Lecture 5a: Performance Analysis

CS 70: Data Structures and Program Development

Tuesday, February 18, 2020

Learning Goals

1. I can describe strengths and weaknesses of timing code
2. I can describe strengths and weaknesses of counting operations
3. I can describe strengths and weaknesses of asymptotic analysis
4. I can contrast $o(g)$, $O(g)$, $\Theta(g)$, $\Omega(g)$, $\omega(g)$.
5. I can do asymptotic analyses for iterative (looping) functions.

Imagine this case study

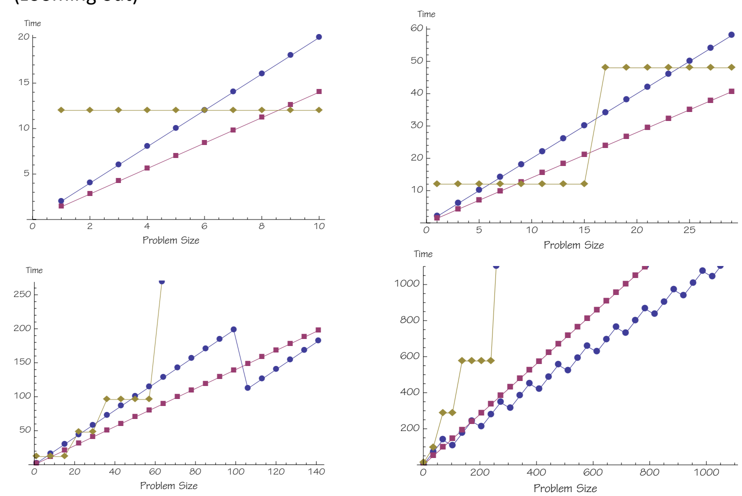
- **At Xcomp '15, Professor Bauer of UCNY announces that the new sorting algorithm WildSort takes only 0.16 seconds to sort a list of 100,000 names.**

- **Later, at the same conference, Professor Taylor of SUNY SD reports the new algorithm SneakerSort takes only 0.03 seconds to sort a list of 100,000 names.**

Your conclusion(s)? Which is better?

Which Program is “Fastest” ?

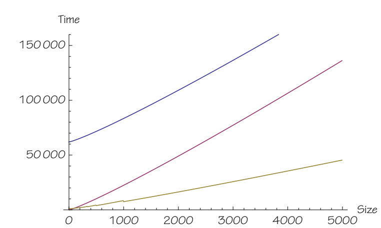
(zooming out)



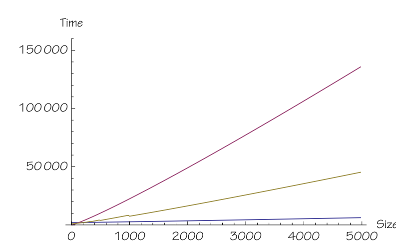
We can directly measure (benchmark)

- a specific algorithm
- written in a specific language
- as a specific program
- compiled using a specific version of a specific compiler
- with specific compiler flags settings (e.g., –g or not)
- running on a specific data set
- on a specific computer
- with a specific cpu (or cpus), memory, bus, hard drive, network card, …
- under a specific version of a specific operating system
- while specific other programs run in the background.

Another example — but blue is in Python



Switch blue to C++ for a 31x speedup:



Counting Steps
Outputs? Comparisons? Additions? Multiplications?

```
int original[NUM_ROWS * NUM_COLS];

// ...load the digits into the array...

for (size_t row = 0; row < NUM_ROWS; ++row) {
  for (size_t col = 0; column < NUM_COLS; ++col) {
    cout << original[row * NUM_COLS + col];
  }
  cout << endl;
}
```

Concern: Sum of operation times != total run-time

Asymptotic Analysis (Big-O, Big-θ, etc.)

- **Answers an abstract question about an algorithm:**
  - How do costs *scale* as input sizes become arbitrarily large?
- **Not**
  - How many seconds do we need for an input of size n
  - How many bytes of memory are required?
  - How many additions are performed?
  - How easy is it to implement the algorithm?
  - Is this the best algorithm for my problem?
- **"Cost" might refer to**
  - Time spent (today's focus)
  - Bytes of memory required
  - Bits transmitted over the network
  - Watts of electricity consumed
  - Etc.

Asymptotic Analysis

Answers an abstract question about an algorithm:

- How do costs *scale* as input sizes become arbitrarily *large*?

Suppose a function with input size n takes 63n steps when it runs. What is the ratio

$$\frac{\text{\# steps for some input } \textit{\textbf{twice}} \textit{ as large}}{\text{\# steps for some input}}$$

Suppose a function with input size n takes $5n^3$ steps when it runs. What is the ratio

$$\frac{\text{\# steps for some input } \textit{\textbf{three times}} \textit{ as large}}{\text{\# steps for some input}}$$

Asymptotic Analysis

Answers an abstract question about an algorithm:

- How do costs *scale* as input sizes become arbitrarily *large*?

Suppose a function with input size n takes $n^3 + 17$ steps when it runs. What is the ratio

$$\frac{\text{\# steps for some input } \textit{\textbf{three times}} \textit{ as large}}{\text{\# steps for some input}}$$

as n gets very large?

Consequences

- If we care only about *scalability* for arbitrarily large inputs: constant factors don't matter
  - 0.01n and n and 9999n scale linearly
  - $0.01n^2$ and n^2 and $9999n^2$ scale quadratically
  - ln n and $\log_2 n$ and $\log_{10} n$ scale logarithmically
- "Small" summands can be ignored
  - $(n^2 + 100n + 1000000) \approx n^2$ for very large n
  - $(n^n + n! + 2^n) \approx n^n$ for very large n

Simplify

1. $O(3n^2 + 2n + 2 + \cos(\pi n))$
2. $O(\log_{10}(n^3))$
3. $O(5n^{1.5} + 2n \log n)$
4. $O(n^2 + 2^n)$

More Consequences

We can count "steps" rather than "instructions" or "clock cycles"
- Both give the same asymptotic result
- A step can be a lot of work, as long as it's bounded by a constant.

```
const int ARR_LENGTH = 80;
for (size_t i = 0; i < 50; ++i) {
  size_t arr[ARR_LENGTH]{i};
  for (size_t j = 0; j < ARR_LENGTH; ++j) {
    cout << arr[j] << " ";
  }
  cout << endl;
}
```


Be careful about identifying single steps!

```
string output;
for(size_t i = 0; i < n; ++i) {
 output += " " + to_string(i);
}
cout << output << endl;
```

Coarse-Grained Group: O(1)

- Takes 6 steps
- Takes 1 (big) step
- No more than 4000 steps
- Somewhere between 2 and 47,000 steps, depending on the input

Coarse-Grained Group: O(n)

- Takes 100n + 3 steps
- Takes n/20 + 10,000,000 steps
- Anywhere from 3 to 68 steps per item, for n items

Coarse-Grained Group: O(n^2)

- Takes $2n^2 + 100n + 3$ steps
- Takes $n^2/17$ steps.
- Somewhere between 1 and 40 steps per item, for n^2 items
- Anywhere between 1 and 7n steps per item, for n items

Making Life Simpler

If there's any one step that dominates (asymptotically), we can ignore everything else, e.g.,

```
for (int i = 0;  i < n;  ++i) {
  sum += 2;
}
```

Asymptotic notation, intuitively.

$f \in o(g)$ if f grows strictly less fast than g $\quad (<)$

$f \in O(g)$ if f grows no faster than g $\quad (<=)$

$f \in \Theta(g)$ if f grows at the same rate as g $\quad (=)$

$f \in \Omega(g)$ if f grows at least as fast as g $\quad (>=)$

$f \in \omega(g)$ if f grows strictly faster than g $\quad (>)$

Asymptotic notation, more formally

$f \in o(g)$ if f stays below every multiple of g, eventually.

$f \in O(g)$ if f stays below some multiple of g, eventually.

$f \in \Theta(g)$ if f stays between two multiples of g, eventually.

$f \in \Omega(g)$ if f stays above some multiple of g, eventually,

$f \in \omega(g)$ if f stays above every multiple of g, eventually.

Asymptotic notation, even more formally

$f \in o(g)$ if $\forall c > 0.\ \exists N \geq 0.\ \forall n \geq N.\ f(n) \leq c \cdot g(n)$

$f \in O(g)$ if $\exists c > 0.\ \exists N \geq 0.\ \forall n \geq N.\ f(n) \leq c \cdot g(n)$

$f \in \Theta(g)$ if $\exists c_1, c_2 > 0.\ \exists N \geq 0.\ \forall n \geq N.\ c_1 \cdot g(n) \leq f(n) \leq c_2 \cdot g(n)$

$f \in \Omega(g)$ if $\exists c > 0.\ \exists N \geq 0.\ \forall n \geq N.\ f(n) \geq c \cdot g(n)$

$f \in \omega(g)$ if $\forall c > 0.\ \exists N \geq 0.\ \forall n \geq N.\ f(n) \geq c \cdot g(n)$

Exercises

1. Is $3n \in O(n)$?
2. Is $3n \in \Omega(n)$?
3. Is $3n \in \Theta(n)$?
4. Is $3n \in O(n^2)$?
5. Is $3n \in \Omega(n^2)$?
6. Is $3n \in \Theta(n^2)$?
7. Is $3n \in O(2^n)$?

Which should we avoid saying?

1. This algorithm isn't scalable because it takes $O(n^3)$ time.
2. This algorithm isn't scalable because it takes $\Theta(n^3)$ time.
3. This algorithm isn't scalable because it takes $\Omega(n^3)$ time.
4. I'm surprised Radix Sort sorts n `ints` in $O(n)$ time!
5. I'm surprised Radix Sort sorts n `ints` in $\Theta(n)$ time!
6. I'm surprised Radix Sort sorts n `ints` in $\Omega(n)$ time!

Warning!

Many programmers say O(g) when they mean θ(g)

- "It's too slow; it's O(n^3)"

and further assume that hidden factors are always small

- "If you double the input size of an O(n^2) algorithm, it will take four times as long."

Calculating Asymptotically

$$O(f) + O(g) = O(f + g).$$

$$O(f) \cdot O(g) = O(f \cdot g)$$

$$O(max\{f, g\}) = O(f + g).$$

Common Summations

$$1 + 2 + 3 + \cdots + n \ \in O(n^2)$$

$$1 + 2 + 4 + \cdots + 2^n \ \in O(2^n)$$

$$1 + 2 + 4 + \cdots + n \ \in O(n)$$

Classify T(n), the number of steps required for input n

```
for (int i = 0;  i < n;  ++i) {
  ++sum;
}
```


Classify T(n), the number of steps required for inputs n and m

```
for (int i = 0;  i < 2*n;  ++i) {
  for (int j = 0;  j < m+1;  ++j) {
    ++sum;
  }
}
```

Classify T(n), the steps required for input n

```
for (int i = 0;  i < n;  ++i) {
  for (int j = 0;  j < i+1;  ++j) {
    ++sum;
  }
}
```


Classify T(n,m), steps required for inputs n and m

```
for (int i = 0;  i < n;  ++i) {
  ++sum;
}
for (int j = 0;  j < m;  ++j) {
  ++sum;
}
```


Classify T(n), the steps required for input n

```
for (int i = 0; i < n; ++i) {
  for (int j = 0; j < i; j += 2) {
    a[j+1] += 1;
    if (a[j+1] % 2 == 0) {
      a[j] = 2*a[j];
    }
  }
}
```