McGill Hackathon

Modelling Supernova Blast Waves

Team Fusion Chips

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Abstract

In this paper, the Fusion Chips team presents Supernova Remnant (SNR) Phase Modeller. This program models type II supernova blast shock waves at 4 different stages of their evolution. The program takes stellar mass as an input and produces an animated plot which models the expansion rate of the SNR throughout its free expansion phase, Sedov Taylor phase, Snowplow phase, and its Momentum driven phase. The SNR Phase Modeller is an educational tool to help instructors describe the magnitude of a supernova as well as its evolution over time. In addition, it can be used as a visual aid to explain the interaction between the shock wave and the interstellar medium (ISM).

1 Introduction

As a main sequence star reaches the end of its lifetime, hydrogen burning ceases to occur in the star's core. As a result, there is no longer a sufficient amount of pressure radially outward which causes the star to collapse[1]. If the star's mass is greater than the *Chandrasekar Limit* (≈ 1.4 $M_\odot$), this collapse will result in an immense explosion known as a *type II supernova*[2]. The energy released during a type II supernova is on the order of 1% of the star's mass energy[3].

$$E_{\mathrm{SNR}} \approx \frac{1}{100} M_{\mathrm{S}} c^2 \tag{1}$$

Where E_{SNR} is the energy released in the type II supernova, M_{S} is the mass of the star, and c is the speed of light. With the assumption that this energy is completely converted into kinetic energy, the initial velocity of the blast can be derived (here denoted as V_{FE} to represent velocity during *free expansion*).

$$\frac{1}{100} M_{\mathrm{S}} c^2 = \frac{1}{2} M_{\mathrm{S}} V_{\mathrm{FE}} \implies V_{\mathrm{FE}} = \frac{c}{5\sqrt{2}} \tag{2}$$

This expansion rate remains stable throughout the first phase of the supernova remnant's (SNR) expanion and it is known as the *free expansion phase*. During the free expansion phase, the radius of the SNR is given by the following:

$$R_{\mathrm{FE}} = tV_{\mathrm{FE}} \quad \text{Where t is time.} \tag{3}$$

The free expansion phase comes to an end once the SNR has reached its *sweep-up radius*[4] R_{SW}. From the sweep-up radius, the time in which the SNR undergoes free expansion can be derived.

$$R_{\mathrm{SW}} = \Big(\frac{3M_{\mathrm{S}}}{4\pi\rho_0}\Big)^{1/3} \implies T_{\mathrm{FE}} = \frac{1}{V_{\mathrm{FE}}}\Big(\frac{3M_{\mathrm{S}}}{4\pi\rho_0}\Big)^{1/3} = \Big(\frac{3M_{\mathrm{S}}c^3}{1000\sqrt{2}\pi\rho_0}\Big)^{1/3} \tag{4}$$

Where ρ_0 is the average density of the local interstellar medium (≈ 10 molecules/cm^3)[5] and $\mathrm{T_{FE}}$ is the time at which the SNR reaches its sweep-up radius. At time $\mathrm{T_{FE}}$, the mechanism driving the expansion of the SNR changes from the pressure caused by the inital explosion to the thermal pressure between gas molecules in the SNR[5]. This new phase is known as the *Sedov-Taylor Phase*. Throughout the Sedov-Taylor phase, the energy lost to radiation is insignificant thus, the SNR expands adiabatically[5]. The radius of the SNR as a function of time is given by the following equation [5]:

$$R_{\mathrm{ST}} = \Big(\frac{75E_{\mathrm{SNR}}(\gamma^2-1)}{8\pi(\gamma^2+3)\rho_0}\Big)^{1/5} t^{2/5} \tag{5}$$

Where $\gamma = \frac{5}{3}$ is the adiabatic index. The SNR will remain in the Sedov-Taylor phase until it has cooled to about 10^6K[5]. At this point, the ions recombine into molecules which ceases the adiabatic expansion. The SNR then cools and its rate of expansion is reduced. The following equation relates the mass of the star to the point in time where this transition occurs (the derivation is presented in the Appendix).

$$t \approx \Big(2.16 * 10^{-25}\frac{s}{kg^{1/3}}\Big)M_S^{1/3} \tag{6}$$

Once the SNR has sufficiently cooled, it begins its third phase known as the *Snowplow phase*. In this phase, the SNR drifts through space "plowing" molecules in the ISM. The radius as a function of time of the SNR during the Snowplow phase is given by the following[5]:

$$R = Bt^{\alpha} \quad \text{Where} \quad \alpha = \frac{2}{2\gamma+3} \tag{7}$$

By imposing continuity between the Sedov-Taylor phase and the Snowplow phase, the value of B was determined.

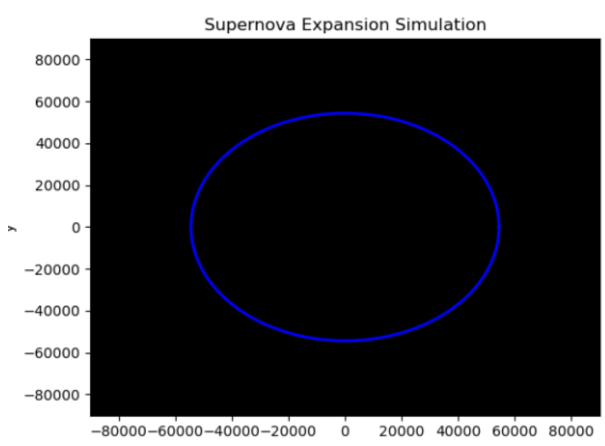
$$B = \left(1.78 * 10^4 \frac{m}{s^{6/19} kg^{13/57}}\right) M_S^{13/57} \tag{8}$$

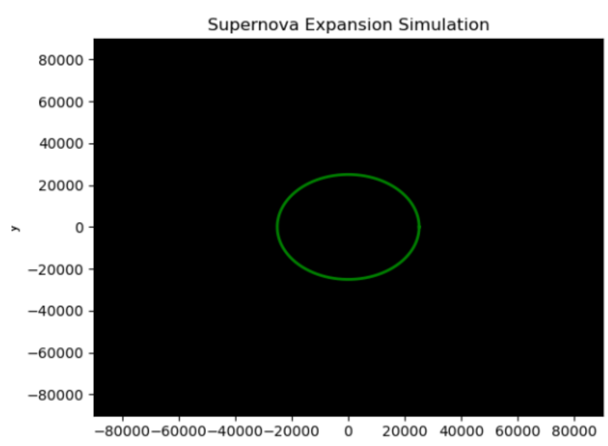
After roughly half a million years[6], the SNR reaches a phase known as the *Momentum driven phase* where the gas particulates gradually merge with the ISM. During this phase, the radius of the SNR is given by [5]:

$$R = \left(\frac{3t}{4\pi\rho_0}\right)^{1/4} \tag{9}$$

2 Results

The Supernova Remnant (SNR) Phase Modeller can be found on Github via the following link: https://github.com/hmathlee/super-sim. There are two modellers in the repository. The *Supernova Final* which models the SNR's expansion over a uniform time scale and the *Time Evolution Diagram* which compresses the time scale allowing the viewer to witness the full evolution of a SNR. Two plots from the Supernova Remnant (SNR) Phase Modeller are shown below. Note that in the animated plot, the SNR blast wave color changes depending on the phase to allow for easier deciphering of the time periods of each phase.





3 Further Studies

The Fusion Chips team plans on producing a 3 dimensional model of the SNR over time to allow for even more intuitive interpretation of how a SNR evolves. In addition, the team hopes to incorporate a heat map in the plot to represent the density at any point within the SNR's radius as well as energy and pressure distribution.

4 Bibliography

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A Appendix A

Since the expansion is adiabatic, the pressure and density are:

$$P = \frac{2\rho_0 V_S^2}{\gamma + 1} \quad \text{and} \quad \frac{\rho_1}{\rho_0} = \frac{\gamma + 1}{\gamma - 1} \tag{10}$$

Assuming the gas is ideal, these equations can be combined with the ideal gas law to obtain:

$$P = \frac{k}{\mu m_u} \rho_1 T \quad \Longrightarrow \quad T = \frac{3\mu m_u}{100k} \left(\frac{25 E_{SNR}}{4\pi\rho_0} \right)^{2/5} t^{-6/5} \tag{11}$$

Where μ, m_u, k, and ρ_0 are all constants, the time at which the temperature reaches 10^6K can be solved for in terms of $\mathrm{M_S}$.

$$T = \frac{3\mu m_u}{100k} \left(\frac{M_S c^2}{1600\pi\rho_0} \right)^{2/5} t^{-6/5} \quad \Longrightarrow \quad T \approx 2.52 * 10^{-24} M_S^{2/5} t^{-6/5} \tag{12}$$

Setting T = 10^6K,

$$10^6 = 2.52 * 10^{-24} M_S^{2/5} t^{-6/5} \quad \Longrightarrow \quad t \approx \left(2.16 * 10^{-25} \frac{s}{kg^{1/3}} \right) M_S^{1/3} \tag{13}$$