The classic birthday problem asks about how many people need to be in a room together before you have better-than-even odds that at least two of them have the same birthday. Ignoring leap years, the answer is, paradoxically, only 23 people fewer than you might intuitively think.

But Joel noticed something interesting about a well-known group of 100 people: In the U.S. Senate, three senators happen to share the same birthday of October 20: Kamala Harris, Brian Schatz and Sheldon Whitehouse.

And so Joel has thrown a new wrinkle into the classic birthday problem. How many people do you need to have better-than-even odds that at least three of them have the same birthday? (Again, ignore leap years.)

The probability that any k people have the same birthday is, ignoring leap years, $\left(\frac{1}{365}\right)^{(k-1)}$, because the first person will have some birthday, and then we want the probability that the other $k-1$ people were all born on that specific day. The probability that, in a room of n people, k of them have the same birthday, is equivalent to 1 minus the probability that no group of k people have the same birthday. That probability is $1-\left(\frac{1}{365}\right)^{(k-1)}$ raised to the power of the number of possible groups of k people, or $\binom{n}{k}$. Thus, the probability of k people sharing a birthday in a room of n people is

$$1-\left(1-\left(\frac{1}{365}\right)^{(k-1)}\right)^{\binom{n}{k}}.$$

In the case where $k=3$, this becomes

$$1-\left(1-\left(\frac{1}{365}\right)^{(2)}\right)^{\binom{n}{3}}.$$

If we set this equal to .5 and solve for n, we get 83.1375, meaning we need at least 84 people in a room before we get better than even odds that 3 of them share a birthday. We can also use the equation above to see that in any given US Senate, the odds of three Senators sharing a birthday are 70.29%.

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