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Desigualdad de Jensen

Proposición 1 (Desigualdad de Jensen). *Sea* $\varphi\colon \mathbb{R} \longrightarrow \mathbb{R}$ *una función convexa y* $\varphi, \varphi(X) \in \mathcal{L}^1(\mu_X)$ *con* $X : \Omega \longrightarrow \mathbb{R}$ *una variable aleatoria* $\implies \varphi(\mathbb{E}[X]) \leq \mathbb{E}[\varphi(X)]$.

***Demostración*:** Vamos a asumir que $\varphi \in \mathcal{C}^1$. Sea A la recta tangente a φ en $t_0 = \mathbb{E}[X]$. Como φ es convexa, $\forall t \in \mathbb{R} : A(t) \leq \varphi(t)$.

$$\begin{aligned}\mathbb{E}[\varphi(X)] &= \int_\Omega \varphi(X(\omega))\, d\mathbb{P}(\omega) \geq \int_\Omega A(X(\omega))\, d\mathbb{P}(\omega) = \mathbb{E}[A(X)] \\ &= \mathbb{E}[aX + b] = a\,\mathbb{E}[X] + b = A(\mathbb{E}[X]) = A(t_0) = \varphi(t_0) = \varphi(\mathbb{E}[X]).\end{aligned}$$

Luego $\mathbb{E}[\varphi(X)] \geq \varphi(\mathbb{E}[X])$. ■

Referenciado en

- `Desigualdad-holder`