Chapter 1

SET THEORY

[Part 2: Operation on Set]

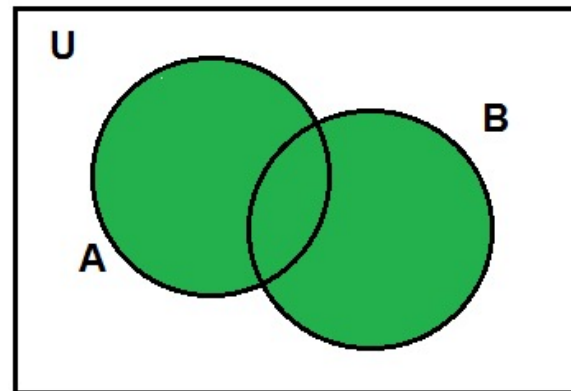Premier
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Union

- The union of two sets A and B, denoted by $A \cup B$, is defined to be the set

$$A \cup B = \{\, x \mid x \in A \text{ or } x \in B\}$$

- The union consists of all elements belonging to either A or B (or both)

Venn diagram of $A \cup B$

Example

$A = \{1, 2, 3, 4, 5\}$, $B = \{2, 4, 6\}$ and $C = \{8, 9\}$

$A \cup B = \{1, 2, 3, 4, 5, 6\}$

$A \cup C = \{1, 2, 3, 4, 5, 8, 9\}$

$B \cup C = \{2, 4, 6, 8, 9\}$

$A \cup B \cup C = \{1, 2, 3, 4, 5, 6, 8, 9\}$

Union

If *A* and *B* are finite sets, the cardinality of $A \cup B$,

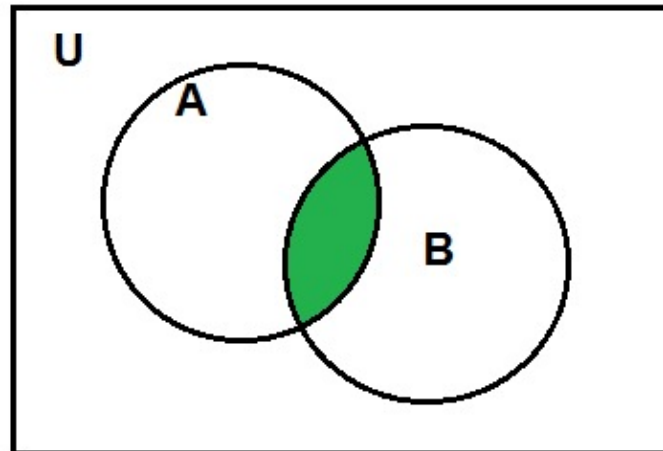$$|A \cup B| = |A| + |B| - |A \cap B|$$

Intersection

- The intersection of two sets A and B, denoted by $A \cap B$, is defined to be the set

$$A \cap B = \{\, x \mid x \in A \text{ and } x \in B \}$$

- The intersection consists of all elements belonging to both A and B.

Venn diagram of $A \cap B$

Example

A={1, 2, 3, 4, 5, 6}, B={2, 4, 6, 8, 10} and C={ 1, 2, 8, 10 }

$A \cap B$ = {2, 4, 6}

$A \cap C$ = {1, 2}

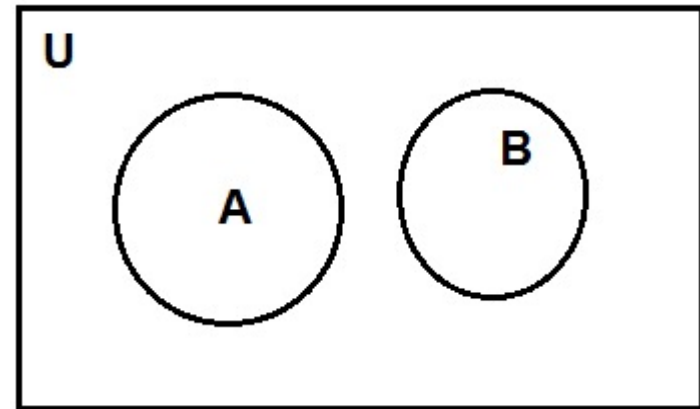$C \cap B$ = {2, 8, 10}

$A \cap B \cap C$ = {2}

Disjoint

Two sets *A* and *B* are said to be disjoint if, $A \cap B = \varnothing$

Venn diagram, $A \cap B = \varnothing$

Example

$$A = \{1, 3, 5, 7, 9, 11\}, B = \{2, 4, 6, 8, 10\}$$

$$A \cap B = \varnothing$$

Difference

The set,

$$A - B = \{x \mid x \in A \text{ and } x \notin B\}$$

is called the difference.

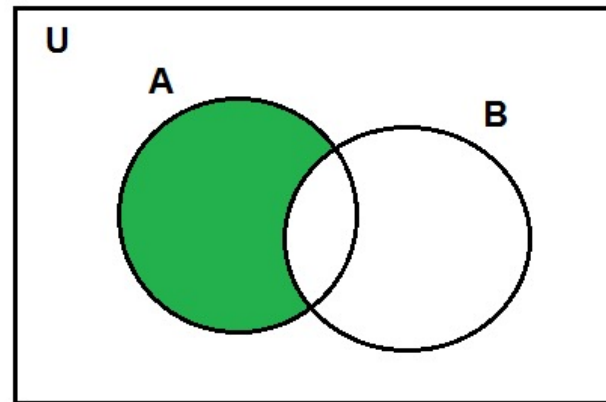The difference $A - B$ consists of all elements in A that are not in B.

Venn diagram of $A - B$

Example

$A = \{1, 2, 3, 4, 5, 6, 7, 8\}$
$B = \{2, 4, 6, 8\}$

$A - B = \{1, 3, 5, 7\}$

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Symmetric Difference

The symmetric difference of set *A* and set *B*, denoted by *A* $\oplus$ *B* is the set **(*A* - *B*) U (*B* - *A*)**

Venn Diagram

$$U = \{0,1,2,3,4,5,6,7,8,9,10\}$$

$$A = \{1,2,3,4,5\}; \; B = \{4,5,6,7,8\}$$

$$A \oplus B = (A - B) \cup (B - A) = \{1,2,3,6,7,8\}$$
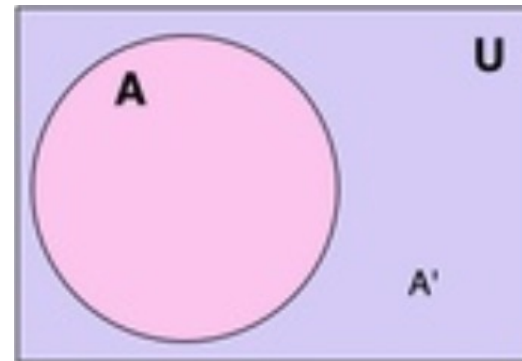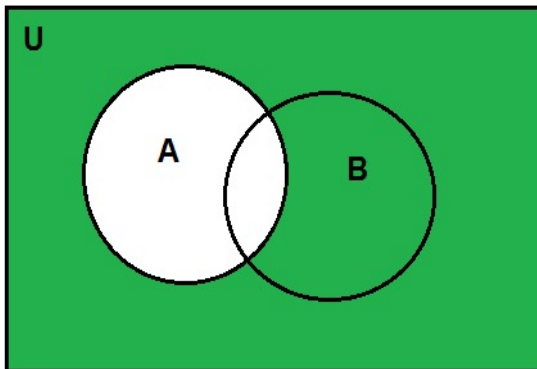
B-A = {6,7,8}

$$A - B = \{1,2,3\}$$

Complement

The complement of a set *A* with respect to a universal set *U*, denoted by *A′* is defined to be

$$A' = \{x \in U | x \notin A\}$$

$$A' = U - A$$

Venn diagram of *A'*

Let *U* be a universal set,

U= { 1, 2, 3, 4, 5, 6, 7 }

A= { 2, 4, 6 }

A′ = U – A = { 1, 3, 5, 7 }

Set Identities (Properties of Set)

- **Commutative laws**

    $A \cap B = B \cap A, \quad A \cup B = B \cup A$

- **Associative laws**

    $A \cap (B \cap C) = (A \cap B) \cap C$

    $A \cup (B \cup C) = (A \cup B) \cup C$

- **Distributive laws**

    $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

    $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Set Identities (Properties of Set)

- **Absorption laws**

$$A \cup (A \cap B) = A$$
$$A \cap (A \cup B) = A$$

- **Idempotent laws**

$$A \cap A = A, \quad A \cup A = A$$

- **De Morgan's laws**

$$(A \cap B)' = A' \cup B'$$
$$(A \cup B)' = A' \cap B'$$

Set Identities (Properties of Set)

- **Complement laws**

$$A \cap A' = \varnothing \qquad A \cup A' = U$$

- **Double complement laws**

$$(A')' = A$$

- **Complement of *U* and $\varnothing$**

$$\varnothing' = U \qquad U' = \varnothing$$

Set Identities (Properties of Set)

- **Properties of universal set**

$$A \cup U = U \qquad A \cap U = A$$

- **Set difference laws**

$$A - B = A \cap B'$$

- **Identity laws**

$$A \cup \varnothing = A \qquad A \cap U = A$$

- **Properties of empty set**

$$A \cup \varnothing = A \qquad A \cap \varnothing = \varnothing$$

Example

- Let *A, B* and *C* denote the subsets of a set *S* and let *C ′* denote a complement of *C* in *S*.

- If $A \cap C = B \cap C$ and $A \cap C' = B \cap C'$, then prove that *A=B*

$$
\begin{aligned}
A &= A \cap S \\
&= A \cap (C \cup C') \\
&= (A \cap C) \cup (A \cap C') \quad \textbf{(distributive)} \\
&= (B \cap C) \cup (B \cap C') \quad \textbf{(the given conditions)} \\
&= B \cap (C \cup C') \quad \textbf{(distributive)} \\
&= B \cap S \\
&= B
\end{aligned}
$$

By referring to the properties of set operations, show that:

$$A - (A \cap B) = A - B$$

$$
\begin{aligned}
A - (A \cap B) \quad &= A \cap (A \cap B)' && \text{[set difference laws]} \\
&= A \cap (A' \cup B') && \text{[De Morgan's laws]} \\
&= (A \cap A') \cup (A \cap B') && \text{[distributive laws]} \\
&= \varnothing \cup (A \cap B') && \text{[complement laws]} \\
&= (A \cap B') \cup \varnothing && \text{[commutative]} \\
&= A \cap B' && \text{[Identity laws]} \\
&= A - B
\end{aligned}
$$

The **union** of a collection of sets is the set that contains those elements that are members of at least one set in the collection.

Notation:

$$\bigcup_{i=1}^{n} A_i = A_1 \cup A_2 \cup \ldots \cup A_n = \left\{ x \in U \,\middle|\, x \in A_i \text{ for at least one } i = 0,1,2,\ldots,n \right\}$$

$$\bigcup_{i=1}^{\infty} A_i = A_1 \cup A_2 \cup \ldots \cup A_n = \left\{ x \in U \,\middle|\, x \in A_i \text{ for at least one nonnegative integer } i \right\}$$

The ***intersection*** of a collection of sets is the set that contains those elements that are members of all the sets in the collection.

Notation:

$$\bigcap_{i=1}^{n} A_i = A_1 \cap A_2 \cap \ldots \cap A_n = \left\{ x \in U \mid x \in A_i \text{ for all } i = 0,1,2,....n \right\}$$

$$\bigcap_{i=1}^{\infty} A_i = A_1 \cap A_2 \cap \ldots \cap A_n = \left\{ x \in U \mid x \in A_i \text{ for all nonnegative integer } i \right\}$$

For i = 1, 2, ……, let $A_i = \{i, i+1, i+2, ...)$. Then,

$$\bigcup_{i=1}^{n} A_i = \bigcup_{i=1}^{n} \{i, i+1, i+2, ...\} = \{1, 2, 3, ...\},$$

and

$$\bigcap_{i=1}^{n} A_i = \bigcap_{i=1}^{n} \{i, i+1, i+2, ...\} = \{n, n+1, n+2, ...\} = A_n$$

Cartesian Product

- Let *A* and *B* be sets. An ordered pair of elements $a \in A$ dan $b \in B$ written (*a*, *b*) is a listing of the elements *a* and *b* in a specific order.

- The ordered pair (*a*, *b*) specifies that *a* is the first element and *b* is the second element.

- An ordered pair (*a*, *b*) is considered distinct from ordered pair (*b*, *a*), unless *a*=*b*.

Example $(1, 2) \neq (2, 1)$

Cartesian Product

- The Cartesian product of two sets A and B, written $A{\times}B$ is the set,

$$A{\times}B = \{(a,b)\mid a{\in}A, b{\in}B\}$$

- For any set A,

$$A{\times}\varnothing = \varnothing{\times}A = \varnothing$$

Example

$A = \{a, b\}$, $B=\{1, 2\}$.

$A{\times}B = \{(a, 1), (a, 2), (b, 1), (b, 2)\}$

$B{\times}A = \{(1, a), (1, b), (2, a), (2, b)\}$

- if $A \neq B$, then $A \times B \neq B \times A$.
- if $|A| = m$ and $|B| = n$, then $|A \times B| = mn$.

Example $A = \{1, 3\}$, $B = \{2, 4, 6\}$.

$A \times B = \{(1, 2), (1, 4), (1, 6), (3, 2), (3, 4), (3, 6)\}$

$B \times A = \{(2, 1), (2, 3), (4, 1), (4, 3), (6, 1), (6, 3)\}$

$A \neq B$, $A \times B \neq B \times A$

$|A| = 2$, $|B| = 3$, $| A \times B | = 2.3 = 6$.

- The Cartesian product of sets $A_1, A_2, …., A_n$ is defined to be the set of all n-tuples

$$(a_1, a_2, … a_n) \text{ where } a_i \in A_i \text{ for } i=1,…,n;$$

- It is denoted $A_1 \times A_2 \times …. \times A_n$

$$|A_1 \times A_2 \times …. \times A_n| = |A_1| . |A_2| …. |A_n|$$

$A = \{a, b\}$, $B = \{1, 2\}$, $C = \{x, y\}$

$A \times B \times C = \{(a,1,x),(a,1,y),\ (a,2,x),\ (a,2,y),$

$(b,1,x),\ (b,1,y),\ (b,2,x),\ (b,2,y)\}$

$|A \times B \times C| = 2.\ 2.\ 2 = 8$