Balloon Flight Analysis

for Balloon Type 1

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Balloon

Lift

$$l_1 := 31.7\,\frac{gm}{ft^3}\cdot\frac{9.8N}{1000gm} \qquad l_1 = 0.311\,\frac{N}{ft^3} \qquad l_1 = 10.97\,\frac{N}{m^3}$$

Weight

$$W_b := 110gm\cdot\frac{9.8N}{1000gm}$$

Gondola & Rigging

$$W_g := 1450gm\cdot\frac{9.8N}{1000gm} \qquad W_g = 14.21\,N$$

Flight String

$$\mu := \frac{350gm}{1000ft}\cdot\frac{9.8N}{1000gm} \qquad \mu = 3.4\times10^{-3}\,\frac{N}{ft} \qquad \mu = 1.1\times10^{-2}\,\frac{N}{m}$$

$$W_s(L) := \mu\cdot L \qquad L := 1000ft \qquad W_s(L) = 3.43\,N$$

Drag Forces

Coefficient of Drag Density of air

$$C_d := 0.5 \qquad \rho := 1.23\,\frac{kg}{m^3} \qquad knot := 0.514\,\frac{m}{s}$$

$$D(v,r) := \frac{1}{2}\cdot C_d\cdot\rho\cdot v^2\cdot\pi\cdot r^2 \qquad r := 2.5ft \quad v := 2knot \qquad D(v,r) = 0.59\,N$$

Free Lift

$$F_u(r) := l_1\cdot\frac{4}{3}\cdot\pi\cdot r^3 - W_b - W_g \qquad r := 2.5ft \qquad F_u(r) = 5.04\,N \qquad W_b = 1.08\,N$$

Tension

$$L := 1000ft \qquad W_g = 14.21\,N$$

$$T_v(r,L) := F_u(r) - W_s(L)$$

$$T_h(v,r) := D(v,r) \qquad T(r,v,L) := \sqrt{T_v(r,L)^2 + T_h(v,r)^2}$$

$$y(L,r,v) := \left(\int_0^L \frac{T_v(r,L)}{T(r,v,L)}\,dL\right)$$

$$x(L,r,v) := \left(\int_0^L \frac{D(v,r)}{T(r,v,L)}\,dL\right)$$

$$R(r) := 3.28\cdot r$$ R is in feet

$$F(L) := 3.28\cdot L$$ F is in feet

$$V(v) := \frac{v}{0.514}$$ V is in knots

$$Y(L,r,v) := y(L,r,v)\cdot3.28$$

$$X(L,r,v) := x(L,r,v)\cdot3.28$$

The Minimum Upward Force and Inflation Radius Needed to lift to 90% of altitude

$$W := W_g + W_b \qquad L := 1000\cdot ft$$

$W = 15.29\,N$

$\mu := \dfrac{350gm}{1000ft} \cdot \dfrac{9.8N}{1000gm}$

Find the minimum radius such that the balloon just reaches 900 ft with 1000 ft of string.

$v := 1 \cdot knot$ $\quad r := 2.43 \cdot ft$ $\quad h := 900 \cdot ft$

$R_m := root(y(L, r, v) - h, r)$ $\quad R_m = 2.43\,ft$

$F_m := F_u(R_m)$ $\quad F_m = 3.29\,N$ $\quad F_m \cdot \dfrac{1000 \cdot gm}{9.8 \cdot N} = 335.2\,gm$

$v := 2 \cdot knot$ $\quad r := 2.5 \cdot ft$ $\quad h := 900 \cdot ft$

$R_m := root(y(L, r, v) - h, r)$ $\quad R_m = 2.44\,ft$

$F_m := F_u(R_m)$ $\quad F_m = 3.65\,N$ $\quad F_m \cdot \dfrac{1000 \cdot gm}{9.8 \cdot N} = 372.5\,gm$

$v := 5 \cdot knot$ $\quad r := 2.5 \cdot ft$ $\quad h := 900 \cdot ft$

$R_m := root(y(L, r, v) - h, r)$ $\quad R_m = 2.72\,ft$

$F_m := F_u(R_m)$ $\quad F_m = 10.9\,N$ $\quad F_m \cdot \dfrac{1000 \cdot gm}{9.8 \cdot N} = 1.1 \times 10^3\,gm$

$v := 10 \cdot knot$ $\quad r := 2.5 \cdot ft$ $\quad h := 900 \cdot ft$

$R_m := root(y(L, r, v) - h, r)$ $\quad R_m = 4.43\,ft$

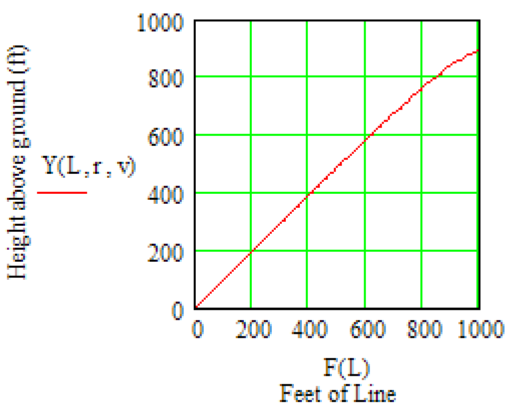
$F_m := F_u(R_m)$ $\quad F_m = 97.75\,N$ $\quad F_m \cdot \dfrac{1000 \cdot gm}{9.8 \cdot N} = 10 \times 10^3\,gm$

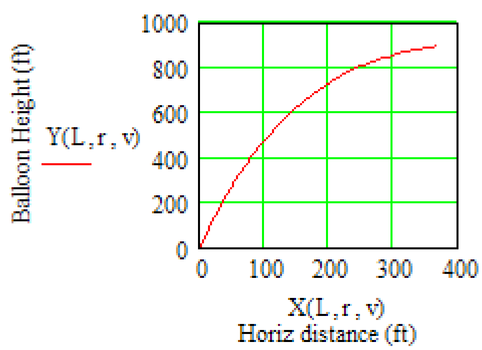
Charts

$r := 2.44 \cdot ft$ $\quad v := 2 \cdot knot$ $\quad L := 0 \cdot ft, 10 \cdot ft .. 1000 \cdot ft$

$F_u(r) = 3.62\,N$

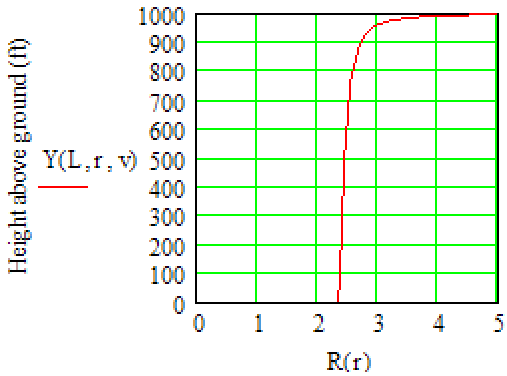
Minimal Lift, Light Air, Line from 0 to 1000 ft

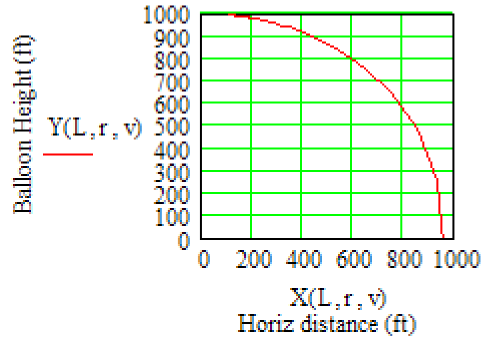




$r := 0.1 \cdot ft, 0.15 \cdot ft .. 5 \cdot ft$ $\quad v := 5 \cdot knot$ $\quad L := 1000 \cdot ft$

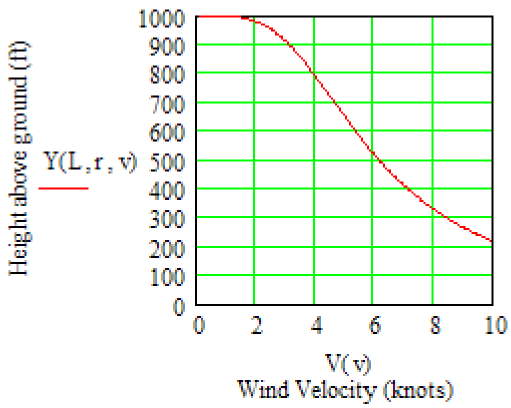
Inflation radius from 0 to 5 ft, Light Breeze, with 1000 ft of line

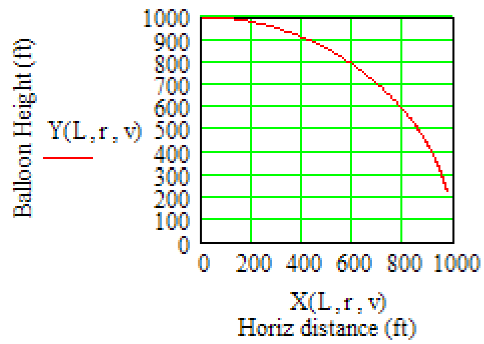




$r := 2.5 \cdot ft$ Infl $v := 0.1 \cdot knot, 0.15 \cdot knot .. 10 \cdot knot$ $L := 1000ft$

Minimum inflation, wind speeds from Calm to Gentle Breeze, and 1000 ft of line





The Angle of the Line from Horizontal at ground level.

$$A(L, r, v) := \frac{360}{2 \cdot \pi} \cdot \operatorname{atan}\left(\frac{T_v(r, L)}{T_h(v, r)}\right)$$

$W = 15.29\,N$

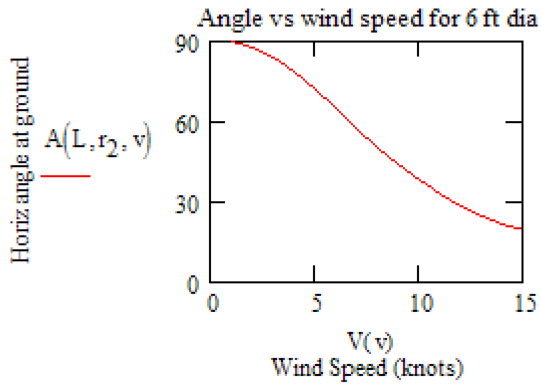
$v := 1 \cdot knot, 2 \cdot knot .. 15 \cdot knot$

$r_1 := 2.5 \cdot ft$ $r_2 := 3 \cdot ft$ $r_3 := 3.5 \cdot ft$

$T_v(r_1, 0 \cdot ft) = 5\,N$ $T_v(r_2, 0 \cdot ft) = 20\,N$ $T_v(r_3, 0 \cdot ft) = 40.5\,N$

Wind 5 ft diameter 6 ft diameter 7 ft diameter
Speed Angle Altitude Angle Altitude Angel Altitude
(Knots) (degrees) (feet) (degrees) (feet) (degrees) (feet)

$v =$		$A(L,r_1,v)$	$y(L,r_1,v) =$		$A(L,r_2,v) =$	$y(L,r_2,v) =$		$A(L,r_3,v) =$	$y(L,r_3,v) =$	
	kn			ft			ft			ft



Lift due to

Density of Helium

$$D_h := .18 \cdot \frac{gm}{liter} \cdot \frac{9.8 \cdot N}{1000 \cdot gm}$$

Density of Air

$$D_a := 1.30 \cdot \frac{gm}{liter} \cdot \frac{9.8 \cdot N}{1000 \cdot gm}$$

Lift per volume

$$L := D_a - D_h \qquad L = 10.98 \frac{kg}{m^2 s^2} \qquad L = 10.98 \frac{N}{m^3}$$

$$L = 0.31 \frac{N}{ft^3}$$