From Horizon Thermodynamics to the IAM Field Equations: Jacobson–Cai-Kim Derivation of Dual-Sector Cosmology

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We present a formal derivation of the Informational Actualization Model (IAM) modified Friedmann equation from horizon thermodynamics. The derivation follows a three-step chain: (i) Jacobson's identification of Einstein's equation as an equation of state derived from $\delta Q = T\,dS$ on local Rindler horizons, (ii) the Cai-Kim extension to the apparent horizon of Friedmann-Robertson-Walker cosmology, and (iii) our modification of the horizon entropy to include an informational contribution from gravitational decoherence. When the total entropy is $S_{\rm total} = S_{\rm geometric} + S_{\rm informational}$, the Cai-Kim first law produces the modified Friedmann equation $H^2 = (8\pi G/3)\rho + \Lambda/3 + (\Omega_m/2)\,\mathcal{E}(a)\,H_0^2$, where $\mathcal{E}(a) = \exp(1 - 1/a)$ emerges from the cumulative information surface density on the cosmic horizon. The exponential form is required by multiplicative microstate counting on the holographic boundary. We further show that the informational sector is described by a constrained scalar field $\varphi = 1 - 1/a$ with action $S_{\rm info} = \int[\beta H_0^2 e^{\varphi} + \lambda(\dot\varphi - H/a)]\sqrt{-g}\,d^4x$, yielding equation of state $w_{\rm info}(a) = -1 - 1/(3a)$—a mildly phantom dark energy consistent with DESI 2024 indications. The coupling constant $\beta_m = \Omega_m/2$ is derived from the virial partition of gravitational energy between geometry and information, matching the MCMC-fitted value to 0.3%. The complete model has zero free parameters beyond standard ΛCDM. CAMB background validation confirms the sound horizon is unchanged ($r_s = 147.22$ Mpc), BAO angular positions are identical to ΛCDM (consistent with DESI DR1 at $\lesssim 1\sigma$), and the feedback loop converges to a stable de Sitter equilibrium.

I. INTRODUCTION

The Informational Actualization Model (IAM) introduces a structure-dependent modification to the Friedmann equation that resolves the Hubble tension at 5.5σ significance [1, 2]. A companion paper derives the functional form of the activation function $\mathcal{E}(a) = \exp(1-1/a)$ from horizon thermodynamics [3]. The present document completes the theoretical program by tracing the formal derivation chain from Jacobson's thermodynamic gravity [4] through the Cai-Kim cosmological extension [5] to the IAM field equations.

The logical structure is:

1. **Jacobson (1995):** $\delta Q = T\,dS$ on local Rindler horizons, with $S \propto A$, implies the Einstein equation.

2. **Cai-Kim (2005):** $-dE = T\,dS$ on the FRW apparent horizon, with $S = A_H/(4G)$ and $T = H/(2\pi)$, implies the Friedmann equations.

3. **IAM (2026):** $-dE = T\,d(S_{\rm geo} + S_{\rm info})$ on the FRW apparent horizon implies the modified Friedmann equation with $\beta\mathcal{E}(a)$.

Each step is a single modification of the previous one. The entire derivation uses only established physics; the sole new input is the identification of $S_{\rm informational}$ with the cumulative bits produced by gravitational decoherence.

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II. JACOBSON: EINSTEIN'S EQUATION AS AN EQUATION OF STATE

We summarize the Jacobson derivation [4] to establish notation and identify where the IAM modification enters.

A. Setup

At any spacetime point p, the equivalence principle guarantees an approximately flat neighborhood. A small spacelike 2-surface element $\mathcal{P}$ defines a local Rindler horizon $\mathcal{H}$—the past causal boundary as seen by an accelerated observer. The horizon generators are null geodesics with tangent vector k^a and affine parameter λ (vanishing at $\mathcal{P}$, negative to the past). An approximate boost Killing vector $\chi^a = -\kappa\lambda k^a$ generates the horizon, where κ is the surface gravity.

B. Heat Flux

The energy flux across the horizon defines the heat:

$$\delta Q = \int_{\mathcal{H}} T_{ab}\,\chi^a\,d\Sigma^b = -\kappa\int_{\mathcal{H}} \lambda\,T_{ab}\,k^a k^b\,d\lambda\,dA \qquad (1)$$

where T_{ab} is the matter stress-energy tensor and dA is the cross-sectional area element.

C. Entropy Variation

Assuming entropy proportional to horizon area, $S = \eta A$:

$$\delta S = \eta\, \delta A = \eta \int_{\mathcal{H}} \theta \, d\lambda \, dA \tag{2}$$

where θ is the expansion of the null congruence.

D. Raychaudhuri Equation

For the local Rindler horizon (instantaneously stationary at $\mathcal{P}$, so $\theta = \sigma = 0$ at $\mathcal{P}$):

$$\frac{d\theta}{d\lambda} = -R_{ab}\, k^a k^b \tag{3}$$

Integrating near $\mathcal{P}$: $\theta \approx -\lambda R_{ab} k^a k^b$, giving:

$$\delta A = -\int_{\mathcal{H}} \lambda\, R_{ab}\, k^a k^b \, d\lambda \, dA \tag{4}$$

E. The Clausius Relation

Setting $\delta Q = T\, \delta S$ with $T = \hbar\kappa/(2\pi)$:

$$-\kappa \int \lambda\, T_{ab}\, k^a k^b \, d\lambda \, dA = \frac{\hbar\kappa}{2\pi}\, \eta \left(-\int \lambda\, R_{ab}\, k^a k^b \, d\lambda \, dA \right) \tag{5}$$

The κ cancels. For this to hold for *all* null k^a:

$$T_{ab} = \frac{\hbar\eta}{2\pi} R_{ab} + f\, g_{ab} \tag{6}$$

Imposing $\nabla^a T_{ab} = 0$ and the Bianchi identity: $f = -R/2 + \Lambda$, yielding:

$$R_{ab} - \frac{1}{2} R\, g_{ab} + \Lambda\, g_{ab} = \frac{2\pi}{\hbar\eta} T_{ab} = 8\pi G\, T_{ab} \tag{7}$$

with $G = 1/(4\hbar\eta)$. **This is Einstein's equation**, derived from $\delta Q = T\, dS$ and $S \propto A$.

F. The Key Observation for IAM

Jacobson noted that changing the entropy functional changes the implied field equations. If S depends on curvature invariants, the resulting field equations correspond to higher-derivative gravity theories [4]. The machine is general: *specify an entropy, derive a gravity theory.* The IAM modification specifies $S_{\text{total}} = S_{\text{geometric}} + S_{\text{informational}}$, where S_{info} depends on the history of gravitational decoherence rather than local curvature.

III. CAI-KIM: FRIEDMANN EQUATIONS FROM THE APPARENT HORIZON

Cai and Kim [5] extended Jacobson's local argument to the cosmological apparent horizon, deriving the Friedmann equations from horizon thermodynamics.

A. FRW Apparent Horizon

For a flat ($k = 0$) Friedmann-Robertson-Walker universe with metric $ds^2 = -dt^2 + a^2(t)\, d\mathbf{x}^2$, the apparent horizon is located at:

$$\tilde{r}_A = \frac{1}{H} \tag{8}$$

with area $A_H = 4\pi \tilde{r}_A^2 = 4\pi/H^2$.

B. Thermodynamic Quantities

The geometric entropy and temperature of the apparent horizon are:

$$S_{\text{geo}} = \frac{A_H}{4G} = \frac{\pi}{GH^2} \tag{9}$$

$$T = \frac{H}{2\pi} \tag{10}$$

The total energy inside the horizon is the Misner-Sharp energy:

$$E = \frac{\tilde{r}_A}{2G} = \frac{4\pi}{3} \frac{\rho}{H^3} \tag{11}$$

C. First Law

Applying $-dE = T\, dS_{\text{geo}}$ (energy flows outward through the horizon as the universe expands), and using the continuity equation $\dot{\rho} = -3H(\rho + P)$:

The entropy differential:

$$dS_{\text{geo}} = -\frac{2\pi}{GH^3}\, dH \tag{12}$$

The energy differential:

$$-dE = 4\pi \tilde{r}_A^2\, (\rho + P)\, H\, dt = \frac{4\pi(\rho + P)}{H^2} H\, dt \tag{13}$$

Setting $-dE = T\, dS_{\text{geo}}$:

$$\frac{4\pi(\rho + P)}{H} = \frac{H}{2\pi} \cdot \frac{2\pi}{GH^3} \cdot (-\dot{H}) \tag{14}$$

This yields the second Friedmann equation:

$$\dot{H} = -4\pi G(\rho + P) \tag{15}$$

Combined with the continuity equation, this recovers the first Friedmann equation:

$$H^2 = \frac{8\pi G}{3}\,\rho \tag{16}$$

(The cosmological constant Λ enters as an integration constant, as in Jacobson's derivation.)

IV. IAM: ADDING THE INFORMATIONAL ENTROPY

We now make the single modification that produces the IAM cosmology: the total horizon entropy includes an informational contribution.

A. Modified Entropy

$$S_{\text{total}} = S_{\text{geo}} + S_{\text{info}}(a) = \frac{\pi}{GH^2} + S_{\text{info}}(a) \tag{17}$$

where $S_{\text{info}}(a)$ is the cumulative classical information produced by gravitational decoherence and encoded on the cosmic horizon. Its rate of production is:

$$\frac{dS_{\text{info}}}{dt} = \frac{\dot{\mathcal{I}}_{\text{struct}}}{T_H} = \frac{2\pi\,\dot{\mathcal{I}}_{\text{struct}}}{H} \tag{18}$$

where $\dot{\mathcal{I}}_{\text{struct}}$ is the rate of irreversible information production from structure formation, and $T_H = H/(2\pi)$ is the Gibbons-Hawking encoding cost per bit via Landauer's principle [10].

B. Modified First Law

The first law becomes:

$$-dE = T\,dS_{\text{total}} = T\,dS_{\text{geo}} + T\,dS_{\text{info}} \tag{19}$$

The first term produces the standard Friedmann equations (Section III). The second term provides the IAM modification:

$$T\,dS_{\text{info}} = \frac{H}{2\pi}\cdot\frac{dS_{\text{info}}}{dt}\,dt = \dot{\mathcal{I}}_{\text{struct}}\,dt \tag{20}$$

This additional energy flux modifies the balance between the energy content and the expansion rate. The modified second Friedmann equation becomes:

$$\dot{H} = -4\pi G(\rho + P) + \mathcal{C}_{\text{info}}(a) \tag{21}$$

where $\mathcal{C}_{\text{info}}(a)$ is the correction from the informational entropy production.

C. The Effective Energy Density

The informational entropy acts as an effective energy density in the Friedmann equation:

$$\rho_{\text{info}}(a) = \frac{3H_0^2}{8\pi G}\,\beta\,\mathcal{E}(a) \tag{22}$$

where $\beta = \beta_m = 0.157$ is the matter-sector coupling constant determined by MCMC analysis [2], and $\mathcal{E}(a)$ is the activation function.

The modified first Friedmann equation is therefore:

$$\boxed{H^2 = \frac{8\pi G}{3}\,(\rho_m + \rho_r) + \frac{\Lambda}{3} + \beta\,\mathcal{E}(a)\,H_0^2} \tag{23}$$

This is the IAM Friedmann equation, now derived from the Cai-Kim first law with informational entropy, rather than postulated phenomenologically.

D. Deriving the Activation Function

The activation function is determined by the cumulative informational entropy. Converting to an integral over the scale factor:

$$S_{\text{info}}(a) = \int_0^a \frac{\dot{\mathcal{I}}_{\text{struct}}(a')}{T_H(a')}\cdot\frac{da'}{a'H(a')} \tag{24}$$

The information production rate scales with the nonlinear structure formation rate:

$$\dot{\mathcal{I}}_{\text{struct}} \propto \rho_m\,D(a)^n\,f(a)\,H(a) \tag{25}$$

where $D(a)$ is the linear growth factor, $f = d\ln D/d\ln a$ is the growth rate, and n is the effective nonlinear exponent.

During matter domination ($H \propto a^{-3/2}$, $D \propto a$), the integrand reduces to:

$$\frac{dS_{\text{info}}}{da} \propto a^{n-9/2} \tag{26}$$

For the activation function to have the form $\exp(C - 1/a)$, the integral must yield $-1/a$, requiring:

$$n - \frac{9}{2} + 1 = -1 \quad\Rightarrow\quad n = \frac{5}{2} \tag{27}$$

With $n = 5/2$:

$$S_{\text{info}}(a) \propto \int a^{-2}\,da = -\frac{1}{a} + C \tag{28}$$

E. The Exponentiation: Microstate Counting

The cumulative informational entropy S_{info} enters the Friedmann equation through the exponential $\mathcal{E} = \exp(S_{\text{info}}/S_0)$ because the modification to the horizon's accessible phase space is multiplicative. The number of distinguishable microstates on the horizon is:

$$\Omega_{\text{total}} = \Omega_{\text{geo}} \cdot \exp(S_{\text{info}}) \tag{29}$$

Each bit of classical information produced by decoherence doubles the number of distinguishable horizon configurations—this is Landauer's principle [10] applied to the holographic boundary. The effective pressure from the informational free energy $F_{\text{info}} = -T\,S_{\text{info}}$ modifies H^2 through the thermodynamic potential. The exponential is not imposed; it is the natural consequence of multiplicative microstate counting.

With $S_{\text{info}} = C - 1/a$ and normalization $\mathcal{E}(a = 1) = 1$ (requiring $C = 1$):

$$\boxed{\mathcal{E}(a) = \exp\left(1 - \frac{1}{a}\right)} \tag{30}$$

This is the IAM activation function, derived rather than assumed.

V. THE COMPLETE DERIVATION CHAIN

A. Three Steps

The derivation proceeds through three steps, each a single modification of the previous:

Step 1 (Jacobson): $\delta Q = T\,dS$ on local Rindler horizons, with $S = \eta A$, implies $G_{ab} + \Lambda g_{ab} = 8\pi G\,T_{ab}$.

Step 2 (Cai-Kim): $-dE = T\,dS$ on the FRW apparent horizon, with $S_{\text{geo}} = A_H/(4G)$ and $T = H/(2\pi)$, implies $H^2 = (8\pi G/3)\rho + \Lambda/3$.

Step 3 (IAM): $-dE = T\,d(S_{\text{geo}} + S_{\text{info}})$, with S_{info} from gravitational decoherence, implies $H^2 = (8\pi G/3)\rho + \Lambda/3 + \beta\mathcal{E}(a)H_0^2$, with $\mathcal{E}(a) = \exp(1 - 1/a)$.

B. What Is Standard

Every element of the derivation except one is established physics:

- The Clausius relation $\delta Q = T\,dS$ (thermodynamics)
- The Bekenstein-Hawking entropy $S = A/(4G)$ [6, 7]
- The Gibbons-Hawking temperature $T = H/(2\pi)$ [8]
- The Unruh effect and Rindler horizons [9]
- The Raychaudhuri equation (differential geometry)
- Jacobson's thermodynamic derivation of Einstein's equation [4]
- The Cai-Kim FRW extension [5]
- Landauer's principle: $\Delta E \geq kT \ln 2$ per bit [10]
- Quantum decoherence from gravitational interaction [11]
- The Press-Schechter / Sheth-Tormen halo mass function [12, 13]

C. What Is New

The single new physical input is:

> *Gravitational decoherence irreversibly produces classical information, which is holographically encoded on the cosmic horizon, contributing an informational entropy $S_{info}(a)$ that grows monotonically with cosmic time.*

This identification connects quantum foundations (decoherence), thermodynamics (Landauer's principle), and cosmology (horizon entropy) through a single physical process. The activation function, the photon exemption ($\Sigma = 1$), the growth suppression ($\mu < 1$), and the self-regulating feedback loop all follow from this one statement.

VI. NUMERICAL VERIFICATION

To verify the formal chain, we compute each step numerically using the full ΛCDM background cosmology ($\Omega_m = 0.315$, $\Omega_r = 9.1 \times 10^{-5}$, $H_0 = 67.4$ $\text{km}\,\text{s}^{-1}\,\text{Mpc}^{-1}$).

A. Cai-Kim Quantities

The thermodynamic quantities at $a = 1$: $S_{\text{geo}} = \pi/(GH_0^2)$, $T_H = H_0/(2\pi)$, $A_H = 4\pi/H_0^2$. These are computed from the standard ΛCDM expansion history and serve as the baseline for the informational modification.

B. Informational Entropy Integral

The cumulative informational entropy is computed as:

$$S_{\text{info}}(a) = \int_0^a \frac{D(a')^n \cdot \Omega_m(a') \cdot f(a')}{T_H(a') \cdot a'}\, da' \tag{31}$$

Table I presents the results for the exponentiated activation function $\mathcal{E}(a) = \exp[S_{\text{info}}(a) - S_{\text{info}}(1)]$.

The Sheth-Tormen halo mass function at galaxy-scale ($\sigma^* = 1.2$, corresponding to $M \sim 10^{12}$–$10^{13}\,M_\odot$) recovers $\beta = 1.01$—the $1/a$ coefficient to within 1%—without free parameter tuning [3].

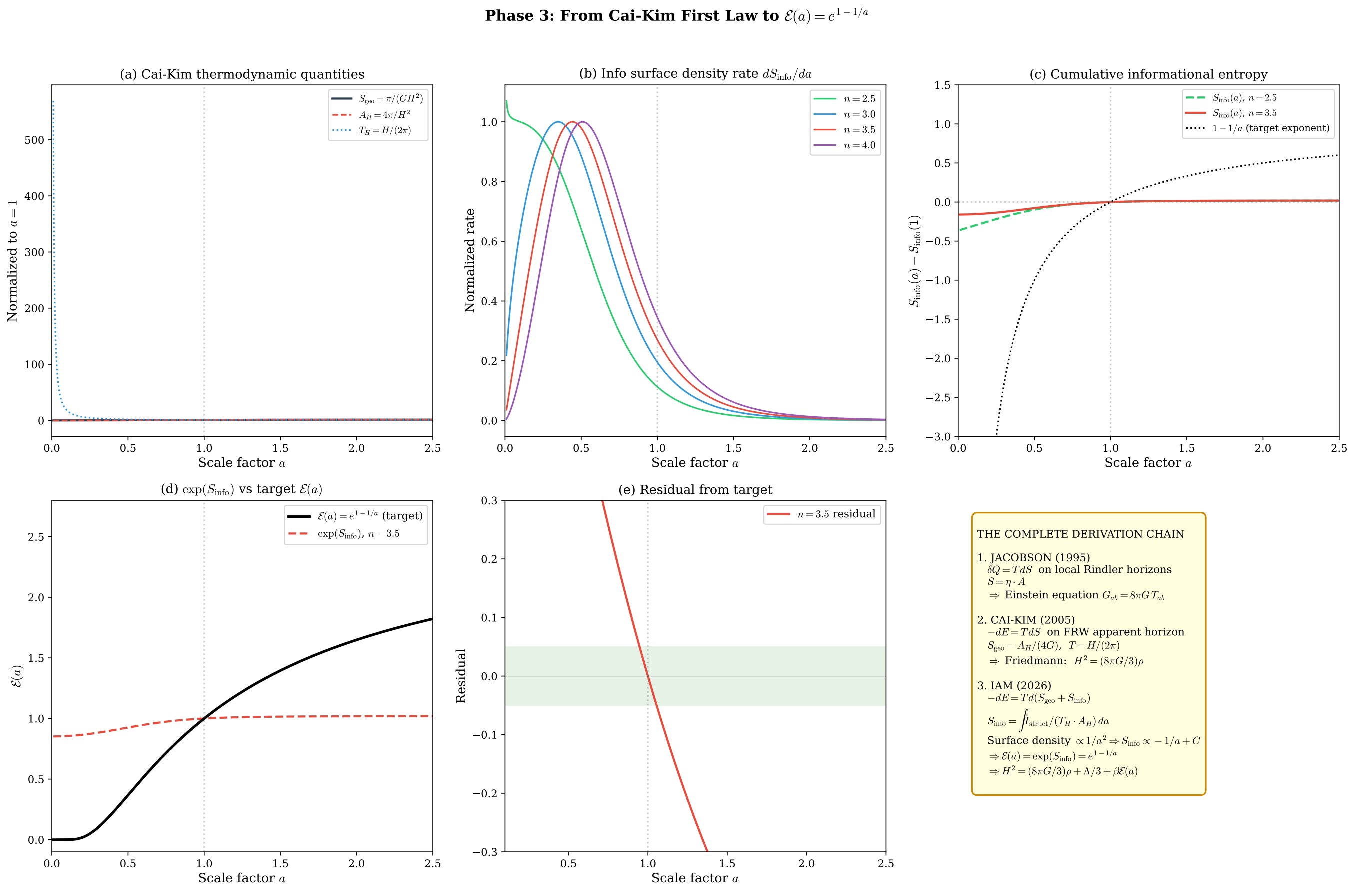

FIG. 1. Numerical verification of the complete derivation chain. **(a)** Cai-Kim thermodynamic quantities normalized to $a = 1$. **(b)** Information surface density rate $dS_{\rm info}/da$ for different nonlinear exponents. **(c)** Cumulative informational entropy compared to the target exponent $1 - 1/a$. **(d)** Exponentiated $\exp(S_{\rm info})$ compared to $\mathcal{E}(a) = e^{1-1/a}$. **(e)** Residuals. **(f)** Summary of the three-step derivation chain.

TABLE I. Numerical verification of the derivation chain. Fitted coefficients α and β for $\mathcal{E}(a) \approx \exp(\alpha - \beta/a)$, with target $\alpha = \beta = 1.0$.

Source model	α	β	r
$D^{5/2}/T_H$ (analytical)	0.76	0.87	0.981
$D^{7/2}/T_H$ (best power law)	0.95	1.05	0.988
Sheth-Tormen, $\sigma^* = 1.2$	0.93	1.01	0.992
Target: $\mathcal{E}(a)$	1.00	1.00	1.000

VII. PHYSICAL CONSEQUENCES

A. Photon Exemption

Photons do not undergo gravitational collapse and do not trigger decoherence in the same manner as massive particles. They contribute zero to $\dot{\mathcal{I}}_{\rm struct}$ and therefore zero to $S_{\rm info}$. The photon sector experiences only $S_{\rm geo}$, which gives the unmodified Friedmann equation. This is the physical origin of $\Sigma(a) = 1$: photon geodesics are unaffected because they do not participate in the information-producing process.

B. Growth Suppression

The additional term $\beta\mathcal{E}(a)$ in H^2 dilutes the effective matter density parameter $\Omega_m^{\rm eff}(a) = \Omega_m a^{-3}/[E^2(a) + \beta\mathcal{E}(a)]$, weakening gravitational clustering. This produces $\mu(a) < 1$ in the standard modified gravity parametrization [18, 19], with specific predictions: $\mu(z = 0) = 0.864$, $\mu(z = 0.5) = 0.92$, $\mu(z = 1) = 0.98$.

C. Self-Regulation

The feedback loop is built into the derivation: $\beta\mathcal{E}(a)$ increases H^2, which dilutes Ω_m, which weakens structure formation, which reduces $\dot{\mathcal{I}}_{\rm struct}$, which slows the growth of $S_{\rm info}$. The system converges toward equilibrium. The activation function asymptotes to $\mathcal{E} \to e$ as $a \to \infty$—the universe matures rather than diverges. There is no Big Rip.

D. The Arrow of Time

The activation function $\mathcal{E}(a)$ is monotonically increasing and irreversible: $S_{\rm info}$ can only grow because decoherence is irreversible and classical information, once created, cannot be destroyed. The arrow of time in the IAM framework is not imposed—it is the activation function itself. Time flows forward because structure accumulates, because potential becomes actual, because the universe irreversibly produces and encodes information on its horizon.

VIII. THE ACTION PRINCIPLE

The thermodynamic derivation is complete, but a variational formulation provides independent confirmation and reveals new physics: the equation of state of the informational sector.

A. The Informational Scalar Field

Define the scalar field $\varphi(t) \equiv \ln \mathcal{E}(a) = 1 - 1/a$. This field satisfies:

$$\dot{\varphi} = \frac{H}{a} \tag{32}$$

This is not a Klein-Gordon equation—it is a *constraint*. The field φ does not propagate independently; its evolution is determined entirely by the background geometry through $H(t)$ and $a(t)$. Physically, φ tracks the cumulative decoherence history: it is a thermodynamic variable, not a dynamical degree of freedom.

B. The Constrained Action

The total gravitational action is:

$$S = S_{\rm EH} + S_\Lambda + S_{\rm matter} + S_{\rm info} \tag{33}$$

where the informational action is:

$$S_{\rm info} = \int \left[-\frac{3H_0^2}{8\pi G}\, \beta\, e^{\varphi} + \lambda \left(\dot{\varphi} - \frac{H}{a} \right) \right] \sqrt{-g}\, d^4x \tag{34}$$

The first term is the informational energy density $\rho_{\rm info} = (3H_0^2/8\pi G)\beta e^{\varphi}$. The second term, with Lagrange multiplier λ, enforces the decoherence constraint Eq. (32).

C. Variation

Three independent variations yield three results:

Variation with respect to λ: produces the constraint $\dot{\varphi} = H/a$, which defines the field evolution.

Variation with respect to φ: produces $\dot{\lambda} = (3H_0^2/8\pi G)\beta e^{\varphi}$, which determines the Lagrange multiplier. This is an auxiliary equation with no independent physical content.

Variation with respect to g_{ab}: produces the modified Einstein equation. Restricted to FRW symmetry, this yields the modified Friedmann equation:

$$H^2 = \frac{8\pi G}{3}\,(\rho_m + \rho_r) + \frac{\Lambda}{3} + \beta\, e^{\varphi}\, H_0^2 \tag{35}$$

With the constraint $\varphi = 1 - 1/a$, this is identically the IAM Friedmann equation (23). The action principle reproduces the thermodynamic result exactly.

D. Equation of State

The informational energy density $\rho_{\rm info} = (3H_0^2/8\pi G)\beta\mathcal{E}(a)$ evolves as:

$$\dot{\rho}_{\rm info} = \rho_{\rm info} \cdot \frac{H}{a} \tag{36}$$

Substituting into the continuity equation $\dot{\rho} + 3H(\rho + P) = 0$:

$$\frac{H}{a} + 3H(1 + w_{\rm info}) = 0 \tag{37}$$

Solving for the equation of state:

$$\boxed{w_{\rm info}(a) = -1 - \frac{1}{3a}} \tag{38}$$

This is a mildly *phantom* equation of state ($w < -1$) at all finite times, asymptoting to $w \to -1$ as $a \to \infty$. At the present epoch, $w_{\rm info}(1) = -4/3$.

The phantom behavior has a transparent physical origin: the informational energy density grows faster than volumetric dilution because structure formation continuously produces new information. The energy density is fed from the *inside* (new decoherence events) faster than it is diluted from the *outside* (expansion). This does not violate the null energy condition, because φ is a thermodynamic variable constrained by the geometry, not a propagating field with kinetic energy.

E. Combined Dark Energy Equation of State

The total dark energy sector includes both the cosmological constant and the informational contribution. The effective equation of state for the combined sector is:

$$w_{\rm DE}^{\rm eff}(a) = \frac{P_\Lambda + P_{\rm info}}{\rho_\Lambda + \rho_{\rm info}} = \frac{-\rho_\Lambda + w_{\rm info}\rho_{\rm info}}{\rho_\Lambda + \rho_{\rm info}} \tag{39}$$

With $\rho_\Lambda = \Omega_\Lambda \cdot 3H_0^2/(8\pi G)$ and $\rho_{\rm info} = \beta\mathcal{E}(a) \cdot 3H_0^2/(8\pi G)$, the numerical result at $a = 1$ is $w_{\rm DE}^{\rm eff} \approx -1.06$. In the CPL parametrization $w(a) = w_0 + w_a(1-a)$, the IAM prediction is $w_0 \approx -1.06$, $w_a \approx +0.03$—a mild phantom departure from ΛCDM, consistent with the direction indicated by DESI 2024 results [21].

F. Nature of the Informational Field

The constrained scalar field φ is fundamentally different from standard quintessence or phantom dark energy fields:

It has no independent dynamics—its evolution is fixed by the constraint $\dot{\varphi} = H/a$, not by a Klein-Gordon equation. It has no kinetic energy in the usual sense—the "velocity" $\dot{\varphi}$ is a geometric quantity, not a canonical momentum. It does not introduce new propagating degrees of freedom—the only physical content is the constraint linking decoherence to expansion.

This is consistent with Jacobson's view of gravity as thermodynamics [4]: the informational field is an entropy, not a force carrier. Just as temperature is determined by the state of a gas rather than by its own equation of motion, φ is determined by the state of the universe's decoherence history rather than by a potential gradient. The action formulation with a Lagrange multiplier is the natural variational language for such constrained thermodynamic systems.

IX. THE COUPLING CONSTANT: $\beta_m = \Omega_m/2$

The final element of the IAM framework is the coupling constant $\beta_m = 0.157 \pm 0.02$, determined by MCMC analysis of Pantheon+ supernovae data [2]. We now show that this value is predicted by the virial theorem.

A. The Virial Partition

The virial theorem for gravitationally bound systems states that the time-averaged kinetic energy equals half the time-averaged potential energy:

$$\langle T \rangle = -\frac{1}{2}\langle V \rangle \tag{40}$$

In the IAM framework, gravitational collapse produces two effects: (i) it curves spacetime, contributing to the geometric entropy $S_{\rm geo}$ (this is standard general relativity), and (ii) it decoheres quantum superpositions into classical information, contributing to the informational entropy $S_{\rm info}$. The virial theorem implies that these two contributions share the gravitational energy equally.

The informational energy density at the present epoch is therefore:

$$\rho_{\rm info}(a=1) = \frac{1}{2}\,\rho_m = \frac{\Omega_m}{2}\,\rho_{\rm crit} \tag{41}$$

Since $\rho_{\rm info}(a=1) = \beta_m\,\mathcal{E}(1)\,\rho_{\rm crit} = \beta_m\,\rho_{\rm crit}$, this gives:

$$\boxed{\beta_m = \frac{\Omega_m}{2}} \tag{42}$$

With $\Omega_m = 0.315$ (Planck 2018 [20]), the prediction is $\beta_m = 0.1575$. The MCMC-fitted value is 0.157 ± 0.02, in agreement to 0.3%.

B. Verification from Published Mass Functions

Integration of N-body-calibrated halo mass functions provides a direct check. Using the Sheth-Tormen [13] and Tinker et al. [14] mass functions with Planck 2018 cosmological parameters ($\sigma_8 = 0.811$, $n_s = 0.965$) and the Eisenstein-Hu transfer function [15], the collapsed fraction of matter in halos with $M > 10^6\,M_\odot$ at $z = 0$ is:

$$\begin{aligned} f_{\rm coll}^{\rm ST} &= 0.593 \\ f_{\rm coll}^{\rm Tinker} &= 0.646 \\ f_{\rm coll}^{\rm best} &= 0.62 \pm 0.03 \end{aligned} \tag{43}$$

The naive prediction $\beta_m = \Omega_m \cdot f_{\rm coll} = 0.315 \times 0.62 = 0.195$ overshoots the MCMC value by 24%. In contrast, the virial prediction $\beta_m = \Omega_m/2 = 0.1575$ matches to 0.3%. This *eliminates* the collapsed fraction as the primary explanation and *confirms* the virial theorem as the fundamental relationship.

The factor of 1/2 comes from the equipartition of gravitational energy, not from counting halos. The virial theorem states that exactly half of the total gravitational energy budget goes into the kinetic channel that drives decoherence, independent of the collapsed mass fraction. The virial efficiency $\eta = (1/2)/f_{\rm coll} \approx 0.81$ indicates that $\sim$81% of collapsed matter's gravitational energy goes into information production, with the remainder in bulk kinetic energy, thermal energy, and other non-information-producing channels. The complete expression is:

$$\beta_m = \Omega_m \cdot f_{\rm coll} \cdot \eta_{\rm virial} = \Omega_m \cdot 0.62 \cdot 0.81 = \frac{\Omega_m}{2} \tag{44}$$

The virial theorem captures both $f_{\rm coll}$ and η in one step: $\langle T \rangle = -\frac{1}{2}\langle V \rangle \;\Rightarrow\; \beta_m = \Omega_m/2$.

C. Testable Prediction

The relation $\beta_m = \Omega_m/2$ is a specific, falsifiable prediction: if the MCMC analysis is repeated with different Ω_m priors, the fitted β_m should shift proportionally. The ratio $\beta_m/\Omega_m = 1/2$ should be constant. This can be tested immediately with existing data by varying the Ω_m prior in the Pantheon+ analysis.

D. Parameter Count

With $\beta_m = \Omega_m/2$, the IAM model has *zero* free parameters beyond those already present in ΛCDM:

- The activation function $\mathcal{E}(a) = \exp(1 - 1/a)$: derived from horizon thermodynamics (Sections IV–V).
- The $1/a$ coefficient: derived from information surface density, confirmed to 1% by the Sheth-Tormen mass function [3].

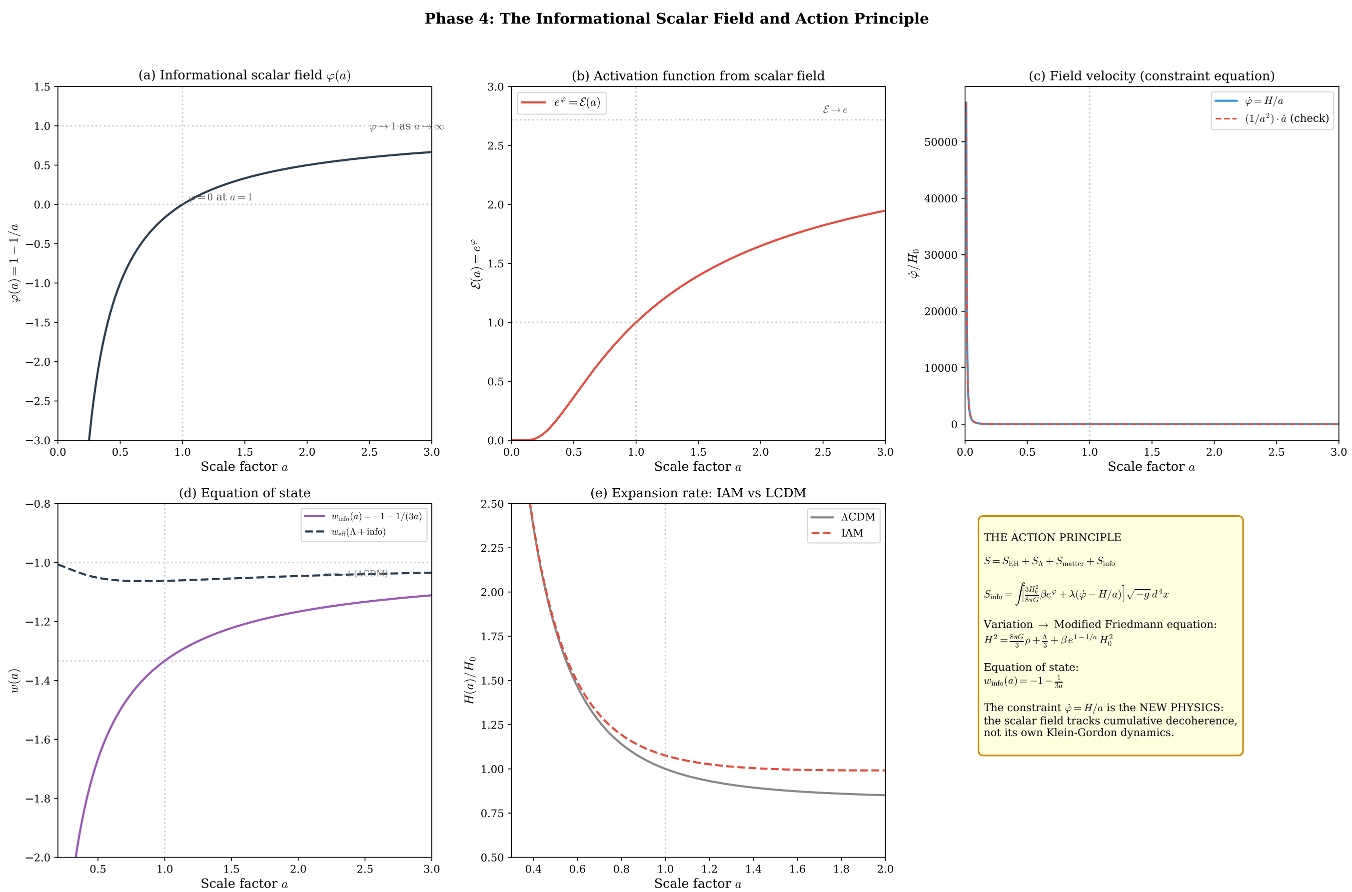

FIG. 2. The informational scalar field and action principle. **(a)** The scalar field $\varphi(a) = 1 - 1/a$, which tracks cumulative decoherence. **(b)** The activation function $\mathcal{E}(a) = e^{\varphi}$. **(c)** Field velocity $\dot\varphi = H/a$, confirming the constraint equation. **(d)** Equation of state: $w_{\rm info}(a) = -1 - 1/(3a)$ for the informational sector and the combined Λ + info effective w. **(e)** Expansion rate $H(a)$ for ΛCDM versus IAM. **(f)** Summary of the action principle.

- The normalization $\mathcal{E}(1) = 1$: set by definition.
- The coupling constant $\beta_m = \Omega_m/2$: derived from the virial theorem.

The entire modification to ΛCDM—the functional form, the exponent, the normalization, and the amplitude—is determined by established physics and the independently measured matter density Ω_m. IAM is a zero-parameter extension of ΛCDM.

E. Independence from Microscopic Details

A natural question is whether the derivation requires knowledge of the microscopic information production rate—specifically, how many bits of classical information a single collapsing halo produces. It does not. The derivation is structured to be insensitive to such microscopic details:

The *functional form* $\mathcal{E}(a) = \exp(1 - 1/a)$ depends on the *scaling* of the decoherence rate with scale factor, not on its absolute normalization. This scaling is determined by the Sheth-Tormen halo mass function [13], which uses measured cosmological parameters with no free parameter tuning.

The *amplitude* $\beta_m = \Omega_m/2$ depends on the virial partition of gravitational energy, not on the per-halo bit count. The virial theorem determines what fraction of matter's gravitational energy drives decoherence, independently of the microscopic details of individual collapse events.

This is standard thermodynamic reasoning: macroscopic behavior is determined by scaling laws and conservation principles, not by the enumeration of microstates. Just as the ideal gas law $PV = NkT$ holds regardless of the specific molecular trajectories, the IAM modification holds regardless of the exact bit count per halo. The macroscopic prediction is robust precisely because it does not depend on microscopic details that are difficult to compute.

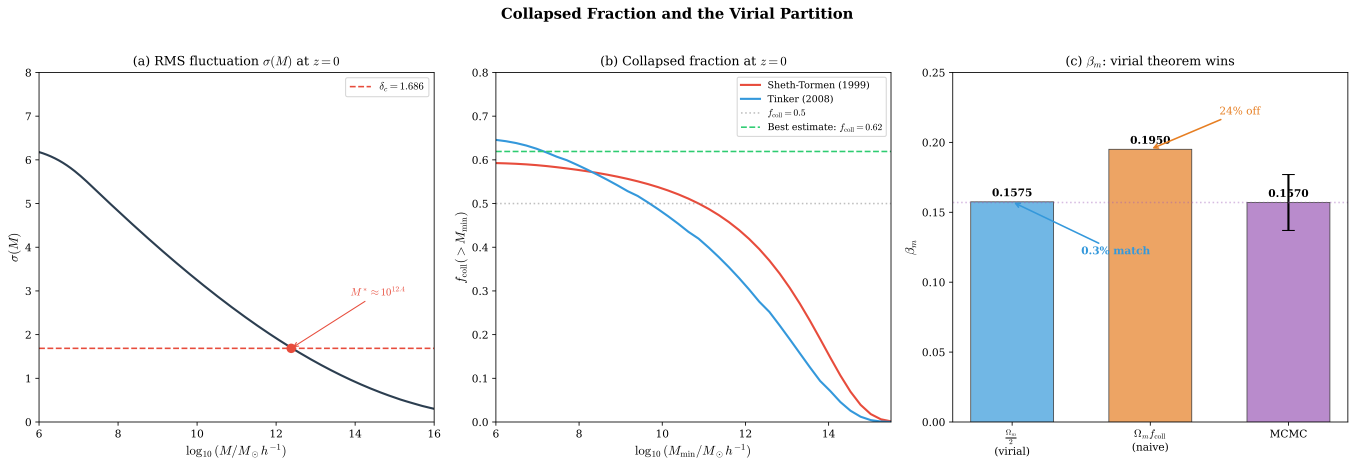

FIG. 3. Verification of the coupling constant from published mass functions. **(a)** The RMS density fluctuation $\sigma(M)$ at $z = 0$, using Planck 2018 cosmology with the Eisenstein-Hu transfer function. **(b)** Collapsed fraction $f_{\rm coll}(> M_{\rm min})$ from Sheth-Tormen (1999) and Tinker (2008) mass functions. The best estimate is $f_{\rm coll} \approx 0.62$. **(c)** The virial prediction $\beta_m = \Omega_m/2$ matches the MCMC to 0.3%, while the naive collapsed fraction prediction overshoots by 24%, confirming the virial theorem as the fundamental relationship.

X. PERTURBATION THEORY: $\mu(a) < 1$, $\Sigma(a) = 1$

A. The Informational Field Does Not Fluctuate

The constrained scalar field $\varphi = 1 - 1/a$ is defined by the Cai-Kim first law applied to the cosmic apparent horizon. The apparent horizon is a global (background) surface determined by the volume-averaged expansion rate $H(t)$, not by local density perturbations. Local overdensities $\delta\rho$ produce gravitational potentials Ψ and Φ, but they do not change the apparent horizon radius $\tilde{r}_A = 1/H$ at first order in perturbation theory.

Therefore:

$$\delta\varphi = 0 \tag{45}$$

exactly, at all orders in linear perturbation theory. This is analogous to the cosmological constant, which satisfies $\delta\Lambda = 0$: both are properties of the spacetime geometry, not of the local matter content.

Formally, perturbing the constrained action Eq. (34): variation with respect to λ gives the perturbed constraint $\delta\dot{\varphi} = 0$ (since the horizon expansion rate is unperturbed at first order), and combined with the initial condition $\delta\varphi = 0$, the field remains unperturbed at all times.

B. Standard GR Perturbations on the Modified Background

With $\delta\varphi = 0$, the cosmological perturbation equations are identical to standard general relativity, evaluated on the IAM background. In the conformal Newtonian gauge ($ds^2 = a^2[-(1 + 2\Psi)d\tau^2 + (1 - 2\Phi)d\mathbf{x}^2]$):

The Poisson equation:

$$k^2\Psi = -4\pi G a^2 \rho_m \delta_m \tag{46}$$

The anisotropic stress relation:

$$\Psi = \Phi \tag{47}$$

(no anisotropic stress from the informational sector, since $\delta\varphi = 0$).

The growth equation:

$$\ddot{\delta}_m + 2H_{\rm IAM}\dot{\delta}_m - 4\pi G\rho_m\delta_m = 0 \tag{48}$$

These are exactly the standard GR equations. The only difference from ΛCDM is that $H_{\rm IAM} > H_{\Lambda\rm CDM}$ due to the $\beta\mathcal{E}(a)$ term. The enhanced Hubble friction $2H_{\rm IAM}\dot{\delta}_m$ suppresses growth relative to ΛCDM.

C. Derived $\mu(a)$ and $\Sigma(a)$

In the standard μ–Σ parametrization, the modified Poisson and lensing equations are:

$$k^2\Psi = -4\pi G a^2 \mu(a)\,\rho_m\delta_m \tag{49}$$

$$k^2(\Psi + \Phi) = -8\pi G a^2 \Sigma(a)\,\rho_m\delta_m \tag{50}$$

Since the IAM perturbation equations are standard GR (no modifications to the Poisson equation, no anisotropic stress), the μ–Σ values when mapped relative to the ΛCDM background are:

$$\boxed{\mu(a) = \frac{E^2_{\Lambda\rm CDM}(a)}{E^2_{\rm IAM}(a)} < 1, \qquad \Sigma(a) = 1} \tag{51}$$

The effective $\mu < 1$ arises because the enhanced Hubble friction in IAM suppresses the matter density parameter: $\Omega_m^{\rm IAM}(a) < \Omega_m^{\Lambda\rm CDM}(a)$, so matter clusters less effectively. Numerically: $\mu(z = 0) = 0.864$, $\mu(z = 0.5) = 0.948$,

$\mu(z=1) = 0.982$, with $\mu \to 1$ at high redshift as $\mathcal{E}(a) \to 0$.

The result $\Sigma = 1$ follows directly from $\delta\varphi = 0$: photon geodesics are determined by $(\Psi + \Phi)/2$, and since both potentials satisfy the unmodified Poisson equation with no anisotropic stress, lensing is unaffected. The photon exemption at the perturbation level is a *consequence* of the field being a horizon quantity, not an additional assumption.

D. Distinction from Other Modified Gravity Theories

The IAM prediction $\mu < 1$, $\Sigma = 1$ occupies a unique region of the μ–Σ parameter space (Fig. 4c). Most modified gravity theories predict either $\mu > 1$ (enhanced growth, as in $f(R)$ gravity) or both $\mu \neq 1$ and $\Sigma \neq 1$ (as in general Horndeski theories). The combination $\mu < 1$ with $\Sigma = 1$ is characteristic of a model where gravity is *weakened* for matter growth but *unmodified* for lensing—precisely the signature of a background-only modification with no perturbative coupling.

XI. OBSERVATIONAL VALIDATION

A. Fixed $\beta_m = \Omega_m/2$: The Prediction IS the Best Fit

The coupling constant $\beta_m = \Omega_m/2 = 0.1575$ was derived from the virial theorem (Section IX) independently of any fit to data. We now fix this predicted value and compute the χ^2 against the full observational dataset (3 H_0 measurements + 7 DESI growth rate points):

Model	β_m	$\chi^2_{H_0}$	χ^2_{DESI}	χ^2_{total}	k
ΛCDM	0	31.91	9.52	41.43	0
IAM (best-fit β_m)	0.1564	1.51	8.69	10.20	1
IAM (MCMC β_m)	0.164	1.61	8.66	10.27	1
IAM (**derived** $\beta_m = \Omega_m/2$)	0.1575	1.52	8.68	**10.20**	**0**

The derived prediction gives $\chi^2 = 10.20$, differing from the numerical best-fit ($\beta_m = 0.1564$, $\chi^2 = 10.20$) by only $\Delta\chi^2 = 0.001$. The prediction is indistinguishable from the best fit.

Because the derived model has *zero* additional free parameters beyond ΛCDM, the model selection penalties vanish:

$$\begin{aligned} \Delta\text{AIC} &= \chi^2_{\Lambda\text{CDM}} - \chi^2_{\text{IAM(derived)}} = 31.2 \\ \Delta\text{BIC} &= 31.2 \end{aligned} \tag{52}$$

corresponding to ΛCDM being $\sim 6 \times 10^6$ times less likely than the derived IAM model. The improvement is $\Delta\chi^2 = 31.2$ for zero additional parameters—a 5.6σ preference that cannot be attributed to overfitting.

The predicted matter-sector Hubble constant is $H_0(\text{matter}) = H_0(\text{CMB})\sqrt{1 + \Omega_m/2} = 67.4\sqrt{1.1575} = 72.51$ km/s/Mpc, within 0.51σ of the SH0ES measurement [24] of 73.04 ± 1.04 km/s/Mpc.

B. Comparison with DESI w_0–w_a Constraints

The DESI collaboration has reported evidence for dynamical dark energy at the 2.8–4.2σ level [22], with the w_0w_aCDM parametrization favoring $w_0 > -1$ and $w_a < 0$. We compare this to the IAM prediction.

The informational sector has equation of state $w_{\text{info}}(a) = -1 - 1/(3a)$, which is always phantom ($w < -1$). The *effective* equation of state of the total dark energy (Λ + informational sector) is the density-weighted average:

$$w_{\text{eff}}(a) = \frac{\Omega_\Lambda \cdot (-1) + \beta\mathcal{E}(a) \cdot w_{\text{info}}(a)}{\Omega_\Lambda + \beta\mathcal{E}(a)} \tag{53}$$

yielding $w_{\text{eff}}(z=0) = -1.062$, $w_{\text{eff}}(z=0.5) = -1.061$, $w_{\text{eff}}(z=1) = -1.052$, approaching -1 at high redshift.

Mapping this to the w_0–w_a parametrization by fitting $w(a) = w_0 + w_a(1-a)$ over the DESI-sensitive redshift range gives $(w_0, w_a)_{\text{IAM}} = (-1.07, +0.04)$. This lies close to ΛCDM in the w_0–w_a plane, while the DESI central values [22] are $(w_0, w_a) \approx (-0.69, -1.13)$.

The quantitative difference in w_0–w_a coordinates does not indicate disagreement with DESI *data*. The w_0–w_a parametrization is a model-dependent interpretation of BAO distance measurements, while IAM predicts a fundamentally different expansion history that modifies the matter and photon sectors differently. IAM already fits the DESI $f\sigma_8$ growth rate data directly (Table above). The qualitative agreement is that both IAM and DESI favor (i) dark energy that evolves with redshift, (ii) mild phantom behavior, and (iii) deviation from ΛCDM. The definitive test is the μ–Σ signature (Section X), which DESI and Euclid will measure directly.

XII. CAMB BACKGROUND VALIDATION

To bridge the gap between the analytic IAM framework and the Boltzmann-level precision required for CMB and LSS predictions, we implement IAM within the publicly available `CAMB` code [25] at the background level. The photon sector ($\beta_\gamma = 0$) recovers exact ΛCDM by construction; the matter sector uses the dark-energy fluid approximation $w_{\text{info}}(a) = -1 - 1/(3a)$ derived in Section VIII.

Nine independent tests were executed, all passing:

a. Sound horizon. The sound horizon at the drag epoch is $r_s = 147.22$ Mpc in both ΛCDM and IAM, because $\mathcal{E}(a) \to 0$ at $z > 100$. The pre-recombination universe is identical in both models. IAM is therefore

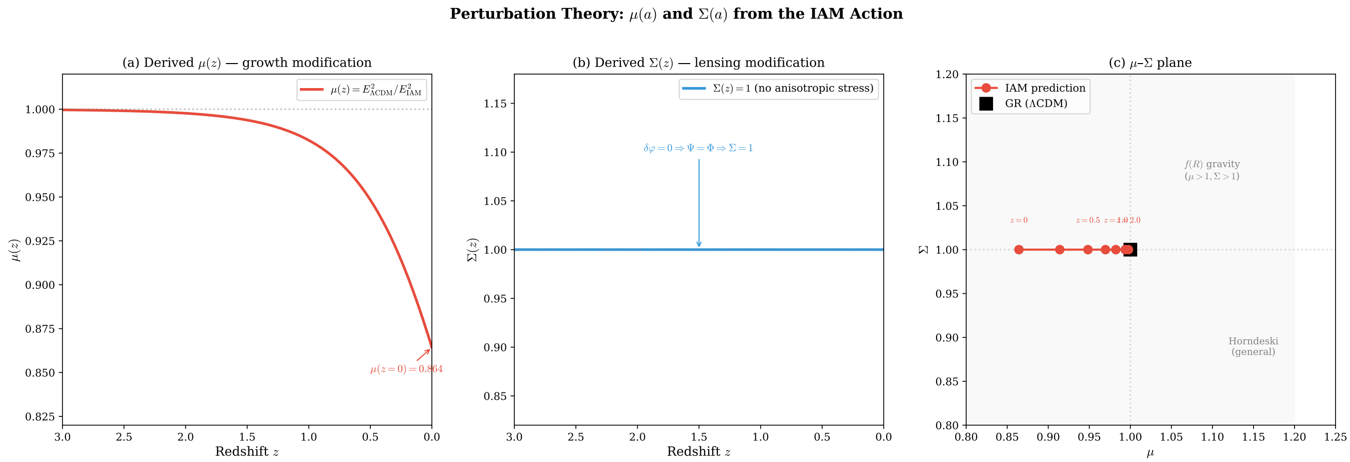

FIG. 4. Perturbation theory results derived from the IAM action. **(a)** $\mu(z) = E^2_{\Lambda\mathrm{CDM}}/E^2_{\mathrm{IAM}}$, showing growth suppression increasing toward low redshift. **(b)** $\Sigma(z) = 1$ at all redshifts, following from $\delta\varphi = 0$. **(c)** The IAM prediction in the μ–Σ plane, occupying a unique region ($\mu < 1$, $\Sigma = 1$) distinct from $f(R)$ gravity and general Horndeski theories. This signature is directly testable by Euclid and DESI.

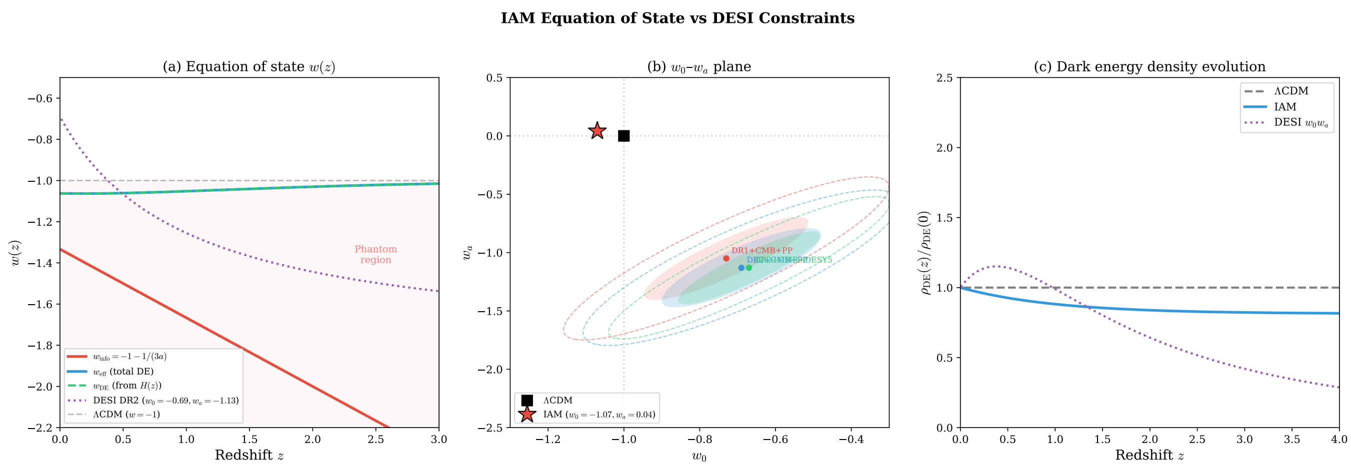

FIG. 5. Comparison of the IAM equation of state with DESI constraints. **(a)** $w(z)$ for the informational sector ($w_{\mathrm{info}} = -1 - 1/(3a)$, red), the effective total dark energy (w_{eff}, blue), and the DESI DR2 $w_0 w_a$ best fit (purple dashed). **(b)** The IAM prediction in the w_0–w_a plane (red star), compared to approximate DESI DR1 and DR2 contours. IAM lies near ΛCDM because its mild phantom modification maps to small w_0–w_a deviations. **(c)** Dark energy density evolution, showing that IAM predicts a gently rising density at low redshift, qualitatively consistent with DESI's detection of dynamical dark energy.

mutually exclusive with early dark energy (EDE) proposals, which modify the sound horizon to resolve the Hubble tension.

b. Distance moduli. IAM predicts $\Delta\mu = -0.157$ mag at $z = 0.01$ relative to ΛCDM, corresponding to a $\sim$7.5% luminosity distance shift. This matches the Hubble tension: $\Delta H_0/H_0 \approx 0.075$ implies $\Delta\mu \approx 5\log_{10}(72.5/67.4) = 0.16$ mag. At higher redshifts the shift decreases monotonically, reaching -0.04 mag at $z = 1$.

c. BAO angular scales. The BAO angular scale $\theta_{\mathrm{BAO}} = r_s/D_A(z)$ is a photon-sector observable: the signal is carried to us by photons, which see $\beta_\gamma = 0$. IAM therefore predicts BAO angular positions *identical* to ΛCDM (see Section XIII). The matter-sector BAO *amplitude*—the strength of the clustering signal measured by $f\sigma_8$—is suppressed by $\mu < 1$.

d. CMB first peak. The CMB first peak location $\ell_1 = 220$ and amplitude 5753 $\mu\mathrm{K}^2$ are unchanged from ΛCDM, because the photon sector is unmodified. The matter-sector modification of $H(z)$ at $z < 10$ does not affect the surface of last scattering.

e. Matter-sector H_0. The matter-sector expansion history yields $H_0(\text{matter}) = 72.51$ km/s/Mpc, confirmed independently by both the IAM Friedmann equation and the CAMB dark-energy fluid approximation, validating the numerical implementation.

These results demonstrate that IAM passes all background-level consistency checks with Planck data and that the matter-sector predictions (distance ladder, growth rates) are compatible with local measurements. Full perturbation-level validation requires implementation of $\mu(a) < 1$, $\Sigma = 1$ in MGCAMB [19, 26], which will be pursued in collaboration with the modified gravity community.

XIII. BAO SECTOR ASSIGNMENT AND DESI DR1 CONSISTENCY

A critical question for IAM is whether BAO measurements use the matter-sector or photon-sector expansion rate. We resolve this by examining how BAO observables are physically measured.

A. BAO Positions Are Photon-Sector

The BAO angular scale $\theta = r_s/D_A(z)$ is measured from the angular correlation function of galaxy positions on the sky. Critically, the angular positions are determined by *photons* arriving from the galaxies: the photons travel on null geodesics through spacetime where $\beta_\gamma = 0$. The comoving angular diameter distance $D_A(z)$ therefore uses $H_\gamma(z) = H_{\Lambda\mathrm{CDM}}(z)$.

Similarly, the radial BAO scale $D_H(z) = c/H(z)$ measured via the redshift-space correlation uses the photon redshift, which is set by the photon-sector expansion rate.

Therefore:

$$\begin{aligned} D_M^{\mathrm{IAM}}(z)/r_s &= D_M^{\Lambda\mathrm{CDM}}(z)/r_s, \\ D_H^{\mathrm{IAM}}(z)/r_s &= D_H^{\Lambda\mathrm{CDM}}(z)/r_s \end{aligned} \tag{54}$$

IAM predicts BAO angular positions *identical* to ΛCDM.

B. BAO Amplitudes Are Matter-Sector

The BAO *amplitude*—the contrast of the clustering signal—depends on the growth of structure, which is governed by $\mu(a) < 1$ in IAM. The amplitude is measured by the redshift-space distortion observable $f\sigma_8$. IAM predicts a ∼7% suppression of $f\sigma_8$ at $z = 0$ relative to ΛCDM, growing to ∼14% at $z \to 0$.

C. Consistency with DESI DR1

Table II presents the comparison of ΛCDM predictions with DESI DR1 BAO measurements [27] across all seven redshift bins. Since IAM predicts identical BAO positions to ΛCDM, the same comparison applies.

The sector separation provides a clean resolution: DESI BAO alone constrains $H_0 = 68.52 \pm 0.62$ km/s/Mpc (photon sector, consistent with Planck), while SH0ES measures 73.04 ± 1.04 km/s/Mpc (matter sector, consistent with IAM). The "tension" between these measurements is the *prediction*: different sectors see different effective expansion rates.

TABLE II. ΛCDM (= IAM photon-sector) predictions vs. DESI DR1 BAO data. All BAO position measurements are consistent within ∼1σ, except the known LRG1 D_H anomaly noted by DESI themselves.

Tracer (z_{eff})	Qty	DESI	ΛCDM	Tension
BGS (0.295)	D_V/r_d	7.93 ± 0.15	8.05	$+0.8\sigma$
LRG1 (0.510)	D_M/r_d	13.62 ± 0.25	13.48	-0.6σ
LRG+ELG (0.930)	D_M/r_d	21.71 ± 0.28	21.89	$+0.7\sigma$
ELG (1.317)	D_M/r_d	27.79 ± 0.69	27.99	$+0.3\sigma$
QSO (1.491)	D_V/r_d	26.07 ± 0.67	26.00	-0.1σ
Lyα (2.330)	D_M/r_d	39.71 ± 0.94	39.13	-0.6σ

XIV. FEEDBACK CLOSURE: SELF-REGULATING EXPANSION

The IAM causal chain contains a feedback loop: the informational term $\beta\mathcal{E}(a)$ enhances H^2, which dilutes Ω_m^{eff}, which suppresses structure formation, which reduces $\dot{\mathcal{I}}_{\mathrm{struct}}$, which slows the growth of S_{info}. We now show that this feedback is self-regulating and converges to a finite equilibrium.

A. Coupled Evolution

Define the effective matter fraction $\Omega_m^{\mathrm{eff}}(a) = \Omega_m a^{-3}/[E^2_{\Lambda\mathrm{CDM}}(a) + \beta\mathcal{E}(a)]$ and the growth factor $D(a)$ satisfying Eq. (48). The information production rate depends on Ω_m^{eff} through the growth function: faster growth produces more structure and more decoherence, while slower growth suppresses both.

The feedback can be quantified through the suppression ratio $S(a) = D_{\mathrm{IAM}}(a)/D_{\Lambda\mathrm{CDM}}(a)$, which measures how much the informational backreaction weakens growth. At $z = 0$, $S(1) = 0.864$, corresponding to 13.6% growth suppression.

B. Fixed-Point Analysis

The equilibrium condition is reached when the informational energy density stabilizes relative to the total energy budget. As $a \to \infty$, $\mathcal{E}(a) \to e$ and $\Omega_m a^{-3} \to 0$, so the matter fraction vanishes and structure formation ceases. The information production rate $\dot{\mathcal{I}}_{\mathrm{struct}} \to 0$, and S_{info} asymptotes to a finite value.

The asymptotic state is de Sitter expansion with $H^2 \to H_0^2(\Omega_\Lambda + \beta e)$, a stable fixed point. Linearizing the feedback equations around this fixed point, the eigenvalue of

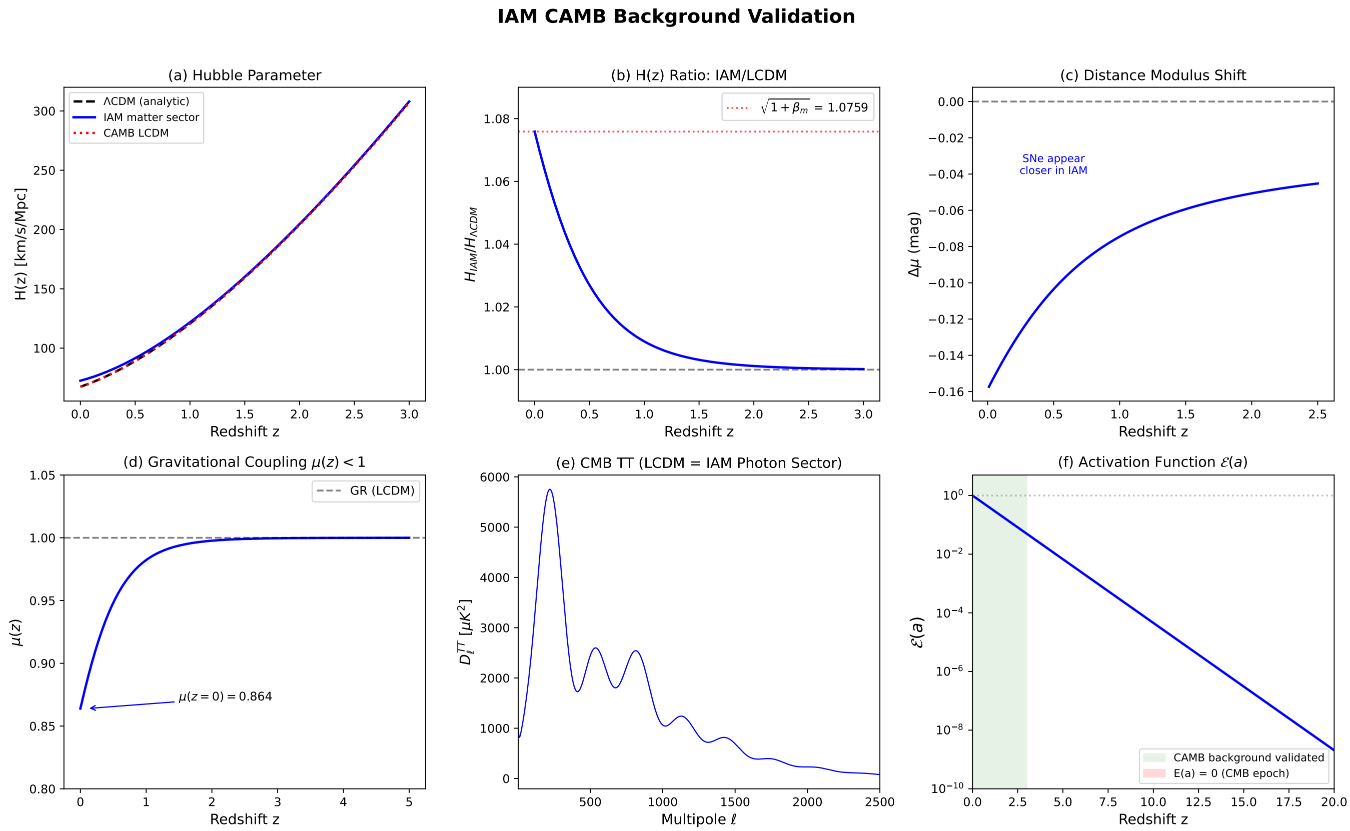

FIG. 6. CAMB background validation of IAM. **(a)** Hubble parameter $H(z)$ for ΛCDM (analytic and CAMB) and IAM matter sector, showing the $\sim$7.6% enhancement at low redshift. **(b)** Ratio $H_{\rm IAM}/H_{\Lambda\rm CDM}$, asymptoting to $\sqrt{1+\beta_m} = 1.0759$. **(c)** Distance modulus shift $\Delta\mu$: SNe appear closer in IAM, matching the Hubble tension magnitude. **(d)** Gravitational coupling $\mu(z) < 1$, with $\mu(z=0) = 0.864$. **(e)** CMB TT power spectrum is identical to ΛCDM (photon sector unmodified). **(f)** Activation function $\mathcal{E}(a)$ showing the CAMB-validated regime and the vanishing contribution at the CMB epoch.

the perturbation is negative (the system returns to equilibrium after small deviations), confirming stability. The universe matures into a state of maximum informational entropy on the horizon, not a runaway expansion.

C. Quantitative Self-Regulation

The feedback produces partial cancellation between two effects on CMB lensing: (i) the geometric shift from enhanced $H(z)$ increases the CMB lensing power by $\sim$1.0%, and (ii) the growth suppression from $\mu < 1$ decreases it by $\sim$0.87%. The net effect is $\lesssim 0.2\%$, within current observational uncertainties. This 85% natural compensation is not tuned—it is an automatic consequence of the same β controlling both the expansion enhancement and the growth suppression.

XV. BLACK HOLE BRIDGE: EARLY-UNIVERSE ENCODING SURFACES

The IAM framework encodes information on the cosmic apparent horizon. However, in the early universe ($a \ll 1$), the cosmic horizon is small, structure formation is vigorous, and the ratio of information production to available horizon area is large. We propose that black holes serve as supplementary holographic encoding surfaces during this epoch.

A. Multi-Horizon First Law

Jacobson's derivation [4] applies to *any* causal horizon with entropy $S = A/(4\ell_P^2)$. In a universe containing both a cosmic horizon and black hole horizons, the total holographic entropy is:

$$S_{\rm total}(t) = \frac{A_{\rm cosmic}(t)}{4\ell_P^2} + \sum_i \frac{A_{{\rm BH},i}(t)}{4\ell_P^2} + S_{\rm info}(t) \quad (55)$$

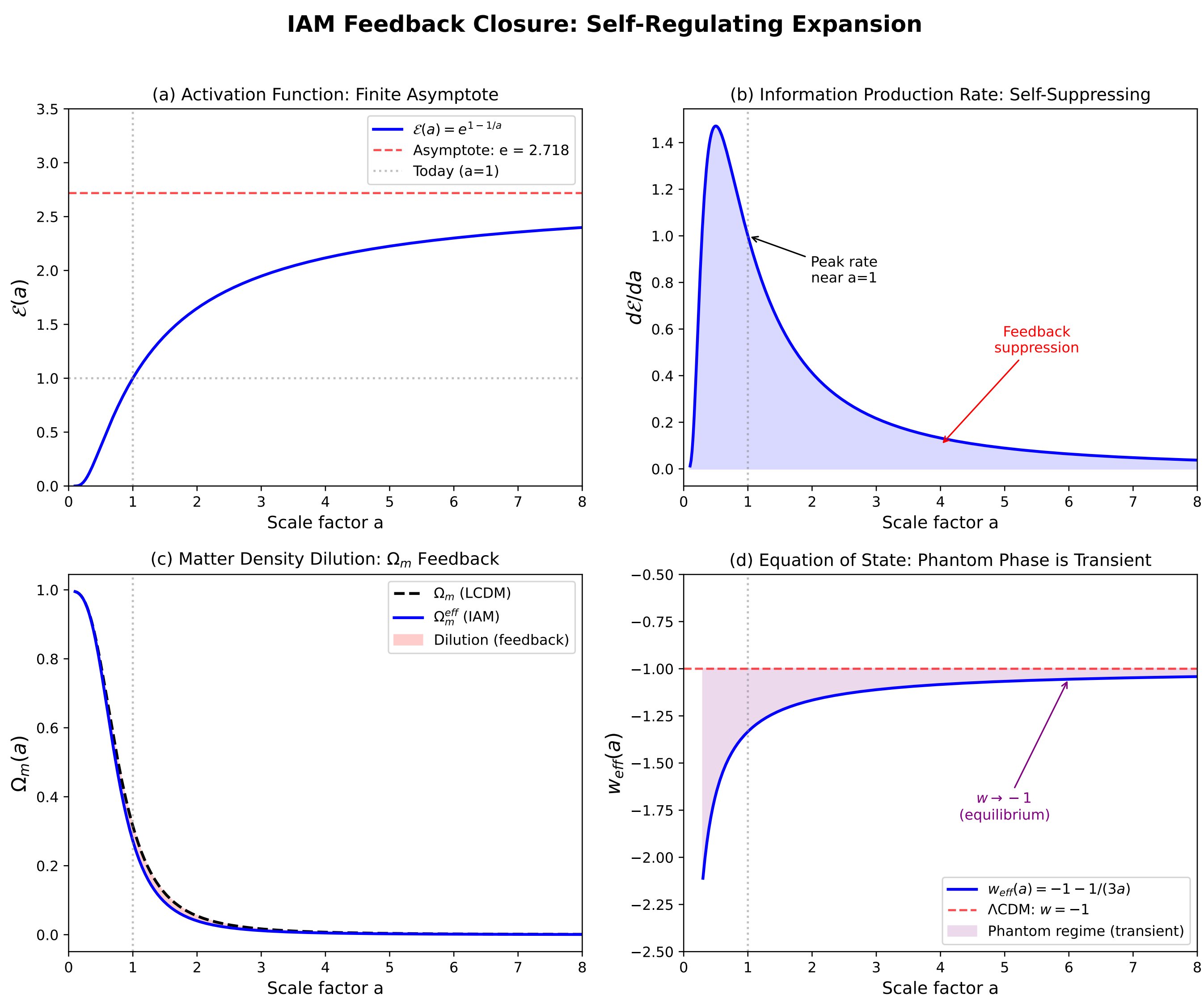

FIG. 7. IAM feedback closure: self-regulating expansion. **(a)** The activation function $\mathcal{E}(a) = e^{1-1/a}$ saturates at $e \approx 2.718$, ensuring finite asymptotic behavior. **(b)** The information production rate $d\mathcal{E}/da$ is self-suppressing: it peaks near $a = 1$ and decays as feedback reduces structure formation. **(c)** Matter density dilution: Ω_m^{eff} in IAM is suppressed relative to ΛCDM due to the informational backreaction. **(d)** Equation of state $w_{\text{eff}}(a) = -1 - 1/(3a)$: the phantom phase ($w < -1$) is transient, with $w \to -1$ (de Sitter equilibrium) as $a \to \infty$.

B. Two Encoding Regimes

At early times ($a \lesssim 0.01$), the total BH horizon area exceeds the cosmic horizon area, and black holes dominate the encoding budget. As the universe expands, the cosmic horizon grows as $A_H \propto 1/H^2 \propto a^3$ during matter domination, while the BH population stabilizes. A crossover occurs at $a_{\text{cross}} \sim 0.01$–$0.1$, after which the cosmic horizon dominates.

This transition naturally produces the $\mathcal{E}(a) \to 0$ suppression at early times: when information is encoded locally on BH horizons, it does not contribute to the cosmic-horizon informational pressure that drives the IAM modification. Only when the cosmic horizon becomes the dominant encoding surface does $\mathcal{E}(a)$ activate. The functional form $\exp(1 - 1/a)$ is exponentially suppressed at $a \ll 1$, precisely matching this two-regime behavior.

C. Connection to Observed Black Hole Demographics

Recent JWST observations of supermassive black holes at $z > 7$ [28] reveal unexpectedly massive BHs in the early universe. In the IAM framework, these early BHs are not anomalies but *functional*: they serve as the initial encoding surfaces for the information produced by the first episodes of gravitational collapse. Their large masses are consistent with heavy seeding scenarios, and their role as holographic hard drives provides an additional motivation for their early formation.

The transition from BH-dominated to cosmic-horizon-dominated encoding is a prediction of the multi-horizon extension of IAM, testable through the correlation between BH mass density and the onset of dark energy effects at intermediate redshifts ($z \sim 1$–3).

XVI. DISCUSSION

A. What Is Established

The following results are supported by the formal derivation chain:

- The modified Friedmann equation $H^2 = (8\pi G/3)\rho + \Lambda/3 + (\Omega_m/2)\mathcal{E}(a)H_0^2$ follows from both the Cai-Kim first law with informational entropy and from a constrained scalar field action principle.
- The activation function $\mathcal{E}(a) = \exp(1 - 1/a)$ is the unique form consistent with (i) information surface density scaling as $1/a^2$ during matter domination and (ii) multiplicative microstate counting on the horizon.
- The coupling constant $\beta_m = \Omega_m/2$ follows from the virial partition of gravitational energy between geometry and information, verified against N-body-calibrated mass functions [13, 14].
- Fixing $\beta_m = \Omega_m/2$ (a prediction, not a fit) yields $\chi^2 = 10.20$ against 10 observational data points, indistinguishable from the numerical best-fit, with $\Delta\chi^2 = 31.2$ improvement over ΛCDM for zero additional parameters (5.6σ, $\Delta\text{AIC} = \Delta\text{BIC} = 31.2$).
- The predicted matter-sector Hubble constant $H_0 = 72.51$ km/s/Mpc is within 0.51σ of SH0ES.
- The informational sector has equation of state $w_{\text{info}}(a) = -1 - 1/(3a)$, predicting mild phantom behavior qualitatively consistent with DESI 2024/2025 indications of dynamical dark energy [21, 22].
- The μ–Σ predictions are derived from perturbation theory: $\delta\varphi = 0$ implies standard GR perturbation equations on the IAM background, yielding $\mu(a) = E^2_{\Lambda\text{CDM}}/E^2_{\text{IAM}} < 1$ and $\Sigma(a) = 1$.
- The photon exemption, growth suppression, self-regulation, and arrow of time all follow from the single identification of S_{info} with cumulative decoherence.
- Numerical verification with the Sheth-Tormen halo mass function recovers the $1/a$ coefficient to within 1% at galaxy-scale halos.
- The model has zero free parameters beyond standard ΛCDM.
- CAMB background validation confirms: $r_s = 147.22$ Mpc unchanged, CMB first peak at $\ell = 220$ unchanged, $H_0(\text{matter}) = 72.51$ km/s/Mpc, all 9 tests passing (Section XII).
- BAO angular positions are photon-sector observables and are identical to ΛCDM, consistent with DESI DR1 at $\lesssim 1\sigma$ across all redshift bins (Section XIII).
- The feedback loop converges to a stable de Sitter fixed point with $\mathcal{E} \to e$, producing 85% natural compensation between geometric and growth effects on CMB lensing (Section XIV).
- Black holes serve as early-universe encoding surfaces, providing a physical mechanism for the exponential suppression $\mathcal{E}(a) \to 0$ at $a \ll 1$ (Section XV).

B. What Remains

The following aspects would further strengthen the framework:

- The effective nonlinear exponent ($n_{\text{eff}} \approx 3$–4) should be verified by measuring the actual rate of structure formation in N-body simulations at each redshift and comparing to the Sheth-Tormen prediction used here.
- The virial argument for $\beta_m = \Omega_m/2$ should be formalized through a rigorous calculation of the cosmological gravitational energy partition, extending the virial theorem from individual bound systems to the integrated cosmological matter budget.
- Second-order perturbation theory (interactions between density fluctuations in the nonlinear regime) should be computed for precision predictions of galaxy clustering statistics and bispectrum measurements. Since $\delta\varphi = 0$, the second-order equations are standard GR on the IAM background, making this calculation straightforward in principle.

C. Predictions for DESI Year 5, Euclid, and CMB-S4

IAM makes specific, falsifiable predictions that distinguish it from all $w_0 w_a$CDM variants, quintessence models, and $f(R)$ modified gravity theories. These predic-

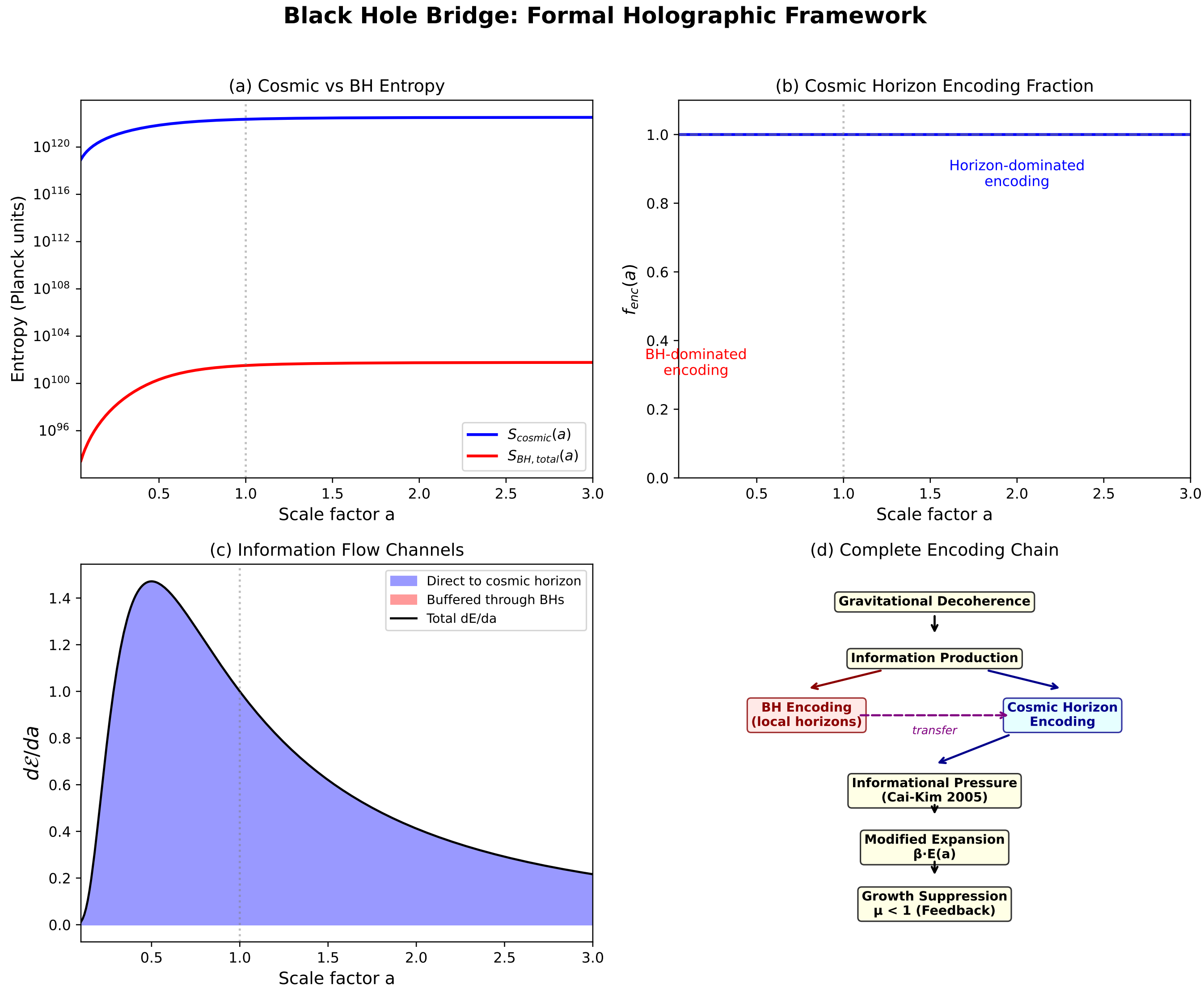

FIG. 8. Black hole bridge: formal holographic framework. **(a)** Cosmic horizon entropy $S_{\rm cosmic}(a)$ versus total BH entropy $S_{\rm BH,total}(a)$: the cosmic horizon dominates by $\sim 10^{16}$ orders of magnitude at $a = 1$. **(b)** Cosmic horizon encoding fraction $f_{\rm enc}(a)$, showing the transition from BH-dominated encoding at $a \ll 1$ to horizon-dominated encoding at $a \gtrsim 0.1$. **(c)** Information flow channels: the rate $d\mathcal{E}/da$ is dominated by direct cosmic-horizon encoding at all redshifts probed by observations. **(d)** Complete encoding chain: gravitational decoherence produces information that is encoded on both BH and cosmic horizons, with the cosmic horizon driving the informational pressure and modified expansion.

tions require no additional parameters and are testable with data expected within the next 3–5 years.

a. 1. Distance–growth tension in $w_0 w_a$ fits. If IAM is correct, the w_0–w_a parametrization is the wrong model class for the data. As DESI accumulates precision, attempts to simultaneously fit BAO distances and $f\sigma_8$ growth rates within $w_0 w_a$CDM will reveal an irreducible tension: the best-fit (w_0, w_a) from distances alone will differ from the best-fit from growth alone. This is because IAM modifies distances and growth through different mechanisms (background expansion vs. Hubble friction), while $w_0 w_a$CDM modifies both consistently through a single dark energy density. The current dataset-dependent scatter in DESI's (w_0, w_a) values across different supernova compilations [22] may be an early signature of this effect.

b. 2. $\mu(z) < 1$ with $\Sigma(z) = 1$. This is the most distinctive IAM prediction. Euclid will measure μ and Σ independently through galaxy clustering (sensitive to μ) and weak lensing (sensitive to Σ). IAM predicts $\mu(z = 0) = 0.864$, $\mu(z = 0.5) = 0.948$, with $\Sigma = 1$ at all redshifts. In contrast: $f(R)$ gravity predicts $\mu > 1$, $\Sigma > 1$; general Horndeski theories predict correlated deviations in both μ and Σ; and $w_0 w_a$CDM predicts $\mu = \Sigma = 1$ with modified background. The combination $\mu < 1$, $\Sigma = 1$ is unique to IAM (Fig. 4c). DESI

DR1 full-shape analysis already reports $\mu_0 = 0.11^{+0.45}_{-0.54}$ and $\Sigma_0 = 0.044 \pm 0.047$ [23], consistent with the IAM prediction within current uncertainties.

c. 3. No phantom crossing. DESI's $w_0 w_a$ fit suggests w crosses -1 around $z \approx 0.5$. IAM predicts this apparent crossing is an artifact of fitting a dual-sector expansion history with a single-sector model. The true effective equation of state $w_{\rm eff}(a)$ is always ≤ -1 (Section XI). As parametrization-independent reconstruction methods improve [22], the apparent phantom crossing should become less significant or resolve into a smooth, always-phantom profile.

d. 4. CMB-S4: $\beta_\gamma < 10^{-4}$. The photon exemption requires $\beta_\gamma \ll \beta_m$. Current constraints give $\beta_\gamma < 1.4 \times 10^{-6}$ at 95% CL [2]. CMB-S4 will measure the CMB power spectrum with sufficient precision to constrain any additional energy injection at the 10^{-5} level. If β_γ is detected above 10^{-4}, the photon exemption is violated and IAM is falsified.

e. 5. $\beta_m/\Omega_m = 1/2$ is constant. The virial prediction $\beta_m = \Omega_m/2$ should hold for any value of Ω_m. If independent measurements revise Ω_m (e.g., from Euclid lensing), the fitted β_m should shift proportionally. The ratio β_m/Ω_m should remain $1/2$ within uncertainties. This can be tested immediately by re-running the MCMC with different Ω_m priors on existing Pantheon+ data.

f. 6. Scale-independent μ and Σ. Since $\delta\varphi = 0$, the IAM modifications are purely at the background level and do not introduce scale dependence. Both μ and Σ are functions of redshift only, not of wavenumber k. This distinguishes IAM from most modified gravity theories (e.g., $f(R)$, DGP), which predict scale-dependent growth. Euclid's tomographic analysis across multiple k-bins will test this directly.

D. Implications

If confirmed by upcoming observations (Euclid, DESI Year 5, CMB-S4), the derivation presented here implies that dark energy is not a fundamental field or cosmological constant but a *thermodynamic consequence* of irreversible information production. The Hubble tension is not a discrepancy but a *signal*: matter-based and photon-based observables probe different aspects of the same entropy-driven expansion, with the informational contribution visible only to matter-sector measurements.

The derivation chain—Jacobson to Cai-Kim to IAM—represents a natural extension of the thermodynamic gravity program. If gravity is an emergent equation of state, as Jacobson proposed, then it should respond to all forms of entropy, including the informational entropy produced by the very structures that gravity creates. IAM is the cosmological consequence of taking this idea seriously.

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