Lecture 8a: 2-3-4 and Red-Black Trees

CS 70: Data Structures and Program Development

Tuesday, March 10, 2020

Learning Targets

1. I can explain the fundamental idea behind Red-Black trees.
2. I can explain the fundamental idea behind 2-3-4 trees.
3. I can explain the mapping between 2-3-4 and Red-Black trees.

The Story So Far

Lookup and insert with n nodes:

- naive BST: worst-case **O(n)**
- randomized BST: expected **O(log n)** and worst-case **O(n)**

Future preview

- splay tree: amortized O(log n) and worst-case O(n)

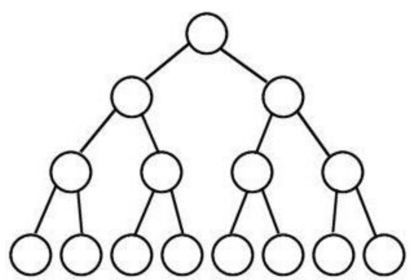
How can we get worst-case O(log n)?

- Need to ensure the tree has height O(log n)?

A Perfect Tree

All levels are full

Has 2^{h+1} - 1 nodes, where h is height.



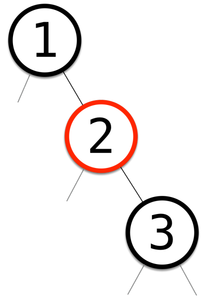
Red-Black Trees (definition 1)

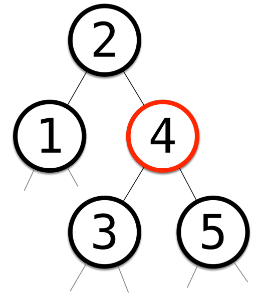
A red-black tree is a BST such that:

- Every node is red or black
- The root is black
- No red parent has a red child
- Every path from the root to `nullptr` passes through the same number of black nodes.

Claim: an n-node red-black tree has height O(log n)

Valid Red-Black Trees?





Visualization

- **https://www.cs.csubak.edu/~msarr/visualizations/RedBlack.html**

Red-Black Trees have O(log n) height.

Suppose every path to `nullptr` hits b black nodes.

1. The height is less than 2b.
2. The tree has at least 2^b-1 nodes, i.e., $2^b-1 <= n$.
3. $h < 2b <= 2\log_2(n+1)$, so $h \in O(\log n)$.

The Problem

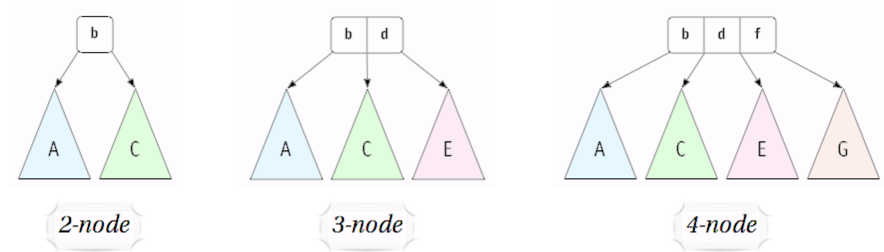
Red-Black Trees can be a pain to implement.

- Lookup is easy - same as in a BST
- But how to update the colors when we insert?
  - Not immediately obvious…

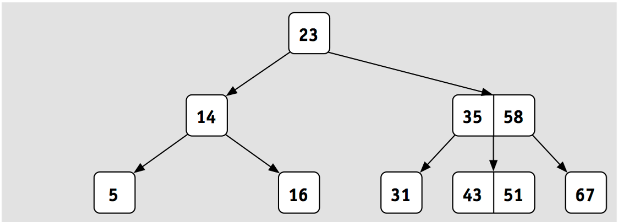
Maybe we can find a balanced tree that is more intuitive?

Introducing the 2-3-4 Tree

1. Allow a node to store more than one key.
2. All leaves at the same depth



2-3-4 tree: example and insertion



Insertion: rules for insert-as-leaf

- Always keep the leaves at the same level
- If leaf is a 2 node: Make it a 3 node
- If leaf is a 3 node: Make it a 4 node
- If we encounter a 4 node on the way down, "promote" the middle (why)?

2-3-4 trees: insertion

- **Insert 1,2,3,4,5,6,…**

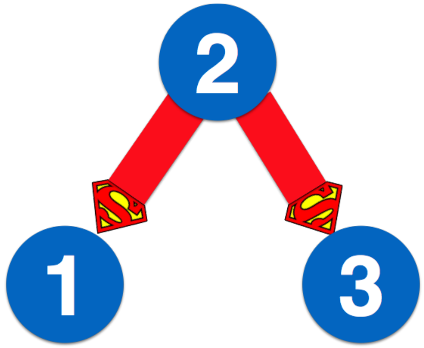
Advantages and Disadvantages

Advantages of 2-3-4 Trees

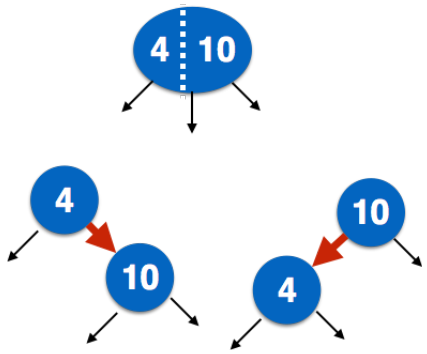
- The tree is always "balanced"
- Worst case for insert and lookup is O(log n) for a tree with n values
- Simple algorithm; no rotations required.
- Smaller height than a typical binary tree. (why?)

Disadvantages?

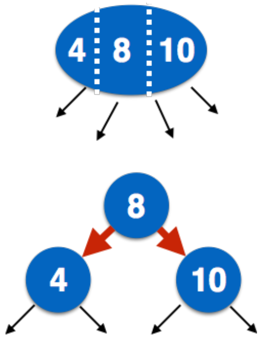
Encoding 2-3-4 Trees as Binary… Super Links!



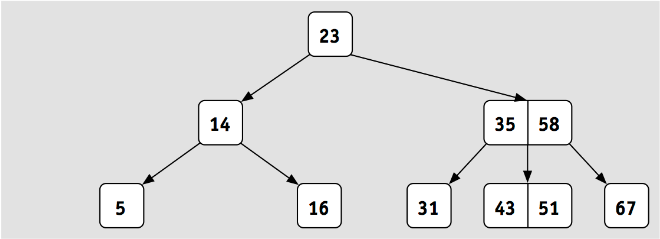
3-Nodes with Super links



4-nodes with Super links



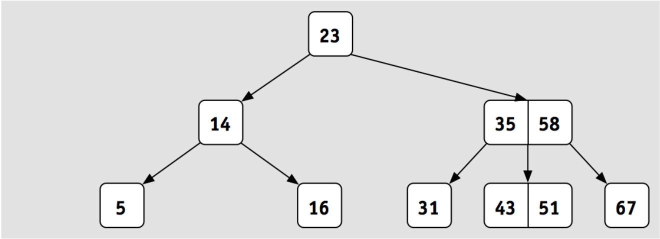
Exercise: Convert to Binary + Superlinks



Coloring *pointers* is weird…

Idea: color the node, not the (incoming) edge!

Exercise: Convert to Red-black tree



Red-black trees in practice: Word of Warning

- Red-Black Trees are efficient and commonly used (e.g., `std::set`)
- But it's easy to write convoluted, messy, confusing, scary Red-Black Tree implementations!
- When coding, think about what's going on. Draw diagrams. Refer back to the 2-3-4 implementation.

Advantages & Disadvantages

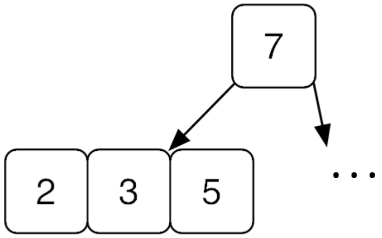
Advantages of Red-Black Trees

- All the balance properties of 2-3-4 Trees
- Only one extra bit of color per node needed over a standard BST
- Only one node type, so easier to implement than 2-3-4 Trees

Disadvantages of Red-Black Trees

- Can be messy if you don't draw diagrams and think about the 2-3-4 equivalents during implementation.
- Rotations trickier conceptually than 2-3-4 tree operations.

Exercise: Promote the 3... 2-3-4 vs. Red-black



1. Draw the Red-Black equivalent before promotion
2. Do the promotion on the original 2-3-4 tree
3. Draw the equivalent Red-Black tree after the promotion

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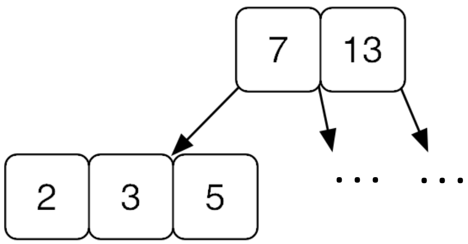

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