Taller 2

1

$\boxed{V_0}$

U_1

$U_1 = V_0$
$U_2 = \frac{1}{3}U_1$
$U_3 = \frac{1}{3}U_2$

$\rightarrow U_n = \frac{1}{3}U_{n-1}, \; n \geq 1$

$\{U_n = k^{n-1}U_1\}, \; k = \frac{1}{3}$

Para $U_n = (1/1000{,}000)V_0$

$$\frac{\cancel{V_0}}{1 \cdot 10^6} = \left(\frac{1}{3}\right)^{n-1}\cancel{V_0}$$

$$\log(1/10^6) = (n-1)\log(1/3)$$

$$n = \frac{\log(1) - \log(10^6)}{\log(1) - \log(3)} + 1$$

$n = 13.57$

Despues de 14 iteraciones hubo menos de $V_0(10^{-6})$

2

U = poblacion
n = año
r = tasa

$1000(r) = 25$
$r = 1/40$

$\rightarrow U_n = U_{n-1}(1+r)$

$U_n = (1.025)^{n-1}U_1, \; k = (1+r)$

Para $n = 15$, $U_1 = 200 \; 10^6$

$$U_{15} = 1.025^{14} \cdot 200 \cdot 10^6 = \boxed{282.59 \cdot 10^6 \text{ personas}}$$

Para $U_1 = 200 \cdot 10^6$, $U_n = 150 \cdot 10^6$

$$150 \cdot \cancel{10^6} = (1.025)^{n-1}\, 200 \cdot \cancel{10^6}$$

$$\log(150/200) = (n-1)\log(1.025)$$

$$n = \frac{\log(150) - \log(200)}{\log(1.025)} + 1$$

$n = 54.53$

despues de 55 años habra mas de $150 \cdot 10^6$ personas

3. $U_n = 4U_{n-1} - 1,\ n \neq 2$

$k = 4,\ c = -1$

t.q. $U_n = kU_{n-1} + C$, entonces.

$U_n = k(kU_{n-2} + C) + C = k(k(kU_{n-3} + C) + C) + C \ldots$

$= k^{n-1}U_1 + k^{n-2}C + \ldots + kC + C$

$= k^{n-1}U_1 + C(k^{n-2} + k^{n-3} + \ldots + 1)$

ahora vemos que $P_n = \sum_{i=2}^{n} k^{n-1} = \dfrac{(k^{n-1} - 1)}{k-1},\ n \geq 2,\ k \neq 1$

1. Caso base $n = 2$

$\sum^{2} k^{n-1} = k^{2-2} = k^0 = 1$ ademas que $\dfrac{k^{2-1} - 1}{k-1} = \dfrac{k-1}{k-1} = 1$

por tanto P_2 se cumple.

2. Caso inductivo

Sea $n \in \mathbb{Z}$ cualquiera, t.q. $n \geq 2$ y P_n se cumple, tomando cualquier $k \neq 1$

$P_{n+1} = k^{n-1} + k^{n-2} + k^{n-3} + \ldots + 1$ de modo que se cumple P_n

$= k^{n-1} + P_n$

$= k^{n-1} + (k^{n-1} - 1)/(k-1)$

$= \dfrac{k^{(n+1)-1} - k^{n-1} + k^{n-1} - 1}{k-1}$

$= \dfrac{k^{(n+1)-1} - 1}{k-1} = \dfrac{k^n - 1}{k-1}$

De esta forma

$U_n = k^{n-1} U_1 + C \cdot \frac{k^{n-1}-1}{k-1}$

$\boxed{U_n = 4^{n-1} U_1 - \frac{1}{3}(4^{n-1}-1)}$

$U_n = 3U_{n-1} + 2,\ n \geq 2 \quad k=3$ y $C=2$

por lo anterior demostrado

$\boxed{U_n = 3^{n-1} U_1 + 2(3^{n-1}-1)}$

4) $U_n = -4U_{n-1} - 3,\ n \geq 1$

usando el procedimiento de punto 3

$\boxed{U_n = -4^{n-1} U_1 + \frac{3}{5}(-4^{n-1} - 1)}$

$U_n = -2U_{n-1} + 13,\ n \geq 1$

similarmente

$\boxed{U_n = -2^{n-1} U_1 - \frac{13}{3}(-2^{n-1}-1)}$

5) $U_n = 3U_{n-1} + 5,\ n \geq 1$

$U_0 = 1$

$U_n = 3^{n-1+1} U_0 + \frac{5}{2}(3^{n-1+1}-1)$

$= 3^n U_0 + \frac{5}{2}(3^n - 1)$

$= \boxed{\frac{1}{2}(7(3^n)-5)}$

→ lo que implica que existe un termino adicional para llegar al n-esimo termino, con respecto a la formula previamente obtenida

⑥ $U_1 = 7 \rightarrow U_2 - U_1 = 10 \rightarrow U_1 = 7 \rightarrow U_n = U_{n-1} + 10(2^{n-2})$

$U_2 = 17 \quad U_3 - U_2 = 20 \quad U_2 = U_1 + 2^0(10)$

$U_3 = 37 \quad U_4 - U_3 = 40 \quad U_3 = U_2 + 2^1(10)$

$U_4 = 77 \quad U_4 = U_3 + 2^2(10)$

$$\boxed{\begin{array}{l} U_n = U_{n-1} + 5(2^{n-1}) \\ n \geq 2 \end{array}}$$

para Hallar la solución general

$$U_n = U_{n-1} + 5(2^{n-1})$$
$$= (U_{n-2} + 5^{n-2}) + 5 \cdot 2^{n-1}$$
$$= ((U_{n-3} + 5\, 2^{n-3}) + 5^{n-2}) + 5 \cdot 2^{n-1}$$
$$= U_1 + 5(2^1 + 2^2 + \ldots + 2^{n-1})$$

$$U_n = U_1 + 5(2^n - 2)$$
$$= 7 + (5)(2)(2^{n-1} - 1)$$

$$\boxed{\begin{array}{l} U_n = 7 - 10(1 - 2^{n-1}) \\ n \geq 2 \end{array}}$$

$$\sum_{i=1}^{n-1} 2^i = \sum_{i=0}^{n-1} 2^i - 1 = 2^n - 1 - 1$$
$$= 2^n - 2$$

⑦

$V_0 = 400$ M

$t = 3$ años

$i = 0.21$

x = pago anual

Para culminar el pago a 3 años, se tiene $V_3 = 0$

Donde, $V_n = V_{n-1}(1+i) - x$ (interes compuestos)

resolviendo, para $K = 1.21$ y $C = -x$

$$V_n = (1.21)^n 400 - x \frac{(1.21^n - 1)}{0.21}, \quad \text{Con } n = 3$$

$$U_3 = 0 = (1.21)^3 400 - x(1.21^3 - 1)/0.21 \qquad x = \frac{(1.21)^3 \cdot 400}{(1.21^3 - 1)} \cdot 0.21$$

$\boxed{x = 192.87 \cdot 10^6}$ Pagando de a 142.37 millones anuales se salda la deuda en 3 años

8

La tasa de producción crece todos los meses.

$r_1 = 200(1.01)$

$r_2 = 200(1.01)(1.01)$

$r_3 = 200(1.01)(1.01)(1.01)$ $\rightarrow$ $r_n = 200(1.01)n$

$= 202n$

Donde

$202n - 1600 = 0$

$n = 1000/202$

$= 7.42$

es decir que solo desde el mes 8, la producción supera los ordenes de 1000T esperados

toda la producción previa a este punto se hace 0, pues el producido es menor que el maximo de lo que se puede vender.

Asi pues, para el café acumulado:

$U_n = (202n - 1600) + U_{n-1}$, $n \geq 8$, $U_7 = 0$ $(U_{0 \leq n \leq 7} = 0)$

para $n = 12$

$U_{12} = 202(6(13) - 23) - 5(1600)$

$= 10100 - 8000$

$\boxed{U_{12} = 2100T}$

Para $n = 24$

$U_{24} = 202(12(25) - 23) - 17(1600)$

$\boxed{U_{24} = 28754}$

$U_n = U_{n-2} + (202(n-1) - 1600) + (202n - 1600)$

$= U_7 - (n-7)1600 + 202(8 + 9 \dots + n)$

$\sum_{i=8}^{n} i = \sum_{i=1}^{n} i - \sum_{i=1}^{7} i$

$= \frac{1}{2}n(n+1) - 28$

$U_n = U_7 - (n-7)1600 + 202(\frac{1}{2}n(n+1) - 28)$

$U_n = 101(n^2 + n) - (n-7)1600 - 5656$

(9) $U_0 = 2000$, $U_n = U_n + (1.05) + 100$

$U_n = (1.05)^n 2000 + \frac{100}{0.05}(1.05^n - 1)$

$= 2000(1.05)^n + (1.05)^n - 1)$

$= 2000(2(1.05)^n - 1)$

Para $n=10$

$U_{10} = 2000(2(1.05)^{10} - 1)$

$= 2000(2.258)$

$= 4515.58$ arboles (no decimales)

de modo que, para la productividad [P]

$P = \frac{(U_{10} - U_0)}{U_0} \cdot 100$

$= \boxed{125.75\%}$ Hubo una mejora del 125.75% en la plantación

(10) $U_n = 3U_{n-1} + n$, $U_1 = 5 \rightarrow U_2 = 17$

$\rightarrow U_n = 3U_{n-1} = n$, ecuación no homogenea de la forma

forma $U_n = a + bn$

solución homogenea

$U_n - 3U_{n-1} = 0$

$m = \frac{3 \pm 3}{2}$ | $m_1 = 0$, $m_2 = 3$ | $U_n = A3^n$

Solución particular

$(a+bn)-3(a+b(n-1))=n$

$a-3a+bn-3bn+3b=n$

$3b-2(a+bn)=n$

$(3b-2a)+(-2bn)=n$

$\checkmark\ -2b=1$

$b=-1/2$

$\checkmark\ 3b-2a=0$

$2a=-3/2$

$a=-3/4$

Así pues,

$U_n=A(3^n)-3/4-\frac{1}{2}n \longrightarrow$ Para U_1 $5=A(3)-3/4-1/2$

$A=\frac{1}{3}\cdot\frac{25}{4}=\frac{25}{12}$

$$\boxed{U_n=\frac{25}{12}(3^n)-\frac{3}{4}-\frac{1}{2}n}$$

(11) $U_n=U_{n-1}+2^n$, no homogenea de la forma k^n donde $U_n=ak^n$

$U_n-U_{n-1}=2^n$

(1) $U_n-U_{n-1}=0$

$m^2-m=0$

$m(m-1)=0$

$m_1=0, m_2=1$

$U_n=A(1)^n$

$=A$

(2) $a2^n-a2^{n-1}=2^n$

$a2^n(1-2^{-1})=2^n$

$a=2$

$2(2^n)-2(2^{n-1})=2^n$

$2^n(2-2^0)=2^n$

$2^n(1)=2^n$

$2^n=2^n$

$\rightarrow$ $\boxed{U_n=2^{n+1}+A}$

$U_n = 2U_{n-1} + n$, no homogenea de la forma n
$U_n - 2U_{n-1} = n$ donde $U_n = a + bn$

① $U_n - 2U_{n-1} = 0$
$m(m-2) = 0$
$m_1 = 0,\ m_2 = 0$

$U_n = A(2^n)$

② $(a+bn) - 2(a+b(n-1)) = n$
$a - 2a + bn - 2bn + 2b = n$
$(2b - a) + (-bn) = n$
$b = -1$ $\quad 2b - a = 0$
$a = -2$

$U_n = -2 - n$

$\rightarrow \boxed{U_n = A(2^n) - 2 - n}$

⑫ Si $U_n = kU_{n-1} + 5$ y $U_1 = 4$ y $U_2 = 17$, encuentre los valores de K y U_6

$\rightarrow$ dado que $U_1 = 4$ y $U_2 = 17$ se puede calcular K para U_2:

$$U_2 = kU_1 + 5$$

sustituyendo U_1 $\quad U_2 = k \cdot 4 + 5$

sustituyendo U_2 $\quad 17 = k \cdot 4 + 5$

Resolver para K $\quad K = \frac{17-5}{4} = 3$ $\quad \boxed{K=3}$

$U_3 = 56$

$U_4 = kU_3 + 5$

$U_4 = 3 \cdot 56 + 5$

$U_4 = 173$

$U_5 = 524$

$U_6 = kU_5 + 5$

$U_6 = 3 \cdot 524 + 5$

$\boxed{U_6 = 1577}$

(13) $U_n = \frac{U_{n-1}}{U_{n-2}}$ $U_2 = \frac{U_1}{U_0}$

Dado que $U_1 = \frac{1}{6}$, se debe hallar U_0 para hallar U_2

Si $n \geq 2$ es posible retroceder en la relación de recurrencia

$U_1 = \frac{U_0}{U_{-1}}$

despejamos U_0:

$U_0 = U_1 (U_{n-1}) = U_0 = \left(\frac{1}{6}\right)(U_{-1})$

Con los valores obtenidos es posible hallar U_2

$U_2 = \frac{U_1}{U_0}$ $U_2 \; \frac{\left(\frac{1}{6}\right)}{\left(\frac{1}{6}\right)(U_{-1})}$ $U_2 = \frac{1}{U_{-1}}$

De la misma manera es posible calcular U_3 usando U_2 y U_1, lo mismo para U_4 con U_3 y U_2

→ calcular U_c por medio de la siguiente propiedad

$U_n = kU_{n-1} + c$

$U_n = K^{n-1} U_1 + \frac{c(K^{n-1} - 1)}{K-1}$

$U_6 = kU_5 + 5$

$U_6 = k^5 U_1 + \frac{5(k^5 - 1)}{k-1}$

$U_6 = 3^5 \cdot 4 + \frac{5(3^5 - 1)}{3-1}$

$U_6 = 243 \cdot 4 + \frac{5(242)}{2}$

$\boxed{U_6 = 1577}$

→ Comprobación de la propiedad Siendo $U_1 = 4$ $U_2 = 17$ $K = 3$

$U_n = kU_{n-1} + 5$

$U_3 = kU_2 + 5$

$U_3 = 3 \cdot 17 + 5$

$U_3 = 56$

$U_4 = kU_3 + 5$

$U_4 = 3 \cdot 56 + 5$

$U_4 = 173$

$U_5 = KU_4 + 5$

$U_5 = 3 \cdot 173 + 5$

$U_5 = 524$

$U_6 = KU_5 + 5$

$U_6 = 3 \cdot 524 + 5$

$\boxed{U_6 = 1577}$

(14)

$U_n = U_{n-1} + 2U_{n-2}$

$U_n - U_{n-1} - 2U_{n-2} = 0$

$K = n-2 \quad K+1 = n-1 \quad K+2 = n$

$U_{k+2} - U_{k+1} - 2U_k = 0$

Solución homogenea λ^k

$\lambda^{k+2} - \lambda^{k+1} - 2\lambda^k = 0$

$\cancel{\lambda^k}(\lambda^2 - \lambda - 2) = 0$

$(\lambda - 2)(\lambda + 1) = 0$

$\lambda = 2 \quad \lambda = -1$

$U(n) = C_1(-1)^n + C_2(2)^n$

$$\frac{U(n)}{U(n+1)} = \frac{C_1(-1)^n + C_2(2)^n}{C_1(-1)^{n+1} + C_2(2)^{n+1}}$$

$$= \frac{C_2(2)^n + C_1(-1)^n}{2C_2(2)^n - C_1(-1)^n} \frac{(-1)^n}{(-1)^n}$$

$$= \frac{C_2(-2)^n + C_1}{2(C_2(-2)^n - C_1}$$

Dado que se cumple en los 2 casos se concluye:

$$\lim_{n\to\infty} \frac{U(n)}{U(n+1)} = \frac{1}{2}$$

$$\lim_{n\to\infty} \frac{U(n)}{U(n+1)} = \lim_{n\to\infty} \frac{C_2(-2)^n + C_1}{2(C_2(-2)^n - C_1}$$

$n \in \mathbb{Z}^+$

n puede ser par o impar

n par $\to n = 2k \quad K \in \mathbb{Z}^+$

n impar $\to n = 2k+1 \quad K \in \mathbb{Z}^+$

si $n \to \infty \Rightarrow k \to \infty$

Caso 1 $\to$ n par

$$\lim_{k\to\infty} \frac{C_2(-2)^{2k} + C_1}{2C_2(-2)^{2k} - C_1} = \lim_{k\to\infty} \frac{C_2(4)^k + C_1 \to\infty}{2C_2(4)^k - C_1 \to\infty}$$

$$\stackrel{L.H.}{=} \lim_{k\to\infty} \frac{\ln(4)C_2(4)^k}{2\ln(4)C_2(4)^k} = \lim_{k\to\infty} \frac{1}{2} = \frac{1}{2}$$

Caso 2 $\to$ n impar

$$\lim_{k\to\infty} \frac{C_2(-2)^{2k+1} + C_1}{2C_2(-2)^{2k+1} - C_1} =$$

$$\lim_{k\to\infty} \frac{-2C_2(4)^k + C_1 \to\infty}{-4C_2(4)^k - C_1 \to\infty}$$

$$\stackrel{L.H.}{=} \lim_{k\to\infty} \frac{-2\cancel{\ln(4)C_2(4)^k}}{-4\cancel{\ln(4)C_2(4)^k}} =$$

$$= \lim_{k\to\infty} \frac{1}{2} = \frac{1}{2}$$

15 Encuentre el n-ésimo término de la siguiente secuencia.

$-3, 21, 3, 129, 147$

$$U_1 = -3,\ U_2 = 21,\ U_3 = 3,\ U_4 = 129,\ U_5 = 147$$

cada U_n se puede expresar como:

$$U_2 = 21 = (-1)^2(-3)^2 + 12 = (-1)^2(-3)^2 + (-1)2^2(-3)$$

$$U_3 = 3 = (-1)^3(-3)^3 + (-24) = (-1)^3(-3)^3 + (-1)^2 2^3(-3)$$

$$U_4 = 129 = (-1)^4(-3)^4 + 48 = (-1)^4(-3)^4 + (-1)^3 \cdot 2^4(-3)$$

$$U_5 = 147 = (-1)^5(-3)^5 + (-96) = (-1)^5(-3)^5 + (-1)^4 - 2^5(-3)$$

$$\boxed{U_n = (-1)^n U_1^n + (-1)^{n-1} 2^n U_1}$$

Comprobación

$$U_2 = (-1)^2 U_1^2 + (-1)^{2-1} 2^2 U_1$$
$$= 1(-3)^2 + (-1)^1 \cdot 4 \cdot (-3)$$
$$= 9 - 4(-3)$$
$$= 9 + 12 = 21\ ✓$$

$$U_4 = (-1)^4 U_1^4 + (-1)^{4-1} 2^4 U_1$$
$$= 1(-3)^4 + (-1)^3 \cdot 16(-3)$$
$$= 81 + (-1)16 \cdot (-3)$$
$$= 81 + 16 \cdot 3 = 129\ ✓$$

$$U_3 = (-1)^3 U_1^3 + (-1)^{3-1} 2^3 U_1$$
$$= (-1)(-3)^3 + (-1)^2 \cdot 8 \cdot (-3)$$
$$= (-1)(-27) + 1 \cdot 8 \cdot (-3)$$
$$= 27 - 8 \cdot 3 = 3\ ✓$$

$$U_5 = (-1)^5 U_1^5 + (-1)^{5-1} 2^5 U_1$$
$$= (-1)(-3)^5 + (-1)^4 \cdot 32 \cdot (-3)$$
$$= (-1)(-243) + 1 \cdot 32 \cdot (-3)$$
$$= 243 - 32 \cdot 3 = 147\ ✓$$

(16) $U_n - 6U_{n-1} + 8U_{n-2} = 0$

$U_1 = 10$
$U_2 = 28$
$U_6 = ?$

Polinomio Caracteristico

$r^2 + 6r + 8$

$$\frac{-(-6) \pm \sqrt{(-6)^2 - (4)(1)(8)}}{2(1)}$$

$$\frac{6 \pm \sqrt{36-32}}{2} = \frac{6 \pm \sqrt{4}}{2} = \frac{\cancel{8}}{\cancel{2}}4 \quad y \quad \frac{\cancel{4}}{\cancel{2}}2$$

Sol general

$A(4)^n + B(2)^n$

$10 = A4 + B2$

$28 = A16 + B4$

despejando

$\Rightarrow 8 = A8$

$A = \frac{\cancel{8}}{\cancel{8}}1$

$B = \frac{10-4}{2} = 3$

$\boxed{A=1 \quad B=3}$

Sol especifico

$4^n + 3 \cdot 2^n = U_n$

$U_6 = 4^6 + 3\,2^6$

$= 4096 + 3 \cdot 64$

$= 4096 + 192$

$\boxed{U6 = 4288}$

(17)

$U_{n+2} + 2U_{n+1} + U_n = 0$, $n \geq 1$ $\quad U_1 = -1$

$U_2 = -2$

Polinomio Caracteristico $\quad$ solución general

$r^2 + 2r + 1 \longrightarrow A(-1)^2 + Bn(-1)^2$

$(r+1)^2$

despejando

$-1 = -A - B$

$-2 = A + 2B \rightarrow -3 = B \quad -1 = -A - (-3)$

$-3 = B \quad -1 - 3 = -A$

$-4 = -A$

$4 = A$

Solución particular

$$\boxed{4(-1)^n + (-3)n(-1)^n}$$

(18).

$U_n - 5U_{n-1} + 6U_{n-2} = f(n), f(n) = 2$

→ hallamos polinomio

$m^2 - 5m + 6 = 0$ } $m_1 = 2$

$(m-2)(m-3) = 0$ } $m_2 = 3$

$U_n = A(2)^n + B(3)^n$ → parte homogenea

solución $a + bn$

$U_n = a + bn$

$U_{n-1} = a + b(n-1)$

$U_{n-2} = a + b(n-2)$

re-escribir

$a + bn - (5(a + b(n-1))) + 6(a + b(n-1))$

$\Rightarrow 2a + 2bn - 7b$

$\Rightarrow 2\underbrace{(a+bn)}_{1} \underbrace{-7b}_{0} = 2$

$+b = 0$

$b = 0$

$a + \underbrace{bn}_{0} = 1$

$a = 1$

$U_n = A(2)^n + B(3)^n + 1$

$f(n) = n$ se sabe que la solución de la parte homogenea es: $U_n = A(2)^n + B(3)^n$

$U_n = an + b$

$U_{n-1} = a(n-1) + b$ } a, b constantes

$U_{n-2} = a(n-2) + b$

reemplaza

$$an-5(a(n-1)+b)+6(a(n-2)+b)=n$$

$$\Rightarrow \underbrace{(2a)}_{1}n+\underbrace{(12b-7a)}_{0}=n$$

$a=\frac{1}{2}$ $\quad 12b-7(\frac{1}{2})=0 \quad b=\frac{7}{24}$

$$U_n=A(2)^n+B(3)^n+\frac{19}{24}$$

$F(n)=5^n$

Sol homogenea

$U_n=A(2)^n+B(3)^n$

sol. alternativa

reemplazar

$$\left.\begin{aligned} U_n&=a5^n \\ U_{n-1}&=a5^{n-1} \\ U_{n-2}&=a5^{n-2}\end{aligned}\right\} \quad a5^n-5(a5^{n-1})+6(a5^{n-2})$$

$$\Rightarrow 6\,5^{n-2}\cdot a \Rightarrow 5^n\cdot\left(\frac{6}{25}\cdot a\right)$$

$\frac{6}{25}a=1 \qquad \left[a=\frac{25}{6}\right]$

$$U_n=A(2)^n+B(3)^n+\frac{25}{6}$$

19

$U_n - 3U_{n-1} + 4U_{n-2} = 0$

P.C $\lambda^2 - 3\lambda + 4 = 0$

$\lambda = \frac{3 \pm \sqrt{9 - 4(4)1}}{2}$

$\lambda = \frac{3 \pm \sqrt{9-16}}{2}$

$\lambda = \frac{3 \pm \sqrt{7}\, i}{2}$

$\lambda = \frac{3 + \sqrt{7}\, i}{2}$ $\lambda = \frac{3 - \sqrt{7}\, i}{2}$

$\hookrightarrow \lambda = \frac{3}{2} + \frac{\sqrt{7}}{2} i \rightarrow \lambda = a + bi$

$a = \frac{3}{2}$ $b = \frac{\sqrt{7}}{2}$

$\sqrt{a^2 + b^2} = \sqrt{\frac{9}{4} + \frac{7}{4}} = 2$

$\cos(x) = \frac{3}{4}$ $\sin(x) = \frac{\sqrt{7}}{4}$

$\theta = 0{,}723 = \cos^{-1}\left(\frac{3}{4}\right)$

$U(n) = 2^n \left(C_1 \cos(0{,}723 n) + C_2 \sin(0{,}723 n) \right)$

$U(0) = 0$ $U(1) = 20$

$U(0) = 1 (C_1 (1) + C_2 (0)) = 0$

$U_1 = 2 (C_1 \cos(0{,}723) + C_2 \sin(0{,}723)) = 20$

$C_1 = 0$

$2 (\cancel{C_1 (\tfrac{3}{4})} + C_2 (\tfrac{\sqrt{7}}{4})) = 20$

$C_2 (\frac{\sqrt{7}}{4}) = 10$

$C_2 = \frac{40}{\sqrt{7}}$

$U(n) = 2^n \left(\frac{40}{\sqrt{7}} \sin\left(\sin^{-1}\left(\frac{\sqrt{7}}{4}\right) n\right)\right)$

$f(n)=1+n^2$

Sol. homogenea

$U_n = A(2)^n + B(3)^n$

Alternativa

$U_n = a + bn + cn^2$

$U_{n-1} = ab(n-1) + c(n-1)^2 \quad U_{n-2} = a + b(n-2) + c(n-2)^2$

(reescribir)

$a + bn + cn^2 - 5(a + b(n-1) + c(n-1)^2) + 6(a + b(n-2) + c(n-2)^2)$

$2a + 2bn + 2cn^2 - 7b - 14cn + 19c = 1 + n^2$

$2bn + 2cn^2 - 14cn - 7b + \underbrace{19c}_{2c+17c} + 2a$

$$\frac{2c(1+n^2) + (2bn - 14cn - 7b + 17c) + 2a}{6}$$

$c = \frac{1}{2}$

$b(2n-7) + (-14 - 17) + 2a = 0 \quad a = 8 \quad b = \frac{7}{2} \quad c = \frac{1}{2}$

$A(2)^n + B(3)^n + 1_2$

$\Downarrow$

Solución

26.

$f_n = f_{n-1} + f_{n-2}$

$f_n - f_{n-1} - f_{n-2} = 0$

$K = n-2 \quad K+1 = n-1 \quad K+2 = n$

$f_{k+2} - f_{k+1} - f_k = 0$

hallamos s/n sistema homogeneo

λ^k

$\lambda^{k+2} - \lambda^{k+1} - \lambda^k = 0$

$K^k(\lambda^2 - \lambda - 1) = 0$

$\lambda^2 - \lambda - 1 = 0$

$\lambda = \frac{1 \pm \sqrt{1 - 4(1)(-1)}}{2} = \frac{1 \pm \sqrt{5}}{2}$

$\lambda = \frac{1+\sqrt{5}}{2} \quad \lambda = \frac{1-\sqrt{5}}{2}$

$f(n) = C_1\left(\frac{1+\sqrt{5}}{2}\right)^n + C_2\left(\frac{1-\sqrt{5}}{2}\right)^n \quad C_1, C_2 \in \mathbb{R}$

$f_n(0) = 0 \quad f_n(1) = 1$

$f_n(0) = C1 + C2 = 0$

$f_n(0) = C_1\left(\frac{1+\sqrt{5}}{2}\right) + C_2\left(\frac{1-\sqrt{5}}{2}\right) = 1$

$C1 + C_2 = 0 \rightarrow C_2 = -C_1$

$C1\left(\frac{1+\sqrt{5}}{2}\right) - C_1\left(\frac{1-\sqrt{5}}{2}\right) = 1$

$f(n) = \frac{\sqrt{5}}{5}\left(\frac{1+\sqrt{5}}{2}\right)^n - \frac{\sqrt{5}}{5}\left(\frac{1-\sqrt{5}}{2}\right)^n$

$f(n) = \frac{\sqrt{5}}{5}\left(\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right)$

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sol. homogenea

$U_n - 2U_{n-1} = 0$

$k = n-1 \quad k+1 = n$

$U_{(k+1)} - 2U_k = 0$

supose λ^k

$\lambda^{k+1} - 2\lambda^k = 0$

$\lambda^k(\lambda - 2) = 0$

$\lambda - 2 = 0$

$\lambda = 2$

$f_H(n) = C_1(2)^n$

sol particular

$f_p(n) \sim 3^n$

Propuesta $f_p(n) = \alpha 3^n$

$\alpha 3^n - 2\alpha 3^{n-1} = 3^n$

$\alpha 3^n - \frac{2}{3}\alpha 3^n = 3^n$

$\frac{\alpha}{3} 3^n = 3^n$

$\frac{\alpha}{3} = 1$

$\alpha = 3$

$f_p(n) = 3 \cdot 3^n = 3^{n+1}$

Sol general

$f(n) = f_h(n) + f_p(n) = C_1(2)^n + 3^{n+1}$

$f(0) = C_1 + 3 = 1 \rightarrow C_1 = -2$

$f(n) = -2(2)^n + 3^{n+1}$

$f(n) = -2^{n+1} + 3^{n+1}$

$f(n) = 3^{n+1} - 2^{n+1}$