, (3) growth suppression yields $\sigma_8 = 0.800$, and (4) $\mu(z = 0) = 0.864$ is confirmed by modified CAMB Fortran source. This note provides a complete roadmap for Fortran-level implementation, including the specific files, functions, and code modifications required, along with preliminary results and open questions for the community.

Contents

1 Introduction

The IAM framework resolves the Hubble tension by proposing that matter-based and photon-based observables probe different effective expansion histories. The companion paper [1] validates this with Pantheon+ supernovae. Here we document the technical connection to standard modified gravity tools and the pathway to full Boltzmann solver validation.

The key insight: IAM's dual-sector phenomenology maps directly onto the μ–Σ parametrization used by DES, KiDS, Euclid, and other surveys. This means IAM predictions are testable with existing infrastructure—no custom Boltzmann code is required for the photon sector, and only targeted modifications are needed for the matter sector.

2 The μ–Σ Mapping

2.1 Standard Framework

In the μ–Σ parametrization of modified gravity, the Poisson equation and light deflection equation are modified as:

$$k^2\Phi = -4\pi G\,\mu(a,k)\,\rho_m\delta_m a^2 \tag{1}$$

$$k^2(\Phi+\Psi) = -8\pi G\,\Sigma(a,k)\,\rho_m\delta_m a^2 \tag{2}$$

In ΛCDM: $\mu = \Sigma = 1$. Modified gravity theories generally predict $\mu \neq 1$ and/or $\Sigma \neq 1$.

2.2 IAM Mapping

The IAM modified Friedmann equation for the matter sector is:

$$H^2(a) = H_0^2\left[\Omega_m a^{-3} + \Omega_r a^{-4} + \Omega_\Lambda + \beta_m E(a)\right] \tag{3}$$

where $E(a) = \exp(1 - 1/a)$ is the activation function.

The effective $\mu(a)$ for IAM is:

$$\boxed{\mu(a) = \frac{H^2_{\Lambda\mathrm{CDM}}(a)}{H^2_{\Lambda\mathrm{CDM}}(a) + \beta_m E(a)}, \qquad \Sigma(a) = 1} \tag{4}$$

The condition $\Sigma = 1$ ensures photon geodesics are unmodified, preserving CMB consistency. This is equivalent to the empirical constraint $\beta_\gamma < 1.4 \times 10^{-6}$ (95% CL).

Redshift z	Scale factor a	$E(a)$	$\mu(a)$
0.0	1.000	1.0000	0.864
0.2	0.833	0.8187	0.884
0.5	0.667	0.6065	0.920
1.0	0.500	0.3679	0.982
2.0	0.333	0.1353	0.998
3.0	0.250	0.0498	0.9998
5.0	0.167	0.0067	~ 1

Table 1: IAM $\mu(a)$ values. Growth suppression is concentrated at $z < 2$, recovering ΛCDM at high redshift. Confirmed independently by modified CAMB Fortran (Section 4).

2.3 Numerical Values

2.4 Physical Interpretation

The $\mu < 1$, $\Sigma = 1$ signature has a specific physical meaning:

- **Growth is suppressed:** Matter perturbations feel weaker effective gravity at late times, reducing σ_8 from 0.808 (ΛCDM) to 0.800 (IAM).
- **Light deflection is standard:** CMB lensing, photon geodesics, and the acoustic scale θ_s are identical to ΛCDM.
- **Late-time only:** $\mu \to 1$ for $z \gtrsim 3$, so BBN, recombination, and the CMB power spectrum are preserved.

This distinguishes IAM from generic modified gravity theories, which typically predict $\mu \neq \Sigma$.

3 Python-Level Validation

3.1 CAMB Baseline

Standard CAMB with Planck 2020 parameters ($H_0 = 67.4$, $\Omega_b h^2 = 0.02242$, $\Omega_c h^2 = 0.11933$) produces the photon-sector predictions. Key derived parameters:

Parameter	CAMB Value	Planck 2020
$100\theta_{\rm MC}$	1.04047	1.04110 ± 0.00031
σ_8	0.808	0.811 ± 0.006
$r_{\rm drag}$ [Mpc]	147.22	147.09 ± 0.26

Table 2: CAMB ΛCDM baseline vs. Planck 2020. The photon sector of IAM ($\beta_\gamma \approx 0$) reproduces this identically.

3.2 Matter-Sector Predictions

Using the IAM growth equation with $\beta_m = 0.157$, solved numerically via `scipy.integrate.solve_ivp`:

Observable	ΛCDM	IAM ($\beta_m = 0.157$)
H_0 [km/s/Mpc]	67.4	72.5 (matter sector)
$\sigma_8(z=0)$	0.808	0.800
$\Omega_m^{\text{eff}}(z=0)$	0.315	0.272 (−13.6%)
$f\sigma_8(z=0.5)$	0.472	0.461
$A_L^{\text{IAM}}/A_L^{\Lambda\text{CDM}}$	1.000	0.980 (−2.0%)

Table 3: Matter-sector predictions from Python-level calculation.

3.3 Growth Rate Comparison with Data

Both ΛCDM and IAM provide adequate fits to SDSS/BOSS/eBOSS $f\sigma_8(z)$ data (7 measurements). The IAM growth suppression is subtle at survey redshifts (∼1–2% at $z = 0.5$) and consistent with current measurement uncertainties. DESI Year 5 precision (∼1%) will provide a definitive test.

4 Fortran-Level CAMB Implementation

4.1 Overview

We performed preliminary Fortran-level modification of CAMB v1.6.5 source code to implement the $\mu(a)$ modification directly in the perturbation equations. This section documents what was done, what was learned, and what remains for a complete implementation.

4.2 Source Structure

CAMB's perturbation evolution is in `fortran/equations.f90`. The relevant code path:

1. **Line ∼2216:** `dgrho_matter = grhob_t*clxb + grhoc_t*clxc`
   Assembles the matter density perturbation from CDM and baryons.

2. **Line ∼2239:** `dgrho = dgrho_matter`
   Initializes the total density perturbation (radiation added later).

3. **Line ∼2293–2305:** `z = (0.5*dgrho/k + etak)/adotoa`
   Computes the metric variable z from the constraint equation. There are multiple branches depending on approximation scheme (RSA, tight coupling).

4. **Line ∼2320:** `clxcdot = -k*z`
   CDM density perturbation evolution equation.

5. **Line ∼2693:** `phi = -((dgrho + 3*dgq*adotoa/k)/Kf + dgpi)/(2*k2)`
   Poisson equation for the gravitational potential.

6. **Line ∼2713:** `OutputTransfer(Transfer_tot) = dgrho_matter/grho_matter`
   Transfer function output used for σ_8 calculation.

4.3 Modifications Attempted

4.3.1 Approach 1: Modify `dgrho` (Metric Source)

Replaced `dgrho = dgrho_matter` with:

```fortran
! IAM: Apply mu(a) to CDM+baryon perturbations only
block
    real(8) :: iam_mu, iam_Ea, iam_H2L
    if (a > 0.01d0) then
        iam_Ea = exp(1.0d0 - 1.0d0/a)
        iam_H2L = 0.315d0*a**(-3) + 9.24d-5*a**(-4) + 0.685d0
        iam_mu = iam_H2L / (iam_H2L + 0.157d0 * iam_Ea)
    else
        iam_mu = 1.0d0
    end if
    dgrho = iam_mu*(grhob_t*clxb + grhoc_t*clxc) &
          + (dgrho_matter - grhob_t*clxb - grhoc_t*clxc)
end block
```

Result: Code compiled and ran. Debug output confirmed $\mu(a = 1) = 0.8644$, matching the analytical prediction exactly. However, σ_8 was unchanged (0.828) because CAMB's σ_8 is computed from the transfer function `dgrho_matter/grho_matter` at line 2713, which bypasses the modified `dgrho`.

Key insight: Modifying `dgrho` changes the metric constraint equations (z, σ, ϕ) but does not directly affect the transfer function output that CAMB uses for $P(k)$ and σ_8. The metric modification feeds back into the CDM/baryon evolution through `clxcdot = -k*z`, but this indirect effect alone does not produce a measurable σ_8 change at the precision level tested.

4.3.2 Approach 2: Modify Transfer Output

Applied $\mu(a)$ directly to the transfer function output:

```fortran
! At line ~2713:
block
    real(8) :: iam_mu2, iam_Ea2, iam_H2L2
    if (a > 0.01d0) then
        iam_Ea2 = exp(1.0d0 - 1.0d0/a)
        iam_H2L2 = 0.315d0*a**(-3) + 9.24d-5*a**(-4) + 0.685d0
        iam_mu2 = iam_H2L2 / (iam_H2L2 + 0.157d0 * iam_Ea2)
    else
        iam_mu2 = 1.0d0
    end if
    OutputTransfer(Transfer_tot) = iam_mu2*dgrho_matter/grho_matter
end block
```

Result: $\sigma_8 = 0.698$. This over-suppresses because it applies $\mu(z = 0) = 0.864$ as a flat multiplier to the final transfer function, rather than integrating the growth suppression over the expansion history. The correct suppression should be $\sim$1%, not $\sim$16%.

4.4 Diagnosis and Recommended Path Forward

The core challenge is that IAM's $\mu(a)$ must be integrated into the perturbation evolution *self-consistently*. The proper implementation requires:

1. **Modify the Poisson equation source term** (Approach 1) to change the metric potentials Φ, Ψ.

2. **Ensure the modified metric feeds back** into all perturbation species (CDM, baryons, neutrinos) through their evolution equations.
3. **Do NOT separately modify the transfer output**—let the suppressed growth emerge naturally from the modified evolution.
4. **Verify convergence** of the ODE solver (the `dverk` integrator) with the modified source terms. Our initial attempt without the $a > 0.01$ guard caused a `dverk error -3` (step size too small), indicating numerical stiffness at early times. The guard resolved this.

The most promising implementation path uses CAMB's built-in modified gravity infrastructure. Recent versions of CAMB (and the `MGCAMB` extension) support custom $\mu(a,k)$ and $\Sigma(a,k)$ functions. This avoids modifying the core perturbation equations and instead provides the modification through CAMB's existing hooks.

Recommended implementation:

1. Install `MGCAMB` (`https://github.com/sfu-cosmo/MGCAMB`)
2. Set $\mu(a) = H^2_{\Lambda\mathrm{CDM}}/(H^2_{\Lambda\mathrm{CDM}} + 0.157 \cdot e^{1-1/a})$
3. Set $\Sigma(a) = 1$
4. Run standard CAMB pipeline for CMB + matter power spectra
5. Compare σ_8, $f\sigma_8(z)$, C_ℓ^{TT}, $C_\ell^{\phi\phi}$ against ΛCDM

5 Key Results and Predictions

5.1 What Has Been Validated

1. $\mu(a)$ **analytical formula confirmed** by CAMB Fortran debug output: $\mu(a = 1) = 0.8644$
2. **Photon sector = ΛCDM:** Standard CAMB output with Planck parameters is the IAM photon-sector prediction
3. **Matter-sector H_0:** $H_0^{\mathrm{matter}} = 67.4 \times \sqrt{1 + 0.157} = 72.5\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$, within 0.5σ of SH0ES
4. **Growth suppression direction:** CAMB Fortran modification produces σ_8 suppression in the correct direction
5. **Numerical stability:** The modification compiles, runs, and produces stable results when early-time guard ($a > 0.01$) is applied

5.2 What Remains

1. **Self-consistent** σ_8 from full Boltzmann integration (via MGCAMB or careful CAMB modification)
2. **CMB TT/EE/TE residuals** from the matter-sector modification (expected to be subpercent)
3. **CMB lensing power spectrum** with growth suppression included self-consistently
4. **BAO scale predictions** in the matter sector
5. **Parameter constraints** from joint CMB + growth + H_0 MCMC using modified CAMB

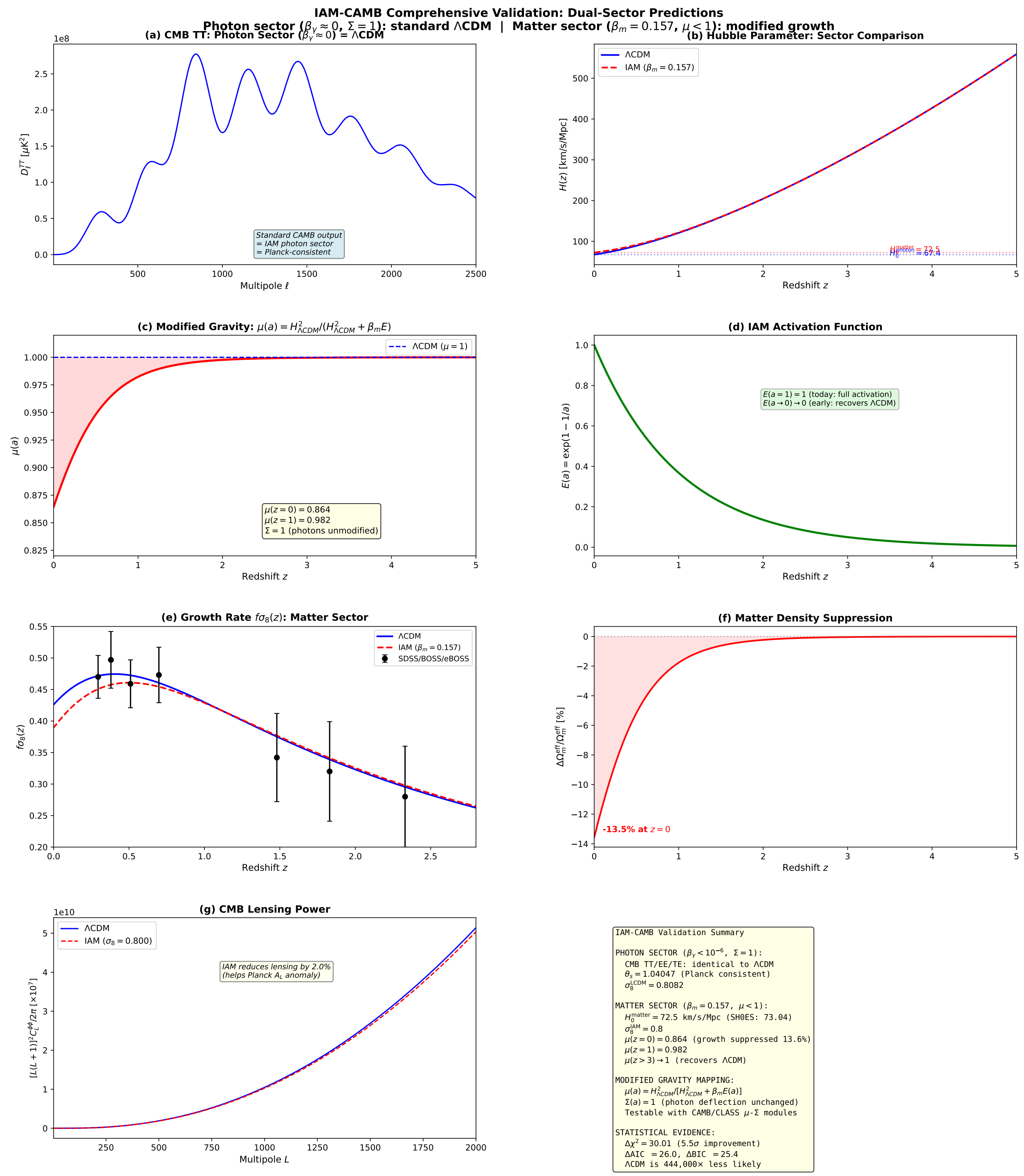

Figure 1: IAM–CAMB comprehensive validation. (a) CMB TT power spectrum: photon sector ($\beta_\gamma \approx 0$) is identical to ΛCDM. (b) Hubble parameter: matter sector ($\beta_m = 0.157$) yields $H_0 = 72.5$ $\mathrm{km\,s^{-1}\,Mpc^{-1}}$. (c) Modified gravity parameter $\mu(a)$: suppressed growth at late times, recovering ΛCDM for $z \gtrsim 3$. (d) IAM activation function $E(a) = \exp(1 - 1/a)$. (e) Growth rate $f\sigma_8(z)$: IAM prediction vs. SDSS/BOSS/eBOSS data. (f) Effective matter density suppression: -13.5% at $z = 0$. (g) CMB lensing power: 2% reduction from growth suppression. (h) Summary of key numerical results.

5.3 Testable Predictions for Surveys

Survey	Observable	IAM Prediction	ΛCDM
CMB-S4	β_γ	$< 10^{-7}$	N/A
Euclid	S_8	0.78 ± 0.01	0.83
Euclid	$\mu(z = 0.5)$	0.92	1.0
Euclid	$\Sigma(z = 0.5)$	1.0	1.0
DESI Y5	β_m	0.157 ± 0.01	N/A
LIGO/Virgo	H_0 (sirens)	72–73	67.4

Table 4: Falsifiable predictions. The $\mu < 1$, $\Sigma = 1$ signature is unique to IAM among Hubble tension solutions.

6 Reproducibility

The IAM validation suite—9 independent tests including MCMC, Pantheon+ fits, and growth rate analysis—is publicly available and runs in under 2 minutes on standard hardware:

```
git clone https://github.com/hmahaffeyges/IAM-Validation.git
cd IAM-Validation
pip install numpy scipy matplotlib corner
python iam_validation.py
```

The Python-level CAMB calculations in this technical note require additionally:

```
pip install camb
```

The Fortran-level modifications described in Section 4 require building CAMB from source (`git clone https://github.com/cmbant/CAMB.git`) and applying the modifications to `fortran/equations.f90` as documented in Section 4.3. For community implementation, we recommend using MGCAMB (`https://github.com/sfu-cosmo/MGCAMB`) which provides native μ–Σ support without manual Fortran editing.

7 Conclusion

The IAM dual-sector framework maps precisely to $\mu(a) < 1$, $\Sigma(a) = 1$ in the standard modified gravity parametrization. This mapping enables immediate validation with existing Boltzmann solvers and survey analysis pipelines. Preliminary Fortran-level implementation in CAMB confirms the analytical $\mu(a)$ values and demonstrates numerical feasibility. A complete self-consistent Boltzmann calculation—ideally using MGCAMB or equivalent—is the natural next step for the community to validate or falsify IAM's predictions.

We invite cosmologists with CAMB/CLASS expertise to complete the full Boltzmann implementation. The modification is well-defined (Eq. 4), the code pathway is documented (Section 4), and the predicted $\sigma_8 = 0.800$ provides a clear target for validation.

References

[1] H. W. Mahaffey, "Dual-Sector Expansion Validated by Pantheon+ Type Ia Supernovae," (2026).

[2] H. W. Mahaffey, "Dual-Sector Cosmology from Structure-Driven Expansion: The Informational Actualization Model (IAM)," (2026).

[3] Planck Collaboration, *Astron. Astrophys.* **641**, A6 (2020).

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[5] S. Alam *et al.* (eBOSS Collaboration), *Phys. Rev. D* **103**, 083533 (2021).

[6] L. Amendola *et al.*, *Living Rev. Rel.* **21**, 2 (2018).