

Name: \_\_\_\_\_

Today's Date: \_\_\_\_\_

## Today's Goals

- Explain what the *amortized* cost of a function or algorithm is.
- Explain when amortized cost is a useful metric.
- Describe an algorithm for which amortized cost is a useful metric.

## Today's Question(s)

What is the *expected* cost of the following algorithm? You should use “number of rolls” as your cost metric.

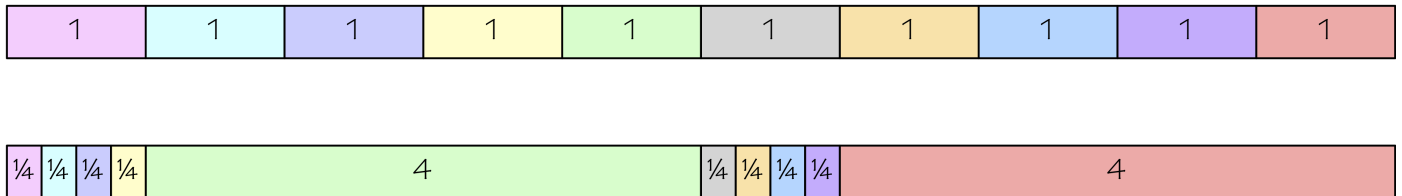
```
while true:
    roll a die
    if the value is 6: break
```

## Lingering Questions



Reminder: Time Complexity

# Visualizing Time Usage

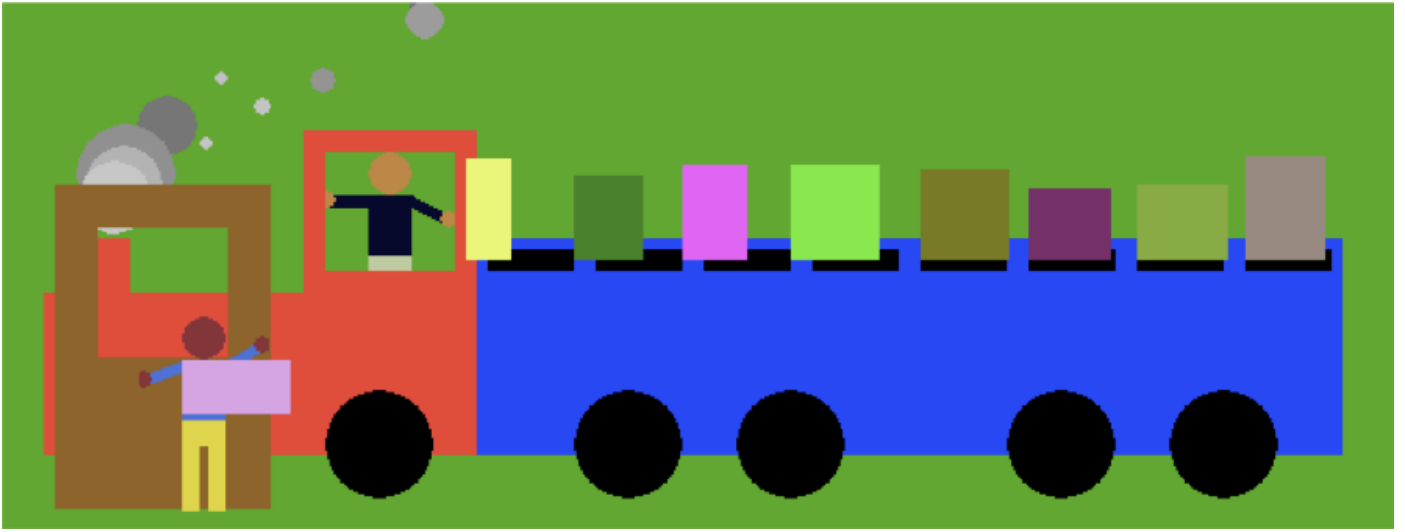


## Bounding time for a sequence of operations

What's the worst case time for:

```
std::vector<int> v;for (int i = 0; i != m; ++i) {  
    v.push_back(n);  
}
```

# Quinn's Story



**Quinn's Problem:** No idea how many packages will show up.

## Scheduling Quinn's Time

We need to know how long it will take Quinn to load up all of the packages. We could:

- ▶ Let each customer wait as long as their package takes.
- ▶ Tell every customer to expect the longest possible loading time.
- ▶ Who will this work for? Who won't it work for?

# Better Estimates

We can make a better estimate of the average time per customer by...

because ...

## Class Exercise: Growable Arrays

Quinn's train doubled when it ran out of room.

- ▶ What else could we have done?
- ▶ How would that affect how much work Quinn had to do?

## A Better Worst-Case Estimate

Let  $f$  be an operation we will carry out many times (like loading a new package)

Time to complete  $f$  for the  $i$ th time:

To perform  $f$   $k$  times:

A “typical” instance will take:

Amortized cost is:

## Banker's method

It's like insurance, because:

# Bounding Time for a Sequence of Operations

```
std::vector<int> v;  
for (int i = 0; i != m; ++i) {  
    v.push_back(n);  
}
```

What is the worst case time for...

- ▶ Just the last iteration?
- ▶ The entire loop?

## Warning: Correctness

Summing amortized times only works when you start from “empty”

```
// Start with n empty strings  
vector<string> ss{n};  
// Let's do just a single push_back  
ss.push_back("wow"); // Not worst-case O(1)!
```



