Lecture 7a: Trees!

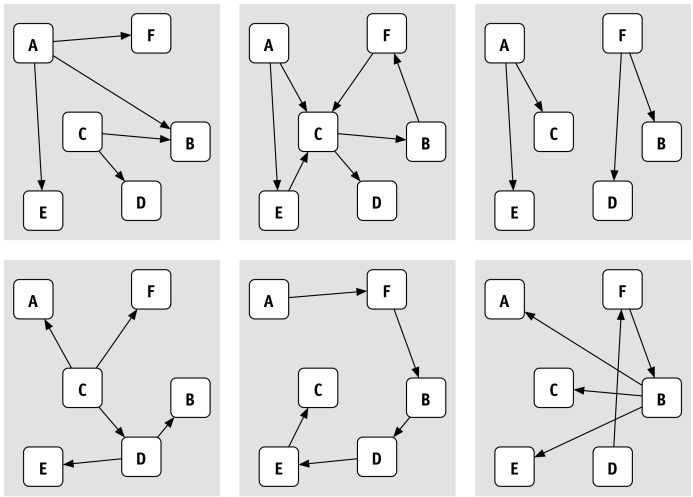
CS 70: Data Structures and Program Development
Tuesday, March 3, 2020

Learning Goals

1. Given a tree, I can tell whether it's a valid BST.
2. I can simulate BST lookup, insert, and delete (on paper)
3. I can simulate left and right rotations (on paper)
4. I can simulate `insertAtRoot` (on paper)
5. I can simulate Randomized Binary Tree insertion.

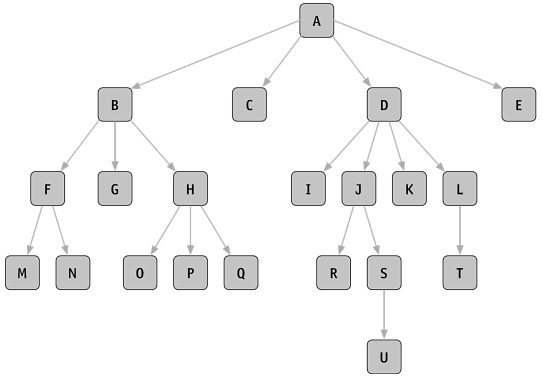
What is a tree?

Which of these are trees?

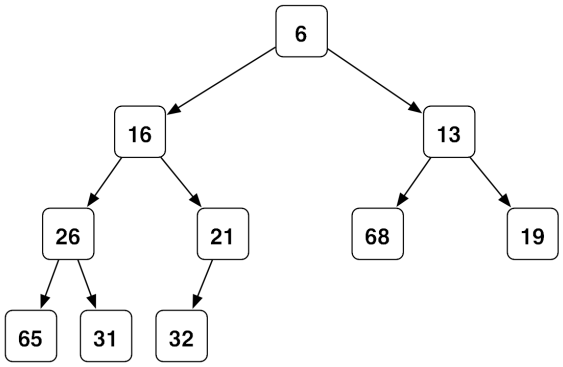


Tree Terminology

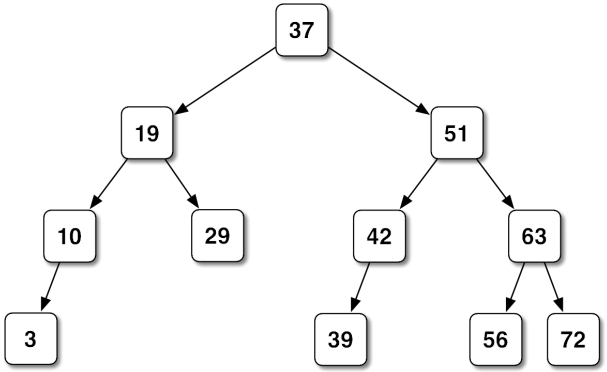
- node, edge
- root, leaf, tree, subtree
- parent, child, ancestor
- height, balanced tree
- binary tree, ordered binary tree



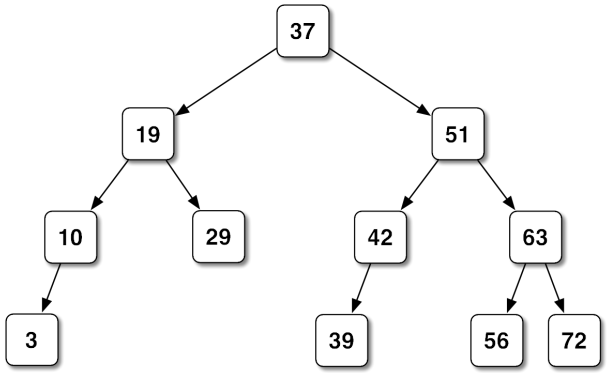
Binary Tree



Binary Search Tree (a.k.a. Ordered Binary Tree)



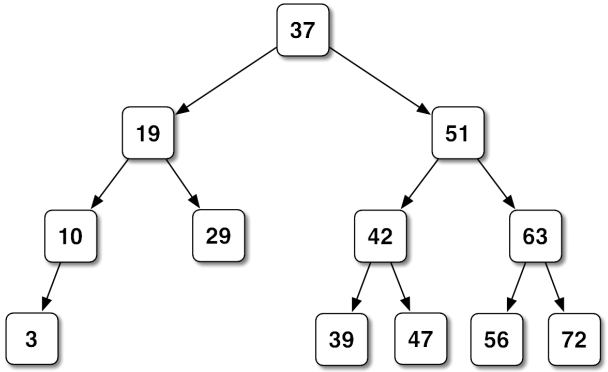
Find 56; find 35; insert 47



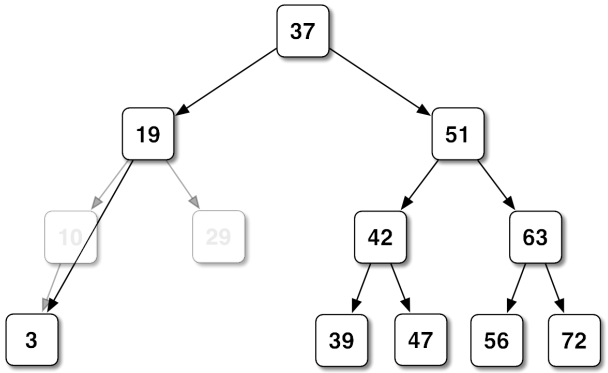
`insert` pseudocode

```
insert(tree, x):
   if tree is empty:
      make x its new root.

   else if x < tree's root:
      insert(left subtree, x)

   else if tree's root < x:
      insert(right subtree, x)
```

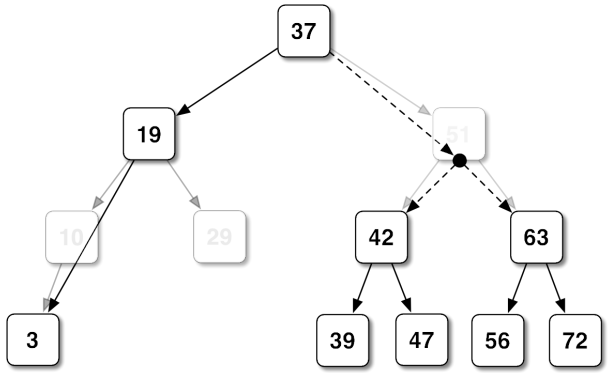

Delete 29, then delete 10



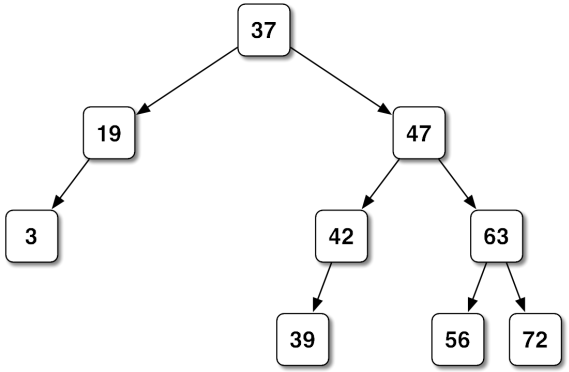
After deleting 29 then 10



Let's try deleting 51… oops!



After deleting 29, 10, 51



Exercise

What tree results from the following sequences of inserts?

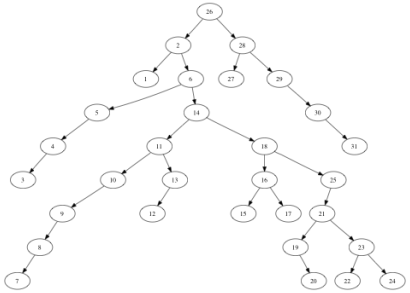
- A, B, C, D, E, F, G
- D, C, A, B, E, F, G

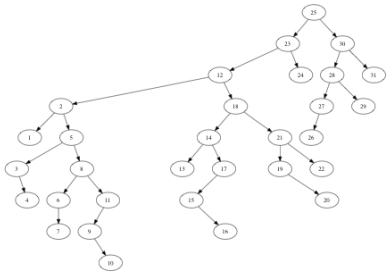
Suppose we have a BST with n nodes

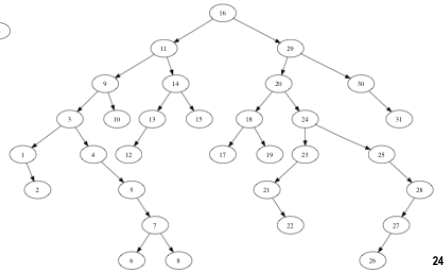
What is the worst-case running time for `find` (and `insert`)

- if we have a really terrible tree?
- if we have a really nice tree?
- if we have a "random" tree?

Random trees average 39% worse than perfect



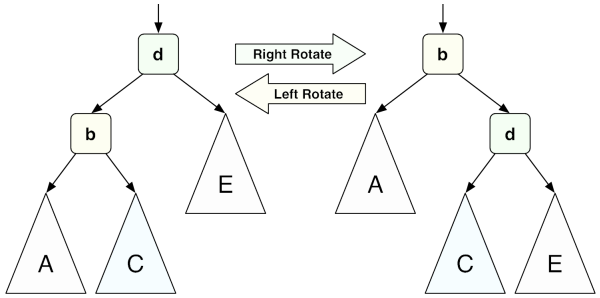




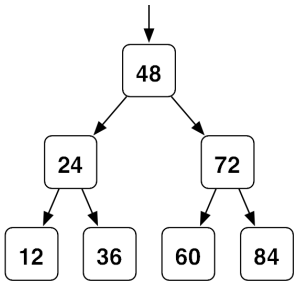
Building better trees: Off-line algorithm

1. Take the inputs we want to put in the tree.
2. Randomly shuffle them.
3. Build tree by inserting in *shuffled* order.

Tree Rotations



Insert 40. Rotate left at 36, left at 24, right at 48.



`insertAtRoot` pseudocode

```
insertAtRoot(tree, x):
   if tree is empty:
      make x its new root.

   else if x < tree's root:
      insertAtRoot(left subtree, x)
      do right rotation at tree's root.

   else if tree's root < x:
      insertAtRoot(right subtree, x)
      do left rotation at tree's root.
```


Building better trees: Randomized Binary Trees

Idea: insert each new key "randomly" into the tree-so-far

- Maybe it should become the new root
- Maybe put it somewhere below the existing root

But how often to do each?

Answer: If the tree has `n` nodes before the insert,

- do insert-at-root with probability `1 / ( n+1 )`
- otherwise, insert randomly into the appropriate child.