Optimizing Clasp by Exploiting Program Structure via Tree Decompositions

Towards D-FLAT0: Motivation and Ideas

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Joint work with SW & JF

TU Wien

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Outline

Introduction

Background

D-FLAT

D-FLATO

Conclusion

Why Tree Decompositions (TDs)?

- Many problems are (comput.) hard on graphs, but simpler on trees
- ► Is there a way to generalize from trees to complex graphs and still retain the good properties?

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- The treewidth of a graph is defined in terms of tree decompositions . . .
- but we can also use



Cops and Robber Games

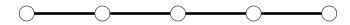
- Round-based board (graph) game between cops and a robber.
- ▶ The **cops** are placed (and removed) freely on the vertices of *G*.
- ► The **robber** starts the game and can only be moved along paths in *G* (which currently are not "blocked" by a cop).

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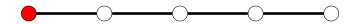
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- ► The **robber** starts the game and can only be moved along paths in *G* (which currently are not "blocked" by a cop).
- ► The **cops** win if a cop is placed on the robbers position and the robber can not move away; otherwise the **robber** wins.

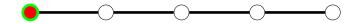
Treewidth

The number of cops required to capture the robber on G equals the treewidth of G (plus one).

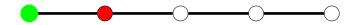


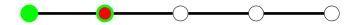










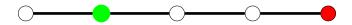
















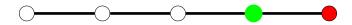


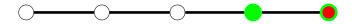


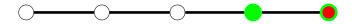


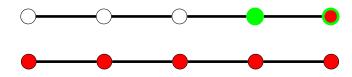


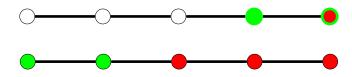


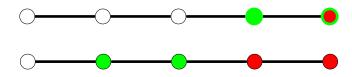


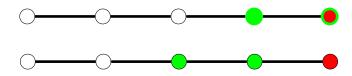


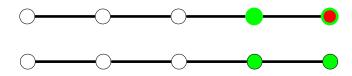


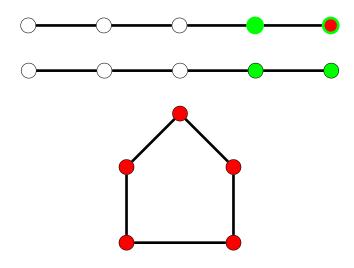


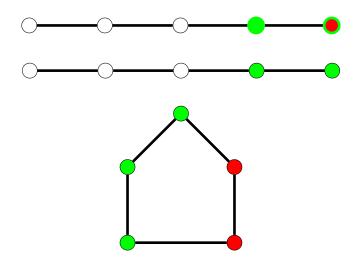




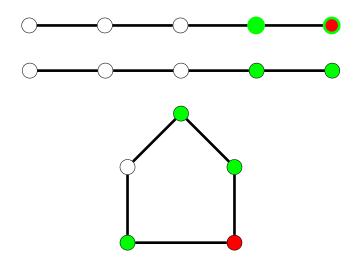




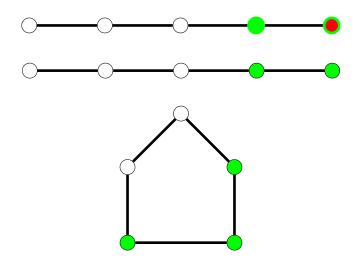


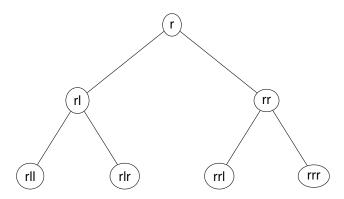


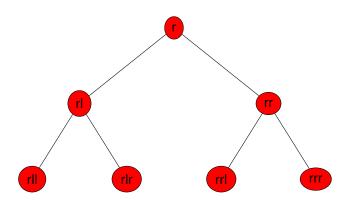
Examples

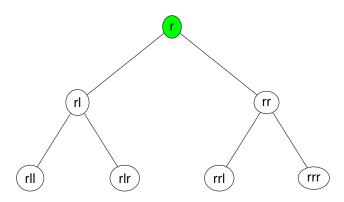


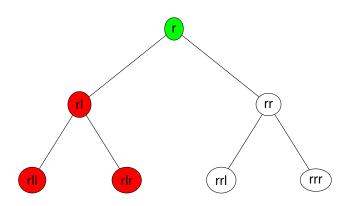
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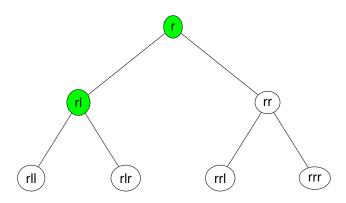


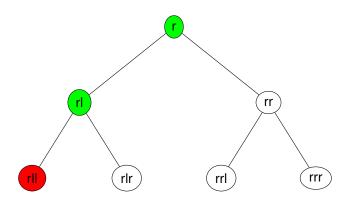


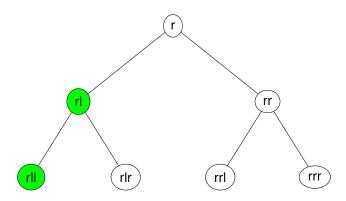


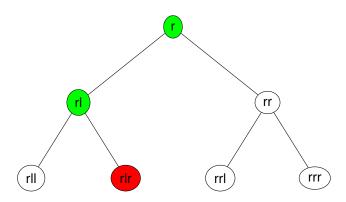


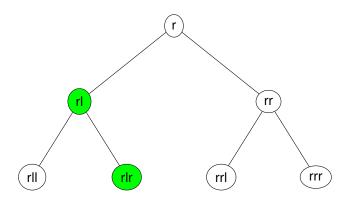


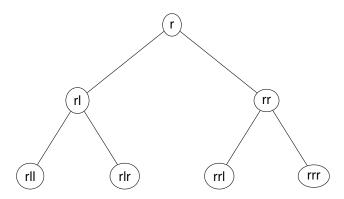












Using Tree Decompositions (TDs)

Task: Solve NP-complete problem instances \mathcal{I}

Problem: Standard approaches require in general $\mathcal{O}(2^{|\mathcal{I}|})$ runtime

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Idea: Dynamic Programming (DP) on Tree Decompositions

lacktriangle Exploit the structure of the problem instance $\mathcal I$

Confine complexity to the parameter "treewidth"

► Goal: Fixed-parameter tractability (FPT) w.r.t. treewidth t, i.e. solvability in time

$$f(t) \cdot |\mathcal{I}|^{\mathcal{O}(1)}$$

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The D-FLAT Approach

- 1. Decompose instance \mathcal{I}
- 2. Specify DP algorithm via ASP
- 3. Combine results of subsequent calls to Clasp

Why This Approach Is Reasonable

- It works for many hard problems
- Real-world applications often have small treewidth
- ▶ In many cases, we can thus avoid the explosion of runtime!

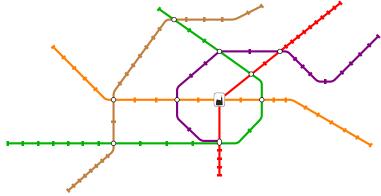
Example: Treewidth 3 (i.e., runtime $\mathcal{O}(2^{(3+1)} \cdot |\mathcal{I}|)$).



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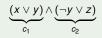
Conclusion

Example: SAT





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Methodology:

1. Decompose instance

$$n_{7} \emptyset$$

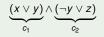
$$n_{6} \{y, c_{2}\}$$

$$n_{3} \{y, c_{2}\} \{y, c_{2}\} n_{5}$$

$$n_{2} \{y\} \{y, z, c_{2}\} n_{4}$$

$$n_{1} \{x, y, c_{1}\}$$

Example: SAT

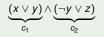




- 1. Decompose instance
- 2. Solve partial problems

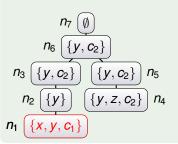
$$n_7$$
 \emptyset
 n_6 $\{y, c_2\}$
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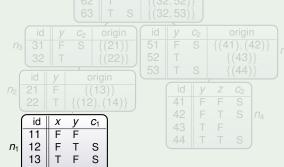
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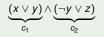


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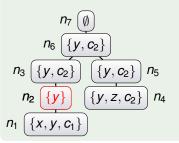


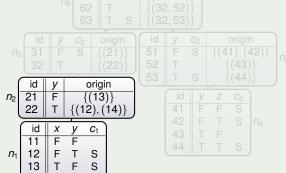
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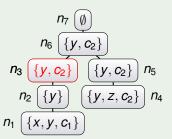


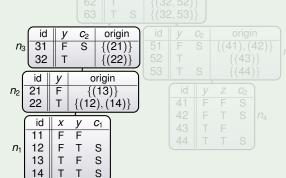
Example: SAT

$$\underbrace{\left(x\vee y\right)}_{c_1}\wedge\underbrace{\left(\neg y\vee z\right)}_{c_2}$$

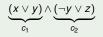


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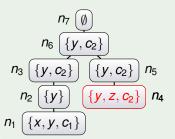


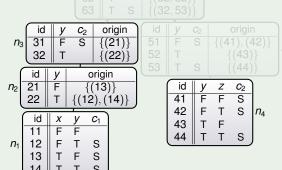
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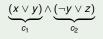


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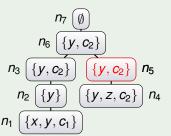


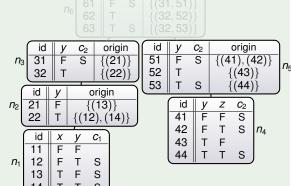
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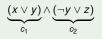


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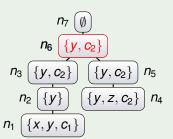


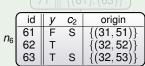
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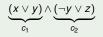
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03 1 3	{(32	2, 55	13	
$ id y c_2 origin $	id	y	C ₂	origin
n ₃ 31 F S {(21)}	51	F	S	{(41), (42)}
32 T {(22)}	52	T		{(43)}
id y origin	53	T	S	{(44)}
n_2 $\begin{bmatrix} 13 & y & \text{Singility} \\ 21 & F & \{(13)\} \\ 22 & T & \{(12), (14)\} \end{bmatrix}$		id 41	y F	<i>Z C</i> ₂ F S
id x y c ₁		42 43	F	T S n_4
n ₁ 11 F F n ₁ 12 F T S		44	T	T S
13 T F S				
[14 T T S]				

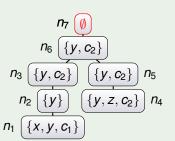
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Methodology:

- 1. Decompose instance
- 2. Solve partial problems



			id	<i>y</i>	c_2	OI	rigin	1	
		n.	61	F	S	{(31	, 51)}	
		n_6	62	T		(32	2, 52)}	
			63	T	S	(32	2, 53	$)$ }	
ſ	id	у	<i>c</i> ₂	origi	n)	id	y	<i>C</i> ₂	origi
n_3	31	F	S	{(21)}	51	F	S	{(41),(
l	32	Т		{(22)}	52	T		{(43

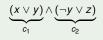
n₇ origin {(61), (63)

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	id	X	У	C_1	1
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id	<i>y</i>	Z	C ₂	1
41	F	F	S	1
42	F	Т	S	n_4
43 44	Т	F		
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{(44)

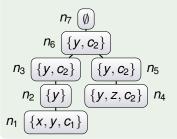
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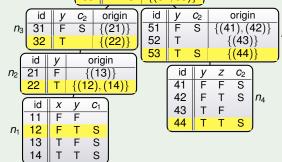


Methodology:

- 1. Decompose instance
- 2. Solve partial problems
- 3. Combine solutions

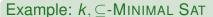


y	Z	<u></u>	71 {(61), (63)}					
		id	y	C ₂	origin			
	n	61	F	S	{(31, 51)}			
	n	⁶ 62	Т		{(32, 52)}			
		63	Т	S	{(32,53)}			
	$ id v c_2 origin id v c_2 $							



*n*₇ origin

Solving 2nd-level Problems







	id	y	c_2	origin	Г
	61	F	S	{(31,51)}	Ø
n-	62	F	S	{(31, 52)}	{61}
n_6	63	T		{(32,53), (33,53)}	Ø
	64	T	S	{(32, 54)}	Ø
	65	T	S	{(33, 54)}	{61,64}
	$\overline{}$		$\overline{}$	$\overline{}$	

						Ī
1	id	У	<i>c</i> ₂	origin	Г	`
n-	31	F	S	{(21)}	Ø	
n ₃	32 33	Т		{(22)}	Ø	
	33	Т		{(23)}	{31,32}	
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ıd	y	c_2	origin	
51	F	S	{(41)}	Ø
52	F	S	{(42)}	{51}
53	Т		{(43)}	{51}
54	Т	S	{(44)}	{51}

	"	y	Ungin	
n	21	F	{(13)}	
"	² 22 23	T	{(12)}	
	23	T	{(14)}	{21,22}
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	11	F	F	Ø
n ₁	12	F	T S	{11}
	13	T	F S	{11}
	14	T	T S	{11, 12, 13}

iu	y		U2	
41	F	F	S	Ø
42	F	Т	S	{41}
43 44	T	F		{41}
44	T	Т	S	{41, 42, 43}
	1			1 6 / / 3

 n_5

 n_4

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D-FLAT

What's that? How does it work?

D-FLATO

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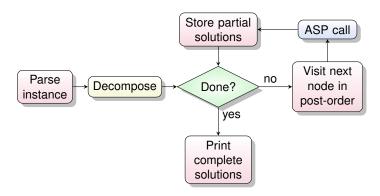
D-FLAT

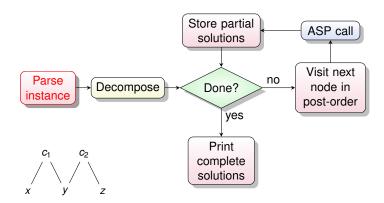
Dynamic Programming Framework with Local Execution of ASP on Tree Decompositions¹

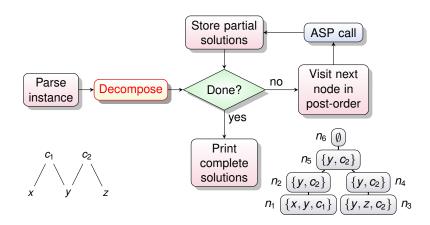
What does it do?

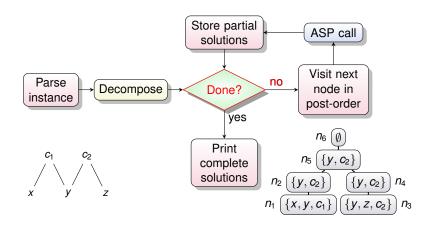
- 1. Constructs a tree decomposition of the input structure
- 2. In each node: Executes user-supplied program with ASP solver
 - Stores partial solutions specified by the answer sets
- 3. Decides the problem (or materializes solutions)
- Users only need to write an ASP program

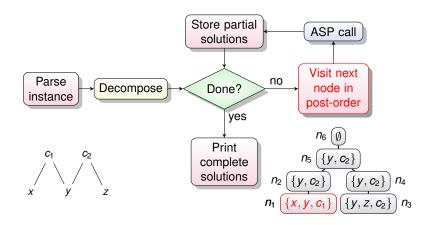
¹D-FLAT is open source: https://www.github.com/bbliem/dflat

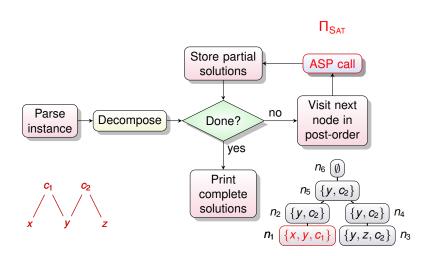


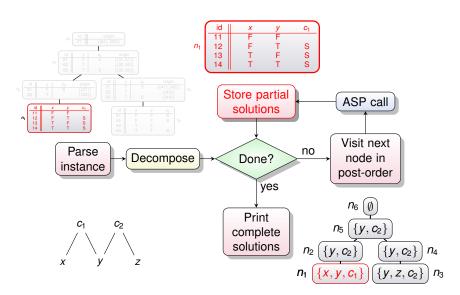


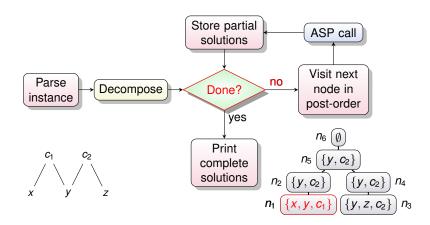


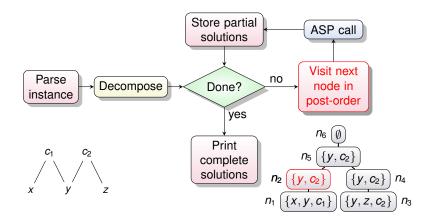


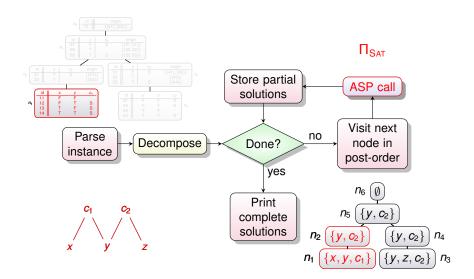


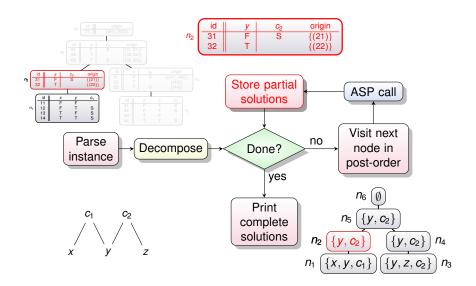


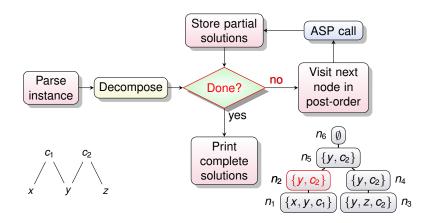


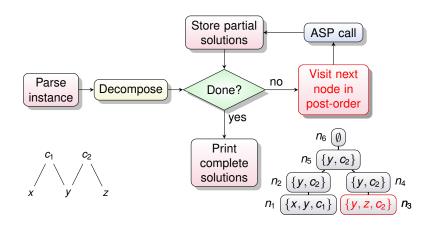


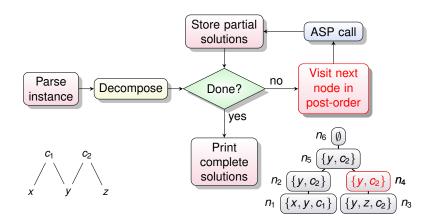


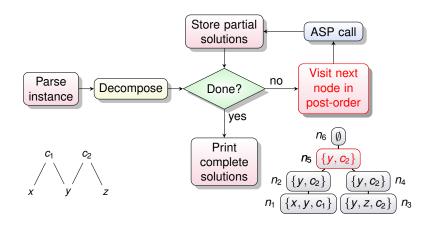


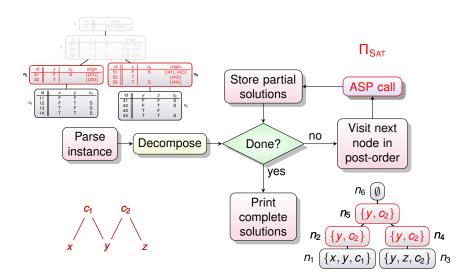


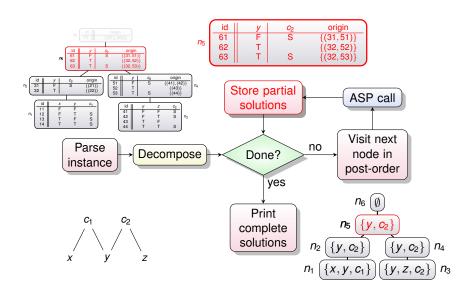


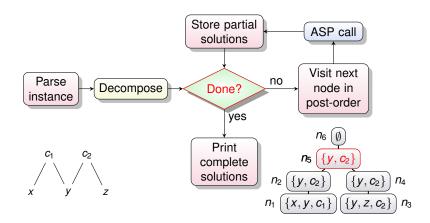


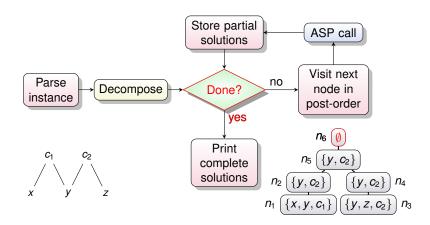


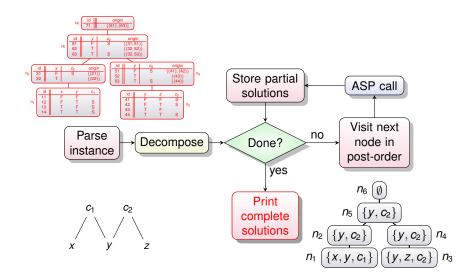












Computing Partial Solutions via ASP

Illustrated by means of SAT

```
User-supplied program \Pi_{SAT}
1 { extend(R) : childRow(R, N) } 1 \leftarrow childNode(N).
                                                               Current table
{ item(A) : atom(A), introduced(A) }.
item(X) \leftarrow extend(R), childItem(R,X), current(X).
item(C) \leftarrow current(C;A), pos(C,A), item(A).
item(C) \leftarrow current(C;A), neg(C,A), not item(A).
                                                               Answer sets
\leftarrow extend(R), clause(C), removed(C), f(R,C).
\leftarrow extend(X;Y), atom(A), childItem(X,A), f(Y,A).
                                                                ASP solver ← Bag
f(R,X) \leftarrow childRow(R,N), bag(N,X), not childItem(R,X).
       Instance
                                                        Child rows
                                                                         Child rows
       atom(a;b;c;d). clause(c1;c2;c3).
       pos(c1,a). neg(c1,b). pos(c2,c).
       neg(c2,a). neg(c3,d).
                                                                       nth child table
                                                       1<sup>st</sup> child table
```

Solving 2nd-level Problems via D-FLAT

Demonstration of user-supplied program $\Pi_{k,\subseteq \text{-MINIMAL SAT}}$

```
length(2). level(1...2). or(0). and(1).
extend(0,R) \leftarrow root(R).
1 { extend(L+1,S) : sub(R,S) } 1 \leftarrow extend(L,R), L<2.
{ item(2,A;1,A) : atom(A), introduced(A) }.
auxItem(L,C) \leftarrow current(C;A), pos(C,A), item(L,A), level(L).
auxItem(L,C) \leftarrow current(C;A), neq(C,A), not item(L,A), level(L).
item(L,X) \leftarrow extend(L,R), childItem(R,X), current(X), level(L).
auxItem(L,C) \leftarrow extend(L,R), childAuxItem(R,C), current(C), level(L).
false(S,X) \leftarrow atNode(S,N), childNode(N), bag(N,X), sub(,S), not childItem(S,X).
unsat(S,C) \leftarrow atNode(S,N), childNode(N), bag(N,C), sub(,S), not childAuxItem(S,C).
unsat(R) \leftarrow clause(C), removed(C), unsat(R,C).
← extend(L, X; L, Y), atom(A), childItem(X, A), false(Y, A), level(L).
\leftarrow extend(L,R), unsat(R), level(L).
reject ← final, extend(1,R), sub(R,S), childAuxItem(S,smaller), not unsat(S).
accept ← final, not reject.
auxItem(2,smaller) ← extend(2,S), childAuxItem(S,smaller).
auxItem(2, smaller) \leftarrow atom(A), item(1, A), not item(2, A).
\leftarrow atom(A), item(2,A), not item(1,A).
```

```
Example: User-supplied program \Pi_{k, \subset -MINIMAL SAT}^2
1 { extend(R) : childRow(R, N) } 1 \leftarrow childNode(N).
{ item(A) : atom(A), introduced(A) }.
item(X) \leftarrow extend(R), childItem(R,X), current(X).
item(C) \leftarrow current(C;A), pos(C,A), item(A).
item(C) \leftarrow current(C; A), neg(C, A), not item(A).
\leftarrow extend(R), clause(C), removed(C), f(R,C).
\leftarrow extend(X;Y), atom(A), childItem(X,A), f(Y,A).
f(R,X) \leftarrow childRow(R,N), bag(N,X), not childItem(R,X).
optItem(X) \leftarrow item(X), atom(X).
```

²D-FLAT² is available at https://www.github.com/hmarkus/dflat-2.

Outline

Introduction

Background

D-FLAT

D-FLAT0

Conclusion

Motivation

Lessons Learnt: D-FLAT

- is well suited for instances of small bounded treewidth
- ► can solve many problems
- ▶ was extended to a competitive tool for 2nd-level DP algorithms [1]
- ▶ is now capable of deriving solutions in a *lazy* fashion [2]
- Bliem, Charwat, Hecher and Woltran: D-FLAT^2: Subset Minimization in Dynamic Programming on Tree Decompositions Made Easy, ASPOCP 2015
- Bliem, Kaufmann, Schaub and Woltran: ASP for Anytime Dynamic Programming on Tree Decompositions, IJCAI 2016

Motivation & Idea

However ...

- it is well suited only for instances of small, bounded treewidth
- it has quite some overhead
 - (Re-)grounding in every TD node
 - Several calls to Clasp (destroys learning process)
 - Finding the first solution is effectively enumerating
 - Actual materialization of partial, intermediate results
 - **.** . . .

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 - Several calls to Clasp (destroys learning process)
 - Finding the first solution is effectively enumerating
 - Actual materialization of partial, intermediate results
 - ▶ ...

Idea

- Reduce overhead by putting a little bit of D-FLAT into Clasp ;-)
- Improve applicability of the result

Possible Approaches

. . . .

Step by step ...

Let's just merge both worlds?

- 1. Improve Clasp by exploiting the structure of problem instance Π_I via extracting properties of some (heuristically computed) TD T_{Π_I}
- 2. ... and let Clasp do all the work ;-)

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Goals

- Derive a method, which goes beyond a simple variable ordering
- Maybe even get better performance with (still decomposable!) instances of higher treewidth

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Goals

- Derive a method, which goes beyond a simple variable ordering
- Maybe even get better performance with (still decomposable!) instances of higher treewidth
- Result shall be useable in a portfolio-based solver
- Exploit performance of Clasp

What has been going on?

Current prototype

works for graph instances^a (decompose input graph)

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- works for graph instances^a (decompose input graph)
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- uses only a static initial modification of heuristic values

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```
\begin{array}{c}
1 \overline{(v, w, y, c_3)} \\
2 \overline{(y, c_2)} \\
5 \overline{(y, c_2)} \overline{(y, c_2)} 3
\end{array}

\begin{array}{c}
-h (true(x), init, 7). -h (true(y), init, 7). \\
7 \overline{(x, y, c_1)}
\end{array}
```

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```
V, W, Y, C_3
                        _h(true(x), init, 3). _h(true(y), init, 3).
\{y, c_2\}
          \{y, c_2\}
                        _h(true(z), init, 2).
          \{y, z, c_2\} \mid 4
                         _h(true(v), init, 1). _h(true(w), init, 1).
```

Troubles

Why this is not so easy ...

- Known problems
 - Runtime depends on the computed TD
 - ▶ TD of smallest width is not always the best one
 - Abseher, Dusberger, Musliu and Woltran: Improving the Efficency of Dynamic Programming on Tree Decompositions via Machine Learning, IJCAI 2015
 - Modifying level via _h is too much
- 2. Does the TD actually help?
 - Same effect by arbitrary variable ordering?
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Why we require your expertise

- ► Look into Clasp to find out why some heuristic configurations work well and others don't
- Also add dynamics by modifying factor

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Why we require your expertise

- Look into Clasp to find out why some heuristic configurations work well and others don't
- Also add dynamics by modifying factor
- Maybe later even modify Clasps interna

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Summary

- D-FLAT allows to specify DP algorithms on tree decompositions using ASP
- Combinatorial explosion is bounded by the treewidth
- The idea is to guide Clasp towards DP on TDs
 - via heuristics
 - 2. via direct implementation
 - 3. ...
- It might help to take a deep breath and look inside Clasp

Thanks to Sebastian Ordyniak for his slides!