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Operadores de Wirtinger

Definición 1 (Operadores de Wirtinger). Sea $f\colon \Omega \subset \mathbb{C} \longrightarrow \mathbb{C}$, con Ω un dominio, $\partial f, \overline{\partial} f\colon \Omega \longrightarrow \mathbb{C}$ son los operadores de Wirtinger

$$\iff \forall x+yi \in \Omega : \begin{cases} \partial f(x+yi) = \partial_z f(x+yi) = \dfrac{1}{2}\left(\dfrac{\partial f}{\partial x}(x,y) - i\dfrac{\partial f}{\partial y}(x,y)\right) \\ \overline{\partial} f(x+yi) = \partial_{\overline{z}} f(x+yi) = \dfrac{1}{2}\left(\dfrac{\partial f}{\partial x}(x,y) + i\dfrac{\partial f}{\partial y}(x,y)\right) \end{cases}$$

donde $\partial_x f = u_x + iv_x$ y $\partial_y f = u_y + iv_y$.

Estos operadores provienen de aplicar la regla de la cadena. Como $x = \dfrac{z+\overline{z}}{2} \wedge y = \dfrac{z-\overline{z}}{2i}$

$$\implies \partial_z x = \partial_{\overline{z}} x = \frac{1}{2} \wedge \partial_z y = -i\partial_{\overline{z}} y = \frac{1}{2i}.$$

Por lo tanto, $\partial f = \partial_x f \partial_z x + \partial_y f \partial_z y = \partial_x f \dfrac{1}{2} + \partial_y f \dfrac{1}{2i} = \dfrac{1}{2}\left(\partial_x f - i\partial_y f\right)$ y

$$\overline{\partial} f = \partial_x f \partial_{\overline{z}} x + \partial_y f \partial_{\overline{z}} y = \partial_x f \frac{1}{2} + \partial_y f \frac{-1}{2i} = \frac{1}{2}\left(\partial_x f + i\partial_y f\right).$$