# PROBLEM SET 1

1. Mathematical Formulation
    - Decision Variables
        - Space allocated for 'Cardio': xC
        - Space allocated for 'Strength': xS
        - Space allocated for 'Group Fitness': xG
        - Space allocated for 'Pool': xP
        - Space allocated for 'Café': xF
        - Space allocated for 'Waiting Area': xW
        - Space allocated for 'Locker Room': xL
    - Objective Function
        - Maximize 58xC+68xS+52xG+47xP+85.5xF
    - Constraints
        - Total Space Available: xC+xS+xG+xP+xF+xW+xL <= 25000
        - Minimum Space Required for Group Fitness: xG >= 800
        - Minimum Space Required for Pool: xP >= 1800
        - Minimum Combined Space Required for Cardio & Strength: (xC+xS) - 0.6(xC+xS+xG+xP+xF+xW+xL) >= 0
        - Maximum Space Available for Cardio: xC - 0.55(xC+xS) <= 0
        - Maximum Space Available for Strength: Xs - 0.55(xC+xS) <= 0
        - Maximum Space Available for Café: xF - 0.05(xC+xS+xG+xP+xF+ xW+xL) <= 0
        - Maximum Space Available for Waiting Area: xW - 0.15xG <= 0
        - Maximum Space Available for Locker Room: xL – 0.20xP = 0
        - Non-Negativity: xC, xS, xG, xP, xF, xW, xL ≥ 0

2. Linear programming model was solved using excel.
    - The total optimal value of annual revenue generated is **$1545620**
    - Optimal Space Allocated to each facility type:

| Facility Type | Optimal Space Allocated (sq. ft.) |
|---|---|
| Cardio | 9301.5 |
| Strength | 11368.5 |
| Group Fitness | 800 |
| Pool | 1800 |
| Café | 1250 |
| Waiting Area | 120 |
| Locker Room | 360 |

    - Annual revenue generated by each facility type:

| Facility Type | Annual Revenue Generated (in $) |
|---|---|
| Cardio | 539487 |
| Strength | 773058 |
| Group Fitness | 41600 |
| Pool | 84600 |
| Café | 106875 |
| | |

3. Additional Revenue calculation without re-solving LP (Using Shadow Price from Sensitivity Report)

Constraints

| Cell | Name | Final Value | Shadow Price | Constraint R.H. Side | Allowable Increase | Allowable Decrease |
|---|---|---|---|---|---|---|
| $I$11 | Total space LHS | 25000 | 64.6 | 25000 | 1E+30 | 16200 |
| $I$12 | Group Fitness minimum LHS | 800 | -21.025 | 800 | 4930.434783 | 800 |
| $I$13 | Pool minimum LHS | 1800 | -29.2 | 1800 | 4725 | 1800 |
| $I$14 | Combined space for Cardio/Strength LHS | 5670 | 0 | 0 | 5670 | 1E+30 |
| $I$15 | Cardio Max LHS | -2067 | 0 | 0 | 1E+30 | 2067 |
| $I$16 | Strength Max LHS | 0 | 10 | 0 | 9301.5 | 2067 |
| $I$17 | Café Max LHS | 2.30926E-13 | 22 | 0 | 5670 | 1250 |
| $I$18 | Waiting Area LHS | 0 | -63.5 | 0 | 5670 | 120 |
| $I$19 | Locker Room LHS | 0 | -63.5 | 0 | 5670 | 360 |

The shadow price of 64.6 shows that for every additional square foot of space, the total annual revenue increases by $64.6 *(sensitivity report snip attached above)*

Additional Space = (30000-25000) sq. ft. = 5000 sq. ft.

Additional Revenue= Shadow Price * Additional Space = 64.6*5000 = 323000

New Total Revenue=Original Revenue + Additional Revenue = 1545620+323000 = 1868620

With the increased space, Summit Fitness can expect an additional $323000 in annual revenue, bringing the new estimated total revenue to $1868620.

4. Group Fitness Annual Revenue Analysis

Variable Cells

| Cell | Name | Final Value | Reduced Cost | Objective Coefficient | Allowable Increase | Allowable Decrease |
|---|---|---|---|---|---|---|
| $B$6 | Cardio | 9301.5 | 0 | 58 | 10 | 40.62801932 |
| $C$6 | Strength | 11368.5 | 0 | 68 | 40 | 10 |
| $D$6 | Group Fitness | 800 | 0 | 52 | 21.025 | 1E+30 |
| $E$6 | Pool | 1800 | 0 | 47 | 29.2 | 1E+30 |
| $F$6 | Café | 1250 | 0 | 85.5 | 1E+30 | 22 |
| $G$6 | Waiting Area | 120 | 0 | 0 | 63.5 | 1E+30 |
| $H$6 | Locker Room | 360 | 0 | 0 | 146 | 1E+30 |

The sensitivity analysis report *(snip attached above)* shows that the allowable increase for the objective coefficient of Group Fitness is 21.025. This means the optimal space allocation remains unchanged until the profit per square foot reaches 73.025 (52 + 21.025).

Additional Revenue Required=(New Profit per Sq. Ft.− Old Profit per Sq. Ft.) × Current Group Fitness Space

=(73.025−52)*800

=16820

Summit Fitness should increase Group Fitness revenue per square foot from $52 to at least $73.025 before expanding its space. This means the total revenue from Group Fitness must rise by about $16820 per year for a space change to be beneficial.

5. Pool Size Requirement Analysis

Constraints

| Cell | Name | Final Value | Shadow Price | Constraint R.H. Side | Allowable Increase | Allowable Decrease |
|---|---|---|---|---|---|---|
| $I$11 | Total space LHS | 25000 | 64.6 | 25000 | 1E+30 | 16200 |
| $I$12 | Group Fitness minimum LHS | 800 | -21.025 | 800 | 4930.434783 | 800 |
| $I$13 | Pool minimum LHS | 1800 | -29.2 | 1800 | 4725 | 1800 |
| $I$14 | Combined space for Cardio/Strength LHS | 5670 | 0 | 0 | 5670 | 1E+30 |
| $I$15 | Cardio Max LHS | -2067 | 0 | 0 | 1E+30 | 2067 |
| $I$16 | Strength Max LHS | 0 | 10 | 0 | 9301.5 | 2067 |
| $I$17 | Café Max LHS | 0 | 22 | 0 | 5670 | 1250 |
| $I$18 | Waiting Area LHS | 0 | -63.5 | 0 | 5670 | 120 |
| $I$19 | Locker Room LHS | 0 | -63.5 | 0 | 5670 | 360 |

From the sensitivity report, the shadow price for the Pool Minimum with Locker Room constraint is -29.2, which means that reducing the pool size requirement by 1 sq. ft. increases total profit by $29.2.

Total reduction in required pool space = 1800 -1500 sq. ft. = 300 sq. ft.

Profit Increase = Shadow Price * Reduction in Pool Requirement

$=(-29.2)\times(-300) = 8760$

Hence, if Summit Fitness reduces the minimum pool size requirement from 1,800 sq. ft. to 1,500 sq. ft., the optimal annual profit will increase by $8760.

6. Mathematical Formulation
   - Decision Variables
     - Amount spent on 'Social Media' (in $): xSM
     - Amount spent on 'TV'(in $): xTV
     - Amount spent on 'Email'(in $): xE
     - Amount spent on 'Print'(in $): xP
   - Objective Function
     - Maximize 10xSM+6xTV+8xE+5xP
   - Constraints
     - Total Budget Availability: xSM+xTV+xE+xP <= 100000
     - Minimum Budget Constraint for TV: xTV – 0.2(xSM+xTV+xE+xP) >= 0
     - Maximum Budget Availability for Print: xP – 0.3(xSM+xTV+xE+xP) <= 0
     - Minimum Budget Constraint for Email: xE – 0.5(xSM) >= 0
     - Maximum Total Reach Constraint for Social Media: 10xSM – 0.5(10xSM+6xTV+8xE+5xP) <= 0
     - Maximum Total Reach Constraint for TV: 6xTV – 0.5(10xSM+6xTV+8xE+5xP) <= 0
     - Maximum Total Reach Constraint for Email: 8xE – 0.5(10xSM+6xTV+8xE+5xP) <= 0
     - Maximum Total Reach Constraint for Print: 5xP – 0.5(10xSM+6xTV+8xE+5xP) <= 0
     - Minimum Spending Constraint for Social Media: xSM >= 15000
     - Minimum Spending Constraint for TC: xTV >= 15000
     - Minimum Spending Constraint for Email: xE >= 15000
     - Minimum Spending Constraint for Print: xP >= 15000
     - Non-Negativity: xSM, xTV, xE, xP ≥ 0
7. Linear programming model was solved using R.
   - The total optimal value of reach is $794444.44
   - Amount spent on each channel in optimal solution (in $) is shown in table below:

| Channel | Amount Spent (in $) |
|---|---|
| Social Media | 39722.22 |
| TV | 20000 |

| | |
|---|---|
| Email | 25277.78 |
| Print | 15000 |

8. Solution for addition $10000 on Total Reach

```
> lp1.solution$objval
[1] 794444.4
> lp1.solution$solution
[1] 39722.22 20000.00 25277.78 15000.00
>
> # Store the optimal DV values as a named object (for future use)
> dv.values <- lp1.solution$solution
>
>
> # Solve the LP with sensitivity
> lp1.solution <- lp(direction = "max", obj.func.coeffs, constraint.coeffs,
+                    constraint.dir, constraint.rhs, compute.sens = TRUE)
> lp1.solution$sens.coef.from
[1]  8.000000e+00 -3.622222e+01  5.500000e+00 -1.000000e+30
> lp1.solution$sens.coef.to
[1] 1.000000e+30 8.222222e+00 1.000000e+01 8.333333e+00
> lp1.solution$duals
 [1]  8.4444444 -2.2222222  0.0000000  0.0000000  0.1111111  0.0000000  0.0000000  0.0000000  0.0000000  0.0000000  0.0000000 -3.3333333
[13]  0.0000000  0.0000000  0.0000000  0.0000000
```

From the *lp1.solution$duals* in R-Console Output *(snip attached above),* we can see that the shadow price for total budget constraint is $8.444444 (~8.4), which means that for every additional $1 added to the budget, the total reach increases by 8.4444.

For extra 10000$ budget, the additional reach = shadow price * additional budget

= 8.4444 * 10000

= 84444.44

Therefore, an extra $10,000 would increase reach by 84444.44 .