Question 1

Let U be the set $\mathbb{Z}$ of all integer numbers and let A, B, C are the subset of U. Given $A = \{x \text{ is a solution of } x^2 + x - 6 = 0\}$, $B = \{-3, 1, 2, 4\}$ and C
$C = \{x \in \mathbb{Z} \mid 2 \leq x < 5\}$. Compute :

a) $A' = \{x \in \mathbb{Z} \mid x \neq 2, x \neq -3\}$

$A = \{2, -3\}$
$B = \{-3, 1, 2, 4\}$
$C = \{2, 3, 4\}$

b) $(B - A) \cap C$

$B - A = \{1, 4\}$

$(B - A) \cap C = \{1, 4\} \cap \{2, 3, 4\}$
$= \{4\}$

c) $|P(B \cap C)|$

$B \cap C = \{-3, 1, 2, 4\} \cap \{2, 3, 4\}$
$= \{2, 4\}$

$|P(B \cap C)| = 2^2$
$= 4$

Question 2

Let P, Q and R are sets, prove that $(((P \cup Q) \cap R)' \cup Q')' = Q \cap R$ by showing all laws that used

$(((P \cup Q) \cap R)' \cup Q')'$

$(((P \cup Q) \cap R)'' \cap Q'') \rightarrow$ De Morgan's laws

$((P \cup Q) \cap R) \cap Q \rightarrow$ Double complement laws

$(P \cup Q) \cap (R \cap Q) \rightarrow$ Associative laws

$(P \cup Q) \cap (Q \cap R) \rightarrow$ Commutative laws

$((P \cup Q) \cap Q) \cap R = Q \cap R \rightarrow$ Absorption laws

$\therefore$ shown

Question 3

There are 35 students in the art class and 57 students in the science class. Find the number of students who are either in art or in science class.

a) When two classes meet at different hours and 12 students are enrolled in both activities.

$|A \cap B| = 12$ $\quad$ $|A \cup B| = |A| + |B| - |A \cap B|$

$|A| = 35$ $\quad$ $= 35 + 57 - 12$

$|B| = 57$ $\quad$ $= 80$

b) When two classes meet at the same hour.

$|A \cap B| = 0$

$|A \cup B| = |A| + |B| - |A \cap B|$

$= 35 + 57 - 0$

$= 92$

Question 4

Consider the statement:

"if you try hard and you have talent then you will be rich"

a) Translate the statement into logic symbols. Use p, q, and r to represent the propositions. Clearly state which statement is p, q and r.

p = you try hard

q = you have talent

r = you will get rich

$$(p \wedge q) \rightarrow r$$

b) Suppose you found out that the statement was a lie even you try hard. What can you conclude?

$(p \wedge q) \rightarrow \neg r$

∴ ~~Hence~~, Since the statement is a lie, (false) the antecedent must be true, ~~but t~~ $(p \wedge q)$, but the consequent is false $(\neg r)$. Therefore, "You try hard and you have talent, but you will not be rich".

c) If you are rich but you do not try hard or have talent, was the statement true or false? Support your conclusion.

$\neg(p \wedge q) \rightarrow r$

The statement is true. A conditional statement is false only when $(p \wedge q)$ is true and r is false. Since $(p \wedge q)$ is false (did not occur) and r is true, the conditional statement is true and cannot be proven false.

Question 5

use thruth table to check if the compound propositions A and B are logically equivalent.

$$A = \neg(p \vee (q \wedge (r \rightarrow p)))$$

$$B = \neg p \wedge (q \rightarrow r)$$

p	q	r	$r \rightarrow p$	$q \wedge (r \rightarrow p)$	$p \vee (q \wedge (r \rightarrow p))$	A
T	T	T	T	T	T	F
T	T	F	T	T	T	F
T	F	T	T	F	T	F
T	F	F	T	F	T	F
F	T	T	F	F	F	T
F	T	F	T	T	T	F
F	F	T	F	F	F	T
F	F	F	T	F	F	T

p	q	r	$q \rightarrow r$	$\neg p$	B
T	T	T	T	F	F
T	T	F	F	F	F
T	F	T	F	F	F
T	F	F	T	F	F
F	T	T	T	T	T
F	T	F	F	T	F
F	F	T	T	T	T
F	F	F	T	T	T

$\therefore A \equiv B$

A is logically equivalent to B

Question 6

Proof that if x is an odd integer and y is an even integer then $x^2 - 2y$ is an odd integer using direct proofing

let

$$x = 2n+1 \quad , \quad y = 2m$$

$$x^2 - 2y = (2n+1)^2 - 2(2m)$$

$$= 4n^2 + 4n + 1 - 4m$$

$$= 2(2n^2 + 2n - 2m) + 1$$

let $2n^2 + 2n - 2m = h$

$$= 2h + 1 \quad \therefore \text{odd integer}$$