UTM
UNIVERSITI TEKNOLOGI MALAYSIA

FACULTY OF COMPUTING

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SECI 1013 DISCRETE STRUCTURE

SECTION 02

ASSIGNMENT 2

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Question 1

Let $D = \{1, 3, 5\}$. Define R on D where $x, y \in D$, xRy if $3x + y$ is a multiple of 6.

i) Find the element of R

$3x + y$ = multiple of 6

$3(1) + 1 = 4 \rightarrow \times$
$3(1) + 3 = 6 \rightarrow \checkmark$
$3(1) + 5 = 8 \rightarrow \times$

$3(3) + 1 = 10 \rightarrow \times$
$3(3) + 3 = 12 \rightarrow \checkmark$
$3(3) + 5 = 14 \rightarrow \times$

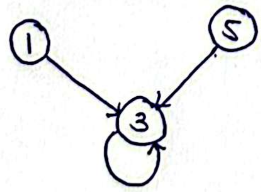
$3(5) + 1 = 16 \rightarrow \times$
$3(5) + 3 = 18 \rightarrow \checkmark$
$3(5) + 5 = 20 \rightarrow \times$

$R = \{(1,3), (3,3), (5,3)\}$

ii) Determine the domain and range of R

Domain = $\{1, 3, 5\}$
Range = $\{3\}$

iii) Draw the digraph of the relation



iv) Determine whether the relation R is assygmetric?

$$\begin{array}{c|c} & \begin{matrix} 1 & 3 & 5 \end{matrix} \\ \begin{matrix} 1 \\ 3 \\ 5 \end{matrix} & \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix} \end{array}$$

$(3,3) \in R$
R is not irreflexive

$$\begin{array}{c|c} & \begin{matrix} 1 & 3 & 5 \end{matrix} \\ \begin{matrix} 1 \\ 3 \\ 5 \end{matrix} & \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix} \end{array}$$

$(1,3) \in R$, $(3,1) \notin R$
R is antisymmetric

$\therefore$ since R is antisymmetric and not irreflexive.
R is not asymmetric.

2. $A = \{x, y, z\}$

$(x, y) \in R$ and $(y, z) \in R$

equivalence so transitive, reflexive and symmetric

since reflexive so $x = y$, $(x, y) \in R$

so reflexive, (x, x), (y, y), $(z, z) \in R$

symmetric $(x, y) \in R$, $(y, x) \in R$

not antisymmetric, $(y, z) \in R$ but $(z, y) \in R$.

transitive,

$(x, y) \in R$, $(y, z) \in R$, $(x, z) \in R$

$(x, z) \in R$, $(z, y) \in R$, $(x, y) \in R$

$(z, y) \in R$, $(y, x) \in R$, $(z, x) \in R$

$(x, y) \in R$, $(y, x) \in R$, $(x, x) \in R$

$(y, x) \in R$, $(x, y) \in R$, $(y, y) \in R$

$(z, x) \in R$, $(x, z) \in R$, $(z, z) \in R$

$$M_R = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$M_R \otimes M_R = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \otimes \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$R = \{(x, y), (x, x), (x, z), (y, y), (y, x), (y, z), (z, z), (z, x), (z, y)\}$

Question 3

Let $B = \{u, v, w, y\}$ and $R = \{(u,u), (u,w), (v,v), (v,w), (w,w), (w,y), (y,u), (y,v), (y,y)\}$.

i) Construct the matrix of relation, M_R for the relation R on B

$$M_R = \begin{array}{c} \\ u \\ v \\ w \\ y \end{array} \begin{array}{c} \begin{array}{cccc} u & v & w & y \end{array} \\ \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix} \end{array}$$

ii) List in-degrees and out-degrees of all vertices.

	u	v	w	y
In - degree	2	2	3	2
Out - degree	2	2	2	3

iii) Determine whether the relation R on the set B is a partial order relation. Check all variance. Justify your answer.

reflexive

$$\begin{array}{c} \\ u \\ v \\ w \\ y \end{array} \begin{array}{c} \begin{array}{cccc} u & v & w & y \end{array} \\ \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix} \end{array}$$

$\therefore$ reflexive because the main diagonal consists entirely of 1's

antisymmetric

$$\begin{array}{c} \\ u \\ v \\ w \\ y \end{array} \begin{array}{c} \begin{array}{cccc} u & v & w & y \end{array} \\ \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix} \end{array}$$

$\therefore$ antisymmetric because for all $x, y \in B$, $(x,y) \in R$ and $(y,x) \notin R$.

Question 4

Let

$$f : [1, \infty) \to [0, \infty), \ f(x) = (x-1)^2.$$

Determine whether the function f is one-one, onto, or bijective. Show full working and justify your answer.

One-to-one

Let $f(x_1) = f(x_2)$, $f(x) = (x-1)^2$

then $\quad (x_1 - 1)^2 = (x_2 - 1)^2$

$$x_1 - 1 = x_2 - 1$$

$$x_1 = x_2$$

$\therefore$ function f is one-to one.

Onto

$$f(x) = y$$

$$(x-1)^2 = y$$

$$x - 1 = \pm\sqrt{y}$$

$$x = 1 + \sqrt{y} \quad \text{or} \quad x = 1 - \sqrt{y}$$

Domain: $[1, \infty)$ Codomain: $[0, \infty)$

Only $x = 1 + \sqrt{y} \geq 1$ lies in the domain.

So, each $y \geq 0$ exists $x = 1 + \sqrt{y} \in [1, \infty)$ with $f(x) = y$.

Thus, f is onto $[0, \infty)$.

transitive

$$M_R \otimes M_R \neq M_R$$

$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix} \otimes \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$\therefore$ not transitive, $M_R \otimes M_R \neq M_R$

Partial Order Relation

- not reflexive
- antisymmetric
- not transitive

$\therefore$ Relation R on the set B is not partial order relation.

Question 5

Let f and g be functions from the positive integers to the positive integers defined by

$$f(x) = 9x + 4, \quad g(x) = \frac{3}{2}x - 1$$

a) Find the inverse of $g(x)$.

$g(x) = \frac{3}{2}x - 1$

Let $(y, x) \in g^{-1}$, $g^{-1}(y) = x$

$y = \frac{3}{2}x - 1$

$y + 1 = \frac{3}{2}x$

$x = \frac{2}{3}(y+1)$

$g^{-1}(y) = \frac{2}{3}(y+1)$

b) Find the composition $(g \circ f)(x)$

$gf(x) = g(9x+4)$

$gf(x) = \frac{3}{2}(9x+4) - 1$

$gf(x) = \frac{27}{2}x + 6 - 1$

$gf(x) = \frac{27}{2}x + 5$

c) Find the composition $(f \circ g)(x)$

$fg(x) = f(\frac{3}{2}x - 1)$

$fg(x) = 9(\frac{3}{2}x - 1) + 4$

$fg(x) = \frac{27}{2}x - 9 + 4$

$fg(x) = \frac{27}{2}x - 5$

d) Find the composition $(f \circ g \circ g)(x)$

$gg(x) = g(\frac{3}{2}x - 1)$

$= \frac{3}{2}(\frac{3}{2}x - 1) - 1$

$= \frac{9}{4}x - \frac{3}{2} - 1$

$= \frac{9}{4}x - \frac{5}{2}$

$fgg(x) = f(\frac{9}{4}x - \frac{5}{2})$

$= 9(\frac{9}{4}x - \frac{5}{2}) + 4$

$= \frac{81}{4}x - \frac{45}{2} + 4$

$= \frac{81}{4}x - \frac{37}{2}$

6. a) $P_0 = 4.0$,

$P_1 = 5.0$,

$P_t = P_{t-1} + \frac{1}{4}P_{t-2}$, $t \geq 2$, $P_0 = 4.0$, $P_1 = 5.0$

b) $P_0 = 4.0$

$P_1 = 5.0$

$P_2 = P_1 + \frac{1}{4}P_0$

$= 5.0 + \frac{1}{4}(4.0)$

$= 6.0$

$P_3 = P_2 + \frac{1}{4}P_1$

$= 6.0 + \frac{1}{4}(5.0)$

$= 7.25$

$P_4 = P_3 + \frac{1}{4}P_2$

$= 7.25 + \frac{1}{4}(6.0)$

$= 8.75$

$P_5 = P_4 + \frac{1}{4}P_3$

$= 8.75 + \frac{1}{4}(7.25)$

$= 10.5625$

$\therefore$ 4.0, 5.0, 6.0, 7.25, 8.75, 10.5625

7.

a) trace the output if n = 4

```
S(n)
{
 if (n=1)
   return 2
 else
   return S(n-1) * S(n-1) - 1
}
```

b) S(4)

n=4

because n≠1

return S(3) * S(3) - 1

S(4)=63

return 8*8 - 1

∴ ~~S(4) = 63~~

∴ answer = 63

S(3)

n=3

because n≠1

return S(2) * S(2) - 1

S(3) = 8

return 3*3 - 1

S(2)

n=2

because n≠1

return S(1) * S(1) - 1

S(2) = 3

return 2*2 - 1

S(1)

n=1

because n=1

return 2