## 3. Distributed Arrays and Map-Reduce

```
February 13, 2014
          addprocs(8)
 In [1]: 8-element Array{Any,1}:
Out [1]:
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          nprocs()
In [2]: 9
Out [2]: dat=map(x-)integer(x*10), rand(10000, 10000))
          10000x10000 Array{Int64,2}:
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         ddat=distribute(dat)
In [4]: 10000x10000 DArray{Int64,2,Array{Int64,2}}:
Out [4]: 7 9
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     map(x->myid(),dat)
In [5]: 10000x10000 Array{Int64,2}:
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     map(x->myid(),ddat)
In [6]: 10000x10000 DArray{Int64,2,Array{Int64,2}}:
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          @time map(sin,dat);
 In [6]: elapsed time: 18.495849369 seconds (4000237720 bytes allocated)
         @time map(sin,ddat);
 In [6]: elapsed time: 3.801187871 seconds (1143612 bytes allocated)
          function intsum(x)
            if x==0
 In [7]:
              0
            else
              x+intsum(x-1)
            end
          intsum (generic function with 1 method)
Out [7]: @time reduce(+, map(intsum, dat))
 In [8]: elapsed time: 13.26386314 seconds (3573678796 bytes allocated)
          1925337529
Out [8]: @everywhere function intsum(x)
            if x==0
 In [9]:
              0
            else
              x+intsum(x-1)
            end
          @time reduce(+, map(intsum, ddat))
In [10]: elapsed time: 6.24797383 seconds (1743872 bytes allocated)
          1925337529
Out [10]: @everywhere function rsumtail(x,n)
            if x==0
In [11]:
              n
            else
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rsumtail (x-1, n+x)
           end
         @everywhere function rsumtail(x)
           rsumtail(x, 0)
         end
         @everywhere function rsum(x)
           if x==0
             \cap
           else
             x+rsum(x-1)
           end
         end
         @time rsumtail(100000)
In [12]: elapsed time: 0.00278938 seconds (42804 bytes allocated)
         5000050000
Out [12]: @time rsum(100000)
In [13]: elapsed time: 0.002768959 seconds (23496 bytes allocated)
         5000050000
Out [13]: @time (s=0;
         for i in dat
In [14]:
          s+=intsum(i)
         end;
         s)
         elapsed time: 26.858735368 seconds (6400048908 bytes allocated)
         1925337529
Out [14]: @time @parallel (+) for i in dat
           intsum(i)
In [21]:
         end
         elapsed time: 30.597089961 seconds (8590437940 bytes allocated)
         1925337529
Out [21]: @time reduce(+, map(intsum, ddat))
In [16]: elapsed time: 7.447626368 seconds (1453340 bytes allocated)
         1925337529
Out [16]: ddat.indexes
In [22]: 2x4 Array{(Range1{Int64}, Range1{Int64}),2}:
Out [22]: (1:5000,1:2500)
                            (1:5000,2501:5000)
                                                         ... (1:5000,7501:10000)
          (5001:10000,1:2500) (5001:10000,2501:5000)
         (5001:10000,7501:10000)
         ddat.chunks
In [23]: 2x4 Array{RemoteRef, 2}:
Out [23]: RemoteRef((2,1,5)) RemoteRef((4,1,7)) RemoteRef((6,1,9))
         RemoteRef(8,1,11)
          RemoteRef(3,1,6) RemoteRef(5,1,8) RemoteRef(7,1,10)
         RemoteRef(9,1,12)
         ddat.pmap
In [24]: 8-element Array{Int64,1}:
Out [24]: 2
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          9
         fetch(@spawnat 2 sum(x->x*x, localpart(ddat)))
In [27]: 418800280
Out [27]: map(fetch, \{@spawnat p sum(map(x->x*x, localpart(ddat)))) for p in ddat.pmap})
In [28]: 8-element Array{Any,1}:
Out [28]: 418800280
          418938675
          418841631
          418547283
          419008882
          418728812
          418788879
          418975117
         @time reduce(+, map(fetch, {@spawnat p sum(map(x->x*x,localpart(ddat))) for p in procs(d
In [29]: elapsed time: 2.15132193 seconds (663104 bytes allocated)
         3350629559
Out [29]: @time reduce(+, map(x->x*x, ddat))
In [39]: elapsed time: 3.018259173 seconds (1472940 bytes allocated)
         3350629559
Out [39]: @time sum(map(x->x*x,dat))
In [42]: elapsed time: 8.165484936 seconds (2399993056 bytes allocated)
         3350629559
Out [42]:
```