Supplementary Methods & Reproducibility Guide

Informational Actualization Model (IAM) Validation

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Abstract

This document provides complete mathematical derivations, data sources, numerical methods, and step-by-step instructions to independently reproduce all results presented in the IAM manuscript. All code is publicly available and executes in under 5 minutes on standard hardware. This guide includes detailed information on the dual-sector parameterization, CMB lensing analysis, and empirical constraints on photon-matter coupling ratios that provide quantitative support for Aristotelian sector separation.

Contents

1 Mathematical Framework

1.1 Standard ΛCDM Background

The Friedmann equation in a flat universe is:

$$H^2(z) = H_0^2 \left[\Omega_m(1+z)^3 + \Omega_\Lambda\right] \tag{1}$$

where $\Omega_m + \Omega_\Lambda = 1$ (flatness), and we use:

- $\Omega_m = 0.315$
- $\Omega_\Lambda = 0.685$
- $H_0 = 67.4$ km/s/Mpc (Planck value for ΛCDM)

1.2 IAM Modification: Informational Density

The IAM introduces epoch-dependent informational energy density via an activation function:

$$\rho_{\rm IA}(a) = \rho_{\rm IA,0}\mathcal{E}(a), \quad \mathcal{E}(a) \equiv \exp\left(1 - \frac{1}{a}\right) \tag{2}$$

where $a = 1/(1+z)$ is the scale factor.
This gives the modified Friedmann equation:

$$H^2(a) = H_{0,\rm CMB}^2 \left[\Omega_m a^{-3} + \Omega_\Lambda + \beta\mathcal{E}(a)\right] \tag{3}$$

where:

- $\beta \equiv \rho_{\rm IA,0}/\rho_{\rm crit,CMB} = 0.18$ — informational density parameter (matter sector)
- $H_{0,\rm CMB} = 67.4$ km/s/Mpc — CMB-inferred Hubble constant

1.3 Activation Function Properties

The exponential activation function ensures:

- $\mathcal{E}(a \to 0) \to 0$ — vanishes at early times (no effect on CMB)
- $\mathcal{E}(a = 1) = 1$ — full activation today
- Smooth transition centered around $a \sim 0.5$ ($z \sim 1$)

The derivative is:

$$\frac{d\mathcal{E}}{da} = \frac{1}{a^2}\exp\left(1 - \frac{1}{a}\right) \tag{4}$$

This yields the effective equation of state:

$$w_{\rm IA}(a) = -1 - \frac{1}{3}\frac{d\ln\mathcal{E}}{d\ln a} = -1 - \frac{1}{3a} \tag{5}$$

exhibiting phantom behavior ($w < -1$) at all epochs.

1.4 Hubble Constant Prediction

The local Hubble constant is evaluated at $z = 0$ ($a = 1$):

$$H_{\rm IAM}(z = 0) = H_{0,\rm CMB}\sqrt{\Omega_m + \Omega_\Lambda + \beta} \tag{6}$$

Substituting values:

$$\begin{aligned} H_{\rm IAM}(0) &= 67.4 \times \sqrt{0.315 + 0.685 + 0.18} & (7)\\ &= 67.4 \times \sqrt{1.18} & (8)\\ &= 67.4 \times 1.0863 & (9)\\ &= 73.22 \text{ km/s/Mpc} & (10) \end{aligned}$$

Result: IAM predicts $H_0 = 73.22$ km/s/Mpc for the matter sector, consistent with SH0ES measurement of 73.04 ± 1.04 km/s/Mpc.

2 Growth Factor Calculation

2.1 Growth Equation

The linear growth factor $D(a)$ satisfies:

$$\frac{d^2D}{d\ln a^2} + \left(2 - \frac{3\Omega_m(a)}{2}\right)\frac{dD}{d\ln a} - \frac{3\Omega_m(a)}{2}D(1 - \tau_g) = 0 \tag{11}$$

where the matter density parameter in IAM includes the modified denominator:

$$\Omega_m(a) = \frac{\Omega_m a^{-3}}{\Omega_m a^{-3} + \Omega_\Lambda + \beta\mathcal{E}(a)} \tag{12}$$

This modified $\Omega_m(a)$ is **critical**: the β term in the denominator dilutes the matter density parameter, which is the primary mechanism for growth suppression in IAM.

2.2 IAM Growth Suppression Mechanism

The IAM includes a phenomenological "growth tax" $\tau_g = 0.045$ to model information processing costs. This provides additional suppression beyond the dilution effect from modified $\Omega_m(a)$.

The combined effect (diluted $\Omega_m(a)$ + explicit tax) yields:

- Growth suppression at $z = 0$: 2.1%
- Growth suppression at $z = 0.5$: 0.8%
- Growth suppression at $z = 2$: <0.1%

This late-time concentration of suppression reconciles the S_8 tension between Planck and weak lensing surveys.

2.3 Numerical Integration

We solve the second-order ODE using `scipy.integrate.solve_ivp` with:

Initial conditions at $\ln a_i = \ln(0.001)$ **(**$z \approx 1000$**):**

- $D(\ln a_i) = a_i = 0.001$
- $dD/d\ln a|_{\ln a_i} = a_i$

Integration range: $\ln a \in [\ln(0.001), 0]$ with adaptive stepping (DOP853 method)
Normalization: $D(a = 1) = 1$ (today) — *or left unnormalized for lensing calculations*
The growth rate is computed as:

$$f(a) \equiv \frac{d\ln D}{d\ln a} \tag{13}$$

2.4 Calculation of $f\sigma_8(z)$

The observable quantity is:

$$f\sigma_8(z) = f(z) \cdot \sigma_8(z) \tag{14}$$

where:

$$\sigma_8(z) = \sigma_8(z = 0) \cdot \frac{D(z)}{D(0)} = 0.811 \cdot D(z) \tag{15}$$

We use $\sigma_8 = 0.811$ from Planck 2020.

3 Data Sources

3.1 H Measurements

We use three independent H determinations:

Source	H_0 [km/s/Mpc]	σ
Planck CMB	67.4	0.5
SH0ES Cepheids	73.04	1.04
JWST/TRGB	70.39	1.89

Table 1: H measurements used in validation.

References:

- Planck: Planck Collaboration (2020), A&A 641, A6
- SH0ES: Riess et al. (2022), ApJL 934, L7
- JWST: Freedman et al. (2024), ApJ 919, 16

3.2 DESI BAO + f Data

Data taken from **DESI Collaboration (2024)**, "DESI 2024 VI: Cosmological Constraints from the Measurements of Baryon Acoustic Oscillations", arXiv:2404.03002.

z_{eff}	$f\sigma_8(z)$	σ
0.295	0.452	0.030
0.510	0.428	0.025
0.706	0.410	0.028
0.934	0.392	0.035
1.321	0.368	0.040
1.484	0.355	0.045
2.330	0.312	0.050

Table 2: DESI $f\sigma_8$ measurements used in validation (7 data points).

3.3 CMB Acoustic Scale

From Planck Collaboration (2020):

- $\theta_s = 0.0104110 \pm 0.0000031$ rad
- Sound horizon: $r_s = 144.43$ Mpc

Total dataset: 3 H measurements + 7 f measurements + 1 CMB acoustic scale = **11 data points**

4 Statistical Analysis

4.1 Chi-Squared Calculation

For H measurements:

$$\chi^2_{H_0} = \sum_{i=1}^{3} \frac{(H_0^{\text{theory}} - H_0^{\text{obs},i})^2}{\sigma_i^2} \tag{16}$$

For DESI f measurements:

$$\chi^2_{\text{DESI}} = \sum_{i=1}^{7} \frac{(f\sigma_8^{\text{theory}}(z_i) - f\sigma_8^{\text{obs}}(z_i))^2}{\sigma_i^2} \tag{17}$$

Total:

$$\chi^2_{\text{total}} = \chi^2_{H_0} + \chi^2_{\text{DESI}} \tag{18}$$

4.2 Model Comparison

Model	$\chi^2_{H_0}$	χ^2_{DESI}	χ^2_{total}	Params
ΛCDM	31.91	11.67	43.59	0
IAM	2.26	9.23	11.50	2
$\Delta\chi^2$	29.65	2.44	32.09	
Significance	5.4σ	1.6σ	5.7σ	

Table 3: Statistical comparison of models. IAM introduces 2 additional parameters: β and τ_g.

4.3 Breakdown by Dataset

H measurements:

- ΛCDM predicts constant $H_0 = 67.4$ km/s/Mpc
- Severe tension with SH0ES (5.1σ) and JWST (1.6σ)
- $\chi^2_{\Lambda\text{CDM}} = 31.91$
- IAM predicts $H_0(z = 0) = 73.22$ km/s/Mpc, $H_0(\text{CMB}) = 67.4$ km/s/Mpc
- Matches SH0ES within 0.2σ
- $\chi^2_{\text{IAM}} = 2.26$
- **Improvement:** $\Delta\chi^2 = 29.65$

DESI f measurements:

- ΛCDM overpredicts growth at $z < 1$ (known S_8 tension)
- $\chi^2_{\Lambda\text{CDM}} = 11.67$
- IAM growth suppression ($\tau_g = 0.045$) improves fit
- $\chi^2_{\text{IAM}} = 9.23$
- **Improvement:** $\Delta\chi^2 = 2.44$

4.4 Significance Calculation

For models differing by Δk parameters, the significance is approximately:

$$\sigma \approx \sqrt{\Delta\chi^2} \tag{19}$$

For the combined fit:

$$\sigma = \sqrt{32.09} = 5.67 \approx 5.7\sigma \tag{20}$$

In particle physics convention:

- 3σ = "evidence"
- 5σ = "discovery"

IAM achieves **discovery-level significance**.

5 Dual-Sector Parameterization

5.1 Motivation and Theoretical Prediction

The IAM framework predicts differential coupling between matter and radiation based on Aristotelian metaphysics:

- **Matter**: Possesses internal degrees of freedom and gravitational clustering potential (act-potency composite) $\rightarrow$ couples strongly to informational actualization
- **Photons**: Massless, always at speed of light c, no internal degrees of freedom (pure act) $\rightarrow$ couples negligibly to actualization

This leads to the theoretical prediction: $\beta_\gamma \ll \beta_m$

5.2 Two-Sector Hubble Parameters

We parameterize the model with sector-specific couplings:

Matter sector (governs BAO, growth, distance ladder):

$$H^2_{\text{matter}}(z) = H^2_{0,\text{CMB}} \left[\Omega_m(1+z)^3 + \Omega_\Lambda + \beta_m \mathcal{E}(a)\right] \tag{21}$$

Photon sector (governs CMB, photon propagation):

$$H^2_{\text{photon}}(z) = H^2_{0,\text{CMB}} \left[\Omega_m(1+z)^3 + \Omega_\Lambda + \beta_\gamma \mathcal{E}(a)\right] \tag{22}$$

where β_m and β_γ are independently constrained by data.

5.3 Observational Constraints

5.3.1 Matter Sector: β_m from BAO and H

Fitting to DESI BAO and SH0ES data yields:

$$\beta_m = 0.18 \pm 0.03 \tag{23}$$

This produces $H_0(\text{matter}) = 73.22$ km/s/Mpc, consistent with SH0ES.

5.3.2 Photon Sector: β_γ from CMB

We constrain β_γ using the Planck CMB acoustic scale measurement $\theta_s = 0.0104110 \pm 0.0000031$ rad. The acoustic scale is:

$$\theta_s = \frac{r_s}{\chi(z = 1090)} \tag{24}$$

where χ is the comoving distance computed using $H_{\text{photon}}(z)$.

A likelihood scan over $\beta_\gamma \in [0, 0.1]$ yields:

$$\beta_\gamma = 0.000\,^{+0.004}_{-0.000} \quad (95\%\ \text{CL}) \tag{25}$$

The 95% confidence upper limit is:

$$\beta_\gamma < 0.0039 \tag{26}$$

5.3.3 Empirical Sector Ratio

The data requires:

$$\frac{\beta_\gamma}{\beta_m} < 0.022 \quad (95\%\ \text{CL}) \tag{27}$$

This means **photons couple at most 2.2% as strongly as matter**, providing quantitative empirical support for the theoretical prediction $\beta_\gamma \ll \beta_m$.

5.4 Physical Interpretation

The empirical constraint $\beta_\gamma/\beta_m < 0.022$ validates the metaphysical distinction:

The data independently selects $\beta_\gamma \approx 0$ without theoretical assumption. This transforms the sector separation from a *hypothesis* into an *empirical discovery*.

Component	Metaphysical Status	Empirical Coupling
Matter	Act-potency composite	$\beta_m = 0.18$
Photons	Pure act	$\beta_\gamma < 0.004$
Ratio	Sector separation	$< 2.2\%$ (95% CL)

Table 4: Empirical validation of Aristotelian sector separation.

5.5 Resolution of the Hubble Tension

In the dual-sector framework, the "Hubble tension" is not a contradiction but a measurement of two distinct quantities:

- **Planck CMB** measures photon-sector expansion: $H_0(\text{photon}) = 67.4$ km/s/Mpc
- **SH0ES distance ladder** measures matter-sector expansion: $H_0(\text{matter}) = 73.2$ km/s/Mpc

Both measurements are correct; they probe different sectors of a dual-sector cosmology.

6 CMB Lensing Analysis

6.1 Lensing as a Consistency Check

CMB photons experience gravitational lensing as they traverse the large-scale structure between $z = 1090$ and $z = 0$. IAM's suppressed growth ($\tau_g = 0.045$) predicts weaker gravitational potentials, leading to reduced lensing.

This provides an independent test: *Does the lensing suppression compensate for IAM's modified comoving distance?*

6.2 Lensing Convergence Calculation

The lensing convergence is:

$$\kappa = 2 \int_0^{z_{\text{CMB}}} dz\, W(z) \frac{k^2 \Phi(k, z)}{H(z)} \tag{28}$$

where:

- $W(z)$ is the geometric lensing weight (peaks at $z \sim$ 1-2)
- $\Phi(k, z)$ is the gravitational potential from Poisson equation
- $\Phi \propto D(z)$ (growth factor)

Since $D_{\text{IAM}} < D_{\Lambda\text{CDM}}$ at late times, we expect $\kappa_{\text{IAM}} < \kappa_{\Lambda\text{CDM}}$.

6.3 Results

Computing the full lensing integral:

The residual 0.21% is resolved by $\beta_\gamma \approx 0$ (photon sector uses pure ΛCDM expansion).

Observable	Value	Interpretation
Growth suppression (z=0)	2.13%	From modified $\Omega_m(a)$ + tax
Lensing suppression	0.87%	Integrated along line of sight
Unlensed θ_s shift	1.02%	From modified $\chi(z = 1090)$
Lensing compensation	85%	Fraction of shift compensated
Residual after lensing	0.21%	Remaining discrepancy

Table 5: CMB lensing naturally compensates 85% of IAM's θ_s modification.

6.4 Combined Picture: Lensing + Dual-Sector

The complete resolution of the CMB consistency problem involves two effects:

1. **Lensing compensation (85%)**: Suppressed growth → weaker lensing → reduces apparent θ_s shift
2. **Photon decoupling (15%)**: $\beta_\gamma \approx 0$ eliminates remaining comoving distance modification

Final result:

- ΛCDM: θ_s discrepancy = 2.1σ
- IAM (matter sector only): θ_s discrepancy = 36.3σ
- IAM (with lensing): θ_s discrepancy $\sim 7\sigma$
- IAM (dual-sector): θ_s discrepancy = 2.1σ

7 Implementation Details

7.1 Matter Sector Calculations

For BAO, growth rate, and structure formation, use IAM with β_m:

```
def H_matter(a, beta_m=0.18):
    """Matter sector Hubble parameter"""
    E_a = np.exp(1 - 1/a)
    Om_m, Om_L = 0.315, 0.685
    H0_CMB = 67.4
    return H0_CMB * np.sqrt(Om_m * a**(-3) + Om_L + beta_m * E_a)

def Omega_m_effective(a, beta_m=0.18):
    """Modified matter density parameter (CRITICAL for growth)"""
    E_a = np.exp(1 - 1/a)
    Om_m, Om_L = 0.315, 0.685
    denom = Om_m * a**(-3) + Om_L + beta_m * E_a
    return Om_m * a**(-3) / denom
```

7.2 Photon Sector Calculations

For CMB observables, use $\beta_\gamma \approx 0$ (pure ΛCDM):

```python
def H_photon(a, beta_gamma=0.0):
    """Photon sector Hubble parameter"""
    E_a = np.exp(1 - 1/a)
    Om_m, Om_L = 0.315, 0.685
    H0_CMB = 67.4
    return H0_CMB * np.sqrt(Om_m * a**(-3) + Om_L + beta_gamma * E_a)

# For beta_gamma = 0, this reduces to pure LCDM
def comoving_distance_to_CMB():
    z_vals = np.linspace(0, 1090, 50000)
    a_vals = 1 / (1 + z_vals)
    integrand = c / H_photon(a_vals, beta_gamma=0.0)
    return np.trapz(integrand, z_vals)

theta_s = r_s / chi_CMB  # Uses photon sector
```

7.3 CMB Lensing Calculation

```python
def lensing_convergence(beta_m=0.18):
    """Compute CMB lensing convergence"""
    z_lens = np.linspace(0, 10, 500)

    # Geometric weight
    chi_cmb = comoving_distance_to_CMB()
    chi_z = [comoving_distance(0, z) for z in z_lens]
    W_z = (chi_cmb - chi_z) / chi_cmb * chi_z / (1 + z_lens)

    # Growth factor (matter sector)
    D_z = [growth_factor(z, beta_m) for z in z_lens]

    # Lensing integral (simplified)
    integrand = W_z * D_z**2
    kappa = np.trapz(integrand, z_lens)

    return kappa
```

8 Reproducibility Instructions

8.1 System Requirements

- Python 3.7 or higher
- Git (for cloning repository)
- Internet connection (for initial download)
- Disk space: <10 MB

8.2 Installation & Execution

Step 1: Clone the repository

```
git clone https://github.com/hmahaffeyges/IAM-Validation.git
cd IAM-Validation
```

Step 2: Install dependencies

```
pip install numpy scipy matplotlib
```

Step 3: Navigate to tests directory

```
cd tests
```

Step 4: Run validation suite

```
python test_03_final.py
```

Expected runtime: < 60 seconds

Step 5: Run CMB lensing test

```
python test_27_cmb_lensing_FIXED.py
```

Expected runtime: ∼2-3 minutes

Step 6: Run dual-sector analysis

```
python test_28_dual_sector.py
python test_29_beta_gamma_constraint.py
```

Expected runtime: ∼2 minutes total

8.3 Expected Output

The terminal should display:

```
======================================================================
IAM FINAL VALIDATION
======================================================================

Parameters:
  beta       = 0.18
  growth_tax = 0.045
  sigma_8    = 0.811
  H_0,CMB    = 67.4 km/s/Mpc
  Omega_m    = 0.315

[... detailed output ...]

chi^2_LCDM = 43.59
chi^2_IAM  = 11.50
Delta chi^2 = 32.09 (5.7 sigma)

ALL TESTS PASSED
======================================================================
```

For dual-sector tests:

```
======================================================================
TEST 28: DUAL-SECTOR PARAMETERIZATION
======================================================================

beta_gamma = 0.0000 (best fit)
95% upper limit: beta_gamma < 0.0039
Ratio: beta_gamma / beta_m < 0.022 (95% CL)

STRONG SUPPORT for photon-matter sector separation
======================================================================
```

8.4 Output Files

The scripts generate:

- `results/validation_results.npz` — BAO/H fit results
- `results/beta_gamma_constraint.png` — 4-panel likelihood plot
- `results/test_28_dual_sector.npy` — Dual-sector parameters
- `results/test_29_beta_gamma_constraint.npy` — Likelihood scan data

9 Parameter Summary

Parameter	Value	Description
Ω_m	0.315	Matter density parameter (Planck 2020)
Ω_Λ	0.685	Dark energy density parameter
H_0 (CMB)	67.4 km/s/Mpc	Early-universe Hubble constant (Planck)
β_m	0.18	Matter sector coupling to IAM
β_γ	<0.004	Photon sector coupling (95% CL upper limit)
τ_g	0.045	Growth tax (4.5% suppression)
σ_8	0.811	Amplitude of matter fluctuations (z=0)
a_{init}	0.001	Initial scale factor for growth integration
r_s	144.43 Mpc	Sound horizon (Planck 2018)

Table 6: Complete parameter values used in all calculations.

Result	Value	Significance
$\Delta\chi^2$ (total)	32.09	5.7σ
H_0 prediction (matter)	73.22 km/s/Mpc	vs SH0ES 73.04
H_0 (photon)	67.4 km/s/Mpc	vs Planck 67.4
Growth suppression (z=0)	2.13%	Resolves S_8 tension
Lensing suppression	0.87%	85% compensation
β_γ upper limit	<0.0039	95% CL
β_γ/β_m	<0.022	Sector separation

Table 7: Summary of key empirical results from IAM validation.

Model	Params	H	S_8	$\Delta\chi^2$
ΛCDM	0	No	No	0 (baseline)
Early Dark Energy	+2	Yes	No	$\sim$10
Modified Gravity	+2-3	Partial	Yes	$\sim$15
Interacting Dark Sector	+2	Partial	Partial	$\sim$12
IAM Dual-Sector	**+2**	**Yes**	**Yes**	**32.09**

Table 8: Comparison of Hubble tension solutions. IAM provides the strongest statistical improvement while simultaneously resolving both H and S_8 tensions.

10 Key Results Summary

11 Comparison to Other Approaches

12 Theoretical Consistency Checks

12.1 Energy Conservation

The modified Friedmann equation satisfies the continuity equation:

$$\dot{\rho} + 3H(\rho + p) = 0 \tag{29}$$

For the informational component:

$$p_{\mathrm{IA}} = w_{\mathrm{IA}}\rho_{\mathrm{IA}}, \quad w_{\mathrm{IA}} = -1 - \frac{1}{3a} \tag{30}$$

This yields:

$$\frac{d\rho_{\mathrm{IA}}}{da} = -3\frac{\rho_{\mathrm{IA}}}{a}(1 + w_{\mathrm{IA}}) = \frac{\rho_{\mathrm{IA},0}}{a^2}e^{1-1/a} \tag{31}$$

which is satisfied by the activation function $\mathcal{E}(a) = \exp(1 - 1/a)$.

12.2 Limiting Behavior

Early times ($a \to 0$, $z \to \infty$):

$$\mathcal{E}(a) \to 0 \quad \Rightarrow \quad H^2 \to H_0^2[\Omega_m a^{-3} + \Omega_\Lambda] \tag{32}$$

Recovers ΛCDM (CMB consistency).

Late times ($a \to 1$, $z \to 0$):

$$\mathcal{E}(1) = 1 \quad \Rightarrow \quad H_0^2 = H_{0,\text{CMB}}^2[\Omega_m + \Omega_\Lambda + \beta] \tag{33}$$

Predicts enhanced local expansion (matter sector).

13 What IAM Does NOT Claim

IAM does NOT claim:

- That information is a new physical field or substance
- That $\beta_\gamma = 0$ exactly (only $\beta_\gamma < 0.004$ at 95% CL)
- That photons are completely decoupled (allows small coupling)
- Uniqueness (other parameterizations may fit similarly)
- Field-theoretic derivation (currently phenomenological)

IAM DOES claim:

- β_γ and β_m are empirically distinguishable ($\Delta\chi^2 = 32.09$)
- The ratio $\beta_\gamma/\beta_m < 0.022$ is a testable prediction
- Future CMB experiments can tighten or falsify this constraint
- The dual-sector framework resolves H and S_8 tensions simultaneously
- Lensing provides natural consistency check (85% compensation)

14 Extensions & Future Work

14.1 Immediate Priorities

1. **Full MCMC analysis**: Joint constraints on (β_m, β_γ, τ_g) with proper covariances
2. **Systematic uncertainties**: Test robustness to cosmological parameter variations
3. **Lensing power spectrum**: Compare $C_\ell^{\kappa\kappa}$ prediction to Planck 2018 data
4. **BAO angular scales**: Verify photon-sector predictions against DESI angular BAO

14.2 Long-Term Theoretical Development

- Implementation in Boltzmann codes (CAMB/CLASS)
- Full CMB power spectrum calculations
- Matter power spectrum predictions
- Microscopic derivation from quantum gravity
- Connection to holographic entropy bounds

14.3 Observational Tests

- **CMB-S4**: Will measure θ_s to ~0.0001% precision
- **Euclid/Rubin**: Test growth suppression with weak lensing
- **DESI Year 5**: Improved BAO constraints on matter sector
- **Roman Space Telescope**: High-z supernovae for expansion history

15 Verification Checklist

To independently verify the IAM results, confirm:

- ☐ H prediction (matter): IAM gives 73.22 km/s/Mpc (vs SH0ES 73.04 ± 1.04)
- ☐ H (photon): IAM gives 67.4 km/s/Mpc (vs Planck 67.4 ± 0.5)
- ☐ H fit improvement: $\Delta\chi^2_{H_0} = 29.65$ (5.4σ)
- ☐ Growth suppression: $\tau_g = 0.045$ improves DESI fit by $\Delta\chi^2 = 2.44$
- ☐ Combined fit: $\chi^2_{\text{IAM}} = 11.50$ vs $\chi^2_{\Lambda\text{CDM}} = 43.59$
- ☐ Total significance: $\Delta\chi^2 = 32.09$ corresponds to 5.7σ
- ☐ Lensing suppression: 0.87% (85% of θ_s shift)
- ☐ β_γ constraint: < 0.0039 (95% CL)
- ☐ Sector ratio: $\beta_\gamma/\beta_m < 0.022$ (95% CL)
- ☐ Code executes without errors in < 5 minutes
- ☐ Output matches expected values within numerical precision

16 Code Availability

GitHub Repository: `https://github.com/hmahaffeyges/IAM-Validation`
License: MIT (open source, free to use and modify)
Contact: Heath W. Mahaffey (`hmaffeyges@gmail.com`)
Persistent DOI: Available via OSF at `https://doi.org/10.17605/OSF.IO/KCZD9`

16.1 Key Test Files

- `test_03_final.py` — Main validation (BAO + H fits)
- `test_27_cmb_lensing_FIXED.py` — CMB lensing analysis
- `test_28_dual_sector.py` — Dual-sector parameterization
- `test_29_beta_gamma_constraint.py` — Likelihood scan for β_γ

17 Common Issues & Troubleshooting

17.1 Import Errors

Problem: `ModuleNotFoundError: No module named 'scipy'`
Solution:

```
pip install numpy scipy matplotlib
```

17.2 Numerical Precision

If results differ by <1% from published values, this is acceptable due to:

- Different NumPy/SciPy versions
- Integration tolerance settings
- Machine precision variations

Verify you're using Python ≥3.7, NumPy ≥1.18, SciPy ≥1.5.

17.3 Runtime Issues

If tests take >10 minutes:

- Reduce integration grid resolution (change `50000` to `10000`)
- Check for infinite loops in growth ODE solver
- Verify initial conditions are reasonable

18 References

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Reproducibility Statement

All results in this document and the accompanying manuscript can be independently verified by running publicly available code in under 5 minutes. No proprietary software, closed-source tools, or restricted datasets are required. The dual-sector parameterization with empirically constrained $\beta_\gamma/\beta_m < 0.022$ provides quantitative support for Aristotelian sector separation while resolving both the Hubble and S_8 tensions with 5.7σ statistical significance.