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INSPIRING CREATIVE AND INNOVATIVE MINDS

CHAPTER 5

Finite Automata



Automata

- In computer science, we study different types of computer languages, such as Basic, Pascal, and C++.
- We will discuss a type of a language that can be recognized by special types of machines.
- Automata is a mathematical model of a machine that accepts languages of some alphabet.



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- Deterministic Finite Automaton is a quintuple

$$M = \{ S, I, q_0, f_s, F \}$$

where,

S is a finite nonempty set of states

I is the input alphabet (a finite nonempty set of symbols)

q_0 is the initial state

f_s is the state transition function

F is the set of final states, subset of S.



example

- Let $M = \{ \{q_0, q_1, q_2\}, \{0, 1\}, q_0, f_s, \{q_2\} \}$
where f_s is defined as follows:

$$f_s(q_0, 0) = q_1, \quad f_s(q_1, 1) = q_2$$

$$f_s(q_0, 1) = q_0, \quad f_s(q_2, 0) = q_0$$

$$f_s(q_1, 0) = q_2, \quad f_s(q_2, 1) = q_1$$

- Note that for M :
 $S = \{q_0, q_1, q_2\}$, $I = \{0, 1\}$, $F = \{q_2\}$
 q_0 is the initial state

S is a finite nonempty set of states
 I is the input alphabet (a finite nonempty set of symbols)
 q_0 is the initial state
 f_s is the state transition function
 F is the set of final states, subset of S .

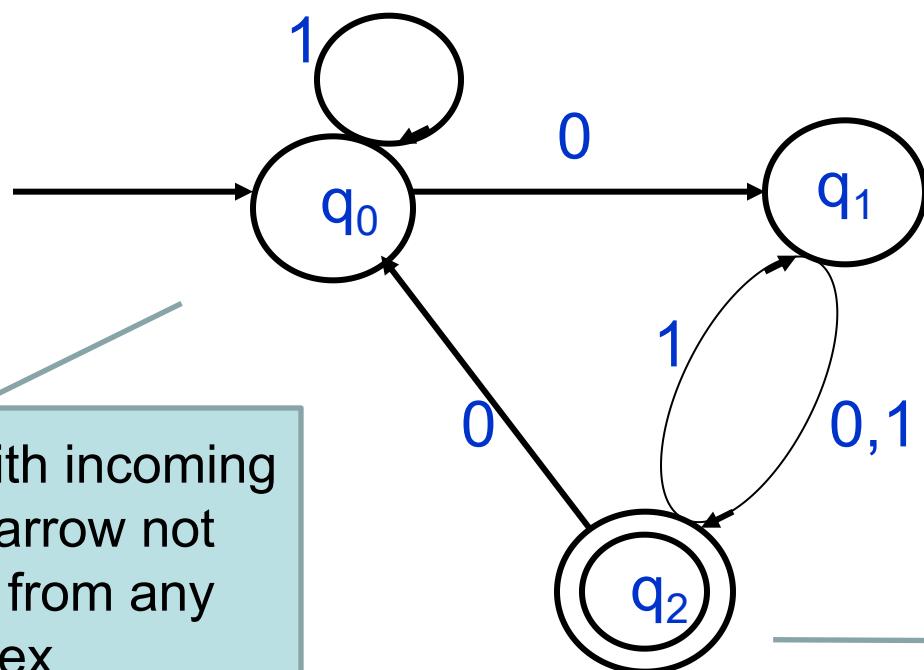
example

- The state transition function of a DFA is often described by means of a table, called a **transition table**.

f_s	0	1
q_0	q_1	q_0
q_1	q_2	q_2
q_2	q_0	q_1

example

- The transition diagram of this DFA is,



Initial state with incoming unlabeled arrow not originating from any vertex

Each state represented by a small circle labeled with the state

Final state with a double circle



example

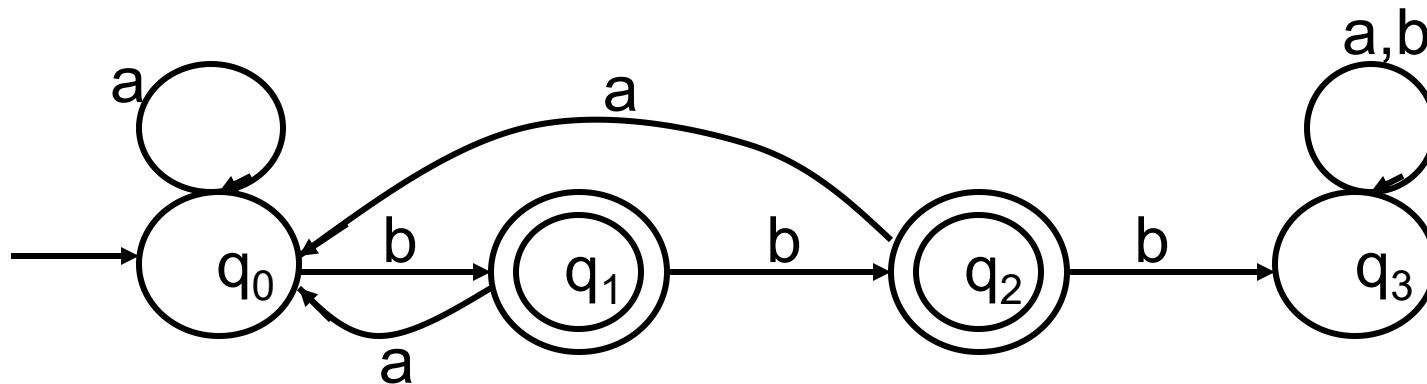
Let $M = (\{q_0, q_1, q_2, q_3\}, \{a, b\}, q_0, f_s, \{q_1, q_2\})$
where f_s is given by the table

f_s	a	b
q_0	q_0	q_1
q_1	q_0	q_2
q_2	q_0	q_3
q_3	q_3	q_3

- S is a finite nonempty set of states
- I is the input alphabet (a finite nonempty set of symbols)
- q_0 is the initial state
- f_s is the state transition function
- F is the set of final states, subset of S.

example

- The transition diagram of this DFA is,

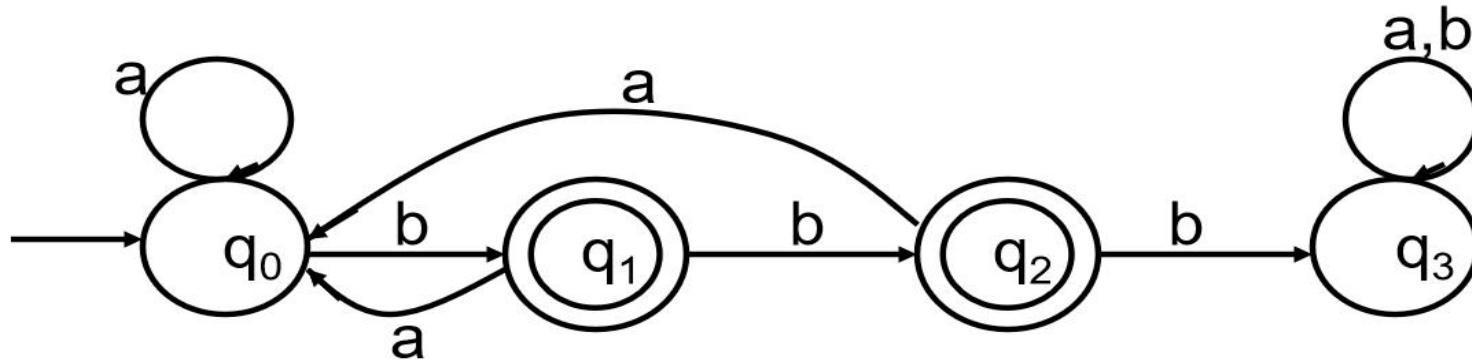




example

- Let $M = \{ S, I, q_0, f_s, F \}$ be a DFA and w is an input string,
- w is said to be accepted by M if $f_s^*(q_0, w) \in F$
- f_s^* - extended transition function for M

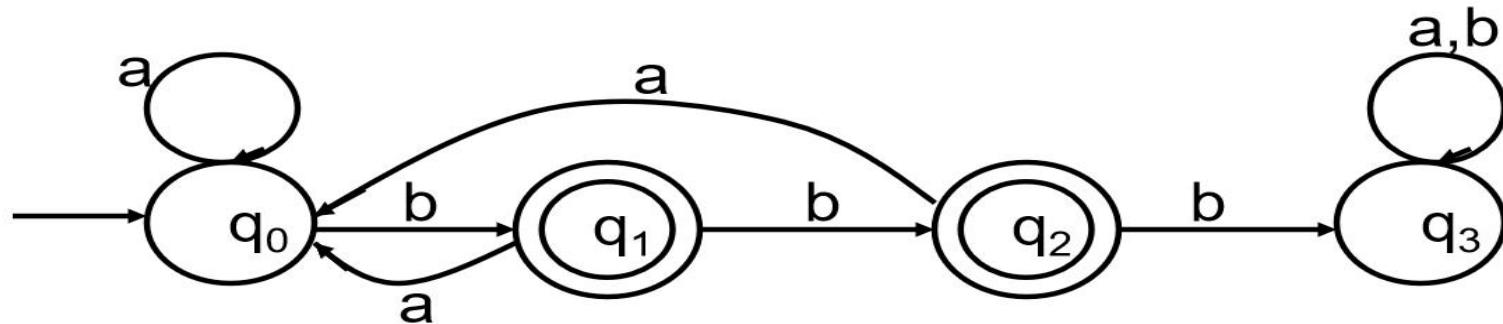
w= abb



$q_0 \xrightarrow{a} q_0 \xrightarrow{b} q_1 \xrightarrow{b} q_2$

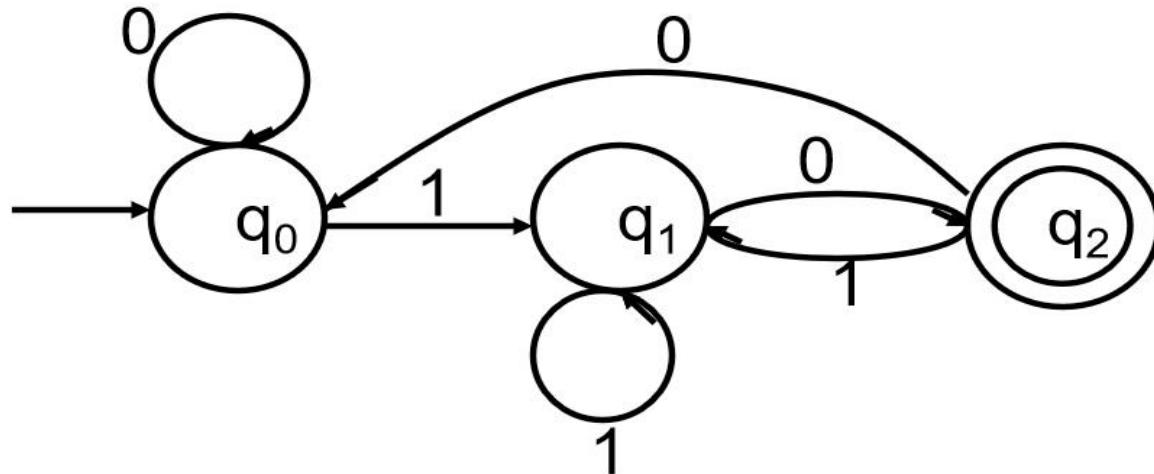
accepted
by M

w= abba


$$q_0 \xrightarrow{a} q_0 \xrightarrow{b} q_1 \xrightarrow{b} q_2 \xrightarrow{a} q_0$$

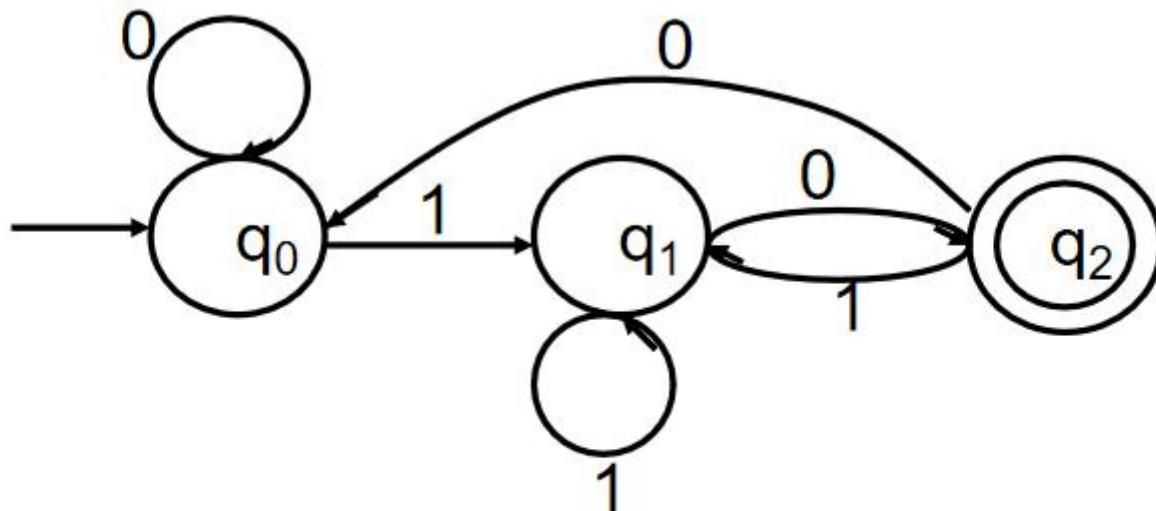
not accepted by M

example



- What are the states of M? q_0, q_1, q_2
- Write the set of input symbols. $I = \{0, 1\}$
- Which is the initial state? q_0

example

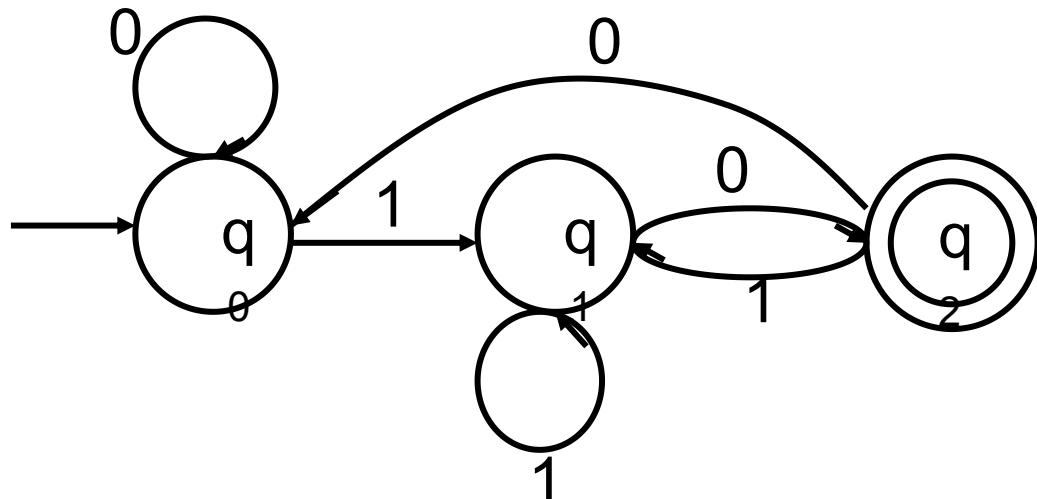


- Write the set of final states.

$$F = \{q_2\}$$

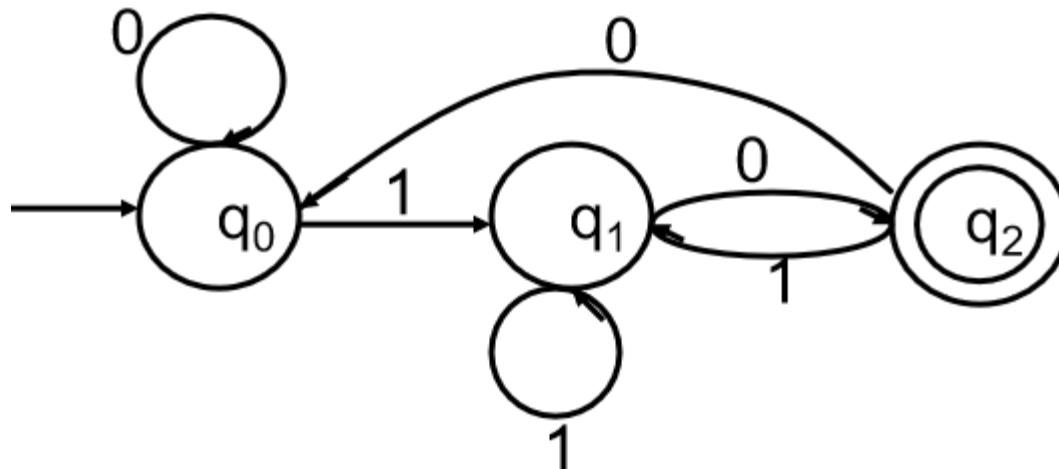
example

Write the transition table for this DFA



f_s	0	1
q_0	q_0	q_1
q_1	q_2	q_1
q_2	q_0	q_1

example



Which of the strings are accepted by M?
0111010, 00111, 111010, 0100, 1110



example

Construct a state transition diagram of a DFA that accepts on $\{a,b\}$ that contain an even number of a's and an odd number of b's.

Example of accepted strings:

aab, baa, baaabba



states and input

q_0	even num. of a's & even num. of b's.
q_1	even num. of a's & odd num. of b's.
q_2	odd num. of a's & odd num. of b's.
q_3	odd num. of a's & even num. of b's.

set of states, $S = \{q_0, q_1, q_2, q_3\}$

initial state, q_0

final state, q_1

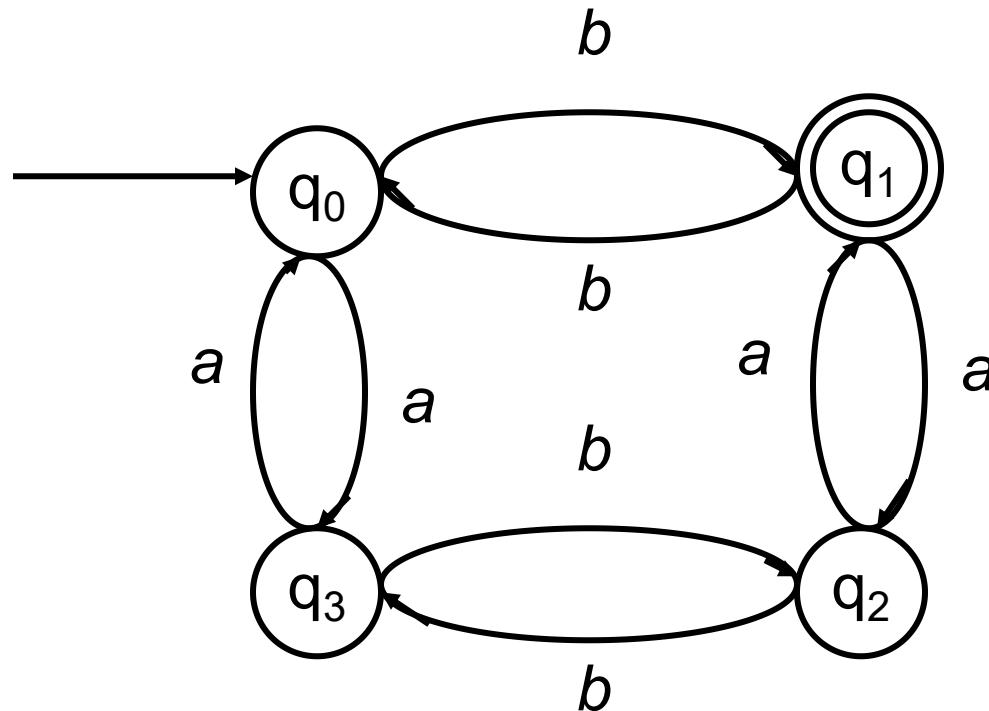
set of input symbols, $I = \{a, b\}$



state transition function

f_s	a	b
q_0	q_3	q_1
q_1	q_2	q_0
q_2	q_1	q_3
q_3	q_0	q_2

State transition diagram





exercise

Let $M=(S, I, q_0, f_s, F)$ be the DFA such that $S=\{q_0, q_1, q_2\}$, $I=\{a, b\}$, $F=\{q_2\}$, q_0 =initial state, and f_s is given by transition table below.

f_s	a	b
q_0	q_0	q_1
q_1	q_2	q_1
q_2	q_2	q_0

- Draw the state diagram of M .
- Which of the strings **abaa**, **bbbabb**, **bbbaa** dan **bababa** are accepted by M ?



Finite State Machines (FSM)

- Automata with input as well as output.
- Every state has an input and corresponding to the input the state also has an output.
- These types of automata are commonly called **finite state machines**.
- A finite state machine is a sextuple,

$M = \{ S, I, O, q_0, f_s, f_o \}$, where

S is a finite nonempty set of states

I is the input alphabet

O is the output alphabet

q_0 is the initial state

f_s is the state transition function

f_o is the output function.



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example

- Let $M = \{ S, I, O, q_0, f_s, f_o \}$ be the FSM
- where,

$$S = \{q_0, q_1, q_2\},$$

$$I = \{a, b\},$$

$$O = \{0, 1\},$$

q_0 = initial state,



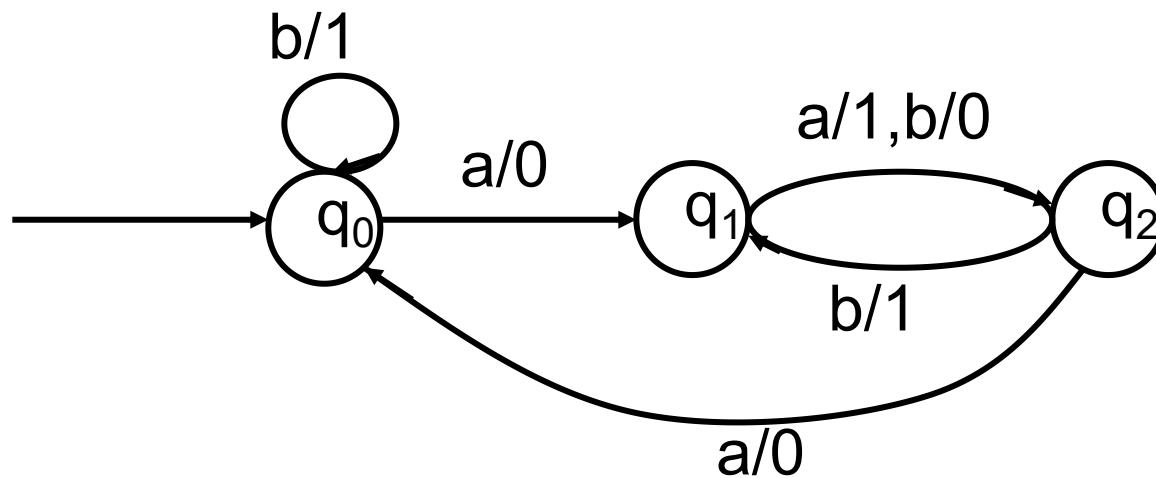
FSM state transition table

f_s and f_o

	f_s		f_o	
	a	b	a	b
q_0	q_1	q_0	0	1
q_1	q_2	q_2	1	0
q_2	q_0	q_1	0	1

example

Given the transition digaram of FSM below.
Find the output string and output when input
string **bbab** and **bababaa**





example

- Let $M = \{ S, I, O, q_0, f_s, f_o \}$ be the FSM
- where,
 - $S = \{q_0, q_1, q_2, q_3\}$,
 - $I = \{a, b\}$,
 - $O = \{0, 1\}$,
 - q_0 = initial state,

example

- Given the f_s and f_0
- Draw the transition diagram of M.
- What is the **output string** and the **output** if the input string is ***abbabab***?

	f_s		f_0	
	a	b	a	b
q_0	q_0	q_2	0	1
q_1	q_1	q_2	1	0
q_2	q_3	q_1	1	1
q_3	q_3	q_3	1	1

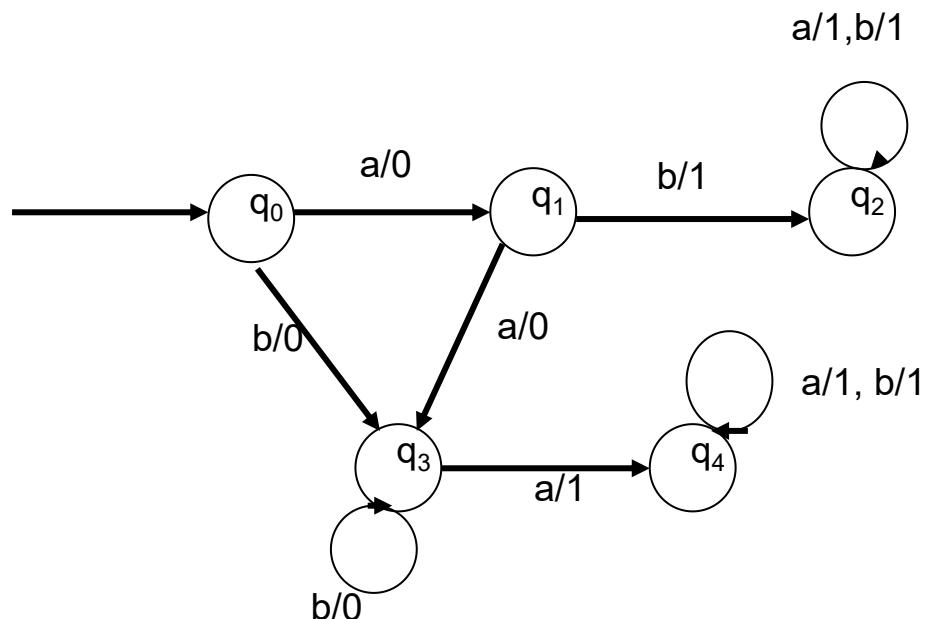


example

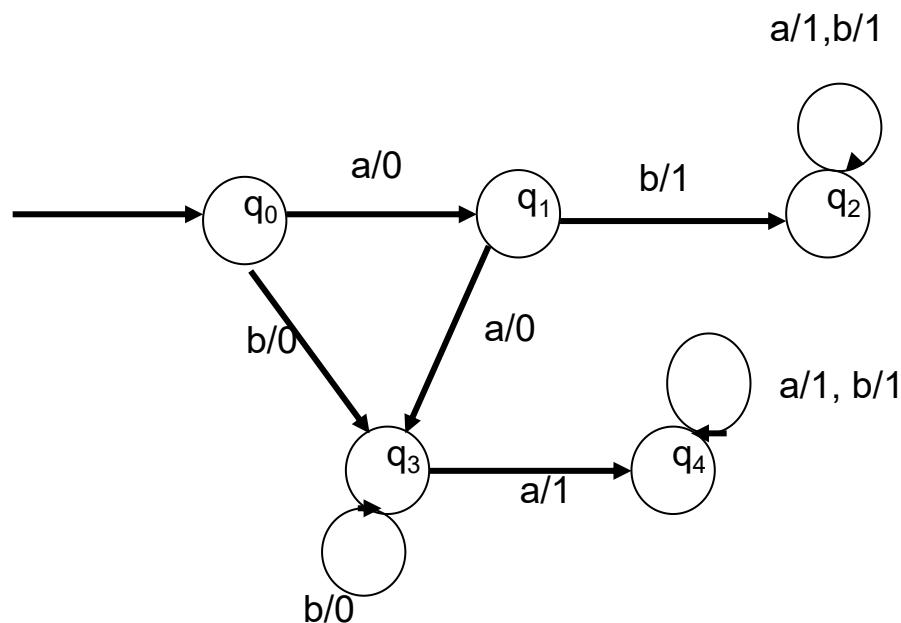
- Let M be a FSM.
- Let x be a nonempty string in M .
- We say that x is accepted by M if and only if the output of x is 1.

example

- Write the transition table of M.
- What is the output string if the input string is *aaabbbbb*?
- What is the output if the input string is *bbbaaaa*?
- Is the string *aaa* accepted by M?



The transition table of M.



	f_s	b	f_o	a	b
	a	b	a	b	
q_0	q_1	q_3	0	0	
q_1	q_3	q_2	0	1	
q_2	q_2	q_2	1	1	
q_3	q_4	q_3	1	0	
q_4	q_4	q_4	1	1	



exercise

Let $M = \{ S, I, O, q_0, f_s, f_o \}$ be a FSM
where,

$$S = \{q_0, q_1, q_2\},$$

$$I = \{a, b\},$$

$$O = \{0, 1\},$$

q_0 = initial state,

	f_s		f_o	
	a	b	a	b
q_0	q_2	q_1	1	1
q_1	q_2	q_2	0	0
q_2	q_1	q_2	1	1

- Draw the transition diagram of M .
- What is the output string if the input string is aabbb?
- What is the output string if the input string is ababab?
- What is the output if the input string is abbbaba?
- What is the output if the input string is bbbbababa?



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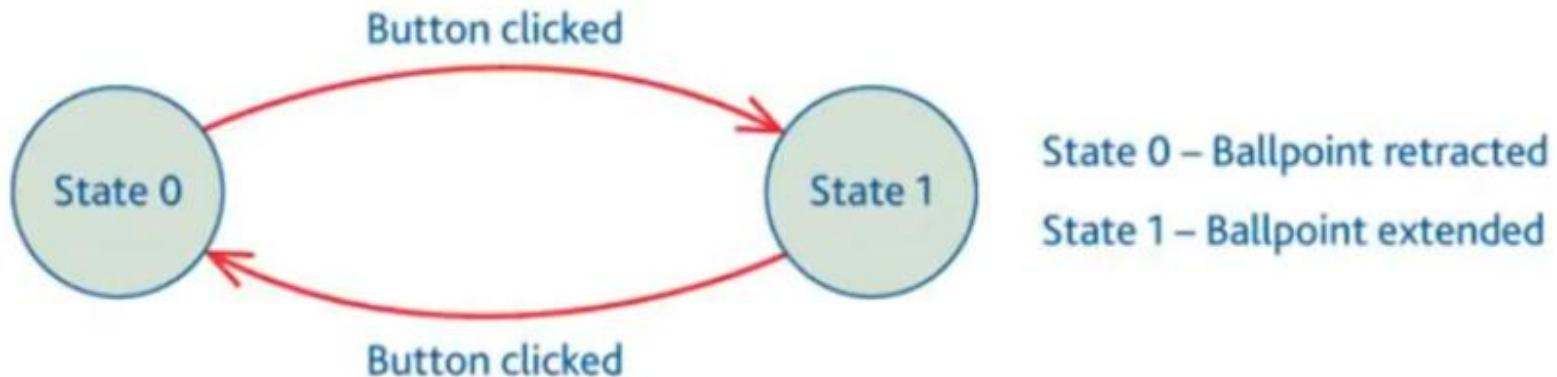
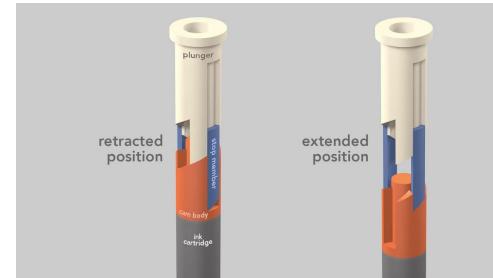
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APPLICATIONS OF FINITE AUTOMATA

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Ballpoint pen

- Finite States:
 - Ballpoint retracted, Ballpoint extended
- Inputs: Click the button



Combintion Lock

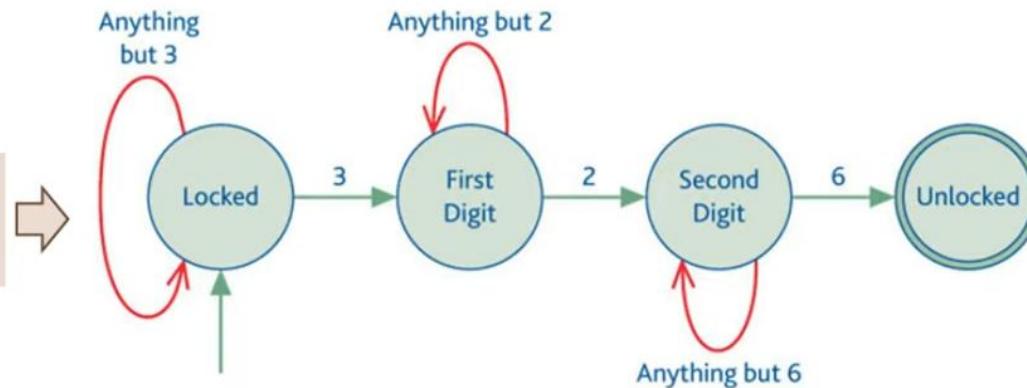
- Unlocked when the correct input, 326, is entered.

State
transition
table



Input	Current State	Next state
Anything but 3	Locked	Locked
3	Locked	First Digit
Anything but 2	First Digit	First Digit
2	First Digit	Second Digit
Anything but 6	Second Digit	Second Digit
6	Second Digit	Unlocked

State
transition
diagram



Vending Machine

- The vending machine will dispense a candy priced at 1.25.

