

# INTERPROCEDURAL PATH COMPLEXITY ANALYSIS

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# INTRODUCTION & MOTIVATION

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- **Motivation:** Software verification and testing is focused on exploring all states of the program and traversing all paths. By finding an approximate for number of paths, we can find how difficult it is to cover all the paths.

- **Motivation:** Software verification and testing is focused on exploring all states of the program and traversing all paths. By finding an approximate for number of paths, we can find how difficult it is to cover all the paths.
- **(Asymptotic) Path Complexity:** Introduced in Lucas et al.'s work on intra-procedural program analysis. Gives a measure of how complex a program is and how long the task of analysing a program will take.
- We improve upon Bang et al.'s work by doing inter-procedural program analysis and extract path complexity while taking recursion into account.

- **Chomsky-Schutzenberger enumeration theorem:** If  $L$  is a context-free language and  $a_k := |L \cap \Sigma^k|$  is the number of words in  $L$  with length  $k$ ,  $G(x) = \sum_{k=0}^{\infty} a_k x^k$  is a power series.

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- If we have a grammar, we can replace each rule with a function and solve the function to obtain an approximation of power series.
- Example:
  - $S \rightarrow M \mid U$        $S = M + U$
  - $M \rightarrow 0M1M \mid \varepsilon$        $M = M^2 x^2 + 1$
  - $U \rightarrow 0S \mid 0M1U$        $U = Sx + MUx^2$



# INTERPROCEDURAL PATH COMPLEXITY ANALYSIS

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- **Control Flow Extraction:** We use Java bytecode analysis tools (Soot) to extract the Control flow graph.

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- Our grammar contains a single character (k) in the alphabet.
- We convert the statements which don't invoke another function to k.
- We convert the statements which invoke functions we are analysing to their corresponding rule name.
- We convert the if statements to |(or) in the rules.

- **Generating and Solving Equations:** According to Chomsky-Schutzenberger theorem, we convert our rules to functions by replacing  $|$  with  $+$ , characters with variable  $x$ , rules with function names.

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- **Generating Function:** We solve the generated function and expand the root to a power series with Taylor expansion.
- $$g(x) = g(0) + \frac{x * g'(0)}{1!} + \frac{x^2 * g''(0)}{2!} + \dots + \frac{x^n * g^{(n)}(0)}{n!} + \dots$$
- Coefficients of  $x^k$  denote number of paths in length  $k$

```
public static int fibonacci(int n){  
    if (n == 0 || n == 1){  
        return 1;  
    } else {  
        return fibonacci(n-1) + fibonacci(n-2)  
    }  
}
```



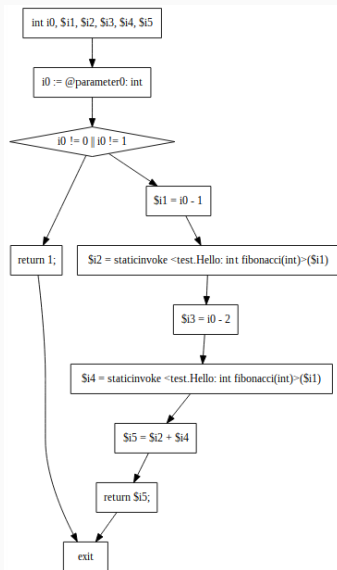
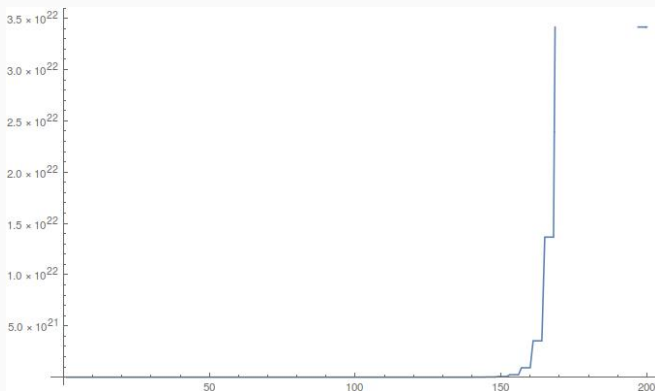


Figure: Fibonacci in CFG form

## CONTEXT-FREE GRAMMAR FOR PROGRAM

- $F = x \mid x^4 * F^2 \rightarrow F(x) = x + x^4 * F^2(x)$  (Conversion to equations)
- $F(x) = \frac{1 + \sqrt{1 - 4x^5}}{2x^4}$  (Finding the root)



**Figure:** X axis denotes length of words, Y axis denotes number of words with length X or less

## FUTURE WORK

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- **Improvement for Overapproximation:** For Quicksort, our analysis gives an exponential upper bound like  $k^n$  which is an overestimate given its complexity is  $n^2$  in worst case.

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- Unnecessary paths may be pruned out or variables can be simulated to reduce overapproximation.
- **Comparing our estimations:** We claim this metric is useful in estimating execution time of testing for full path coverage. We can compare our results to running times of testing tools and check the correlation.

# CONCLUSION

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We have presented an analysis method built upon automata theory and word counting to estimate path complexity.

Although some improvements can be done, we believe it demonstrates some theoretical way to analyse the control flow graphs.



- Bang, Lucas, Abdulbaki Aydin, and Tevfik Bultan. "Automatically computing path complexity of programs." Proceedings of the 2015 10th Joint Meeting on Foundations of Software Engineering. ACM, 2015.
- Lam, Patrick, et al. "The Soot framework for Java program analysis: a retrospective." Cetus Users and Compiler Infrastructure Workshop (CETUS 2011). Vol. 15. 2011.

## QUESTIONS?

