Boolean Function Simplification

Tabular Method

Prepared by
Abdullah Al Fahad(1805033)
and
Hasan Masum(1805052)

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Table of Contents

Introduction

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- No limit on the number of input variables.
- Can be programmed and implemented in a computer.
- Based on the concept of prime implicants.

Terminology

Implicants

Implicant is a product/minterm in Sum of Products (SOP) form or sum/maxterm in Product of Sums (POS) form of a Boolean function. E.g., consider a boolean function, $F = AB + ABC + BC$. Implicants are AB, ABC and BC.

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Prime implicants

A prime implicant of a function is an implicant that cannot be covered by a more general, (more reduced with fewer literals) implicant.

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Prime implicants

A prime implicant of a function is an implicant that cannot be covered by a more general, (more reduced with fewer literals) implicant.

Essential prime implicants

Essential prime implicants (aka core prime implicants) are prime implicants that cover an output of the function that no combination of other prime implicants is able to cover.

How it works?

This tabular method is useful to get the prime implicants by repeatedly using the following Boolean identity.

$$xy + xy' = x(y + y') = x.1 = x$$

- Two major steps
  - Identify prime imlicants(implicant tables)
  - Identify essential prime implicants(cover tables).

Example

Example

$Y(A, B, C, D) = \sum_m(2, 6, 8, 9, 10, 11, 14, 15)$

we will minimize it using *Tabuler Method*

Implication table

Step-1: Group the minterm according to the number of 1's

Group	Minterm	binary
1	m_2	0 0 1 0
	m_8	1 0 0 0

Implication table

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	m_8	1 0 0 0
2	m_6	0 1 1 0
	m_9	1 0 0 1
	m_{10}	1 0 1 0

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1	m_2	0 0 1 0
	m_8	1 0 0 0
2	m_6	0 1 1 0
	m_9	1 0 0 1
	m_{10}	1 0 1 0
3	m_{11}	1 0 1 1
	m_{14}	1 1 1 0

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	m_{10}	1 0 1 0
3	m_{11}	1 0 1 1
	m_{14}	1 1 1 0
4	m_{15}	1 1 1 1

Implication table

Step-2: Merge minterms from adjacent groups to form a new implication table.

Group	Min Term	A	B	C	D
1	m_2	0	0	1	0
	m_8	1	0	0	0
2	m_6	0	1	1	0
	m_9	1	0	0	1
	m_{10}	1	0	1	0
3	m_{11}	1	0	1	1
	m_{14}	1	1	1	0
4	m_{15}	1	1	1	1

Implication table continue

Step-2: Merge minterms from adjacent groups to form a new implication table.

Group	Minterm	A	B	C	D
1	m_2 ✓	0	0	1	0
	m_8	1	0	0	0
2	m_6 ✓	0	1	1	0
	m_9	1	0	0	1
	m_{10}	1	0	1	0
3	m_{11}	1	0	1	1
	m_{14}	1	1	1	0
4	m_{15}	1	1	1	1

Group	Minterm	Binary			
1	m_2, m_6	0	-	1	0

Implication table continue

Step-2: Merge minterms from adjacent groups to form a new implication table.

Group	Minterm	A	B	C	D
1	m_2 ✓	0	0	1	0
	m_8	1	0	0	0
2	m_6 ✓	0	1	1	0
	m_9	1	0	0	1
	m_{10} ✓	1	0	1	0
3	m_{11}	1	0	1	1
	m_{14}	1	1	1	0
4	m_{15}	1	1	1	1

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	m_2, m_{10}	-	0	1	0

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	m_{10} ✓	1	0	1	0
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	m_{14}	1	1	1	0
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1	m_2, m_6	0	-	1	0
	m_2, m_{10}	-	0	1	0
	m_8, m_9	1	0	0	-

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2	m_6 ✓	0	1	1	0
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Group	Minterm	Binary			
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	m_2, m_{10}	-	0	1	0
	m_8, m_9	1	0	0	-
	m_8, m_{10}	1	0	-	0

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2	m_6 ✓	0	1	1	0
	m_9 ✓	1	0	0	1
	m_{10} ✓	1	0	1	0
3	m_{11} ✓	1	0	1	1
	m_{14} ✓	1	1	1	0
4	m_{15}	1	1	1	1

Group	Minterm	Binary			
1	m_2, m_6	0	-	1	0
	m_2, m_{10}	-	0	1	0
	m_8, m_9	1	0	0	-
	m_8, m_{10}	1	0	-	0
2	m_6, m_{14}	-	1	1	0
	m_9, m_{11}	1	0	-	0
	m_{10}, m_{11}	1	0	1	-
	m_{10}, m_{14}	1	-	1	0

Implication table continue

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	m_8 ✓	1	0	0	0
2	m_6 ✓	0	1	1	0
	m_9 ✓	1	0	0	1
	m_{10} ✓	1	0	1	0
3	m_{11} ✓	1	0	1	1
	m_{14} ✓	1	1	1	0
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Group	Minterm	Binary			
1	m_2, m_6	0	-	1	0
	m_2, m_{10}	-	0	1	0
	m_8, m_9	1	0	0	-
	m_8, m_{10}	1	0	-	0
2	m_6, m_{14}	-	1	1	0
	m_9, m_{11}	1	0	-	0
	m_{10}, m_{11}	1	0	1	-
	m_{10}, m_{14}	1	-	1	0
3	m_{11}, m_{15}	1	-	1	1
	m_{14}, m_{15}	1	1	1	-

Implication table continue

Step-3: Repeat step 2 until no more merging is possible.

Group	Minterm	A	B	C	D
1	m_2, m_6	0	-	1	0
	m_2, m_{10}	-	0	1	0
	m_8, m_9	1	0	0	-
	m_8, m_{10}	1	0	-	0
2	m_6, m_{14}	-	1	1	0
	m_9, m_{11}	1	0	-	0
	m_{10}, m_{11}	1	0	1	-
	m_{10}, m_{14}	1	-	1	0
3	m_{11}, m_{15}	1	-	1	1
	m_{14}, m_{15}	1	1	1	-

Group	Minterm	Binary			
1	m_2, m_6, m_{10}, m_{14}	-	-	1	0
	m_2, m_{10}, m_6, m_{14}	-	-	1	0
	m_8, m_9, m_{10}, m_{11}	1	0	-	-
	m_8, m_{10}, m_9, m_{11}	1	0	-	-

Implication table continue

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1	m_2, m_6	0	-	1	0
	m_2, m_{10}	-	0	1	0
	m_8, m_9	1	0	0	-
	m_8, m_{10}	1	0	-	0
2	m_6, m_{14}	-	1	1	0
	m_9, m_{11}	1	0	-	0
	m_{10}, m_{11}	1	0	1	-
	m_{10}, m_{14}	1	-	1	0
3	m_{11}, m_{15}	1	-	1	1
	m_{14}, m_{15}	1	1	1	-

Group	Minterm	Binary			
1	m_2, m_6, m_{10}, m_{14}	-	-	1	0
	m_2, m_{10}, m_6, m_{14}	-	-	1	0
	m_8, m_9, m_{10}, m_{11}	1	0	-	-
	m_8, m_{10}, m_9, m_{11}	1	0	-	-
2	$m_{10}, m_{11}, m_{14}, m_{15}$	1	-	1	-
	$m_{10}, m_{14}, m_{11}, m_{15}$	1	-	1	-

Implication table continue

Group	Minterm	A	B	C	D
1	m_2, m_6, m_{10}, m_{14}	-	-	1	0
	m_2, m_{10}, m_6, m_{14}	-	-	1	0
	m_8, m_9, m_{10}, m_{11}	1	0	-	-
	m_8, m_{10}, m_9, m_{11}	1	0	-	-
2	$m_{10}, m_{11}, m_{14}, m_{15}$	1	-	1	-
	$m_{10}, m_{14}, m_{11}, m_{15}$	1	-	1	-

Implication table continue

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1	m_2, m_6, m_{10}, m_{14}	-	-	1	0
	m_2, m_{10}, m_6, m_{14}	-	-	1	0
	m_8, m_9, m_{10}, m_{11}	1	0	-	-
	m_8, m_{10}, m_9, m_{11}	1	0	-	-
2	$m_{10}, m_{11}, m_{14}, m_{15}$	1	-	1	-
	$m_{10}, m_{14}, m_{11}, m_{15}$	1	-	1	-

The reduced table after removing the redundant rows is shown below:

Group	Minterm	Binary			
1	m_2, m_6, m_{10}, m_{14}	-	-	1	0
	m_8, m_9, m_{10}, m_{11}	1	0	-	-
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	m_8, m_{10}, m_9, m_{11}	1	0	-	-
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No more merging possible

Implication table continue

Group	Minterm	Binary			
1	m_2, m_6, m_{10}, m_{14}	-	-	1	0
	m_8, m_9, m_{10}, m_{11}	1	0	-	-
2	$m_{10}, m_{11}, m_{14}, m_{15}$	1	-	1	-

There are three rows in the above table. So, each row will give one prime implicant. Therefore, the prime implicants are CD', AB' & AC.

Cover table

Min terms/ Prime Imlicants	2	6	8	9	10	11	14	15
CD'	1	1			1		1	
AB'			1	1	1	1		
AC					1	1	1	1

- The min terms 2 and 6 are covered only by one prime implicant CD'. So, it is an essential prime implicant.
- The min terms 8 and 9 are covered only by one prime implicant AB'. So, it is an essential prime implicant.
- The min terms 8 and 9 are covered only by one prime implicant AB'. So, it is an essential prime implicant.

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Min terms/ Prime Imlicants	2	6	8	9	10	11	14	15
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Smplified Expression

We got three prime implicants and all the three are essential. Therefore, the simplified Boolean function is,

$$Y(A, B, C, D) = CD' + AB' + AC$$

Summary of the method

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- Step-2: Compare and merge the min terms present in successive groups.
- Step-3: Repeat step-2 with newly formed terms till we get all prime implicants.
- Step-4: Formulate prime implicants table(cover table) and reduce it removing the row of each essential prime implicant and the columns corresponding to the min terms.

References

- Tutorials point
- https://en.wikipedia.org/wiki/Implicant