GRE Notes

Hannah Massenbauer

October 2024

Contents

1 Number facts

Numbers are

- rational, if they can be represented in a fraction
- irrational, if they cannot be represented in a fraction (π $\sqrt{2}$)
- 0 = even integer, neither positive nor negative
- Every number is a factor of 0, even 0 is a factor of 0, and 0 is a multiple of every number
- Adding / Subtracting numbers from each other result in an even result if both numbers are even / odd. As soon as one number is even and one is odd, the subtraction / addition will result in an odd number
- Multiplying numbers result only in an odd result if ALL factors are odd
- Division rules:
$$\frac{even}{odd} = even$$
$$\frac{odd}{odd} = odd$$
$$\frac{even}{even} = ?$$
- If an integer N divided by an integer X results in an integer then dividing the integer N by any factor of X results again in an integer
- Integers = - ∞ ; ∞
- Positive numbers = N > 0
- Whole numbers = nonnegative integers (incl. 0)

1.1 Primes

- 15 prime number until 50: 2,3,5,7,11,13,17,19,23,29,31,37,41,43,47
- Every prime factorization is unique $\rightarrow$ no two integers have the same prime factorization (unique ID)
- Total number of prime factors = number of factors eg $6 = 2 * 3$ has two factors, while $4 = 2 * 2$ exhibits also 2 factors
- Total number of unique factors = how many different factors exist: $6 = 2*3$ has 2 factors, while $4 = 2 * 2$ has 1 unique factor
- There are 25 primes between 1 to 100, 21 primes between 101 to 200 and 16 between 201 to 300

- Test if it is a prime number:
  Take the root of the number and if it is NOT divisible by any prime number that is smaller than the number is a prime number

- A positive integer which has exactly two divisors must be the square of a prime

- Prime numbers that are 1 greater than a multiple of 4 can always be written as the sum of two squares.

- Goldbach's Conjecture = asserts that every even integer greater than 2 can be expressed as the sum of two prime numbers

- Two consecutive integers dont share any prime

1.2 Factorials

- Factor or divisor is a positive integer that divides evenly into that number

- Products of the factorials count as well as a factor of a number. E.g.:
  $8! = 8 * 7 * 6 * 5 * 4 * 3 * 2 * 1$
  And also 56,30 it 1440 are factors of 8!

- 8! has 96 positive factors and its prime factorization form is $2^7 \cdot 3^2 \cdot 5^1 \cdot 7^1$

- 20! has 41.040 positive factors

- Number of factorials:
  $\frac{100!}{3^x}$ = integer $\rightarrow$ only if the denominator is a prime
  Then divide the fraction's numerator by the denominator, take the quotient, and divide again by the denominator. Repeat as long as division is no longer possible. Then add the quotients together to receive the solution for x.

- Factor = number that leaves no remainder (0 is never a factor)

- Prime Factorization and each possible combination of the parts yield all possible factors

- Number of positive factors:

  1. Prime factorization
  2. Add +1 to each exponent
  3. Multiply the exponents (+1) with each other
  4. The result of the multiplication is the number of positive factors

- Number of odd positive factors:
  The same approach as above but adding +1 only to the odd prime factors. And then multiply only the odd factors with each other.

- Number of even factors:
  Calculate the total number of positive factors, subtract the odd number = the even number
- GCF = Greatest common factor:
  1. Find the prime factorization of both numbers.
  2. Multiply the prime factors with the smallest exponents with each other, which occur in **both** prime factorizations.
  3. If there is no common prime factor than GCF = 1
- LCM = Least common multiple:
  Find prime factors of both numbers and take the number <u>with</u> their highest exponents and multiply the numbers out.
- If N>2 then it is sufficient to take the common biggest prime factor that occurs for at least 2/N (no repetition necessary). The unique factors are used as before
- If LCM and GCF are given but only one of the numbers than

$$LCF * GCF = x * y$$

- LCM delivers all unique PF of the numbers having the LCM in common
- If z is divisible by x and y such that it results in an integer, then z can be evenly divided as well by the LCM of x and y

1.3 Divisibility

- Divisibility rules:
  - $\frac{x}{3}$: if the sum of the integers can be divided by 3
  - $\frac{x}{4}$: if the last two digits can be divided by 4
  - $\frac{x}{6}$: if the integer is divisible by 2 and 3
  - $\frac{x}{8}$: if the last three digits are divisible by 8
  - $\frac{x}{9}$: if the sum of the integers´ digits can be divided by 9
  - $\frac{x}{11}$: in the case of three digits: left digit + right digits - middle digit
  - If it is unclear what the e.g numerator must be perform a prime factorization in order to determine which factors are still missing to be evenly divided by a certain denominator
  - Division

$$\frac{x}{y} = Q + \frac{r}{y}$$
$$x = Qy + r$$
$$r = x - Qy$$

- remainder is never negative. If the number is negative go to the next lowest quotient and then **add** the remainder
- When only a decimal number is given, it is known what the reminder is. Reduce the decimal to the simplest fraction
- When we have several terms we can divide each term by denominator and then add up remainder. If the sum of the remainder exceeds the denominator then the sum is divided by the denominator to obtain the real reminder

- Trailing 0s: $\frac{270!}{10^x}$
  First, split the denominator into its primes: $2 \cdot 5$.
  As 5 is the larger number divide 270 by 5.
  Repeat as long as the resulting quotient cannot be divided anymore by 5.
  Add all quotients up to receive the number of trailing 0s.
  - If the 5s and 2s are not tantamount than use the factor that occurs less often
  - Looking at the 5 * 2 pairs work as any 5*2 pair can be extracted to get a $x * 10n$ where n stands for the number of 2*5 pairs
  - Any factorial larger 4! has for sure a trailing 0 as 5! contains a 5*2
- Leading 0s = the number of 0s right to the comma point if $0 < |N| < 1$

$$\frac{3}{100} = 0.03$$

  $\rightarrow$ tantamount to 1 leading 0
- If perfect square, N of leading 0 = N of exponents of 10 + digits of number not power of 10
- If not perfect squares: again split into powers of 10 and the other PF, then count as above the number of digits but subtract 1
- Consecutive numbers:
  - A multiple of 3 is always included among the three integers,
  - The product of the three integers is always a multiple of 3
  - The sum of the three integers is also a multiple of 3
  - Product of n consecutive elements will always be divisible by n! (as well as by all factors of n!)
  - With n **even** consecutive numbers the product of these numbers will always be divisible by

$$2^n * n$$

1.4 Random facts

- Perfect squares
  - can end with a 0,1,4,5,6, or 9.
  - cannot end with a 2,3,7, or 8.
  - If n != 0,1 then all of the PF must have an even exponent
  - All PF must have an even exponent
- Number of digits =
  1. Extract number of trialing 0s
  2. Count leftovers power exponents
  3. Add up the number of trailing 0s and the leftovers powers
- Weird 0:
  - 0^0 = undefined
  - 0! = 1

2 Roots and powers

2.1 Roots:

$$\sqrt{a} = x \text{ where } x > 0$$
$$\sqrt{x^2} = |x| = \pm x$$

if root even

$$\sqrt{a} = |x|$$

if root odd

$$3\sqrt{-x^3} = -x$$

- $\sqrt{}$ = radical
  Radicals with the same root can multiply the radicands with each other
  $$m\sqrt{ab} = m\sqrt{a}m\sqrt{b}$$
- a = radicand
- Like radicals = $a\sqrt{x} + b\sqrt{x} = (a+b)\sqrt{x}$

2.2 Powers:

- Exponents can be distributed over multiplication and division only $(x^2)^2 = x^{2\cdot 2}$
- 0^n: always 0
- 1^n: always 1
- 2^n: $2, 4, 8, 6$ (repeats in blocks of 4)
- 3^n: $3, 9, 7, 1$ (repeats in blocks of 4)
- 4^n: $4, 6$ (repeats in blocks of 2)
- 5^n: always 5
- 6^n: always 6
- 7^n: $7, 9, 3, 1$ (repeats in blocks of 4)
- 8^n: $8, 4, 2, 6$ (repeats in blocks of 4)
- 9^n: $9, 1$ (repeats in blocks of 2)
  $\rightarrow$ for numbers larger than 9 the pattern repeats in the same manner as for unit digits

- $a^x = a^2 \rightarrow x = 2$
  This only holds if $a \neq -1, 0, 1$
- $x^{ab} = (x^a)^b$
- $(x^a)(y^a) = (xy)^a$
- This formula simplifies nested radicals using exponents. When simplifying, ensure that exponents follow the rules of multiplication.

$$\sqrt[a]{\sqrt[b]{x}} = \left(x^{\frac{1}{b}}\right)^{\frac{1}{a}} = x^{\frac{1}{b}\cdot\frac{1}{a}} = x^{\frac{1}{ab}}$$

- How to handle exponents when base and exponents are same:

$$x^{\frac{1}{a}} \cdot x^{\frac{1}{a}} = x^{\frac{2}{a}}$$

- if $0 < x < 1$ then $x^2 < x < \sqrt{x}$

Attention!
GRE just use positive number from root, BUT uses positive and negative solution for e.g. x^2

- perfect square = if root is taken, then integer is solution

- Every´s PF exponent must be even
- Perfect square: 1,4,9,16,25,36,49,64,81,100,121,144,169,225..

- perfect cube = if cube root is take, then integer is solution
  e.g. 0,1,8 (2),27 (3),64 (4),125 (5),216 (6),343 (7),512 (8)
  Its exponents are divisible by 3
- $(\sqrt{x})^2 = x$ even for $(\sqrt{x-2})^2 = x - 2$
- $\sqrt{x}\sqrt{y} = \sqrt{xy}$
- $\frac{\sqrt{x}}{\sqrt{y}} = \sqrt{\frac{x}{y}}$
- To find out how many roots an equation has, check whether the discriminant: $b^2 - 4ac$
  - > 0 then 2 Solutions
  - $= 0$ then 1 Solutions
  - < 0 then 0 Solutions
- Sum of two roots = $\frac{-b}{a}$
- Product of two roots = $\frac{c}{a}$

3 Formulas

1. If the Sequence Starts at 1

$$S = \frac{n(n+1)}{2} \tag{1}$$

where n is the last number in the sequence.

2. If the Sequence Does Not Start at 1:

The number of terms n is determined by:

$$n = \frac{\text{greatest number} - \text{smallest number}}{\text{step size}} + 1 \tag{2}$$

- Step size: The interval between consecutive terms (e.g., 2 for even numbers).
- Greatest/Smallest number: The largest/smallest integer divisible by the given step rule.

$$S = \frac{n\# \cdot (n_N + n_1)}{2}$$

3.0.1 Sequences

- Recursive sequences is conditional on the previous outcome (n-1)
- Arithmetic sequence (constant steps):

$$a_n = a_1 + (n-1)d$$

- Geometric sequence = ratio between every pair of two consecutive terms is the name

$$a_n = a_1 \cdot r^{n-1}$$

- Never assume something is a sequence unless it is stated as one

Solving Systems with Quadratic Roots

Using the **difference of squares** identity, $P = a^2 - b^2 = (a-b)(a+b)$, the roots can be factored. Or use quadratic formula

There are 3 identities:

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

$$a^2 - b^2 = (a-b)(a+b)$$

Quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

3.1 Inequalities

Absolute value = how far is x from 0?
Example: x + y = 100, xy = 2475 $\implies$ $|x-y| = 10$.

- Sign reverses if divided or multiplied by a negative number
- If the signs direct in the same direction, then the equations can be solved like with regular linear equations.
- Be aware of unknown signs. If there is e.g. only x, I don't know whether $x < 0$ or $x > 0$. The inequality sign would flip in case of $x < 0$.

$$|a+b| \leq |a| + |b|$$

- if
$$|a+b| = |a| + |b|$$
then a and b have the same sign
$$|a-b| \geq |a| - |b|$$
- if
$$|a-b| = |a| - |b|$$
then a and b must have the same sign and $|a| \geq |b|$
- If an absolute value expression is negative, then there is no solution
- If an absolute value is equal to an unknown variable than compute both possible values $\rightarrow$ plug the values in the initial equation and check which value holds true

4 Word Problems

4.1 Age problems:

- Set up equations with current age
- Add / subtract if problem goes into past / future
- If two rates / wages are made equal because A gives B x then the system of equations look like:
$$\frac{w_A - x}{h_A} = \frac{w_B + x}{h_B}$$
- If we want to determine share that is left after subtracting an amount, it can be stated as
$$x - \frac{x}{y} = \frac{xy}{y} - \frac{x}{y} = \frac{xy - x}{y} = \frac{(1-y)x}{y}$$

4.2 Distance problems:

- Distance = rate $*$ time
- Using a matrix system can help to see what is missing and how to get there
- Units must be compatible
- Total average speed: $\frac{\text{Total Distance}}{\text{Total Time}} = \frac{\text{Miles 1 + Miles 2}}{\text{Time 1 + Time 2}}$

- Different leaving times can be incorporated by adding an unknown "t". The difference in departure is added to the object leaving earlier eg A leaves 30 min earlier than B: $Distance = r_B * (time + 0.5)$ The added time measure must be in the same unit conversion as the rate. E.g the rate here is in hours so 30 min = 0.5 hr

- They are still word problems, so setting up aid equation describing the relation of additional info necessary

- Diverging directions: Distances can be simply added

- Catch up problems:

  1. Set equations equal and add difference
  2. Shortcut formula: Time = $\frac{\Delta\text{Distance}}{\Delta\text{Rate}}$

- Manipulations in the rate (e.g. wind supporting / hindering airplane) can be added / subtracted from the rate.

- If-then problems. The if situation is the aid equation. Use it to express the desired variable explicitly and then plug it into the then situation.
  IF: $30 = rt$
  desired outcome t: $r = \frac{30}{t}$
  Then: $30 = (r \pm \Delta_r)(t - \pm \Delta_t)$ and plug stage 1 in $30 = (\frac{30}{t} \pm \Delta_r)(t - \pm \Delta_t)$.
  Changes are given.

  Stylized facts:

  1. Distance is directly related to time and rate
  2. Rate is inversely proportional to time and directly proportional to distance
  3. Time is inveresly proportional to rate and directly proportional to distance

4.3 Work problems

- Like distance problems

- Rates can be deducted from rate together and one rate of two workers

$$\frac{X}{t_1} + \frac{X}{t_2} = \frac{X}{t}$$

- Ratio of work done by you = $\frac{\text{Work rate i}}{\text{Total work rates combined}}$

- Opposite work force = subtract work rates from each other

4.4 Mixture problems:

1. Dry Problems: Required: Components, Units, Quantity
2. Wet Problems: Required: Components, Concentration, Quantity

4.5 Growth rates:

- Work like functions - set up equations that can be solved

- Consider whether its
  - a constant growth Starting value $+tx$ where x is the constant growth factor
  - b factor growth Starting value $*x^t$

4.6 Tips:

1. Use the proportion trick. Compare the distance to the final solution $\rightarrow$ this reveals the proportion of the mixtures
2. Create a table. This keeps things tidy and then complete the table if only 1 /3 fields is missing
3. For conversions write the unit next to it if unsure

4.7 Ratios:

- Scenario 1: If a ratio is given and you want to have a new ratio between the two elements add/subtract an unknown and solve for the unknown

$$\frac{A}{B} = \frac{2}{3}$$

  But first find ratio multiplier μ, where T is given

$$A\mu + B\mu = T$$

  Then use μ and multiply it with A and B to have the absolute values. If we want to have a new ratio then add the value and finally calculate the number of units it would take to obtain the new ratio

$$\frac{A\mu + x}{B\mu} = 4 : 5$$

- Scenario 2: We are interested in the share in the total amount. Firstly, calculate the ratio multiplier μ and secondly construct a similar equation as above, but (!)

$$\frac{A\mu + x}{T + x} = \frac{4}{5}$$

Here we add the x to both, numerator and denominator.

Alternative:

$$\frac{A}{T+x} = \frac{4}{5}$$

Then reform the equation such that you see in which parts the part must be to the total to comply with the new ratio

- When the question says the ratio of A to B halfs than it means

$$\frac{A}{2B}$$

4.8 Percentage

$$percentage = \frac{part}{whole} * 100 \rightarrow part(A) = whole(P) * (1 + \frac{\text{Percent x}}{100})$$

1. Going from x% to decimal

$$100\% = \frac{100}{100} = 1$$

$$22\% = \frac{22}{100} = .22$$

2. X (numerator) is what percent of Y (denumerator)

$$\frac{X}{Y} 100 = \%$$

3. Percent less than problem:

$$\text{Final value} = \text{Initial value}(1 - \frac{\text{Percent less than}}{100})$$

4. 25% greater than z = 125% of z
   100% greater than z = 200% of z

5. Percentage change =

$$\frac{\text{Difference}}{\text{Original}} * 100 = \frac{\text{New - Old}}{\text{Old}}$$

   If its positive = increase, negative = decrease

4.9 Interest

- simple annual rest = always from the base level

$$A = P(1+p)^t$$

- When GRE writes x percents, then it is necessary to write in the equation $\frac{x}{100}$
- If only the interest is asked then

$$\text{Interest} = \text{Principal} * \text{Rate} * \text{Time}$$

- compound interest rate = bigger than simple annual rest
- compound interest =

$$A = P(1+\frac{r}{n})^{nt}$$

- If the compound interest rate is more than yearly then
  1. divide p by frequency of occurrence
  2. adjust time factor, e.g. 2 years every 6 months, then t = * 2

5 Statistics

- **Venn Diagrams**
  - In a three group Venn Diagram the number of individuals in neither bubble can be calculated as follows:

$$A + B + C - AB - BC - CA + ABC + none$$

  - Number of unique elements

$$N = A + B + C - \text{both} + 2(\text{all } 3) + \text{None}$$

  - In questions like how many more items need group A to represent $\frac{2}{3}$ of the sample than calculate share of sample and then see what is the difference between this number and the current number of group A.
- **Mean, median, standard deviation**
  - Mean = $\frac{Sum}{\text{Number of elements}}$
  - Mean in an even spaced set $\mu = \frac{Last-First}{2}$
  - Mean in an odd space = middle number
  - Mean in an even space = average of two middle numbers
  - Median in an odd space = $\frac{n+1}{2}$

- Median in an even space = $\frac{\frac{n}{2}+\frac{n+2}{2}}{2}$
- Standard deviation = spread of data.
  1. Calculate mean
  2. Subtract mean from each element of the list
  3. Square the difference
  4. Take the sum of the squared difference and divide by the number of elements
- Sum of multiples:

$$(\frac{\text{Biggest - smallest in set}}{2}+1)*(\frac{\text{Biggest + smallest in set}}{2})$$

$$= Number\# * Average$$

- Total number of multiples.
  E.g. number of multiples of 2 and 3. Then number of multiples of 2 plus number of multiples of three minus LCM of 2 and 3 is equal to the unique number of multiples

$$\#(A) + \#(B) - \#LCM(2,3)$$

- Number of multiples excluding overlap.

$$\#(A) + \#(B) - 2 \cdot \#LCM(2,3)$$

  Standard Deviation:

- If a constant x is added, the standard deviation remains unchanged
- If numbers are multiplied with x¿1, sd increases
- If numbers are divided, mean is added, or multiplied (for -1¡x¡1) then sd decreases
- Smallest SD possible is 0
- The **population standard deviation** (σ) measures the spread of data for an entire population and is calculated using the formula:

$$\sigma = \sqrt{\frac{\sum(x_i - \bar{x})^2}{N}}$$

  where N is the total number of data points.
- The **sample standard deviation** (s) estimates the spread for a sample of data and divides by $n-1$ to account for sampling bias. Its formula is:

$$s = \sqrt{\frac{\sum(x_i - \bar{x})^2}{n-1}}$$

  where n is the sample size.

6 Geometry

6.1 Angles

- Acute, obtuse angles
- Supplementary angle = two angles sum up to 180
- Transversal = line that crosses several other lines $\rightarrow$ opposite angles are equal

6.2 Coordination plan

- Slope = $\frac{\Delta y}{\Delta x}$
- Lines can be positivie, negative, 0 (horizontal) or undefined (vertical line)
- The **domain** of a function $f(x)$ is the set of all possible input values (x) for which the function is defined.
- The **range** of a function $f(x)$ is the set of all possible output values ($f(x)$) that result from plugging in the domain values.
- The larger the absolute value the steeper the line
- x-intercept = $\frac{-d}{k}$
- Slopes of two perpendicular slopes = -1
- Reflected over origin = both signs flip
- Reflection
  - f(x)=x - positiion of x,y coordinate switch
  - f(x)=-x - positin of x,y and sign switch
- Perimeter of a figure is the distance between the points
- Area = Distance of longest side to opposing angle (as this resembles the height) and divided by two. it is still the same formula just have to find the longest side
- Asymmetric function = One exponent is even / other odd or if only one variable than this one is odd
- Even function =
  $$f(x) = f(-x)$$
  $\rightarrow$ symmetric to the y-axis

- Odd function:

$$f(-x) = -f(x)$$

$\rightarrow$ symmetric about the origin

- The **distance formula** in a geometric plane is given by:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

where (x_1, y_1) and (x_2, y_2) are the coordinates of two points.

- **Equation of a Circle:** The equation of a circle centered at the point (a, b) is given by:

$$(x - a)^2 + (y - b)^2 = r^2$$

where r is the radius of the circle.

6.3 Triangles

- Interior angles sum up to 180 degrees
- Exterior angle = prolong a side of the triangle and calculate the difference to complete the straight angle (sum to 180)
$\rightarrow$ sum of exterior angle = 360
- Altitude = height (can be outside the triangular)
- Similar triangle = triangle and an enlarged version of it
  1. all sides are multiplied by a constant factor
  2. The two triangles have identical angles
  3. One identical angle and the two adjacent sides are multiples
- Ratio of longer leg:shorter leg[1] remains constant for similar triangles rules:

$$a + b > c > a - b \tag{3}$$

$$b + c > a > b - c \tag{4}$$

$$a + c > b > a - c \tag{5}$$

If the lengths of two sides of a triangle are unequal, the greater angle lies opposite the longer side and vice versa. In the figure above, if ∠A > ∠B > ∠C, then a > b > c.

- Area of a triangle:

$$A = \frac{1}{2} \cdot b \cdot h \tag{6}$$

Base and height must be perpendicular to each other

[1]Dont forget to add the ratio multiplier!

- Special triangles:
  - Isoceles triangle: 2 Legs have same length
    2 same sized legs $\rightarrow$ 2 same sized angles.
    Ration of sides = $x : x : \sqrt{2}x$
  - Equilateral triangles: All legs have the same length
    3 same sized legs $\rightarrow$ 3 same sized angles (60°)

$$\textbf{Area} = \frac{s}{2} \cdot \frac{s}{2} \cdot \sqrt{3} = \frac{s^2 * \sqrt{3}}{4}$$

  - Right triangle: one angle with 90°

$$a^2 + b^2 = c^2$$

  - Right isocles triangle: the legs are the same and the hypothenuses is equal to the legs $\cdot\sqrt{2}$. If only hypothenus is given as an integer and we want to know the leg length then we take the hypothenus $\frac{c}{\sqrt{2}}$
  - Pythagorean triplets:
    1. 3:4:5
    2. 6:8:10 (scale up from prior)
    3. 5:12:13
    4. 8:15:17
    5. 7:24:25
    6. 1:1:$\sqrt{2}$
    7. 1:$\sqrt{3}$:2 (30-60-90 angle triangle)

Good to know things:

- Ratio of two triangles (ABC & DEF), which have the same angles but different lengths:

$$(\frac{AB}{DE})^2$$

6.4 Quadrilaterals

- If 90 degree angles $\rightarrow$ then diagonals congruent (equal length) = rectangle / square
- No 90 degrees e.g. parallelogram, trapezoid $\rightarrow$ angles sum up to 180 in each half (both vertically and horizontally cut)
- Trapezoid doesn't have equal length sides, on 2/4 are

6.4.1 Rectangle

- Diagonal = $\sqrt{l^2 + w^2} = d$
- In a square the diagonal creates two 45:45:90 triangles with $d = s\sqrt{2}$
- Maximum area = square[2]
- Minimum perimeter = square

6.4.2 Parallelogram

- 2 pair of parallel sides
- Diagonal angles are identical
- Diagonals are bisecting each other
- Area of parallelogram: need to find the height! → b * h

equal in length

6.4.3 Trapezoid

- Has two parallel sides, called bases
- Area of trapezoid =
$$\frac{(b1 + b2)}{2} \cdot h$$

equal in length

6.4.4 Polygons

- Diagonal = line segment which connects two nonadjacent vertices (ecken)
- Regular Polygon = Sides are of equal length and interior angles of equal measure (equivalent eines Quadrats order **equilateral triangles**)
- Pentagons can be divided into triangles → useful to determine the interior angle
  E.g.: Pentagon can be divided into 3 triangles, thus sum of interior angles is $3 \cdot 180 = 540$.
- Formula of interior one angle size
$$\text{angle} = \frac{(n-1)180}{n}$$

[2]Generally, regular forms exhibit the largest area possible of its kind

- Formula of total interior angles:

$$\text{total angle } = (n-2) \cdot 180$$

- Quadrilateral: Four sided polygon. Sum of interior angles is 360° (hässliches viereck)
- Rectangle: Quadrilateral with four equal angles, each a right angle (rechteck)
- Square: Quadrilateral with four equal sides and equal angles (Quadrat)
  - All exterior angles must add up to 360 degree
  - The diagonal has the length of $s\sqrt{2}$

  Fun facts about hexagons:

  1. Area of regular hexagon

  $$A_6 = \frac{3 * \sqrt{3}}{2} s^2$$

  2. Consists of 6 equilateral triangles

  $$\text{Number of diagonals} = \frac{n(n-3)}{2}$$

6.5 Circles

- Circles with the same center a called concentric
- Chord = Line segment joining two points on the circle (diameter is the longest chord possible)
- Central angle = angle created by two radii
- π = ratio circles circumference to its diameter = 3,1314 = 3
- Circumfence = $\pi \cdot d = 2 \cdot \pi \cdot r$
- arc = portion of the circumference (Piece of cake):
  - Has same angle as the central angle created by the two radii
  - Minor arc = shorter distance along the circumference between two points.
  - Major arc = longer distance along the circumference between two points

- Length of the arc = fraction of circles circumference as its degree

$$\text{arc length} = \frac{n}{360} \cdot \pi 2r$$

- Area of circle = πr^2
- Sector = portion of circles area

$$\text{area sector} = \frac{n}{360} \cdot \pi r^2$$

- Inscribed polygon = all vertices of polygon lay on circle
- Inscribed triangle = all vertices are allocated on the circle
- Circumscribed polgyon = all sides are tangent to the circle
- The degree measure of an inscribed angle equals half of the degree measure of the arc it intercepts.
- Equilateral inscribed triangles area is sufficient to determine circles area and circumfence. The side length of the triangle allows to determin the radius by exploiting the property of the 30:60:90 triangle than emerges when dividing one side by a half and drawing from the center of one of the sides a line to the center of the circle. The ratio of the sides of the newly created subtriangle is $x : \sqrt{3}x : 2x$
- Two circles with same center

$$A = \pi(R_2 - R_1)$$

- Formular of circle:

$$(x-h)^2 + (y-k)^2 = r^2$$

  - h = x-offset from the origin
  - k = y-offset from the origin

  If the formular is given in another form then 1) complete the perfect squares. Then built the $(x-h)^2$ formel and DONT forget to add the number also on the right hand side (r-side)

6.6 Solids

- Edge = Connects vertices. Cube has 12 edges.
- Face = Polygons that are the boundaries of the solid. Cube has 6 faces.
- Rectangular:
  - Volume = length * width * height

  - Surface area = $2lw + 2lh + 2hw$
  - Diagonal

$$d^2 = x^2 + y^2 + z^2$$
$$d = \sqrt{x^2 + y^2 + z^2}$$

- Cube: Special rectangular with l = w = h
  - Volume = lwh = l^3
  - Surface area = $6s^2$
  - Diagonal

$$d = s\sqrt{3}$$

- Cylinder = soup can
  - Volume = $\pi r^2 h$
  - Lateral surface area = $2\pi rh$
  - Total surface area = $2\pi r^2 + 2\pi rh$
- Traps:
  1. If the volume is give but none of the w,l,h variables, we cannot determine the e.g. height of an inflowing stream after x minutes
  2. Placing items in another spherical form, cannot be determined without some information on the form

7 Probability

$$\frac{choices}{\text{total number}} = [0, 1]$$

- Probabilities (sample space) must add up to 1

Type of Event	**Definition**	**Formula**	**Key Property**
Independent Events	The outcome of one event does not affect the outcome of the other.	$P(A \cap B) = P(A) \cdot P(B)$	The probability of $A \cap B$ is the product of their individual probabilities.
Dependent Events	The outcome of one event affects the probability of the other.	$P(A \cap B) = P(A) \cdot P(B\|A)$	Use conditional probability $P(B\|A)$ to account for the dependency between A and B.
Mutually Exclusive Events	Events that cannot happen at the same time (no overlap between events).	$P(A \cup B) = P(A) + P(B)$	The probability of $A \cup B$ is the sum of their individual probabilities, since $P(A \cap B) = 0$.

Table 1: Comparison of Independent, Dependent, and Mutually Exclusive Events

- If there are **multiple outcomes**, consider: actual number of event x probability. E.g. throw a coin twice and get one head. It can be be either the first or second throw a head. And the probability is both a half so

$$2 * \frac{1}{2} * \frac{1}{2}$$

  If there are more rounds like four rounds and 2 should be head (HHTT), the problem becomes:

$$\text{Possibilities * probability } = \frac{4!}{2 * 2} * \frac{1}{2^4}$$

- **independent** events = A + B - both

- both = just multiply AB
- or add up A and B and subtract middle part (the both result), thus exclude the overlap that has been included in both bits
- AND = multiplying (if independent events)
- **dependent**: Multiply but reduce in each step the number that is left. E.g. Draw heart cards from a staple independent events can occur at the same time
- **Mutually exclusive**:
  - OR (mutually exclusive) = adding up (no overlap so no and
    If the probabilities sum up to something greater than one they cannot be mutually exclusive, because the sum of mutually exclusive sets is equal to one
- If binomial probability than add exponent how often this prob occur
- At least / at most use the opposite if it saves steps of calculations. Instead of how often it appears use how often does it not appear.
- at least = greater than or equal

- ## Probability Using Combination Method

Theory

When solving probability problems with dependent outcomes, the combination method is used to calculate the probability as:

$$P(\text{event}) = \frac{\text{Number of Favorable Outcomes}}{\text{Total Number of Outcomes}}$$

where combinations ($\binom{n}{r}$) count the number of ways to choose r items from n total items without regard to order:

$$\binom{n}{r} = \frac{n!}{r!(n-r)!}.$$

Example

In a veterinary waiting room, there are 5 cats and 3 dogs. If 5 animals are randomly selected, what is the probability that exactly 3 cats and 2 dogs are selected?

Solution

- **Step 1: Total Outcomes**
  Total animals = 5 cats + 3 dogs = 8 animals. Total ways to choose 5 animals:
  $$\binom{8}{5} = \frac{8!}{5!(8-5)!} = \frac{8 \cdot 7 \cdot 6}{3 \cdot 2 \cdot 1} = 56.$$
- **Step 2: Favorable Outcomes**
  To select 3 cats from 5:
  $$\binom{5}{3} = \frac{5!}{3!(5-3)!} = \frac{5 \cdot 4}{2 \cdot 1} = 10.$$
  To select 2 dogs from 3:
  $$\binom{3}{2} = \frac{3!}{2!(3-2)!} = \frac{3 \cdot 2}{2 \cdot 1} = 3.$$
  Total favorable outcomes:
  $$\binom{5}{3} \cdot \binom{3}{2} = 10 \cdot 3 = 30.$$
- **Step 3: Probability**
  The probability is:
  $$P = \frac{\text{Favorable Outcomes}}{\text{Total Outcomes}} = \frac{30}{56} = \frac{15}{28}.$$

The probability of selecting 3 cats and 2 dogs is $\frac{15}{28}$.

- If we must choose out of a group N and select C people, but 1 c is certain and 1 c is for sure excluded one follow this recipe:
  $$\frac{\text{Possible outcome}}{\text{Total outcome}}$$
  - Possible outcome = subtract both the certain included and certain added so -2
  - Reduce C in possible outcome by 1 because it is already taken now by the 1c

7.1 Combinatorics & Permutation

7.1.1 Permutation = order is important

$$5! = 120$$

- If restriction start with reduced number

- If not identical units are pooled, then don't forget to multiply by the number of possible orders within that group
- Number of cases where eg pooling is not the case:
  total number - number where it is the case
  = number where it is not the case
- Choice method can be applied eg

$$\frac{10!}{10!-3!} = \frac{10!}{7!} = 10 \cdot 9 \cdot 8 = 720$$

- Repeated sequences = divide by repetitions: BOOK has 4 letters but two repeat

$$\frac{4!}{2!} = \frac{4 * 3 * 2}{2} = 4 * 3 = 12$$

  $\rightarrow$ count only the number of distinguishable permutation

- Permutation in a circle is total number - 1 as the two borders touch each other

$$(N-1)! = (6-1)! = 5!$$

- Use anchoring if some restriction is give, such as Jack must sit on the first seat
- Bundling together: When two objects must be next to each other bundle them.
  Attention!
  1. Dont forget to consider the position within a bundle. E.g. A and B are together, so its AB in once place, but it could also be BA $\rightarrow$ multiply by 2
  2. The bundle could, if not given otherwise, be placed at any position multiply by the possible positions. E.g. there are 5 people but AB must be together so we act as we have 4 spaces. Thus the permutation requires $\rightarrow$ multiply by 4
- In "cannot" scenarios, as Tim and Bill cannot sit next to each other examples, follow these steps
  1. Calculate the permutation as there were no restriction at all
  2. Calculate the permutation as if they were bundled
  3. Subtract all - bundled to receive the cannot scenario

7.1.2 Combinations = order is not important

- Combinations shortcut

$$\frac{n!}{(n! - r!)r!} = \frac{6!}{3!3!} = \frac{6 \cdot 5 \cdot 4}{3 \cdot 2}$$

If there is a restriction such that out of a group N, two specific items A and B are not allowed to be selected together, the total number of valid combinations can be calculated as:

$$\text{Valid Combinations} = \text{Total Unconstrained Combinations} - \text{Combinations Where Both Are Selected}$$

Example: Group of 6, Selecting 3 - We have a group of 6 people, and we need to select 3. However, A and B cannot be selected together.

Step 1: Total Unconstrained Combinations - The total number of ways to select 3 people out of 6 is:

$$\binom{6}{3} = \frac{6 \times 5 \times 4}{3 \times 2 \times 1} = 20.$$

Step 2: Combinations Where Both A and B Are Selected - If A and B are already selected, only 1 additional person can be chosen from the remaining 4 people:

$$\binom{4}{1} = 4.$$

Step 3: Valid Combinations - Subtract the combinations where both A and B are selected from the total unconstrained combinations:

$$\text{Valid Combinations} = \binom{6}{3} - \binom{4}{1} = 20 - 4 = 16.$$

General Formula For any group of size N, selecting x people with the restriction that two specific items A and B cannot both be selected:

$$\text{Valid Combinations} = \binom{N}{x} - \binom{N-2}{x-2}.$$

Key Explanation - $\binom{N}{x}$: Total combinations without restriction. - $\binom{N-2}{x-2}$: Combinations where A and B are pre-selected, leaving $N - 2$ items to fill $x - 2$ remaining spots.

This method ensures we correctly account for the restriction.

- In more complex scenarios where two choice problems are dependent on each other, e.g. in the first half of the exam a students must have at least 3/5 and in total 7/10 the

1. Multiply $5C3$ times $5C4$
2. Add second scenario $5C4$ times $5C3$
3. Last scenario $5C5$ times $5C2$

$\rightarrow$ Calculate each combination - directly multiply - and then sum the results up

8 General tips

- Quantitative comparisons make a large part
- If A and B are number "The relationship cannot be determined" is never true
- As soon as two solutions are possible, then "The relationship cannot be determined" is true
- Is there are <u>not</u> in the sentence?
- Work problems: is the question asking for rate, work or time?
- QC, setting equations equal can be beneficial
- In ordering of a line dont forget to consider all the possible positions of the pooling
- Inequalities: be meticulous whether its a $<$ or $\leq$ sign
- In QC questions with π dont forget that π is slightly larger than 3